Chapter 15

Economic Monotonicity of Price Index Formulas

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1. Preliminary Remarks

The strict monotonicity test is probably one of the most widely accepted axioms in axiomatic index theory. In a paper included in this volume, Kohli (2010) implements the idea of monotonicity in an economic framework where quantities depend on prices. I show how the notion of strict monotonicity, as defined in the traditional axiomatic index theory approach, is somewhat different from Kohli's notion of economic monotonicity. Specifically, in the traditional approach, it is assumed that quantities and prices are independently determined.

Kohli convincingly demonstrates that, embedded in an economic framework, both the Paasche and Fisher index formulas violate monotonicity. Since the Fisher formula is often advocated as the most appropriate price index and the Paasche formula is widely used for the GDP implicit price deflator, Kohli's findings challenge the "general wisdom of index theory."

The present paper relates Kohli's approach to traditional axiomatic index theory. Building on the notion of monotonicity as defined in axiomatic index theory, I show how the notion of economic monotonicity can be defined in a precise manner.

2. Monotonicity in Axiomatic Index Theory

A price index formula P is a positive function that maps all of the prices and quantities in the base and comparison periods into a single positive number; i.e.,

$$P: R_{++}^{4n} \rightarrow R_{++}, \quad (\mathbf{p_0}, \mathbf{q_0}, \mathbf{p_1}, \mathbf{q_1}) \rightarrow P(\mathbf{p_0}, \mathbf{q_0}, \mathbf{p_1}, \mathbf{q_1})$$

where $\mathbf{p_t} = (p_{1,t}, ..., p_{N,t})^T$ and $p_{i,t} > 0$ denotes the unit price of commodity i ($i=1,2,...,N$) in period t, $\mathbf{q_t} = (q_{1,t}, ..., q_{N,t})^T$ and $q_{i,t} > 0$ is the quantity of commodity i in period t, and where $t=0$ is the base period and $t=1$ is the comparison period.

The traditional axiomatic approach embodies the assumption that there is no causal relationship between prices and quantities. What follows are two monotonicity tests – a weak test and a strict test – set out in the usual context for the axiomatic approach regarding the independence of changes in prices and quantities.

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Weak Monotonicity Test (Olt, 1996): *Suppose that for all commodities we have* $p_{i,1} \geq p_{i,0}$ *and for at least one i the inequality is strict. Then*

$$P(\mathbf{p_0}, \mathbf{q_0}, \mathbf{p_1}, \mathbf{q_1}) > P(\mathbf{p_0}, \mathbf{q_0}, \mathbf{p_0}, \mathbf{q_1}).$$

Strict Monotonicity Test (Eichhorn and Voeller, 1976): *Consider two different scenarios for the comparison period (t =1 and t =1*) and the base period (t =0 and t =0*). Suppose that for all commodities we have* $p_{i,1^*} \geq p_{i,1}$*, and suppose that for at least one i the inequality is strict. Then*

(1) $$P(\mathbf{p_0}, \mathbf{q_0}, \mathbf{p_{1*}}, \mathbf{q_1}) > P(\mathbf{p_0}, \mathbf{q_0}, \mathbf{p_1}, \mathbf{q_1}).$$

Or suppose for all commodities we have $p_{i,0^*} \geq p_{i,0}$*, and suppose that for at least one i this inequality is strict. Then*

(2) $$P(\mathbf{p_{0*}}, \mathbf{q_0}, \mathbf{p_1}, \mathbf{q_1}) < P(\mathbf{p_0}, \mathbf{q_0}, \mathbf{p_1}, \mathbf{q_1}).$$

For the special case where $\mathbf{p_1} = \mathbf{p_0}$, inequality (1) simplifies to the weak monotonicity test case.

3. Monotonicity in Economic Approaches to Index Theory

Kohli's (2010) concept of *economic monotonicity* relates to the strict monotonicity test. Moreover, it relates only to postulate (1) of the strict monotonicity test and not to the postulate expressed by inequality (2). Postulate (1) considers alternative scenarios for the comparison period. In contrast, postulate (2) considers alternative scenarios for the base period. I recommend taking a more symmetric approach – that is, also considering postulate (2) as a necessary condition for monotonicity. I suggest this change as a "friendly" amendment to Kohli's concept of economic monotonicity; i.e., I suggest this is a change that is consistent with and would improve Kohli's contribution.

There is a second important difference between Kohli's notion of economic monotonicity and the strict monotonicity test. In Kohli's economic framework, quantities are functions of prices; i.e., we have $\mathbf{q_0} = \mathbf{q}(\mathbf{p_0})$ and $\mathbf{q_1} = \mathbf{q}(\mathbf{p_1})$. Thus we have $q_{i,t} = q_i(\mathbf{p_t})$ with either

(3a) $\partial q_{i,t} / \partial p_{i,t} \geq 0$ ($i = 1,2,\ldots,N;\ t = 0, 1$), or

(3b) $\partial q_{i,t} / \partial p_{i,t} \leq 0$ ($i = 1,2,\ldots,N;\ t = 0, 1$).

A formal definition of the economic monotonicity axiom (including the symmetric treatment of base and comparison period scenarios) can now be given:

Economic Monotonicity Test: *Let* $\mathbf{q_0} = \mathbf{q}(\mathbf{p_0})$ *and* $\mathbf{q_1} = \mathbf{q}(\mathbf{p_1})$. *Consider two different scenarios for the comparison period (t=1 and t=1*) and the base period (t=0 and t=0*). Suppose that for all commodities we have* $p_{i,1^*} \geq p_{i,1}$ *and for at least one i the inequality is strict. Then*

(4) $$P(p_0, q_0, p_{1*}, q_{1*}) > P(p_0, q_0, p_1, q_1).$$

Or suppose for all commodities we have $p_{i,0*} \geq p_{i,0}$ *and for at least one i the inequality is strict. Then*

(5) $$P(p_{0*}, q_{0*}, p_1, q_1) < P(p_0, q_0, p_1, q_1) .$$

In (4), p_{1*} differs from p_1 and simultaneously q_{1*} is allowed to differ from q_1, whereas in (1) the quantity vector q_1 is kept fixed. Analogously, in (5) p_{0*} differs from p_0 and simultaneously q_{0*} is allowed to differ from q_0, whereas in (2) the quantity vector q_0 is kept fixed. This is the crucial difference between economic monotonicity and strict monotonicity.

4. Laspeyres and Paasche Index

Kohli (2010) has demonstrated that the Paasche and Fisher index formulas violate economic monotonicity. Taking the approach stated in (4) and (5), the Laspeyres index formula also violates economic monotonicity. In order to see *why* the Laspeyres formula violates economic monotonicity too, it is useful to reformulate this index as the weighted arithmetic mean of price ratios, where the weights are "expenditure shares" for the base period:

(6) $$P^L_{1,0} = \frac{\sum_i p_{i,1} q_{i,0}}{\sum_i p_{i,0} q_{i,0}} = \sum_i \frac{p_{i,0} q_{i,0}}{\sum_j p_{j,0} q_{j,0}} \frac{p_{i,1}}{p_{i,0}}$$

Formula (6) violates inequality (5) of economic monotonicity. Suppose, for example, that the price of only one good *i* differs between base period *t=0* and base period $t=0^*$. Inequality (5) postulates that if $p_{i,0} < p_{i,0*}$, then for the Laspeyres index we must have $P^L_{1,0} > P^L_{1,0*}$, which will not necessarily be true. Suppose, for example, that the case defined by relationship (3a) applies – that is, quantities are non-negatively related to prices – then $p_{i,0} < p_{i,0*}$ implies that $q_{i,0} \le q_{i,0*}$. As a consequence, the weight $p_{i,0} q_{i,0} \big/ \sum_j p_{j,0} q_{j,0}$ may be much smaller than $p_{i,0*} q_{i,0*} \big/ \sum_j p_{j,0*} q_{j,0*}$. In formula (6), the (larger) price increase $(p_{i,1} / p_{i,0})$ will receive a *smaller* weight than the (smaller) price increase $(p_{i,1} / p_{i,0*})$. In extreme cases, the changes in the weights will overcompensate the impact of the respective price changes, resulting in a violation of economic monotonicity.

The same line of reasoning can be applied to the Paasche index. This index formula can be reformulated as the weighted harmonic mean of price ratios, where the weights are "expenditure shares" of the comparison period:

(7) $$P^P_{1,0} = \frac{\sum_i p_{i,1} q_{i,1}}{\sum_i p_{i,0} q_{i,1}} = \left[\sum_i \frac{p_{i,1} q_{i,1}}{\sum_j p_{j,1} q_{j,1}} \left(\frac{p_{i,1}}{p_{i,0}} \right)^{-1} \right]^{-1} .$$

This formula violates inequality (4) of economic monotonicity. To see this, suppose, for example, that the price of only one good *i* differs between comparison period $t = 1$ and base period $t = 1^*$. Inequality (4) postulates that for $p_{i,1} < p_{i,1*}$, the Paasche index must produce $P^P_{1,0} < P^P_{1*,0}$. If quantity changes are non-negatively related to price changes, then $p_{i,1} < p_{i,1*} < p_{i,0}$ implies that

$q_{i,1} \leq q_{i,1*}$. As a consequence, the weight $p_{i,1}q_{i,1} \Big/ \sum_j p_{j,1}q_{j,1}$ could be much smaller than $p_{i,1*}q_{i,1*} \Big/ \sum_j p_{j,1*}q_{j,1*}$. In formula (7), the (larger) price decline $(p_{i,1}/p_{i,0})$ will receive a smaller weight than the (smaller) price decline $(p_{i,1*}/p_{i,0})$. In extreme cases, the difference in the weights will overcompensate the difference in the respective price changes, leading to $P^P_{1,0} > P^P_{1*,0}$.

Above, in demonstrating that the Laspeyres and the Paasche index violate the economic monotonicity test, it was assumed that quantities are positively related to prices. However, the case described by relationship (3b), where quantities are negatively related to prices, is standard in the context of the economic theory of consumer demand. In this case, the Laspeyres index still violates (5) and the Paasche index still violates (4).

5. Concluding Remarks

A primary concern of axiomatic index theory is the construction of tests that can provide insight into the properties of index formulas. Many price index formulas in common use violate some of the proposed axioms of index theory. Knowing which axioms are, and are not, satisfied is one important criteria for assessing the appropriateness of a formula for specific uses.

Among the axioms that have been proposed, the strict monotonicity test is one of the most widely accepted. Kohli (2010) has introduced the concept of index monotonicity in an economic framework. The present paper has shown that the notion of economic monotonicity can be formalized along the lines of the traditional strict monotonicity test of the axiomatic approach to index theory.

Kohli has demonstrated that the Paasche and Fisher index formulas violate economic monotonicity. This paper has shown that the same deficiency applies to the Laspeyres index.

References

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