Chapter 1

INTRODUCTION TO INDEX NUMBER THEORY FOR PRICE AND PRODUCTIVITY MEASUREMENT

Bert M. Balk, W. Erwin Diewert and Alice O. Nakamura[1]

Formal index number theory is not needed for measurement when the definition of a measure is obvious and its properties are apparent. However, there is a need to choose among many seemingly appropriate ways that have been proposed to meet important economic measurement needs such as assessing the rates of inflation and productivity growth for a nation and the determinants of changes over time or differences in the standard of living. The papers in this volume attempt to meet needs for theory in price and productivity measurement.

The papers fall in two groups. Part I papers deal with alternative productivity measures and decompositions of productivity growth. Part II papers focus on the properties of alternative index number formulas for price and productivity measurement. This volume is intended for specialists, in contrast to most of the other volumes in this Price and Productivity Measurement series (the "Vancouver Volumes") that should be accessible also for non specialists. The papers have been ordered within each of the two parts of this volume to assist students and others trying to attain a specialist level of understanding in mastering key terms.

PART I Productivity Measures and Decompositions

In **chapter 2**, **Paul Schreyer** of the Organization for Economic Co-operation and Development (OECD) explains that different ways of specifying computable measures of multifactor productivity (MFP) embed different assumptions about the technology and competition on output markets. The author focuses especially on assumptions often invoked in the absence of direct information about the prices and volumes of capital services.

Schreyer develops a very general model which provides a decomposition of traditional Total Factor Productivity or Multifactor Productivity growth (which he calls Apparent Multifactor Productivity, or AMFP, growth) into economic explanatory factors; see his equation (17c). The explanatory factors include:

- Possible nonconstant returns to scale;
- Technical progress (a shift in the production or cost function);
- Possible monopolistic pricing of products, and
- Possible omitted inputs.

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Schreyer's decomposition of apparent productivity growth into explanatory factors generalizes the analysis of Denny, Fuss and Waverman (1981), which developed a similar methodology that included the first three factors listed above but not the fourth.[2] Schreyer goes on to make additional assumptions that will allow statistical agencies to implement his general productivity formula. Schreyer lists five sets of assumptions and develops alternative empirical productivity growth estimates for Canada, France, Japan, and the United States. Empirically, the different assumptions are found to matter. The problem of deciding which set of assumptions is "best" has still not been solved in the literature but Schreyer's chapter should be required reading for statistical agencies contemplating the implementation of a multifactor productivity measure.

Different investigators have chosen different methods for measuring the contributions to industry productivity growth of entering and exiting firms. In **chapter 3**, **W. Erwin Diewert** of the University of British Columbia and **Kevin J. Fox** of the University of New South Wales propose a new formula for decomposing industry productivity growth into terms that reflect the productivity growth of individual production units that operate in both the base and comparison time periods (the "continuing firms") as well as the impacts on industry productivity growth of firm entry and exit. This formula is initially developed for the simplistic case in which each production unit produces a single homogeneous output and uses a single homogeneous input.

Diewert and Fox then take up the problems involved in combining many outputs and many inputs into aggregates. There are some significant index number problems. There is no problem in using normal index number theory to construct output and input aggregates for each *continuing* firm present for both periods under consideration. However, this approach does not work with entering and exiting firms, since there is no natural base or current period observation to use as a standard of comparison for the single period data for these firms. The authors address this problem (which has not been widely recognized in the literature) by applying multilateral index number theory. In this approach, the data for each firm in each period is regarded as if it pertained to a "country" and various multilateral methods are applied. They illustrate their methodology using an artificial data set.

In **chapter 4**, **W. Erwin Diewert** of the University of British Columbia first focuses on a decomposition derived by Tang and Wang (2004) of economy wide labour productivity into sectoral contributions. Diewert reworks the Tang-Wang result so as to provide a more transparent and simple decomposition. He also explores another decomposition approach due to Gini which is a generalization of the Fisher ideal index number methodology to aggregates that are products of three factors: (i) growth in the labour productivity of individual sectors, (ii) changes in sectoral real output prices, and (iii) changes in the allocation of labour across sectors.

Policy makers are also interested in estimates of the contributions to aggregate productivity growth of particular industries. **Marshall Reinsdorf** and **Robert Yuskavage** of the U.S. Bureau of Economic Analysis (BEA) explain in **chapter 5** that the lack of an additive formula for industry contributions to real output growth means that formulas for industry contributions to aggregate productivity growth also generally add up to incorrect totals. The authors observe that the unavailability of exact formulas for industry contributions to aggregate productivity growth has led to reliance on approximate decomposition formulas. They show, however, that the approximate formulas examined can work well. In addition to its

[2] Schreyer's analysis is also related to Balk (2010a) and Diewert and Fox (2008). These last authors do not consider the case where assets are missing and so Schreyer's framework is more general.

methodological contributions, this paper makes an empirical contribution to the literature on the industry sources of the post-1995 rebound in productivity growth. Reinsdorf and Yuskavage note that interest in investigating industry sources of productivity change has been further heightened by the availability for the United States, since June 2004, of data on industry gross output, intermediate inputs and value added resulting from the integration of the GDP-by-industry accounts and the annual I-O accounts. Using these data, the authors find that information technology (IT) producing industries directly account for far less of the post-1995 *speedup* in productivity growth than the wholesale and retail trade industries.

In **chapter 6**, **Ulrich Kohli**, who was chief economist of the Swiss National Bank when this chapter was written and is now with the University of Geneva, points out that most headline productivity measures refer to the *average* product of labor. Kohli notes, however, that a more relevant measure might be the *marginal* product. Nevertheless, as long as the income share of labor remains essentially constant, the two measures give very similar results. In the case of the United States, Kohli observes, the share of labor has been quite steady over 1971-2001 and the paths of both measures have been similar. The stability of the labor share explains why the Cobb-Douglas production function fits U.S. data well. Yet Kohli shows that a more thorough look at the evidence reveals that the historical constancy is the outcome of opposing forces.

Kohli expands the model by adopting the GDP function framework. Using a functional form more flexible than the Cobb-Douglas, he finds on the one hand that the Hicksian elasticity of complementarity between labor and capital has been significantly greater than one. Thus capital deepening has tended to increase the share of labor and raise its marginal product by relatively more than its average product. On the other hand, he finds that technological change has had an offsetting effect. An improvement in the terms of trade and a depreciation of the home currency are also shown to have impacts on average labor productivity. This paper seeks to analytically disentangle these effects and proposes a measurement methodology which is then applied to produce a multiplicative decomposition of the average and marginal U.S. labor productivity over the past three decades. Both econometric and index number methods are used.

In **chapter 7**, **Bert M. Balk** of the Rotterdam School of Management, Erasmus University, and Statistics Netherlands argues that official statistics agencies should adopt a definition for productivity growth that does not embed strong assumptions that are not supported by empirical evidence such as a constant returns to scale technology, competitive input and output markets, optimizing behaviour, and perfect foresight. Instead, Balk urges that total factor productivity growth (which official statistics agencies measure as MFP growth) should be defined as an output quantity index divided by an input quantity index.[3] Balk also provides an alternative framework for measuring productivity growth that is based on a difference approach to index number theory as opposed to the usual ratio approach. Balk shows that this alternative approach has some advantages over the traditional approach. The difference approach to index number theory was originally developed by Montgomery (1937) and Diewert (2005) but has not attracted much attention in the index number literature. However, Balk shows that the difference approach to measuring productivity change has a significant advantage over the traditional ratio approach in that it is invariant to whether output is measured as gross output or real value added.

The paper systematically considers the measurement of productivity change using a KLEMS framework and illustrates how the analysis can be conducted without imposing

[3] Diewert and Nakamura (2007) also advocate this definition of total factor productivity growth.

neoclassical assumptions. The paper also provides a rigorous discussion of the issues relating to the measurement of the cost of capital. When it comes to the explanation of productivity change, Balk explains that there are two main directions. The first is disaggregation: the explanation of productivity change at an aggregate level (economy, sector, industry) by productivity change at lower levels (firm, plant) and other factors subsumed under the heading of re-allocation (expansion, contraction, entry, and exit of production units). The second direction is concerned with the decomposition of productivity change into factors such as technological change, scale effects, input- and output-mix effects, and random chance. "And here come the neoclassical assumptions," writes Balk, "at the end of the day rather than at its beginning."

Balk's contribution also has three valuable appendices. Appendix A gives the reader a brief overview of the axiomatic approach to bilateral index number theory (a ratio approach) and also the axiomatic approach to indicators[4] (a difference approach to the aggregation of prices and quantities). For additional material on the axiomatic approach to bilateral index number theory, see Diewert (1992b); on the axiomatic approach to indicators, see Diewert (2005), and Diewert and Mizobuchi (2009); and on both approaches, see Balk (2008). Appendix B in Balk's contribution shows how value added ratios can be decomposed into price and quantity components. Finally appendix C provides a comprehensive discussion of possible methods for decomposing time series depreciation into revaluation and depreciation or deterioration terms.

PART II Index Number Formulas

Many official statistical agencies state that they use a Laspeyres price index as a conceptual target for their consumer price index. However, in **chapter 8**, **Bert M. Balk** of the Rotterdam School of Management, Erasmus University, and Statistics Netherlands, and **W. Erwin Diewert** of the University of British Columbia note that the headline inflation figure for the Netherlands, for example, in 2007 was obtained as the percentage change between a current month and the corresponding month of the prior year, with 2006 serving as the reference year for the quantity weights. This is *not* a Laspeyres index; they call this a "Lowe index" since the English economist, Joseph Lowe, suggested this type of index in 1823. More specifically, they define a Lowe index to be a fixed basket index where the commodity basket corresponds to household consumption patterns in a base year and this basket is priced out using current month prices in the numerator and base month prices in the denominator of the index. Thus there are two separate bases for this index: a base year for the quantity basket and a base month for the prices. The base year always proceeds the base month. Most CPIs are actually Lowe indexes.

Suppose households had preferences defined over the commodities in the annual basket given by the utility function, $f(q)$ say, where q is a consumption vector. Let the base year consumption vector be q^b. Then a Konüs true cost of living index comparing the cost of achieving utility level $u^q \equiv f(q^b)$ at the month t prices, p^t, to the cost of achieving the same utility level at the base month prices, p^0, is equal to the cost ratio, $C(u^b, p^t) / C(u^b, p^0)$, where $C(u,p)$ is the minimum cost of achieving the utility level u when the household faces the prices p. This true cost of living index can be compared to the corresponding Lowe index,

[4] Diewert (1992a) introduced the term "indicator" into the economics literature as a term to describe the difference counterpart to a bilateral index number formula.

$p^t \cdot q^b / p^0 \cdot q^b$, and the bias in this Lowe index can be estimated. The authors derive first and second order approximations to the substitution bias of a Lowe index. They then make assumptions about price trends and substitution elasticities and develop approximations to the bias in a Lowe index relative to the corresponding true cost of living index.

In **chapter 9**, **T. Peter Hill**, the editor of the international *Consumer Price Index Manual: Theory and Practice* (ILO et al. 2004) explains that this Manual and a 2003 working paper by Balk and Diewert (published as chapter 8 in this volume) are where the term "Lowe Index" was introduced. For a *Lowe price index*, Hill explains, the quantity weights are fixed and predetermined but need not pertain to either time period for which prices are being compared. Hill also introduces the concept of a *Lowe quantity index* in which the price weights are fixed, but need not pertain to either time period for which quantities are being compared. Hill discusses the fact that there are many ways in which the reference quantities or prices might be specified for a Lowe index. His results make it clear that the importance of the Balk-Diewert chapter 8 paper is rooted in the fact that in naming the Lowe index, they also defined a class of indexes, the members of which have valuable properties in common. For example, Lowe indexes are transitive and have additive decompositions, and can be expressed as ratios of Laspeyres indexes. They can also be viewed as chain indexes that link through some other period, country or group of countries. Two classes of indexes used in international comparisons that, in fact, are Lowe indexes are the average quantity methods and the average price methods. Hill reminds the reader, however, that these are not described as "Lowe" PPPs or indexes in earlier literature because the term was only introduced in 2003. Hill also argues that, in the case of temporal price or quantity indexes where the link is some past period, its relevance necessarily diminishes as it recedes into the past. Hill thus favours frequent updating of the reference prices in Lowe quantity indexes and of the reference quantity baskets in Lowe price indexes.

A recurrent theme when measuring aggregate price and quantity change between more than two periods is the choice between the computation of direct or chained index numbers. The impression one gets from the literature is that chained index numbers are closer to the truth than direct index numbers. **Bert M. Balk** of the Rotterdam School of Management, Erasmus University, and Statistics Netherlands rigorously explores this issue in **chapter 10**.

Balk notes that statistical agencies, until the recent past, favoured the use of fixed base or direct indexes, usually of the Paasche or Laspeyres variety (or perhaps of the Lowe type), but now opinion has shifted to the use of chained indexes, at least for annual data. Balk notes that the 2004 CPI Manual recommends the use of chained indexes provided that the prices and quantities of adjacent periods are *more similar* than the prices and quantities of more distant periods. In this circumstance, chaining will tend to reduce the spread between Paasche and Laspeyres indexes and indeed of superlative indexes as well.[5] Balk reviews the arguments for and against chaining as opposed to the use of direct indexes[6] and then he goes on to show mathematically, that it is impossible to reconcile the two approaches.

[5] For an introduction to the use of similarity measures to determine a path for chaining indexes, see section 10 of the Diewert and Fox contribution to this volume (chapter 3), which in turn draws on the work of Robert Hill.
[6] Some of the problems associated with the use of fixed base indexes for measuring price and quantity change for adjacent periods within the sample period are discussed by Balk in section 2 but it should be noted that some of these problems were already anticipated by Hill (1988).

In section 5 of this chapter, Balk relates the question "to chain or not to chain" to the modern theory of revealed preference developed by Afriat, Diewert and Varian. Balk develops criteria that may be helpful in answering the basic question as to whether one should use chained indexes or not. Of particular interest is Balk's section 5.3, where he develops a theory for the cost of living index that is an extension of his earlier theory for Konüs type cost of living indexes when preferences change between the two periods in the comparison.[7]

In section 6, Balk reviews Divisia's continuous time approach to index number theory and notes that this approach provides some justification for the use of chained index numbers over their direct counterparts since "chained index numbers also use the available data for the intermediate periods and map out a segmented path that coincides with the true one at the observation points." In section 7, Balk concludes that chaining is probably preferable to the use of direct indexes provided that quantities (and prices) do not exhibit "cyclical behaviour"; i.e., they do not "bounce" up and down as over time.[8] Balk also provides a useful appendix which looks (somewhat critically) at the index number program proposed by Claude Hillinger in 2002.

In **chapter 11**, **W. Erwin Diewert** of the University of British Columbia provides a selective review of the stochastic approach to index numbers, from its inception in the 1800s through a recent resurrection of interest. This paper was written in 1995 and has been much cited, including by other Vancouver Volume papers. Thus it is included in this volume to aide readers in checking this reference and because it ties in well with material in other papers in this volume.

Diewert explains that the two main competing approaches to index number theory are the test approach and the economic approach. In the test approach, the vectors of prices and quantities for the two periods being compared are regarded as independent variables. In contrast, in the economic approach, the prices are regarded as independent variables but the quantities are viewed as solutions to economic maximization or minimization problems. Diewert goes on to explain that the economic approach to index number theory concentrates on finding functional forms for price indexes that are *exact* for flexible unit cost functions, and on finding functional forms for quantity indexes that are *exact* for flexible linearly homogeneous utility functions. Index number formulas that are exact for flexible functional forms are called *superlative*.

Diewert notes that the traditional test and economic approaches to index number theory do not provide estimates of reliability for index number formulas: a shortcoming that proponents feel the stochastic approach can overcome. Basically, the early proponents of the stochastic approach to index number theory (Jevons and Edgeworth) looked at the price relatives or ratios for commodity i for periods 0 and t, p_{it} / p_{i0} for $i = 1, \dots, N$, and made the assumption that the price relatives have a common mean. This line of reasoning leads to the arithmetic mean index, called the Carli index and written as $P_C \equiv \sum_{i=1}^{N} (1/N)(p_{it} / p_{i0})$, as an estimator of the common trend in prices and an advantage of this estimator (over the economic and test approaches) is that a measure of precision can be attached to it. Edgeworth did not argue for this specification of the problem of measuring the trend in prices: rather he argued it would be more appropriate to

[7] See Balk (1989) for this earlier theory.

[8] This is consistent with Peter Hill's (1988) earlier advice on this topic. The term "price bouncing behavior" was introduced by Szulc (1983), who showed that bad things can happen with chained index numbers if there is price bouncing behavior. For a more recent study that attempts to deal with the chain drift problem with sub-annual data, see Ivancic, Diewert and Fox (2010), who apply multilateral comparison methods to eliminate chain drift.

assume that the logarithms of the price ratios have a common mean, which leads to the Jevons index, $P_J \equiv \prod_{i=1}^{N}(p_{it}/p_{i0})^{1/N}$, as the estimator of the common trend in prices. Diewert reviews these approaches in section 2-4, and in 6 he reviews the more recent approaches. He argues that the more recent approaches are flawed due to their assumptions about the variances of the price relative components.

In the later sections of his paper, Diewert turns from being negative on the stochastic approach to being somewhat positive. Thus in section 5, he reformulates a stochastic model due to Edgeworth (which Irving Fisher thought was totally impractical) and shows that it can be estimated. The essence of this neo-Edgeworthian approach is that the variance for each of the N commodity classes is empirically determined. Thus an empirically determined parameter replaces the assumption that the error variances are known up to a multiplicative constant. In section 7, Diewert changes the focus from an econometric model (with assumptions about error variances) to *descriptive statistics*. He sets up his descriptive statistics framework along the lines suggested by the following quotation from the chapter: “A more direct approach to the reliability of a price index, $P(p^s, p^t, q^s, q^t)$, is to simply look at the variability of the individual price relatives, p_{it}/p_{is}, around the index number “average” value, $P(p^s, p^t, q^s, q^t)$.” Diewert goes on to suggest several alternative measures of the variability of individual price relatives. The approach of Theil (1967; 136-137) to the Törnqvist-Theil index number formula is an example of this descriptive statistics approach. It can also be seen that Diewert’s work in section 7 of the present paper is a precursor to his work on similarity indexes; see Diewert (2009).

As said, this chapter was written in 1995 and there are many further developments associated with the stochastic approach to index numbers. The models have become more complex than the original straightforward approaches proposed by Jevons and Edgeworth. For an up to date, excellent review of these later developments, see Clements et al. (2006).[9]

In **chapter 12**, **W. Erwin Diewert** of the University of British Columbia and **Robert J. Hill** of the University of Graz, Austria and also the University of New South Wales reconsider the fundamental concepts of true and exact indexes, as these concepts are defined in the index number literature. The authors explain that these concepts form the bedrock of the economic approach to index number theory. They review these concepts. In brief, a *true index* is the underlying target; it is a formal statement of the measurement objective. An empirically calculable index is *exact* when, under certain conditions, it exactly equals the true index. Van Veelen and van der Weide (2008) recently introduced alternative definitions of true indexes. These combine the existing literature’s identification of a true index, such as the Allen (1949) quantity index or the Konüs cost of living index, with some of Afriat’s (1981) ideas for checking whether a given set of data are actually consistent with the maximizing or minimizing hypotheses underlying the definitions of the ‘”true” indexes. Diewert and Hill conclude that it would be preferable for those authors to come up with a new term to describe their concept.

In **chapter 13**, **Andrew Baldwin** of Statistics Canada endorses the 1993 System of National Accounts (SNA93) recommendation for chain linking, but not its support for chain Fisher aggregates, nor, as a second-best solution, chain Laspeyres aggregates. Baldwin argues it is feasible and desirable to calculate chain fixed price aggregates that are not vulnerable to chain

[9] See also the summary in Balk (2008, section 1.8).

drift. According to Baldwin, these aggregates can be calculated as direct series for the most recent period so that they are additive over commodities, industries or regions, in contrast to their chain Fisher counterparts. The Edgeworth-Marshall formula is what Baldwin recommends. He notes that it respects the property of transactions equality (i.e., the importance of a transaction in the formula does not depend on the period in which it occurs). Also, it does not discard commodities from a volume aggregate if the price goes to zero from a positive price or *vice versa*, nor does it discard commodities from a price index if the quantity goes to zero from a positive quantity or *vice versa*. Baldwin feels it is unfortunate that the index number theory literature has focused on two formulas -- the Laspeyres and the Fisher -- neither of which is well-suited, in his view, for the calculation of chain aggregates.[10]

Ulrich Kohli who was chief economist of the Swiss National Bank when this chapter was written and is now with the University of Geneva reports in **chapter 15** that several countries have switched – or are about to switch – to chained price and quantity indexes for their national accounts. In particular, the United States and Canada have adopted the chained Fisher indexes, whereas the United Kingdom, Switzerland, Australia and New Zealand have opted for chained Laspeyres indexes for real GDP, and chained Paasche for the implicit price deflator. Yet Kohli notes that the vast majority of countries, including most OECD members, still have not embraced chaining. In these countries the GDP implicit price deflator is still computed as a *direct* (or fixed-base) Paasche price index. Changes in the price level over consecutive periods are measured by the change in the direct Paasche index, a use for which Kohli feels this index is ill suited. Using the economic approach to index numbers, Kohli shows that because of this failure, the price index can register a drop between consecutive periods *even though none of the disaggregate prices has fallen, and some have actually increased*. He notes that a similar result holds for the direct Fisher index. In his view (and our view as well), this provides a powerful argument in favor of chaining, at least for annual data subject to smooth trends. Kohli gives strong arguments against the use of direct indexes in addition to the arguments that were suggested in the earlier work of Hill (1988) and in chapter 10 by Balk in this volume.

Kohli argues that the fact that the direct Paasche GDP deflator is not monotonically increasing in prices makes it a poor indicator of inflation, since it might point at a price increase when prices are actually falling, and *vice versa*. Yet it is widely used in the literature. According to Kohli, there are other reasons why the use of the direct Paasche GDP deflator as a measure of the price level should be avoided, independently of whether chaining takes place or not. GDP price deflators incorporate terms-of-trade changes, which are fundamentally a real – not a price – phenomenon. The problem with the standard procedure becomes apparent if import prices fall, for instance. This will increase the GDP price deflator (since import prices enter the calculation of the GDP deflator with a negative weight), even though this shock is clearly not inflationary.

Ludwig van Auer in **chapter 15** follows up on Kohli's chapter 15. He extends Kohli's results to show that a run of direct Laspeyres indexes can also violate a monotonicity property if there are more than two periods in the run of time periods. This extension provides a more symmetric definition of the basic monotonicity test used by Kohli. The net impact of von Auer's paper is to reinforce the case for using chained indexes when we have annual data.

[10] However, the reader is reminded of Balk's appendix in his chapter 10 where he finds some problems with the use of the Edgeworth-Marshall index number formula.

In **chapter 16**, **W. Erwin Diewert** of the University of British Columbia considers problems involved in collecting and aggregating price and quantity information at the lowest level of aggregation. This chapter was originally written in 1995 and, like chapter 11 above, it was not updated. However, it proved to be an important source on elementary indexes for the 2004 Consumer Price Index Manual. Diewert develops an axiomatic approach for finding an appropriate functional form for an elementary level price index. The axiomatic properties of the Carli (arithmetic average of price relatives), Jevons (geometric average of the price relatives), and Dutot (ratio of average prices) elementary indexes are obtained and the Jevons index emerges as the winner. Finally, Diewert also discusses the problem of price and quantity aggregation at the very first stage of aggregation when individual transactions over a time period must be aggregated into price and quantity aggregates that can be inserted into a bilateral index number formula. It turns out that unit value prices and total quantities transacted (over a set of transactions involving a "homogeneous" commodity) emerge as reasonable aggregates at this first stage of aggregation. Finally, the paper catalogues sources of bias in consumer price indexes and makes rough guesses as to the likely magnitude of the biases. Diewert's rough guesses proved to be very similar to the Boskin Commission's guestimates of biases.

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