Chapter 2

DEALING WITH SEASONAL PRODUCTS IN PRICE INDEXES

W. Erwin Diewert, Paul A. Armknecht and Alice O. Nakamura[1]

1.   The Problem of Seasonal Products

Product prices and sales quantities can change from one month to the next because of seasonal circumstances, such as lower production costs for strawberries in-season (usually from domestic sources) versus out-of-season (often imported). Inflationary pressure can also cause changes in prices and sales quantities from one month to the next. Inflationary pressure is what governments and central banks are interested in trying to control. Thus there is interest in how inflationary changes can best be measured, given that prices for many products also have fluctuations due to season-specific circumstances.

Strongly seasonal products are not available at all in the marketplace during certain seasons. Weakly seasonal products are available all year but have fluctuations in prices or quantities that are synchronized with the time of year.[2] For a country like the United States or Canada, seasonal purchases amount to one-fifth to one-third of all consumer purchases. Strongly seasonal products create the biggest problems for price statisticians. Often these products are simply omitted in price index making. In the case of weakly seasonal products, their calendar related fluctuations are widely viewed as noise.

As of now, neither the Consumer Price Index (CPI) nor the Producer Price Index (PPI) is seasonally adjusted by the national statistics agencies of most nations. This reflects, in part, a reluctance to revise these price series, with revisions being inevitable for the seasonally adjusted series produced using methods such as X11 and X12. Nevertheless, there is interest in finding conceptually acceptable and operationally tractable ways of including seasonal products in consumer and producer indexes without introducing a lot of seasonal fluctuation. These are some of the motivations for the treatment of seasonal products in chapter 22 of both the most recent international *CPI Manual* (ILO et al., 2004) and *PPI Manual* (IMF et al., 2004). This chapter is referred to hereafter simply as the *Manual chapter*. To aid readers in going on to read the Manual

---

[1] Diewert is with the Department of Economics of the University of British Columbia in Vancouver Canada and can be reached at diewert@econ.ubc.ca. Armknecht, who was previously with the International Monetary Fund in Washington DC, is now a private consultant and can be reached at paularmknecht@msn.com. Nakamura is with the University of Alberta School of Business and can be reached at alice.nakamura@ualberta.ca.

[2] This classification corresponds to Balk's (1980a, p. 7; 1980b, p. 110; 1980c, p. 68; 1981) narrow and wide sense seasonal products. Diewert (1998, p. 457) used the terms type 1 and type 2 seasonality. The definition of strongly seasonal products should not be confused with Granger's (1978) definition of strongly seasonal times series.

chapter, the present chapter has the same section headings as in the Manual chapter through section 10, and the Manual chapter equation numbers are shown as well [in square brackets].[3]

In the Manual chapter, the various methods discussed are applied using an artificial data set: the modified Turvey data set which is introduced in section 2. In section 3, year-over-year monthly price indexes are introduced. Strongly seasonal products cause serious problems in conventional month-to-month price indexes. However, these problems are largely resolved by using indexes that compare the prices for the same month in different years.

Year-over-year monthly indexes can be combined to form an annual index. Calendar year annual year-over-year indexes are introduced in section 4, and "rolling year" non-calendar year annual indexes are considered in section 5. Seasonal adjustment factors (SAF) are defined using rolling year indexes, and section 6 presents a rolling year index centered on the current month.

Sections 7-10 explore more conventional month-to-month price index methods that have been proposed for accommodating seasonal products. These methods use different ways of compensating for missing price information for products not available in some months. In section 7, the maximum overlap index is introduced. Approaches for filling in data for the months when products are unavailable are to carry forward the last price that was observed for a product, or to impute the missing price in some other way. The carry forward approach is explored in section 8, and the alternative imputation approach is the subject of section 9. In section 10, yet another method that can be used even with strongly seasonal products is introduced: the Bean and Stine Type C, sometimes also called the Rothwell, index.

In section 11, seasonal adjustment factors (SAF values) that incorporate the year-over-year approach (from section 6) are calculated for the various methods presented in section 7-11. In section 12, we discuss the alternative X-11 and X-12 family approaches for seasonally adjusting times series: approaches that are widely used by official statistics agencies but are only briefly mentioned in the Manual chapter. Section 13 concludes.

2.   A Seasonal Product Data Set

In the Manual chapter, the index number formulas considered are applied using an artificial data set developed by Ralph Turvey and then modified by Erwin Diewert to enhance its value for assessing alternative methods of dealing with seasonal products.[4] The full results are shown in the Manual chapter and the summary results are reported here.

Turvey constructed his original data set for five seasonal products (apples, peaches, grapes, strawberries, and oranges) over four years (1970-1973). Turvey sent this dataset to statistical agencies around the world, asking them to use their normal techniques to construct monthly and annual average price indexes. Turvey (1979, p. 13) summarizes the responses:

> "It will be seen that the monthly indexes display very large differences.... It will also be seen that the indexes vary as to the peak month or year."

---

[3] Readers are referred to the Manual chapter, listed in the references as Diewert and Armknecht (2004).

[4] The modified Turvey data set is tabled in the Manual chapter, and the original Turvey data set is tabled in Turvey's 1979 paper and in Diewert (1983, 2009).

3.   Year-over-Year Monthly Indexes

One way of dealing with seasonal products is to change the focus from short-term month-to-month price indexes to year-over-year comparisons for given months. This approach can accommodate even seasonal products. The formulas for the chained Laspeyres, Paasche and Fisher year-over-year monthly indexes are given in box 1 below.

It has been recognized for over a century that making year-over-year price level comparisons[5] is the simplest method for removing the effects of seasonal fluctuations so that trends in the price level can be measured. For example, Jevons (1863; 1884, p. 3) wrote:

> "In the daily market reports, and other statistical publications, we continually find comparisons between numbers referring to the week, month, or other parts of the year, and those for the corresponding parts of a previous year. The comparison is given in this way in order to avoid any variation due to the time of the year. And it is obvious to everyone that this precaution is necessary. Every branch of industry and commerce must be affected more or less by the revolution of the seasons, and we must allow for what is due to this cause before we can learn what is due to other causes."

The economist Flux (1921, pp. 184-185) also endorsed the idea of making year-over-year comparisons to minimize the effects of seasonal fluctuations:

> "Each month the average price change compared with the corresponding month of the previous year is to be computed.…"

More recently, Zarnowitz (1961, p. 266) endorsed the use of year-over-year monthly indexes:

> "There is of course no difficulty in measuring the average price change between the same months of successive years, if a month is our unit 'season,' and if a constant seasonal market basket can be used, for traditional methods of price index construction can be applied in such comparisons."

Suppose that data are available for the prices and quantities for all products available for purchase each month for two or more years. Then year-over-year monthly chained and fixed base Laspeyres (L), Paasche (P) and Fisher (F) price indexes, defined in box 1, can be used for comparing the prices in some given month for two different years.

The Manual chapter provides and compares tabular results for the year-over-year monthly chained and fixed base Laspeyres, Paasche, and Fisher indexes. All of the resulting monthly series show year-to-year trends that are free of the purely seasonal variation in the modified Turvey data. The chained indexes are found to reduce the spread between Paasche and Laspeyres indexes compared with their fixed base counterparts. Since the Laspeyres and Paasche perspectives both have merit, the Manual chapter recommends as the target measure of inflation the chained year-over-year Fisher index, which is the geometric average of the Laspeyres and Paasche indexes.

The year-over-year monthly indexes defined in box 1 use monthly data for years t and t+1. Many countries collect price information monthly. However, the expenditure data needed for deriving the quantity observations are only available for intermittent years when a household

---

[5] In the seasonal price index context, this type of index corresponds to Bean and Stine's (1924, p. 31) Type D index.

expenditure survey (HES) has been conducted. In the Manual chapter it is argued that monthly expenditure share vectors can be used instead of the current and comparison year monthly expenditure share vectors in an index formula such as one of those in the box 1. This is how the approximate indexes are defined in box 2. When evaluated using the modified Turvey data, the year-over-year chained approximate indexes track their true chained counterparts closely.[6]

Box 1. Definitions for Year-over-Year Monthly Indexes

For each month $m=1,2,\ldots,12$, let $S(m)$ denote the set of products available for purchase that month in all years $t=1,2,\ldots,T$. Let $p_n^{t,m}$ and $q_n^{t,m}$ denote the price and quantity of product n available in month m of year t, and let $p^{t,m}$ and $q^{t,m}$ denote the corresponding month m and year t price and quantity vectors. The year-over-year monthly chained Laspeyres (L), Paasche (P) and Fisher (F) price indexes going from month m of year t to month m of t+1 can now be defined, respectively, as:[7]

(1)[22.4]
$$P_L = \sum_{n\in S(m)} s_n^{t,m}\left(p_n^{t+1,m}/p_n^{t,m}\right), \qquad m=1,2,\ldots,12;$$

(2)[22.5]
$$P_P = \left[\sum_{n\in S(m)} s_n^{t+1,m}\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}, \qquad m=1,2,\ldots,12, \text{ and}$$

(3)[22.6]
$$P_F = \sqrt{P_L P_P} ,$$

where the monthly expenditure share for product $n\in S(m)$ in month m of year t is defined as:

$$s_n^{t,m} = \frac{p_n^{t,m}q_n^{t,m}}{\sum_{i\in S(m)} p_i^{t,m}q_i^{t,m}} .$$

The corresponding fixed base indexes have similar formulas to the chained indexes; the year t observations are simply replaced by the observations for the fixed base year 0.

The approximate year-over-year monthly Laspeyres and Paasche indexes will always satisfy inequalities (7) and (8) of box 2. The first of these inequalities says that the approximate year-over-year monthly Laspeyres index fails the time reversal test with an upward formula bias. The second of these inequalities says that the approximate year-over-year monthly Paasche index fails the time reversal test with a downward formula bias. The approximate Fisher formula is recommended because the upward bias of the Laspeyres index part of the Fisher index will balance out the downward bias of the Paasche index part of the Fisher index.

In general, the approximate year-over-year monthly Fisher index defined by (6) in box 2 will closely approximate the true Fisher index defined by (3) in box 1 when the monthly expenditure shares for the base year 0 are close in value to the corresponding year t and year

---

[6] The approximate Laspeyres index actually equals the original fixed base Laspeyres index.

[7] The numbers in square brackets are the equation numbers in the Diewert and Armknecht (2004) Manual chapter.

$t+1$ values.[8] The approximate Fisher indexes are just as easy to compute as the approximate Laspeyres and Paasche indexes, so it is recommended that statistical agencies make approximate Fisher index values available along with the approximate Laspeyres and Paasche ones.

Box 2. Definitions for Approximate Year-over-Year Monthly Indexes

Suppose that expenditure share data are available for some base year 0. If the base year monthly expenditure share vectors, $s^{0,m}$, is substituted for the current year monthly expenditure share vectors, $s^{t,m}$, in equation (1), and for the year $t+1$ monthly expenditure share vectors, $s^{t+1,m}$, in equation (2), this yields the approximate year-over-year monthly Laspeyres and Paasche indexes:

$$(4)[22.8] \qquad P_{AL} = \sum_{n \in S(m)} s_n^{0,m} \left(p_n^{t+1,m} / p_n^{t,m} \right), \qquad\qquad m=1,2,\ldots,12, \text{ and}$$

$$(5)[22.9] \qquad P_{AP} = \left[\sum_{n \in S(m)} s_n^{0,m} \left(p_n^{t+1,m} / p_n^{t,m} \right)^{-1} \right]^{-1}, \qquad\qquad m=1,2,\ldots,12,$$

where $s_n^{0,m}$ is the base period month m expenditure share for product n. The approximate Fisher year-over-year monthly index is defined by

$$(6)[22.10] \qquad P_{AF} = \sqrt{P_{AL} P_{AP}},$$

where P_{AL} and P_{AP} are defined in (4) and (5), respectively.

The approximate year-over-year monthly Laspeyres and Paasche indexes satisfy the following inequalities:

$$(7)[22.11] \qquad P_{AL}(p^{t,m}, p^{t+1,m}, s^{0,m}) \times P_{AL}(p^{t+1,m}, p^{t,m}, s^{0,m}) \geq 1, \qquad m=1,2,\ldots,12, \text{ and}$$

$$(8)[22.12] \qquad P_{AP}(p^{t,m}, p^{t+1,m}, s^{0,m}) \times P_{AP}(p^{t+1,m}, p^{t,m}, s^{0,m}) \leq 1, \qquad m=1,2,\ldots,12,$$

with strict inequalities holding if the monthly price vectors $p^{t,m}$ and $p^{t+1,m}$ are not proportional to each other.

---

4.   Year-over-Year Annual Indexes

For some policy purposes, it is useful to have a summary measure of annual price level change from year to year in addition to, or as an alternative to, the 12 month-specific measures of year-to-year price level change defined in the previous section. Treating each product in each month as a *separate annual product* is the simplest and theoretically most satisfactory method for dealing with seasonal products when annual price and quantity indexes can be used. Annual measures of price level change can then be defined, as in box 3, as (monthly) share weighted averages of the year-over-year monthly chain linked Laspeyres, Paasche and Fisher indexes. Thus once the year-over-year monthly indexes defined in the previous section have been numerically calculated, it is easy to calculate the corresponding annual indexes.

---

[8] If the monthly expenditure shares for the base year, $s_n^{0,m}$, are all equal, then the approximate Fisher index defined by equation (6) reduces to Fisher's (1922, p. 472) formula 101.

Box 3. Definitions for Annual Indexes

The Laspeyres and Paasche annual chain link indexes which compare the prices in every month of year t with the corresponding prices in year t + 1 can be defined as follows:

(9)[22.16]
$$P_L = \sum_{m=1}^{12} \sum_{n \in S(m)} \sigma_m^t s_n^{t,m} \left(p_n^{t+1,m} / p_n^{t,m} \right) \text{ and}$$

(10)[22.17]
$$P_P = \left[\sum_{m=1}^{12} \sum_{n \in S(m)} \sigma_m^{t+1} s_n^{t+1,m} \left(p_n^{t+1,m} / p_n^{t,m} \right)^{-1} \right]^{-1},$$

where the expenditure share for month m in year t is defined as

$$\sigma_n^t = \frac{\sum\limits_{n \in S(m)} p_n^{t,m} q_n^{t,m}}{\sum\limits_{i=1}^{12} \sum\limits_{j \in S(i)} p_j^{t,i} q_j^{t,i}}, \ m = 1,2,\ldots,12, \qquad\qquad t = 0,1,\ldots,T.$$

The annual chain linked Fisher index P_F, which compares the prices in every month of year t with the corresponding prices in year t + 1, is the geometric mean of the Laspeyres and Paasche indexes, P_L and P_P, defined by equations (9) and (10); i.e.,

(11)[22.18]
$$P_F = \sqrt{P_L P_P} \ .$$

Fixed base counterparts to the formulas defined by equations (9)-(11) can readily be defined: simply replace the data pertaining to period t with the corresponding data pertaining to the base period 0.

The annual chained Laspeyres, Paasche, and Fisher indexes can readily be calculated using the equations (9)-(11) in box 3 for the chain links. For the modified Turvey data, the use of chained indexes is found to substantially narrow the gap between the Paasche and Laspeyres indexes.

When monthly expenditure share data are only available for some base year, approximate annual Laspeyres, Paasche and Fisher indexes can be calculated. The fixed base Laspeyres price index uses only expenditure shares for a base year; consequently, the approximate fixed base Laspeyres index is equal to the true fixed base Laspeyres index. For the modified artificial Turvey data set, the approximate Paasche and approximate Fisher indexes are quite close to the corresponding true annual Paasche and Fisher indexes. Also, the true annual fixed base Fisher is closely tracked by the approximate Fisher index P_{AF} (or the geometric Laspeyres index P_{GL}).[9]

The annual fixed base Fisher index is close to the annual chained approximate Fisher counterpart. This approximate index can be computed using the information usually available to statistical agencies. However, the true annual chained Fisher index is still recommended as the

---

[9] The fixed base geometric Laspeyres annual index, P_{GL}, is the weighted geometric mean counterpart to the fixed base Laspeyres index, which is equal to a base period weighted arithmetic average of the long-term price relative. It can be shown that P_{GL} approximates the approximate fixed base Fisher index P_{AF} to the second order around a point where all of the long-term price relatives are equal to unity.

target index and should be computed when the necessary data are available, and used as a check on the quality of the approximate Fisher index.[10]

5.   Rolling Year Annual Indexes

In the previous section, the price and quantity data pertaining to the 12 months of a current calendar year were compared to the 12 months of some base calendar year. However, there is no need to restrict attention to calendar years. Any two periods of 12 consecutive months can be compared, provided that the January data are compared to the January data, the February data are compared to the February data, and so on.[11] Alterman, Diewert, and Feenstra (1999, p. 70) and Diewert, Alterman and Feenstra (2009) define what they refer to as *rolling year indexes*.[12] The specifics of constructing rolling year indexes are spelled out in the Manual chapter for both the chained and fixed base cases. The rolling year index series constructed using the modified Turvey data are found to be free from erratic seasonal fluctuations. These rolling year indexes offer statistical agencies an objective and reproducible method of incorporating seasonal products into price indexes.

For the rolling year indexes, when evaluated using the modified Turvey data, it is found that chaining substantially narrows the gap between the Paasche and Laspeyres indexes. The chained Fisher rolling year index is thus deemed to be a suitable target seasonally adjusted annual index for cases in which seasonal products are in scope for a price index.[13]

When necessary owing to data availability limitations, the current year weights can be approximated by base year weights,[14] yielding the annual approximate chained and fixed base rolling year Laspeyres, Paasche, and Fisher indexes. When evaluated using the modified Turvey data, these approximate rolling year indexes are found to be close to their true rolling year counterparts. In particular, the approximate chained rolling year Fisher index (which can be computed using just base year expenditure share information along with base and current period information on prices) is close to the preferred target index: the rolling year chained Fisher index.

---

[10] The approach to computing annual indexes outlined in this section, which essentially involves taking monthly expenditure share-weighted averages of the 12 year-over-year monthly indexes, is contrasted in the Manual chapter with the approach that takes the unweighted arithmetic mean of the 12 monthly indexes. The key problem with the latter approach is that months where expenditures are below the average (for example, February) are given the same weight in the annual average as months with above average expenditures (e.g., December).

[11] Diewert (1983) suggested this type of comparison and termed the resulting index a *split year* comparison.

[12] Crump (1924, p. 185) and Mendershausen (1937, p. 245), respectively, used these terms in the context of various seasonal adjustment procedures. The term *rolling year* seems to be well established in the business literature in the United Kingdom. In order to theoretically justify the rolling year indexes from the viewpoint of the economic approach to index number theory, some restrictions on preferences are required. The details of these assumptions can be found in Diewert (1996, pp. 32-34; 1999, pp. 56-61).

[13] Diewert (2002) discusses measurement issues involved in choosing an index for inflation targeting purposes.

[14] These weights are $s_n^{t,m}$ and σ_m^t and $s_n^{t+1,m}$ and σ_m^{t+1} for the chain link equations (9)-(11). The corresponding fixed base formulas can be approximated using the corresponding base year weights, $s_n^{0,m}$ and σ_m^0.

6. Predicting a Rolling Year Index

In a regime where the long-run trend in prices is smooth, changes in the year-over-year inflation rate for this month compared with last month theoretically could give valuable information about the long-run trend in price inflation. This conjecture is demonstrated for the modified Turvey data for the year-over-year monthly fixed base Laspeyres rolling year index. The Laspeyres case is used for showing how indexes of this sort can be used for prediction.

The fixed base rolling year Laspeyres index, P_{LRY}, for month m and current year t is a weighted average of year-over-year price relatives over the 12 months in the current and the base year period. Consider, for example, the December fixed base rolling year Laspeyres index. This index value is a weighted average of year-over-year monthly price relatives for years t and 0 that is centered between June and July of the years being compared. Thus, an approximation of this index value could be obtained by taking the arithmetic average of just the June and July year-over-year monthly indexes for years t and 0.[15] Similarly, this sort of approximation can be made for each month for the rolling year Laspeyres index, P_{LRY} This approximation to the rolling year P_{LRY} index, based on averaging the year-over-year monthly index values for the months at the center for the rolling year for the P_{LRY} index, is denoted by P_{ARY}.

Seasonal adjustment factors, SAF, are defined as the ratios of the P_{LRY} to the P_{ARY} values using the initial 12 months of values for these series. These estimated monthly adjustment factors are assumed to be the same for all subsequent years.[16] Once the seasonal adjustment factors have been defined, the approximate rolling year index, P_{ARY}, can be multiplied by the corresponding seasonal adjustment factor, SAF, to form a seasonally adjusted approximate rolling year index, P_{SAARY}.[17]

With this approach, users could obtain a reasonably accurate forecast of trend inflation. It is not *necessary* to use rolling year indexes in the seasonal adjustment process, but this is recommended as a way of increasing the objectivity and reproducibility of the seasonally adjusted indexes. The method of seasonal adjustment used in this section is crude in that no adjustments have been made for other known factors such as differences for the same month in the numbers of trading days and holiday effects. These refinements would be laborious but straightforward to add.

---

[15] Suppose the middle two months are June and July. Then if an average of the year-over-year monthly indexes for more months such as for May, June, July, and August were taken instead, a better approximation to the annual index could be obtained, and if an average of the year-over-year monthly indexes for April, May, June, July, August, and September were taken, an even better approximation could be obtained, and so on.

[16] If SAF is greater than 1, this means that the two months in the middle of the corresponding rolling year have year-over-year rates of price increase that average out to a number below the overall average of the year-over-year rates of price increase for the entire rolling year. The opposite is true if SAF is less than 1.

[17] The rolling year fixed base Laspeyres index P_{LRY} and the seasonally adjusted approximate rolling year index P_{SAARY} will be identical by construction for the first 12 observations. However, after that, the rolling year index P_{LRY} will differ from the corresponding seasonally adjusted approximate rolling year index.

In the previous sections, the suggested indexes are based on year-over-year monthly indexes and their averages. In sections 7-10, attention is turned to more conventional approaches.

7. Maximum Overlap Month-to-Month Price Indexes

One approach for dealing with strongly seasonal products in a month-to-month price index is the maximum overlap method. The first step for this index is to identify the maximum overlap of products: the products available in each of a pair of months. For this set of products, some index formula such as the Fisher is defined, as in box 4. Thus, the bilateral index number formula is applied only to the subset of products that are present in both months for which prices are being compared.[18] The question now arises: should the comparison and the base months be adjacent (thus leading to a chained index), or should the base month be fixed (leading to a fixed base index)? One reason for preferring chain indexes is that, from one month to the next, new products are introduced and old ones are withdrawn, so fixed base indexes inevitably become unrepresentative over time. Hence the Manual chapter recommends the use of chained indexes that can more closely follow market developments.

The expenditure shares that appear in the maximum overlap month-to-month Laspeyres index, defined by equation (14) in box 4, are given by (12). These are the shares that result from expenditures on seasonal products present in month m of year t *and also* present in the following month. Similarly, the expenditure shares that appear in the maximum overlap month-to-month Paasche index, defined by equation (15) in box 4, are given by (13). These are the shares that result from expenditures on seasonal products that are present in month $m+1$ of year t *and* are also present in the following month. The maximum overlap month-to-month Fisher index, defined by equation (16) in box 4, is the geometric mean of the Laspeyres and Paasche indexes.

For the artificial modified Turvey data set, the maximum overlap index suffers from a significant downward bias. Part of the problem seems to involve the seasonal pattern of prices for peaches and strawberries (products 2 and 4 in the modified Turvey data). For the first month of the year when each of these fruits become available, they are relatively high priced; in subsequent months, their prices drop substantially. The effects of the initially high prices (compared with the relatively low prices in the last month of the previous year) are not captured by the maximum overlap month-to-month indexes, so the resulting indexes build up a tremendous downward bias. The downward bias is most pronounced for the Paasche index, which uses the quantities or volumes of the current month. These volumes are relatively large compared to those of the initial month when the products become available, reflecting the effects of falling prices as the quantity available in the market increases.

---

[18] Keynes (1930, p. 95) called this the highest common factor method for making bilateral index number comparisons. This target index drops those strongly seasonal products that are not present in the marketplace during one of the two months being compared. Thus, the index number comparison is not completely comprehensive. Mudgett (1951, p. 46) called the error in an index number comparison that is introduced by the highest common factor method (or maximum overlap method) the homogeneity error.

Box 4. Definitions for Maximum Overlap Month-to-Month Indexes

Let there be N products that are each available in one or more months of some year and let $p_n^{t,m}$ and $q_n^{t,m}$ denote the price and quantity of product n that is in the marketplace in month m of year t. (If the product is unavailable, $p_n^{t,m}$ and $q_n^{t,m}$ are set equal to 0.) Let $p^{t,m} \equiv [p_1^{t,m}, p_2^{t,m}, \ldots, p_N^{t,m}]$ and $q^{t,m} \equiv [q_1^{t,m}, q_2^{t,m}, \ldots, q_N^{t,m}]$ be the month m and year t price and quantity vectors, respectively. Let $S(t,m)$ be defined as the set of products present in month m of year t *and the following month*.

The expenditure shares of product n in month m and m+1 of year t, using the set of products that are present in month m of year t and the subsequent month, are defined as follows:

(12)[22.23]
$$s_n^{t,m}(t,m) = \frac{p_n^{t,m} q_n^{t,m}}{\sum_{i \in S(t,m)} p_i^{t,m} q_i^{t,m}}, \qquad m=1,2,\ldots,11;\ n \in S(t,m),$$
$$= 0 \text{ otherwise; and}$$

(13)[22.24]
$$s_n^{t,m+1}(t,m) = \frac{p_n^{t,m+1} q_n^{t,m+1}}{\sum_{i \in S(t,m)} p_i^{t,m+1} q_i^{t,m+1}}, \qquad m=1,2,\ldots,11;\ n \in S(t,m),$$
$$= 0 \text{ otherwise,}$$

where $s_n^{t,m+1}(t,m)$ must be distinguished from $s_n^{t,m+1}(t,m+1)$. The expenditure share $s_n^{t,m+1}(t,m)$ is the share of product n in month m+1 of year t with n restricted to the set of products that are present in month m of year t *and* the subsequent month, whereas $s_n^{t,m+1}(t,m+1)$ is the share of product n in month m+1 of year t with n restricted to the set of products that are present in month m+1 of year t *and* the subsequent month.

Using these share definitions, the maximum overlap Laspeyres, Paasche, and Fisher indexes, going from month m of year t to the following month, can be defined as follows:[19]

(14)[22.25]
$$P_L = \sum_{n \in S(t,m)} s_n^{t,m}(t,m)\left(p_n^{t+1,m}/p_n^{t,m}\right), \qquad m=1,2,\ldots,11;$$

(15)[22.26]
$$P_P = \left[\sum_{n \in S(t,m)} s_n^{t,m+1}(t,m)\left(p_n^{t+1,m}/p_n^{t,m}\right)^{-1}\right]^{-1}, \qquad m=1,2,\ldots,11;\ \text{and}$$

(16)[22.27]
$$P_F = \sqrt{P_L P_P},$$

where P_L and P_P are now defined by (14) and (15). Note that P_L, P_P, and P_F depend on the (complete) price and quantity vectors for months m and m + 1 of year t, $p^{t,m}$, $p^{t,m+1}$, $q^{t,m}$, $q^{t,m+1}$, but they also depend on S(t,m), which is the set of products present in both months.

Results are also shown in the Manual chapter using the chained Laspeyres, Paasche, and Fisher indexes with the data for the modified Turvey dataset for just products 1, 3 and 5 (that is, using only the three year-round products). These series are still found to suffer from substantial seasonal variability. For the modified Turvey data, the quantity of grapes (product 3) available in

---

[19] It is important that the expenditure shares that are used in an index number formula add up to unity. The use of unadjusted expenditure shares would lead to a systematic bias in the index number formula. The equations are slightly different for the indexes that go from December to January of the following year. In order to simplify the exposition and convey the main concepts, these equations are left for the reader to work out.

the market varies tremendously over the course of a year, with substantial increases in price for the months when grapes are almost out of season. The price of grapes decreases as the quantity increases during the last half of each year, and then the annual increase in the price of grapes takes place in the first half of the year when the quantities are small. This pattern of seasonal price and quantity changes causes a downward bias.[20] Basically, the monthly varying high volumes are associated with low or declining prices and the low volumes are associated with high or rising prices. These weight effects magnify the seasonal price declines relative to the seasonal price increases using month-to-month index number formulas with monthly varying weights.[21]

All of the month-to-month chained indexes show substantial seasonal fluctuations in prices over the course of a year:[22] probably too much seasonal fluctuation if the purpose of a month-to-month price index is to indicate changes in general inflation.

8.   Annual Basket Indexes with Carry Forward of Unavailable Prices

The various indexes introduced that use monthly expenditure share information can be approximated by indexes where the monthly expenditure shares are replaced by annual expenditure shares. When this substitution is made because monthly expenditure or quantity data are unavailable, this approach is similar in spirit to another common statistical agency practice. Official statistics agencies commonly have price data that are collected monthly, but expenditure data that are collected less frequently. Thus, statistical agencies commonly use some sort of a Lowe index: a type of index that allows for different base periods for the prices and the quantity or expenditure weights.[23] An annual basket Lowe index is defined by (17) in box 5. The annual basket Young index, defined in equation (18) in box 5, could also be used. Yet another annual basket monthly index is the geometric Laspeyres defined in equation (19) in box 5. The geometric Laspeyres index makes use of the same information as the Young index, but a geometric average of the price relatives is taken instead of an arithmetic one.

---

[20] Baldwin (1990) used the original Turvey data to illustrate various treatments of seasonal products. He provides a good discussion of what causes various month-to-month indexes to behave badly. "It is a sad fact that for some seasonal product groups, monthly price changes are not meaningful, whatever the choice of formula" (p. 264).

[21] Another problem with month-to-month chained indexes is that purchases and sales of individual products can become irregular as the time period becomes shorter and shorter and the problem of zero purchases and sales becomes more pronounced. Feenstra and Shapiro (2003, p. 125) find an *upward* bias for their chained *weekly* indexes for canned tuna compared to a fixed base index; their bias was caused by variable weight effects due to the timing of advertising expenditures. In general, these drift effects of chained indexes can be reduced by lengthening the unit time period, so that the *trends* in the data become more prominent than the *high-frequency fluctuations*.

[22] Irregular high-frequency fluctuations will tend to be smaller for quarters than for months. For this reason, chained quarterly indexes can be expected to perform better than chained monthly or weekly indexes. Statistical agencies should check that their month-to-month indexes are at least approximately consistent with the corresponding year-over-year indexes.

[23] See Hill (2009) and Balk and Diewert (2009). Often data for per unit prices come from surveys in retail outlets, and on expenditures come from household expenditure surveys. The quantity estimates are then obtained by dividing the expenditure figures by per unit prices.

Box 5. Lowe, Young and Geometric Laspeyres Annual Basket Indexes

The annual basket monthly Lowe (1823) index for month m is defined by:

$$(17)[22.28] \qquad P_{LO} = \frac{\sum\limits_{n=1}^{N} p_n^m q_n}{\sum\limits_{n=1}^{N} p_n^0 q_n},$$

where $p^0 \equiv [p_1^0, p_2^0, \ldots, p_N^0]$ is the price vector for the price reference period, $p^m \equiv [p_1^m, p_2^m, \ldots, p_N^m]$ is the price vector for the current month m, and $q \equiv [q_1, \ldots, q_N]$ is the quantity weight vector for the quantity (or expenditure) reference year. In the context of seasonal price indexes, this type corresponds to Bean and Stine's (1924, p. 31) Type A index.

The annual basket monthly Young (1812) index is also sometimes used:

$$(18)[22.30] \qquad P_Y = \sum\limits_{n=1}^{N} s_n (p_n^m / p_n^0),$$

where $s \equiv [s_1, \ldots, s_N]$ is the reference year vector of expenditure share weights.

The annual basket monthly geometric Laspeyres index is defined as:

$$(19)[22.32] \qquad P_{GL} = \prod\limits_{n=1}^{N} (p_n^m / p_n^0)^{s_n^0}.$$

In the Manual chapter, the above three annual basket indexes are compared with the fixed base Laspeyres rolling year indexes. However, the rolling year index that ends in the current month is centered five and a half months backwards. Hence the annual basket type indexes are compared with an arithmetic average of two rolling year indexes that have their last month 5 and 6 months forward, respectively. This centered rolling year index is labeled P_{CRY}.[24]

The Lowe, Young, and geometric Laspeyres annual basket indexes display considerable seasonality and do not seem to track their rolling year counterparts well.[25]

Andrew Baldwin's (1990, p. 258) comments on annual basket (AB) type indexes such as the three defined above are also relevant for those considering use of these indexes:

"For seasonal goods, the AB index is best considered an index partially adjusted for seasonal variation. It is based on annual quantities, which do not reflect the seasonal fluctuations in the volume of purchases, and on raw monthly prices, which do incorporate seasonal price fluctuations. Zarnowitz (1961, pp. 256–257) calls it an index of 'a hybrid sort.' Being neither of sea nor land, it does not provide an appropriate measure either of monthly or 12 month price change. The question that an AB index answers with respect to price change from January to February say, or January of one year to January of the next, is 'What would the change in consumer prices have been if there were no seasonality in purchases in the months in question, but prices nonetheless retained their own seasonal behaviour?' It is hard to believe that this is a question that anyone would be interested in asking. On the other hand, the 12 month ratio of an AB index based on

---

[24] The series was normalized to equal 1 in December 1970 for comparability with the other month-to-month indexes.

[25] The four series, P_{LO}, P_Y, P_{GL}, and P_{CRY}, are examined graphically in the Manual chapter.

seasonally adjusted prices would be conceptually valid, if one were interested in eliminating seasonal influences."

Annual basket indexes are of interest to us though because they are used by some statistical agencies.

9.   Annual Basket Indexes with Imputation of Unavailable Prices

The poor performance of the annual basket type of indexes considered in section 8 is likely due to the carry forward of prices of strongly seasonal products into months when the products were not available. This could augment the seasonal movements in the indexes. Hence in this section, the properties are examined of the Lowe, Young, and geometric Laspeyres indexes when a different way of imputing the missing prices is used.[26] In this section, prices in months when they are not observed are assumed to have increased at some given rate.[27] The resulting indexes are compared with the centered rolling year index, P_{CRY}, and are found to be a little less variable.[28] Nevertheless, the Lowe, Young, and geometric Laspeyres annual basket indexes that incorporate imputed prices still display tremendous seasonality when evaluated for the modified Turvey data, and they fail to closely track their rolling year counterparts.

10.   The Bean and Stine Type C or Rothwell Index

The Bean and Stine Type C (1924, p. 31), also called the Rothwell (1958, p. 72), index is defined in box 6. This index makes use of monthly baskets for a base year. The index also makes use of a vector of base year unit value prices. The quantity weights for this index change from month to month. Thus the monthly movements in this index are a mixture of price and quantity changes.[29] The conclusion reached in the Manual chapter based on comparisons using the modified Turvey data is that the Rothwell index has smaller seasonal movements than the Lowe index (defined in box 5) and is less volatile in general.[30] However, there still are large seasonal movements in the Rothwell index.

---

[26] Alternative imputation methods are discussed, for example, by Armknecht and Maitland-Smith (1999) and Feenstra and Diewert (2001).

[27] In the applications for the Manual chapter based on the modified Turvey data, a multiplicative rate of 1.008 is used, except for the last year when this rate is escalated by an additional 1.008.

[28] The imputed indexes are preferred to the carry forward indexes on general methodological grounds. In high inflation environments, the carry forward indexes will be subject to sudden jumps when previously unavailable products become available.

[29] Rothwell (1958, p. 72) showed that the month-to-month movements in the index have the form of an expenditure ratio divided by a quantity index.

[30] In the Manual chapter, the Rothwell index, P_R, is compared to the Lowe index with carry forward of missing prices, P_{LO}. To make the two types of series more comparable, the normalized Rothwell index, P_{NR}, is also presented; this index equals the original Rothwell index divided by its first observation.

Box 6. The Bean and Stine Type C, also Known as the Rothwell, Index

For <u>the Bean and Stine Type C or Rothwell index</u>, the seasonal baskets in the base year are denoted as the vectors $q^{0,m}$ for the months m = 1,2,…,12. The index also makes use of a vector of base year unit value prices, $p^0 \equiv [p_1^0,…,p_5^0]$, where the nth price in this vector is defined as:

$$(20)[22.33] \qquad p_n^0 \equiv \frac{\sum_{m=1}^{12} p_n^{0,m} q_n^{0,m}}{\sum_{m=1}^{12} q_n^{0,m}} \; .$$

The Rothwell price index for month m in year t can now be defined as follows:

$$(21)[22.34] \qquad P_R = \frac{\sum_{n=1}^{N} p_n^{t,m} q_n^{0,m}}{\sum_{n=1}^{N} p_n^0 q_n^{0,m}} \; , \qquad\qquad m=1,…,12 \; .$$

11.   Adjustment of Seasonal Data

The fact that the maximum overlap method [section 7], the annual basket indexes with carry forward of unavailable prices [section 8], and the annual basket indexes with imputation of unavailable prices [section 9] produce price index series that still display considerable seasonality when seasonal products are included has stimulated interest in adjustment of the data for seasonal products prior to incorporating these products into annual basket indexes such as the Lowe, Young, Geometric Laspeyres or Rothwell. The Manual chapter reports results for two alternative ways of carrying out this pre-adjustment. The first is that the modified Turvey data are seasonally adjusted using SAF terms computed as specified in section 6: an approach based on year-over-year month specific comparisons.[31] Second of all, the X-11 method is used to seasonally adjust the data before it is used to evaluate the alternative indexes. Since little is said in the Manual chapter about the X-11 method, this approach is now taken up in the next section.[32]

In the Manual chapter, the predicted values of the month-to-month indexes with pre-adjustment of the data for the seasonal products are fairly close to the corresponding target index

---

[31] More specifically, for each of the price index series -- the Lowe (LO), Young (Y), Geometric Laspeyres (GL), and Rothwell (ROTH) -- a seasonal adjustment factor (SAF) is defined, as in section 6, as the centered rolling year index P_{CRY} divided, respectively, by P_{LO}, P_Y, P_{GL} or P_{ROTH} to produce the first 12 observations, which are then replicated for the other years. The result is four SAF series: SAF_{LO}, SAF_Y, SAF_{GL}, and SAF_{ROTH}, respectively. Seasonally adjusted Lowe, Young, Geometric Laspeyres and Rothwell indexes are computed by multiplying each unadjusted index by the corresponding SAF vector.

[32] X-11 adjusted numerical results are included in the PPI Manual, but not in the CPI Manual, version of chapter 22. In the CPI Manual, X-11 type seasonal adjustment methods are mentioned only in the next to last footnote. In the PPI Manual, X-11 deflation is briefly mentioned, without specifics, and empirical results are presented for an X-11 type method in three columns added into table 22.27 and in the added figure 22.8b (the only figure not included in both the CPI and the PPI versions of the chapter, always with the same figure numbers).

values. It should also be noted that the seasonally adjusted geometric Laspeyres index is generally the best predictor of the corresponding rolling year index evaluated using the modified Turvey data, with the results for the Lowe index being quite similar and with the results for the seasonally adjusted Rothwell index being furthest away.

12. X-11 and X-12 Type Seasonal Adjustment

As already noted, the Manual chapter reports results that utilize the widely used X-11 approach to adjust the price indexes for seasonal patterns. Here, for completeness, we review the origins and essence of the X-11 approach, and how the X-11 approach relates to the X-12 family of methods.

All members of the X-11 family are conceptually based on univariate time series models. The idea that an observed time series could be usefully decomposed into components none of which can be directly observed -- a trend, cycle, seasonal variations and irregular fluctuations -- reportedly comes from astronomy and meteorology.[33] Trends and long cycles are difficult to distinguish with the sorts of data usually available for the macro economy, and hence are usually treated together and referred to collectively as "trends" in the literature on the seasonal adjustment of economic time series. That is, the four component model is collapsed to a three component model.

Research on seasonal adjustment for economic time series was stimulated during the 1920s and 1930s by the work of Persons (1919). He made simple transformations of the data to remove the trend, and then calculated seasonal estimates and used these in analyzing the original data. Persons called this the link-relative method. The method of Persons (1923) assumes the existence of fixed seasonal factors, though he acknowledged that the idea of strictly fixed seasonality is problematic. Indeed, the recognition that the seasonal factors in economic time series are mostly not rigidly tied to the calendar was one reason why Macauley and others chose instead to employ moving average methods in preference to using deterministic explicit functions of time. Moving averages are a type of filter that successively averages a shifting time span of data so as to produce a smoothed estimate of a time series. A filter removes or reduces the strength of certain cycles in the data.

12.1 X-11 development

Precursors of X-11 were developed in the 1950s. These include the U.S. Census Bureau's Census I and Census II methods. Julius Shiskin was a guiding force and key contributor in this development. With many intervening steps, the Census II method evolved into the X-11

---

[33] Nerlove, Grether, and Carvalho (1979) point out that the idea that an observed time series reflects several unobserved components came originally from astronomy and meteorology and became popular in economics in England during the period of 1825-1875. They discuss the work of the Dutch meteorologist Buys Ballot (1847) concerning the early development of seasonal adjustment methods. See also Yule (1921a) on this history.

method.[34] X-11 gave users the choice between additive and multiplicative adjustments.[35] In addition to moving average type filters, the X-11 package also enabled the operator to conveniently adjust for differing numbers of trading days: a feature that greatly contributed to its popularity.

The adjustment methods that were developed (including X-11) were basically modifications of previously used methods that attempted to incorporate automatically the professional judgment that had been required previously to envision and implement these types of adjustments. In addition to making it more feasible for statistical agencies around the world to produce the large volumes of seasonally adjusted data wanted by the policy community, Bell and Hillmer (1984) note that: "This helped lend an air of objectivity to the seasonal adjustment process, so that seasonal adjusters would not be accused of tampering with the data, a consideration that has become even more important in recent years." Dagum (1983) confirms that the design of the X-11 family was shaped by the objective that the method could be encoded in a packaged program that could be used by a statistical technician anywhere, who might have little specialized knowledge of the real world processes generating the series being seasonally adjusted.[36]

X-11-ARIMA was developed at Statistics Canada in the 1970s and entailed the addition of ARIMA (Auto Regressive Integrated Moving Average) features that augmented the capability for extrapolation of observations at the end points of the actual time series to be seasonally adjusted. This capability was also part of the original X-11 process, but in a more rudimentary, less convenient form.[37] The forecasted values are used in the X-11-ARIAMA adjustment process as though they were actual data. Using the extrapolated data *along with the real data* is what permits the use of filters in the seasonal adjustment process for the production of preliminary series that are more like the filters to be used in producing revisions of the seasonally adjusted series.

Shiskin, Young and Musgrave (1967) derived the original asymmetric weights for the Henderson moving average that are used in the X-11 family of adjustment packages. To obtain the weights, a compromise was struck between the two *assumed* objectives that the trend should be able to represent a wide range of curvatures and that it should be as smooth as possible.

The greater the amount of data that is available on each side of what is treated as the current period for a time series that is to be seasonally adjusted, the better the conformity will usually be between the preliminary seasonally adjusted time series released by the statistical agency and the final adjusted time series. *Symmetric moving average filters* are created making use of data both sides of what is taken to be the "current" period for the time series. Each symmetric filter is tailored by software algorithms to the specific time series being seasonally adjusted. In contrast, *asymmetric* filters are pre-made and generic in nature.

---

[34] This was primarily the work of Eisenpress (1956), Marris (1960) and Young (1965, 1968). See U.S. Bureau of the Census (1967) and also Shiskin (1978).

[35] The differences between the results for the additive and the multiplicative versions are sometimes substantial.

[36] For more background, see also Dagum (1975, 1988/1992). Budget limitations probably have also influenced the choices made. Nakamura and Diewert (1996) report on efforts in Canada to protect the reliability of the CPI while reducing the number of price quotes collected.

[37] This was recommended by Macauley (1931, pp. 95-96). Similar approaches were investigated by Geweke (1978) and by Kenny and Durbin (1982). See also Cleveland, W. P., and Tiao, G. C. (1976).

Research showed that use of X-11-ARIMA can drastically reduce the magnitude of subsequent revisions compared with the original X-11-ARIMA. Reports of this finding persuaded a number of statistical agencies to begin using X-11-ARIMA type methods. What statistical agency would not be happy to have smaller changes to report when issuing revisions to important data series! It is important to note, however, that the greater conformity between the original and revised seasonally corrected data probably primarily reflects the fact that the ARIMA based extrapolation that is part of X-11-ARIMA enables greater conformity *between the methods used to produce the original and the revised series*. This finding does not mean that the resulting series do a better job of tracking some agreed on "true" target index.

12.2   The X-11 family spawns the X-12 family

Events and circumstances that are foreseeable but have somewhat irregular calendar dates or represent seasonal irregularities in the calendar itself (e.g., leap year) were recognized early on as important factors to allow for in explaining the short-term movements of economic time series. Once computers were available, their power was used to implement automated ways of allowing for these sorts of foreseeable irregularities.[38] The Bureau of the Census developed an extension of X-11-ARIMA called X-12-ARIMA that had added tools to enable the estimation and diagnosis of a wide range of special effects.

X-12-ARIMA package features are used to make adjustments for the following sorts of calendar circumstances:

(1) Trading day effects. There are different numbers of working and shopping and stock market trading days from month to month, and often for the same month from year to year, due to factors including leap year and official holidays like Christmas and Easter.

(2) Major holidays and other events on fixed calendar dates that are associated with special buying and pricing patterns: e.g., Christmas.

(3) Major holidays and other events associated in known, but not fixed-date, ways with the calendar and that are associated with special buying behavior. The Chinese New Year is a festival of this sort. Some religious and ethnic groups, and some nations, have more holidays that are seasonal but not tied to specific calendar dates. For example, the moving Jewish festivals have always posed special problems for the Israeli CPI program.[39]

---

[38] See, for example, Joy and Thomas (1927), Homan (1933), Young (1965), and Hillmer, Bell and Tiao (1983). According to King (1924), the first to adjust data with varying seasonal factors were Sydenstricker and Britten of the U.S. Public Health Service, while investigating causes of influenza. Their graphical method is briefly described in Britten and Sydenstricker (1922). King (1924) modified Sydenstricker and Britten's method using moving medians, and emphasized the need to account for changing seasonality. Snow (1923) suggested fitting straight lines to each quarter (or month) separately, and checking for varying seasonality by examining the lines to see if they were parallel. Crum (1925) gave a general discussion of varying seasonality and modified Person's link relative method to handle this complication. Mendershausen (1937) reviews early efforts to deal with changing seasonality.

[39] As Burck and Gubman (2003) note, festival date movements in Israel are typical of festivals with dates fixed according to the lunar year, but vary according to the Georgian calendar. Jewish festivals usually move between two consecutive solar months. The date of the Passover festival moves between March and April. The dates of the Jewish New Year, the Day of Atonement (Yom Kippur) and the Feast of Tabernacles (Succoth) move between

X-12-ARIMA also includes packaged hypothesis testing features and special options for detecting and dealing with outliers.[40] The options that are available in the X-12-ARIMA package for making adjustments to reflect foreseeable monthly differences in the numbers of trading days and the timing of major holidays encouraged statistical agencies like the Israeli one to make the switch, at least partially, from using X-11-ARIMA to X-12-ARIMA.

Box 7. The Steps for an X-11-ARIMA Decomposition

The steps for an X-11-ARIMA decomposition (which are similar to the steps for an X-12-ARIMA one) are:

1. An ARIMA model is used to extend the series being adjusted. Extending the series provides forecast and backcast data that help minimize the use of asymmetric filters at the beginning and end of the series.

The next four steps involve three stages of iteration to produce estimates of the three time series components: trend (including cycle), seasonal and irregular.

2. An initial trend estimate is produced using a centered moving average. The estimated trend is removed from the original series to produce an estimate of the combined seasonal and irregular components. Initial seasonal factors are estimated using an iterative process. Stage 1 estimates of trend and seasonal factors are produced.

3. The stage 1 trend estimates are refined in the third step by using the stage 1 seasonal factors in combination with a Henderson filter.

Henderson (1916) derived moving average filters for use in actuarial applications. The Henderson filters are used with all of the X-11 family methods: X-11, X-11-ARIMA, X-11-ARIMA/88 and X-12-ARIMA. They smooth seasonally adjusted estimates and generate an estimated trend. Henderson filters are used in preference to simpler moving averages because they can reproduce polynomials of up to degree 3, thereby capturing trend turning points. Henderson filters can be either symmetric or asymmetric. *As already noted, symmetric moving averages can only be applied at points that are sufficiently far away from the ends of a time series.*

4. The refined trend is used to create refined seasonal factors following essentially the same process used to produce the stage 1 estimates. Stage 2 final estimates of the seasonal component (i.e., the seasonal factors) are obtained.

5. The third stage uses the stage 2 final seasonal factors combined with a Henderson filter to estimate a stage 3 final trend component. Lastly, using the stage 2 and 3 final seasonal and trend estimates, an estimate of the irregular component is produced.

The step 1 insertion into the data set of ARIMA created observations is what enables the greater use of symmetric Henderson filters in both the third and the fifth steps. Symmetric filters are what are also used in an X-11 type revision process as more real data become available.

13.   Concluding Thoughts

A large proportion of products are seasonal. Strongly seasonal products that are not available at all in some months pose the most severe challenges for conventional price index methods. However, weakly seasonal products that are available in all months but have prices and quantities that have seasonal fluctuations can introduce considerable seasonal fluctuation into a consumer or producer price index. The conventional solution to these problems is to omit seasonal products in price index making. However, there are obvious drawbacks to a CPI or PPI

September and October. The Feast of Weeks (Shavuoth) and Independence Day are two other lunar holidays with dates moving between April and May, and between May and June, respectively.

[40] See Findley et al. (1998).

that fails to cover a large share of the products that households consume. These considerations have led to interest in alternative methods for dealing with both strongly and weakly seasonal products in price indexes.

Seasonal and other types of patterned variation can definitely be a source of spurious correlations, and can lead to causally wrong findings.[41] Thus, Granger (1978) argues that, "By using adjusted series, one possible source of spurious relationship is removed." Similarly, Bryan and Cecchetti (1994) argue for the use of trimmed means in the core inflation literature[42] on the grounds that these provide a more robust measure of central tendency than the standard CPI inflation rate, by reducing the influence of transitory price movements that distort the underlying inflationary impulse.

However, others including Bell and Hilmer (1984) and Ghysels (1988, 1990), have raised concerns that seasonal adjustment might lead to mistaken inferences. Orcutt and James (1948), 40 years earlier, noted the nub of concern: the choice of the relevant analogy for use in hypothesis testing:

> "The testing of the significance of a correlation involves a comparison with what would have been obtained between non-related series thought to be analogous to the observed series. And, of course, the significance found for the correlation will depend upon the analogy deemed to be appropriate.... "

Diewert, Alterman and Feenstra (2009) argue that time series methods for the seasonal adjustment of economic price or quantity series are not well suited for helping analysts uncover insights into inflationary developments. Moreover, they argue that, in general, these methods are not suited for producing month-to-month price series that are free of seasonal influences. Diewert, Alterman and Feenstra argue that this criticism of time series seasonal adjustment methods can be better understood by imagining a situation in which each seasonal product in an aggregate is present for only one season of each year. The price for each product would be affected by circumstances in that product market that would not necessarily affect any of the other product markets, and hence the price series for each of the products could have a different evolution over time. Diewert, Alterman and Feenstra argue that, in situations with elements of this nature, no amount of smoothing of the month-to-month price realizations will necessarily be of any use for predicting the future change in the price level going from one month to the next.

However, year-over-year monthly indexes can always be constructed, even for strongly seasonal products. Many users directly need these indexes, and these indexes are also the building blocks as well for annual indexes and for rolling year indexes. Statistical agencies

---

[41] Orcutt (1948) and Cochrane and Orcutt (1949) were early proponents of autoregressive filtering or differencing of the dependent and independent variables prior to testing the significance of regression coefficients in models with autocorrelated errors. On the need and also problems and results for autoregressive filters, see also Orcutt and Winokur (1969), Nakamura and Nakamura (1978), and Nakamura, Nakamura and Orcutt (1976). All tests of significance for apparent relationships among time series are seriously affected when the series themselves, or the error terms for the equations being estimated, contain substantial autoregressive or other non-white noise components. The coefficient estimates of the equation parameters may not be biased, but the estimates of the standard errors will be, thereby seriously distorting the significance test results. See also Durbin (1959, 1960), Jorgenson (1967), and Wallis (1974, 1978).

[42] The best known core inflation indicator is perhaps the CPI with food and energy excluded that Blinder (1982) proposed. See Bloem, Amknecht and Zieschang (2002) on the use of the CPI, PPI and other price indexes for inflation targeting in the conduct of monetary policies.

should compute these indexes. In addition, the Manual chapter argues for the importance of replicability, and year-over-year monthly indexes produce results that can readily be replicated by others given access to the same data.

Thus, we find that insights dating back more than 100 years are still valid today. The statistician Yule (1921b, p. 199) stated:

> "My own inclination would be to form the index number for any month by taking ratios to the corresponding month of the year being used for reference, the year before presumably, as this would avoid any difficulties with seasonal commodities. I should then form the annual average by the geometric mean of the monthly figures."

This approach was also promoted by Mudgett (1955) in the consumer price context and Stone in the producer price context. For example, Stone (1956, pp. 74-75) wrote:

> "The existence of a regular seasonal pattern in prices which more or less repeats itself year after year suggests very strongly that the varieties of a commodity available at different seasons cannot be transformed into one another without cost and that, accordingly, in all cases where seasonal variations in price are significant, the varieties available at different times of the year should be treated, in principle, as separate commodities."

More research needs to be done on the problems associated with the index number treatment of seasonal products. A consensus on what is best practice in this area has not yet formed. However, it appears that 12-month change indexes may offer the best approach for inflation monitoring.

References

Alterman, W.F., W.E. Diewert and R.C. Feenstra (1999), *International Trade Price Indexes and Seasonal Commodities*, Bureau of Labor Statistics, Washington D.C.

Armknecht, P.A. and F. Maitland-Smith (1999), "Price Imputation and Other Techniques for Dealing with Missing Observations, Seasonality and Quality Change in Price Indices," IMF Working Paper No. 99/78, Washington, D.C., June.

Baldwin, A. (1990), "Seasonal Baskets in Consumer Price Indexes," *Journal of Official Statistics* 6 (3), 251-273.

Balk, B.M. (1980a), "Seasonal Products in Agriculture and Horticulture and Methods for Computing Price Indices," *Statistical Studies*, Netherlands Central Bureau of Statistics.

Balk, B.M. (1980b), "Seasonal Commodities and the Construction of Annual and Monthly Price Indexes," *Statistische Hefte* 21, 110-116.

Balk, B.M. (1980c), "A Method for Constructing Price Indices for Seasonal Commodities," *Journal of the Royal Statistical Society Series A* 143, 68-75.

Balk, B.M. (1981), "A Simple Method for Constructing Price Indices for Seasonal Commodities," *Statistische Hefte* 22, 72-78.

Balk, B.M. and W.E. Diewert (2009), "The Lowe Consumer Price Index and Its Substitution Bias," in W.E. Diewert, B.M. Balk, D. Fixler, K.J. Fox, and A.O. Nakamura (eds.), *Price and Productivity Measurement: Volume 6 -- Index Number Theory*, Trafford Press.

Ballot, B.C. H. D. (1847), *Les Changements Criodiques de Tembrature*, Utrecht: Kemink et Fils.

Bean, L.H. and O.C. Stine (1924), "Four Types of Index Numbers of Farm Prices," *Journal of the American Statistical Association* 19, 30-35.

Bell, W.R. and S.C. Hillmer (1984), "Issues Involved with the Seasonal Adjustment of Economic Time Series," *Journal of Business and Economic Statistics,* 2, 291-320.

Blinder, A. (1982), "Double-Digit Inflation in the 1970s", in R.E. Hall (ed.), *Inflation: Causes and Effects*, University of Chicago Press, 261-282.

Bloem, A. M., P.A. Armknecht and K.D. Zieschang (2002), "Price Indices for Inflation Targeting," International Monetary Fund.

Box, G.E.P. and Jenkins, G.M. (1970): *Time Series Analysis Forecasting and Control*, Holden Day.

Britten, R.H. and E. Sydenstricker (1922), "Mortality from Pulmonary Tuberculosis in Recent Years," Public Health Reports 37, U.S. Public Health Service, 2843-2858.

Bryan, M.F. and S.G. Cecchetti (1994), "Measuring Core Inflation," in N.G. Mankiw (ed), *Monetary Policy*, University of Chicago Press, 195-215.

Burck, L. and Y. Gubman (2003), "Pre-adjustment in X12-ARIMA," working paper of the Israel Central Bureau of Statistics. http://www.fcsm.gov/03papers/Burck.pdf

Cleveland, W. P., and Tiao, G. C. (1976), "Decomposition of Seasonal Time Series: A Model for the X-11 Program," *Journal of the American Statistical Association* 71, 581-587.

Cochrane, D., and G.H. Orcutt (1949), "Application of Least Squares Regression to Relationships Containing Autocorrelated Error Terms," *Journal of the American Statistical Association* 44, 32-61.

Crum, W.L. (1925), "Progressive Variation in Seasonality," *Journal of the American Statistical Association* 20, 48-64.

Crump, N. (1924), "The Interrelation and Distribution of Prices and their Incidence Upon Price Stabilization," *Journal of the Royal Statistical Society* 87, 167-206.

Dagum, E.B. (1975), "Seasonal Factor Forecasts from ARIMA Models," *Proceedings of the 40th Session of the International Statistical Institute*, Warsaw, 206-219.

Dagum, E.B. (1983), "The X-11 ARIMA Seasonal Adjustment Method," Statistics Canada Catalogue No. 12-564E.

Dagum, E.B. (1988, revised 1992), "Annex D: The X11ARIMA Seasonal Adjustment Program," *The X11ARIMA/88 Seasonal Adjustment Method - Foundations and User's Manual*, Statistics Canada.

Diewert, W.E. (1983), "The Treatment of Seasonality in a Cost of Living Index," in W.E. Diewert and C. Montmarquette (eds.), *Price Level Measurement*, Statistics Canada, 1019-1045. http://www.econ.ubc.ca/diewert/living.pdf. An excerpt is reprinted as chapter 8 in Diewert, W.E., Balk, B.M., Fixler, D., Fox, K.J. and Nakamura, A.O. (eds.) (2009), *Price and Productivity Measurement, Volume 2: Seasonality*, Trafford Press.

Diewert, W.E. (1995a), "Axiomatic and Economic Approaches to Elementary Price Indexes," Discussion Paper No. 95-01, Department of Economics, University of British Columbia. http://www.econ.ubc.ca/diewert/dp9501p1.pdf and http://www.econ.ubc.ca/diewert/dp9501p2.pdf.

Diewert, W.E. (1995b), "On the Stochastic Approach to Index Numbers," Discussion Paper No. 95-31. http://www.econ.ubc.ca/diewert/9531.pdf. Forthcoming also in Diewert, W.E., Balk, B.M., Fixler, D., Fox, K.J. and Nakamura, A.O. (eds.) (2009), *Price and Productivity Measurement: Volume 4 -- International Comparisons and Trade*, Trafford Press.

Diewert, W.E. (1996), "Seasonal Commodities, High Inflation and Index Number Theory," Discussion Paper 96-06, Department of Economics, University of British Columbia. http://www.econ.ubc.ca/dp9606.pdf

Diewert, W.E. (1998), "High Inflation, Seasonal Commodities and Annual Index Numbers," *Macroeconomic Dynamics* 2, 456-471. http://www.econ.ubc.ca/diewert/highinfl.pdf

Diewert, W.E. (1999), "Index Number Approaches to Seasonal Adjustment," *Macroeconomic Dynamics* 3, 48-68. http://www.econ.ubc.ca/diewert/seasonal.pdf

Diewert, W.E. (2002), "Harmonized Indexes of Consumer Prices: Their Conceptual Foundations," *Swiss Journal of Economics and Statistics* 138 (4), 547-637. http://www.econ.ubc.ca/diewert/harindex.pdf

Diewert, W.E., W.F. Alterman and R.C. Feenstra (2009), "Time Series versus Index Number Methods of Seasonal Adjustment," chapter 3 in W.E. Diewert, B.M. Balk, D. Fixler, K.J. Fox, and A.O. Nakamura (eds.) (2008), *Price and Productivity Measurement: Volume 2 -- Seasonality*, Trafford Press.

Diewert, W.E. and P.A. Armknecht (2004), "The Treatment of Seasonal Products," chapter 22 in ILO et al. (2004) and in IMF et al. (2004). Available for free download at http://www.imf.org/external/np/sta/tegppi/ch22.pdf

Diewert, W.E., Y. Finkel and Y. Artsev (2009), "Empirical Evidence on the Treatment of Seasonal Products: The Israeli CPI Experience," chapter 4 in W.E. Diewert, B.M. Balk, D. Fixler, K.J. Fox, and A.O. Nakamura (eds.), *Price and Productivity Measurement: Volume 2 -- Seasonality*, Trafford Press.

Durbin, J. (1959), "Efficient Estimation of Parameters in Moving Average Models," Biometrika, 46, 306-316. - (1960a), "Estimation of Parameters in Time Series Regression Models," Journal of the Royal Statistical Society Ser. B, 22, 139-153.

Durbin, J. (1960), "The Fitting of Time-Series Models," *Review of the International Statistical Institute*, 28, 233-243.

Eisenpress, H. (1956), "Regression Techniques Applied to Seasonal Corrections and Adjustments for Calendar Shifts," *Journal of the American Statistical Association* 51, 615-20

Feenstra, R.C. and W.E. Diewert (2001), "Imputation and Price Indexes: Theory and Evidence from the International Price Program," U.S. Bureau of Labor Statistics Working paper 335. http://www.bls.gov/ore/pdf/ec010030.pdf

Feenstra, R.C. and M.D. Shapiro (2003), "High Frequency Substitution and the Measurement of Price Indexes," in R.C. Feenstra and M.D. Shapiro (eds.), *Scanner Data and Price Indexes*, NBER Studies in Income and Wealth, University of Chicago Press, 123–146.

Findley, D.F., B.C. Monsell, W.R. Bell, M.C. Otto and B.-C. Chen (1998), "New Capabilities and Methods of the X-12-ARIMA Seasonal Adjustment Program," *Journal of Business and Economic Statistics* 16, 127-177.

Fisher, I. (1922), *The Making of Index Numbers*, Boston, MA: Houghton-Mifflin.

Fisher, A. (1937), "A Brief Note on Seasonal Variation," *Journal of Accountancy* 64, 174-199.

Flux, A.W. (1921), "The Measurement of Price Change," *Journal of the Royal Statistical Society* 84, 167-199.

Geweke, J. (1978), "Revision of Seasonally Adjusted Time Series," SSRI Report No. 7822, Department of Economics, University of Wisconsin.

Ghysels, E. (1988), "A Study towards a Dynamic Theory of Seasonality for Economic Time Series," *Journal of the American Statistical Association* 83, 168-172

Ghysels, E. (1990), "Unit Root Tests and the Statistical Pitfalls of Seasonal Adjustment: the Case of U.S. Post War Real GNP," *Journal of Business and Economics Statistics* 8, 145-152

Granger, C.W.J. (1978), "Seasonality: Causation, Interpretation, and Implications," in Arnold Zellner (ed.), *Seasonal Analysis of Economic Time Series*, U.S. Bureau of the Census, 33-46.

Hardy, G.H., J.E. Littlewood and G. Pólya (1934), *Inequalities*, Cambridge University Press.

Hill, T.P. (2009), "Lowe Indices," in Diewert, W.E., Balk, B.M., Fixler, D., Fox, K.J. and Nakamura, A.O. (eds.), *Price and Productivity Measurement: Volume 6 -- Index Number Theory*, Trafford Press.

Hillmer, S.C., W.R. Bell, and G.C. Tiao (1983), "Modeling Considerations in the Seasonal Adjustment of Economic Time Series," to appear in the Proceedings of the Conference on Applied Time Series Analysis of Economic Data, A. Zellner (ed.), U.S. Department of Commerce, Bureau of the Census.

Homan, J. (1933), "Adjusting for the Changing Date of Easter in Economic Series," *Journal of the American Statistical Association* 28, 328-332.

Henderson, R. (1916): "Note on Graduation by Adjusted Average", *Transactions of the Actuarial Society of America* 17, 43-48.

International Labour Office (ILO), International Monetary Fund, Organisation for Economic Co-operation and Development, Eurostat, United Nations, and The World Bank (2004), *Consumer Price Index Manual: Theory and Practice*. Available for free download in whole or by chapter at http://www.ilo.org/public/english/bureau/stat/guides/cpi/index.htm

International Monetary Fund (IMF), International Labour Office, Organisation for Economic Co-operation and Development, Eurostat, United Nations, and The World Bank (2004), *Producer Price Index Manual: Theory and Practice* (*PPI Manual*). Chapters and whole can be downloaded for free at http://www.imf.org/external/np/sta/tegppi/index.htm.

Jevons, W.S. (1863), "A Serious Fall in the Price of Gold Ascertained and its Social Effects Set Forth," reprinted in *Investigations in Currency and Finance*, Macmillan, 1884, 13-118.

Jorgenson, D.W. (1967), "Seasonal Adjustment of Data for Econometric Analysis," *Journal of the American Statistical Association* 62, 137-140.

Joy A., and W. Thomas (1927), "Adjustments for the Influence of Easter in Department Stores Sales," *Journal of the American Statistical Association* 22, 493-496.

Kenny, P.B. and J. Durbin (1982), "Local Trend Estimation and Seasonal Adjustment of Economic and Social Time Series," *Journal of the Royal Statistical Society*, Series A, 145, l-41.

Keynes, J.M. (1930), *A Treatise on Money in Two Volumes*, Macmillan.

King, W.W.I. (1924), "An Improved Method for Measuring the Seasonal Factor," *Journal of the American Statistical Association* 19, 301-313.

Lowe, J. (1823), *The Present State of England in Regard to Agriculture, Trade and Finance, Second Edition*, Longman, Hurst, Rees, Orme and Brown.

Macauley, F.R. (1931), *The Smoothing of Time Series*, National Bureau of Economic Research.

Marris, S.N. (1960), "The Measurement of Calendar Variation," *Seasonal Adjustment in Electronic Computers*, OECD, 345-60

Mendershausen, H. (1937), "Annual Survey of Statistical Technique: Methods of Computing and Eliminating Changing Seasonal Fluctuations," *Econometrica* 5, 234-262.

Mudgett, B.D. (1951), *Index Numbers*, John Wiley and Sons.

Mudgett, B.D. (1955), "The Measurement of Seasonal Movements in Price and Quantity Indexes," *Journal of the American Statistical Association* 50, 93-98.

Nakamura, A.O. and W.E. Diewert (1996), "Can Canada Afford to Spend Less on National Statistics?" *Canadian Business of Economics* 4 (3), 33-34.

Nakamura, A.O. and M. Nakamura (1978), "On the Impact of the Tests for Serial Correlation Upon the Test of Significance for the Regression Coefficient," *Journal of Econometrics* 7, 199-210.

Nakamura, A.O., M. Nakamura and G.H. Orcutt (1976), "Testing for Relationships between Time Series," *Journal of the American Statistical Association, Theory and Methods Section* 71 (March), 214-222.

Nakamura, E. (2005), "Inflation Forecasting using a Neural Network," *Economic Letters* 86(3), pp. 373-378.

Nerlove, M., Grether, D. M., and Carvalho, J. L. (1979), *Analysis of Economic Time Series: A Synthesis*, Academic Press.

Orcutt, G.H., 1948, "A Study of the Autoregressive Nature of the Time Series Used for Tinbergen's Model of the Economic System of the United States, 1919-1932," *Journal of the Royal Statistical Society*, Series B, 1-45.

Orcutt, G.H., and S.F. James (1948), "Testing the Significance of Correlation between Time Series," *Biometrika* 35, 397-413.

Orcutt, G.H. and H.S. Winokur, Jr. (1969), "First Order Autoregression: Inference, Estimation, and Prediction," *Econometrica* 37 (1) Jan., 1-14.

Persons, W.M. (1919), "Indices of Business Conditions," *Review of Economics and Statistics* 1, 5-107.

Persons, W.M. (1923), "'Correlation of Time Series," *Journal of the American Statistical Association* 18, 713-726.

Rothwell, D.P. (1958), "Use of Varying Seasonal Weights in Price Index Construction," *Journal of the American Statistical Association*, Vol. 53, 66-77.

Shiskin, J. (1957), "Electronic Computers and Business Indicators," *Journal of Business* 30 (October).

Shiskin, J. (1978), "Seasonal Adjustment of Sensitive Indicators," in A. Zellner (ed.), *Seasonal Analysis of Economic Time Series*, U.S. Department of Commerce, Bureau of the Census, 97-103.

Shiskin, J., Young, A.H., and Musgrave, J.C. (1967), "The X-11 Variant of the Census Method II Seasonal Adjustment Program," Technical Paper 15, U.S. Bureau of the Census.

Silver, M. (2007), "Core Inflation: Measurement and Statistical Issues in Choosing Among Alternative Measures," IMF Staff Papers.

Slutzky, E. (1937), "The summation of random causes as the source of cyclical processes," *Econometrica* 5, 105-146.

Snow, E.C. (1923), "Trade Forecasting and Prices," *Journal of the Royal Statistical Society* 86, 332-376.

Stone, R. (1956), *Quantity and Price Indexes in the National Accounts*, OECD.

Turvey, R. (1979), "The Treatment of Seasonal Items in Consumer Price Indices," *Bulletin of Labour Statistics*, *Fourth Quarter*, ILO, 13-33.

U.S. Bureau of the Census (1967), *The X-11 Variant of the Census Method II Seasonal Adjustment Program*. Technical Paper No. 15.

Wallis, K.F. (1974), "Seasonal Adjustment and Relations between Variables," *Journal of the American Statistical Association* 69, 18-32.

Young, A. (1812), *An Inquiry into the Progressive Value of Money in England as Marked by the Price of Agricultural Products*, Hatchard.

Young, A.H. (1965), "Estimating Trading-day Variation in Monthly Economic Time Series," Technical Paper No. 12, U.S. Bureau of the Census.

Young, A.H. (1968), "Linear Approximations of the Census and BLS Seasonal Adjustment Methods," *Journal of the American Statistical Association* 63, 445-471.

Yule, G.U. (1921a), "On the Time Correlation Problem, with Especial Reference to the Variable Difference Correlation Method", *Journal of the Royal Statistical Society* 84, 497-526.

Yule, G.U. (1921b), "Discussion of Mr. Flux's Paper," *Journal of the Royal Statistical Society* 84, 199-202.

Zarnowitz, V. (1961), "Index Numbers and the Seasonality of Quantities and Prices," G.J. Stigler (ed.), *The Price Statistics of the Federal Government*, National Bureau of Economic Research, 233-304.