CHAPTER 2

MEASURING MULTI-FACTOR PRODUCTIVITY WHEN RATES OF RETURN ARE EXOGENOUS

Paul Schreyer[1]

1. Introduction: Gross Operating Surplus and the Remuneration of Capital

Official statistics do not normally provide direct observations on the price and volume of capital services. What is available from the national accounts is a residual measure of gross operating surplus (GOS): a measure often interpreted as profits from normal business activity, including mixed income which is the income of self-employed persons. Thus, the national accounts provide the researcher with data according to the following accounting identity:

(1) $P \cdot Q = wL + GOS$

where $P \cdot Q$ is the sum of current-price output in the economy, $P = [P_1, P_2, ...P_M]$ denotes the vector of prices and $Q = [Q_1, Q_2, ...Q_M]$ denotes the vector of quantities of output. To simplify notation, we use $P \cdot Q \equiv \sum_{i=1}^{M} P_i Q_i$ for the inner product between P and Q. Note, however, that normally the quantities in Q are not directly measured. Output is defined and measured as value-added, and prices are defined and measured at basic prices, i.e., they exclude all product taxes but include subsidies on products. The term wL is the remuneration of labour, with wage component w and volume component L, with the value and price components measured directly. For simplicity, it will be assumed here that mixed income is either zero or is split up between the labour and the GOS components. Thus, the two sides of (1) represent the total production and the total income sides of the national accounts.

The national accounts provide no guidance as to the factors of production that are remunerated through GOS. Fixed assets certainly are among these factors, but there could be others too. The business literature has discussions about the importance of intangible assets, and there are good reasons to argue that such assets account at least for part of GOS. While this may appear a minor point, it calls into question an assumption often made by analysts of productivity and growth, namely that GOS exactly represents the remuneration of the fixed assets recognised in the System of National Accounts (SNA), or the value of the services of these.

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[1] The contact information for the author is: Paul Schreyer, OECD Statistics Directorate, Paul.Schreyer@OECD.org. Opinions expressed in this paper are those of the author and do not necessarily reflect views of the OECD or its Member countries. The author thanks Erwin Diewert, Chuck Hulten and Alice Nakamura for helpful comments. Special thanks go also to Mathilde Mas (University of Valencia and IVIE) for useful discussions on the subject.

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Let $u^* = [u_1^*, u_2^*, ... u_N^*]$ denote a vector of user costs for N types of capital services and let $K = [K_1, K_2, \ldots, K_N]$ denote the corresponding vector of the quantities of capital service flows. The assumption typically made is:

$$(2) \qquad u^* \cdot K = GOS,$$

where $u^* \cdot K$ denotes the inner product of the price and quantity vectors: i.e., where $u^* \cdot K \equiv \sum_{i=1}^{N} u_i^* K_i$. In other words, it is assumed that remuneration of capital services exactly exhausts gross operating surplus. Empirically, the equality $u^* \cdot K = GOS$ is obtained by choosing what is thought to be an appropriate value for the net rate of return on assets, which is part of the user costs.[2] With this formulation, the rate of return is assumed to adjust *endogenously*. This setup is consistent with competitive behaviour on product and factor markets and a production process that exhibits constant returns to scale. Under these conditions, (1) can be restated as

$$(3) \qquad P \cdot Q = wL + u^* \cdot K,$$

since these conditions ensure that the gross operating surplus corresponds exactly to the remuneration of the assets included in K; hence if only fixed assets are assumed to be in K, this is equivalent to assuming that GOS corresponds to the remuneration of fixed assets. Note that this setup also depends on the following being true:

- the set of assets $[K_1, K_2, ... K_N]$ is complete; i.e., all assets are observed by the official statisticians who compile the national accounts and there are only the stated fixed assets;
- the ex-post rate of return on each asset (implicitly observed by the national accountants as part of GOS) equals its ex-ante rate of return, which is the economically relevant part in the user cost of capital services;
- there are no residual profits (or losses) such as might arise in the presence of market power, or with non-constant returns to scale, or owing to the availability of publicly available or any other uncounted or miscounted capital assets.

Several questions arise when some of the above conditions do not hold. For example, when there is independent information about the rates of return to capital services, there is no guarantee that the sum of labour remuneration and the observed capital remuneration will equal measured total value added at current prices. How should multi-factor productivity (MFP) be conceptually defined, computed and interpreted? How should growth accounting exercises be carried out? How should measures of technical change be defined and evaluated? These are the questions explored in this paper. In the rest of this paper, a preference is expressed for a simple

[2] In a simple continuous-time formulation, the user cost or rental price of an asset (Jorgenson and Griliches, 1967) is given by $u^i = q^i(r + \delta^i - d\ln q^i / dt)$ where $q^i(t)$ is the purchase price of a new asset of type i, $r(t)$ is the net rate of return, δ^i is a rate of depreciation, and $d\ln q/dt$ is the rate of change of q^i .

MFP measure that is consistent with index number traditions. Such a measure cannot be interpreted as capturing only, or all, technical change.[3]

2. Why GOS May Differ from Remuneration of Capital

This paper generalizes the formulation of the income-production relationship (3) by allowing for and utilizing independent measures of capital remuneration, $u = [u_1, u_2, ... u_N]$ that may not satisfy condition (2). Under these circumstances, equation (3) is replaced by

(4) $$P \cdot Q - wL = GOS = u \cdot K + M,$$

where the term M denotes the difference obtained by subtracting from current-price output both the remuneration of assets included in K,[4] $u \cdot K$, and the value of the labour input; i.e., it is the observed current price output minus observed factor payments. $C \equiv wL + u \cdot K$ is used as shorthand for observed factor payments. Hence gross operating surplus can be split into a component that reflects observable factor remuneration plus a residual M with several possible interpretations. In principle, there is no restriction on the sign of M. However, if the sign were negative over an extended period of time, this would imply sustained losses. Since this seems economically implausible, in what follows, the non-negativity of M is assumed.[5]

Four possible reasons for nonzero values of M are considered in this paper.

Models of short-run disequilibrium over the business cycle provide a first possible theoretical justification for the existence of nonzero values of M.[6]

A second possibility is that M reflects the existence of pure profits as a consequence of the presence of decreasing returns to scale combined with marginal cost pricing for outputs, or of increasing returns to scale and a positive mark-up over marginal costs. If returns to scale are the key source of non-zero values of M, then the size of M will depend on the degree of competition in output markets: free market entry and competition would be expected to drive mark-ups and prices to a level where total revenues just cover total costs, implying M = 0.

The Lucas-Romer model of endogenous growth (Romer, 1990) provides a third possible justification for non-zero M values. According to this model, at the firm level, returns to scale are constant, but at the aggregate level there are increasing returns to scale due to externalities.

A fourth possibility is that M reflects the existence of unobserved inputs and hence reflects a measurement problem. This situation could arise if not all of the capital inputs that give rise to operating surplus are recognised in the national accounts. In contrast to the second

[3] Aspects of the interpretation and derivations that follow build on Jorgenson and Griliches (1967), Fuss and McFadden (eds.) (1978), Diewert and Nakamura (2007) and Harper et al. (1989).

[4] As the context makes clear, the symbol M is also used sometimes to denote the number of output goods.

[5] In the empirical part of the paper, M is positive for the four countries reviewed (Canada, France, Japan, United States) for most years over 1980-2002. If M were negative over an extended period of time, this would cast doubt on the measures for the remuneration of capital, and in particular on the choice of the exogenous rates of return.

[6] These include models of time-varying capacity utilisation of the sort investigated by Berndt and Fuss (1986) and Hulten (1986). The cyclicality of productivity measures and the relation of these to technical change are dealt with by Basu and Kimball (1997). They find strong effects of variable capacity utilisation on measures of productivity.

interpretation, in this case we would expect M to remain positive even in the longer run because true total costs are higher than what the observed assets would justify and M would cover these.[7]

3. Production Technology and Producer Behaviour

We let Z(t) denote a feasible set of inputs and outputs in period t. We further assume that there is a total cost function TC that shows the minimum costs of production, given a vector Q of quantities $[Q_1, Q_2,...Q_M]$ for the M outputs and given a corresponding set of input prices. Inputs comprise labour L, N types of observed capital services $K_1, K_2,...K_N$ and one unobserved asset D. The corresponding prices are the wage rate, w, the user costs of capital, $u = [u_1, u_2,...u_M]$, and the price of the unobserved input D, ϕ. The total cost function is defined as:

(5) $$TC[Q, w, u, \phi, t] = \min_{L,K,D} \{wL + u \cdot K + \phi D : (Q, L, K, D) \text{ belongs to } Z(t)\}.$$

The cost function is linearly homogenous in input prices and non-decreasing, but not necessarily linearly homogenous in the vector of outputs $[Q_1,...Q_M]$. Thus, there is no assumption of constant returns to scale. However, producers are assumed to minimise total cost, so that actual costs equal minimum costs ($wL + u \cdot K + \phi D = TC(Q, w, u, \phi, t)$). Furthermore, producers are assumed to face competitive factor markets so that Shephard's (1970) conditions for optimality for factor inputs apply:

(6a) $$L = \left(\frac{\partial TC}{\partial w}\right);$$

(6b) $$K_i = \left(\frac{\partial TC}{\partial u_i}\right), \quad i = 1, \ldots, N;$$

(6c) $$D = \left(\frac{\partial TC}{\partial \phi}\right).$$

On the output side, imperfect product markets are allowed for with the sole stipulation that output prices are proportional to marginal costs. No explicit assumption is made about the kind of imperfect competition that prevails or concerning whether producers are profit maximising or not. All that is needed is a relationship between prices and marginal costs so that if the price of output i is P_i and if $1 \leq 1/\mu_i$ is a product-specific, time-varying mark-up factor, producer behaviour on the output side is described by

(7) $$P_i \mu_i = \partial TC / \partial Q_i \quad i = 1,..., M.$$

Next, we follow the literature (e.g., Panzar 1989) and define the local elasticity of cost with respect to scale as

[7] Non-observed inputs and their link to measured MFP growth and technical change have been investigated by Basu et al. (2003). They introduce unobserved intangible organisational capital that they take as complementary to observed investment in information technology. Unlike the present model, however, theirs is a general equilibrium setting that tries to account not only for the unobserved intangible inputs but also for their unobserved production.

(8) $$\varepsilon \equiv \sum_{i=1}^{M} \frac{\partial \ln TC}{\partial \ln Q_i}.$$

Hence, $\varepsilon > 0$ indicates the percentage change in total cost for a given percentage change in all outputs. The inverse of this parameter can readily be interpreted as a measure of local returns to scale for the production unit. For instance, $\varepsilon > 1$ implies that a one percent rise in the quantity of each of the outputs increases total costs by more than one percent, which is tantamount to a situation of decreasing returns to scale. Similarly, $\varepsilon < 1$ and $\varepsilon = 1$ correspond to increasing and constant returns to scale, respectively.[8]

Given (7), the measure of the cost elasticity defined in (8) can be further transformed:

(9) $$\begin{aligned}\varepsilon &\equiv \sum_{i=1}^{M} \frac{\partial \ln TC}{\partial \ln Q_i} = \sum_{i=1}^{M} \frac{P_i Q_i \mu_i}{TC} \\ &= \sum_{i=1}^{M} \frac{P_i Q_i \mu_i}{P \cdot Q} \frac{P \cdot Q}{TC} \\ &= \mu \frac{P \cdot Q}{TC} \quad \text{where } \mu \equiv \sum_{i=1}^{M} \frac{P_i Q_i}{P \cdot Q} \mu_i.\end{aligned}$$

In (9), μ is the economy-wide inverted average mark-up factor – a weighted average of industry-specific mark-ups with simple output shares as weights. Expression (9) can be rearranged as

(10) $$P \cdot Q = (\varepsilon / \mu) TC.$$

Thus, the value of total output revenues equals total costs, adjusted by a mark-up factor $1/\mu$ and ε, the parameter for the scale elasticity.

The equalities in (9) can now be combined with the national accounts information mentioned earlier. In particular, it was pointed out that gross operating surplus is defined as the difference between the value of output and labour income: $GOS = P \cdot Q - wL$. Using the result $P \cdot Q = (\varepsilon / \mu) TC$ in (10), from (4), one obtains

(11) $$GOS = TC(\varepsilon / \mu) - wL.$$

Recall that the difference between GOS and observed capital income has been labelled M: $M = GOS - u \cdot K$. Using the expression (11) for GOS and taking into account the definition of TC now allows us to derive a relation for M that can readily be interpreted:

[8] As we operate with a multi-product cost function, a distinction needs to be made between general economies of scale and product-specific economies of scale. The former – treated here – deals with changes in costs when all outputs are changed by the same proportion. The latter deals with changes in costs as one particular output is increased while holding all other outputs constant. For the latter form of economies of scale, see Panzar (1989).

(12a)
$$
\begin{aligned}
M &= GOS - u \cdot K = PQ - wL - u \cdot K \quad \text{from (4)} \\
&= \left(\frac{\varepsilon}{\mu}\right) TC - wL - u \cdot K \quad \text{from (10)} \\
&= \left(\frac{\varepsilon}{\mu}\right) TC - TC + (TC - wL - u \cdot K) = \left(\frac{\varepsilon}{\mu}\right) TC - TC + \phi D \quad \text{using (5)} \\
&= \left(\frac{\varepsilon}{\mu} - 1\right) TC + \phi D
\end{aligned}
$$

or alternatively

(12b) $$M = P \cdot Q \left(1 - \frac{\mu}{\varepsilon} + \frac{\phi D}{P \cdot Q}\right) \quad \text{using (10).}$$

Expressions (12a) and (12b) show how the difference M between GOS from the national accounts and the sum of payments to observed factors reflects mark-ups and returns to scale (captured by ε / μ) and the influence of unobserved capital inputs (captured by ϕD). The expressions in (12) will be instrumental for the discussion in the following sections.

4. Technical Change

In an environment of constant returns to scale, Hicks-neutral technical change can be defined either as a shift of the production function over time (an output-based measure) or as a shift of the cost function over time (an input-based measure). Here producer behaviour has been described by way of a cost function, so we shall use the input-based approach to derive measures of technical change. One important advantage of the cost-based measure is that no assumptions about profit or revenue maximisation need to be made for the output markets.

If there were an assumption of constant returns to scale, and competitive markets, the choice of the input-based productivity measure would simply be a matter of convenience, with no consequences for results. However, for the moment we have imposed no such a-priori condition, and the input-based measure will in general be different from the output-based measure, as will be shown in section 5.2.5.

Technical change is measured here as a downward shift over time of the total cost function. To derive an analytical expression, TC is differentiated totally and technical change is then defined as the negative of the partial derivative of the cost function with respect to time:

(13)
$$
\begin{aligned}
-\frac{\partial TC}{\partial t}\frac{1}{TC} = -\frac{\partial \ln TC}{\partial t} &= \sum_{i=1}^{M} \frac{\partial \ln TC}{\partial \ln Q_i}\frac{d \ln Q_i}{dt} \\
&-\left(\frac{d \ln TC}{dt} - \frac{\partial \ln TC}{\partial \ln w}\frac{d \ln w}{dt} - \sum_{i=1}^{N} \frac{\partial \ln TC}{\partial \ln u_i}\frac{d \ln u_i}{dt} - \frac{\partial \ln TC}{\partial \ln \phi}\frac{d \ln \phi}{dt}\right).
\end{aligned}
$$

To interpret (13), consider its parts in turn. On the right-hand side, first there is a Divisia-type output quantity change index, $\sum_{i=1}^{N}\frac{\partial \ln TC}{\partial \ln Q_i}\frac{d\ln Q_i}{dt}$, that aggregates the growth rates of the quantities of individual outputs. To find a computable expression for the growth rate of output, use (7) and (9) to obtain:

$$\sum_{i=1}^{M}\frac{\partial \ln TC}{\partial \ln Q_i}\frac{d\ln Q_i}{dt}=\sum_{i=1}^{M}\frac{\partial TC}{\partial Q_i}\frac{Q_i}{TC}\frac{d\ln Q_i}{dt}=\sum_{i=1}^{M}P_i\mu_i\frac{Q_i}{TC}\frac{d\ln Q_i}{dt}$$

(14a)
$$=\sum_{i=1}^{M}\frac{P_iQ_i}{P\cdot Q}\mu_i\frac{P\cdot Q}{TC}\frac{d\ln Q_i}{dt}=\frac{P\cdot Q}{TC}\sum_{i=1}^{M}\frac{P_iQ_i}{P\cdot Q}\mu_i\frac{d\ln Q_i}{dt}$$

$$=\varepsilon\sum_{i=1}^{M}\frac{P_iQ_i\mu_i}{P\cdot Q\mu}\frac{d\ln Q_i}{dt}.$$

Thus, the output aggregate resembles a traditional output aggregate with revenue shares as weights, but the latter are now corrected for the relative mark-ups μ_i/μ and the scale factor ε.

Moving on to the terms in brackets on the right hand side of (13), it can be seen that these measure the difference in the growth rate of total costs and the growth rates of the various types of input prices. In fact, $\left(\frac{\partial \ln TC}{\partial \ln w}\frac{d\ln w}{dt}+\sum_{i=1}^{N}\frac{\partial \ln TC}{\partial \ln u_i}\frac{d\ln u_i}{dt}+\frac{\partial \ln TC}{\partial \ln \phi}\frac{d\ln \phi}{dt}\right)$ is a Divisia index of input prices. This is apparent by invoking the optimality conditions for factor inputs (6a)-(6c) and then inserting them into the above expression which now becomes $\left(\frac{wL}{TC}\frac{d\ln w}{dt}+\sum_{i=1}^{N}\frac{u_iK_i}{TC}\frac{d\ln u_i}{dt}+\frac{\phi D}{TC}\frac{d\ln \phi}{dt}\right)$. Moreover, by construction, the difference between the Divisia index of total costs and the Divisia index of input prices is the Divisia index of input quantities. The term in brackets on the right hand side of (13) can be rewritten as

(14b)
$$\left(\frac{d\ln TC}{dt}-\frac{\partial \ln TC}{\partial w}\frac{d\ln w}{dt}-\sum_{i=1}^{N}\frac{\partial \ln TC}{\partial \ln u_i}\frac{d\ln u_i}{dt}-\frac{\partial \ln TC}{\partial \ln \phi}\frac{d\ln \phi}{dt}\right)$$

$$=\left(\frac{wL}{TC}\frac{d\ln L}{dt}+\sum_{i=1}^{N}\frac{u_iK_i}{TC}\frac{d\ln K_i}{dt}+\frac{\phi D}{TC}\frac{d\ln D}{dt}\right).$$

Hence, the theoretical index (13) becomes:

(15a)
$$-\frac{\partial \ln TC}{\partial t}=\varepsilon\sum_{i=1}^{M}\left(\frac{P_iQ_i}{P\cdot Q}\frac{\mu_i}{\mu}\right)\frac{d\ln Q_i}{dt}-\left(\frac{wL}{TC}\frac{d\ln L}{dt}+\sum_{i=1}^{N}\frac{u_iK_i}{TC}\frac{d\ln K_i}{dt}+\frac{\phi D}{TC}\frac{d\ln D}{dt}\right).$$

Turned around, the 'growth accounting' form of (15a) is:

(15b)
$$\varepsilon\sum_{i=1}^{M}\left(\frac{P_iQ_i}{P\cdot Q}\frac{\mu_i}{\mu}\right)\frac{d\ln Q_i}{dt}=\left(\frac{wL}{TC}\frac{d\ln L}{dt}+\sum_{i=1}^{N}\frac{u_iK_i}{TC}\frac{d\ln K_i}{dt}+\frac{\phi D}{TC}\frac{d\ln D}{dt}\right)-\frac{\partial \ln TC}{\partial t}.$$

Expression (15b) delivers an explicit formula for the change in aggregate inputs and outputs. If there were no unobserved factor D, and if mark-up factors and the local scale elasticity were known, (15b) could readily be implemented. However, with an unobserved factor D, things are more complicated. We start with a proposal for a computable MFP measure and follow with a discussion of its interpretation.

5. Deriving Computable Measures

There are essentially three strategies for the implementation of expression (15b): (i) to introduce additional, and typically restrictive, hypotheses about the size or nature of the unknown variables until an expression emerges that is both computable and that offers a (seemingly) clear interpretation of productivity growth; (ii) to stay away from invoking additional hypotheses, and define a computable measure of productivity growth while allowing for the fact that it may reflect more than pure technology shifts; or (iii) impose the assumptions needed to apply econometric methods to estimate or correct for the unknown factor and construct estimates of the conceptually correct aggregates of outputs, inputs and productivity.

We discard the third possibility simply because it is not a practical way for statistical offices when they have to compute and publish periodic and easily reproducible statistical series. We do, however, acknowledge that this econometric option may be an important one for more research-oriented, one-off projects. As such it may also deliver useful insights concerning the empirical importance of the unobserved factor. Similarly, to assess some of the choices among non-parametric methods as described below, econometric studies (such as Paquet and Robidoux, 2001 or Oliveira-Martins et al., 1996) can be very useful.

5.1 Apparent Multi-Factor Productivity

We first follow avenue (ii) and propose a measure of 'apparent multi-factor productivity'. Then, in the following subsection we consider strategy (i).

For the purpose at hand, let there be an aggregator X that combines the quantities of the *observable* inputs K and L. Specifically, define

(16) $$\frac{d\ln X}{dt} \equiv \sum_{i=1}^{N} \frac{u_i K_i}{C} \frac{d\ln K_i}{dt} + \frac{wL}{C} \frac{d\ln L}{dt}$$

as a Divisia quantity index of observable inputs, noting that the weights correspond to the shares of each input in total *observable* inputs, as $C \equiv wL + u \cdot K$. Next, define the rate of apparent multi-factor productivity growth (AMFP) as the difference between a Divisia quantity index of output and the quantity index of observable inputs as specified above in (16):

(17a) $$\text{AMFP} \equiv \frac{d\ln Q}{dt} - \frac{d\ln X}{dt}.$$

The Divisia output index in (17a) is a 'traditional' one, i.e., an average of rates of change for individual outputs, each weighted by its revenue share: $\frac{d\ln Q}{dt} \equiv \sum_{i=1}^{M} \left(\frac{P_i Q_i}{P \cdot Q} \right) \frac{d\ln Q_i}{dt}$. Note that this Divisia output index differs from the more general output growth index identified in (15). The growth accounting equation that corresponds to (17a) is:

(17b) $$\frac{d\ln Q}{dt} = \sum_{i=1}^{N} \frac{u_i K_i}{C} \frac{d\ln K_i}{dt} + \frac{wL}{C} \frac{d\ln L}{dt} + \text{AMFP}$$

where, in conjunction with (15b), it can be shown that:

(17c) $$\text{AMFP} = -\frac{\partial \ln TC}{\partial t} + \sum_{i=1}^{M} \frac{P_i Q_i}{P \cdot Q}\left(1 - \frac{\varepsilon \mu_i}{\mu}\right)\frac{d \ln Q_i}{dt} + \frac{\phi D}{TC}\left(\frac{d \ln D}{dt} - \frac{d \ln X}{dt}\right).$$

According to (17b), the direct growth contribution of observed capital inputs and labour is given by the rate of change in these variables weighted by their respective average shares in observed costs C. The productivity term AMFP reflects the three factors shown in (17c): pure technical change or the shift of the cost function, a term that captures the effects of mark-ups and non-constant returns, and a term that captures the effects of the non-observed variable D. Consider the following special cases:

- If there is no unobserved input (D=0), the third term in (17c) disappears and AMFP captures technical change plus a term that reflects the non-constant returns and mark-ups – a result similar to the one developed by Denny, Fuss and Waverman (1981). AMFP will exactly correspond to technical change if there are constant returns to scale ($\varepsilon = 1$) and if the same mark-up factor applies throughout the economy ($\mu_i = \mu$).

- If the volume change of the unobserved input equals the volume change of observed inputs, the third term disappears also and AMFP reflects only technical change and the effects of non-constant returns and mark-ups.

We conclude that, whatever the exact nature of the unobserved factor D, the AMFP computation will capture ‘pure’ technical change, the growth contributions of unobserved assets and scale effects, and also the distribution of mark-ups. With the exception of the mark-ups that can be a consequence of market power, these effects are technology-related and could be considered analytically meaningful expressions of productivity growth. These effects are now path independent – they vary with the levels and growth rates of observed and non-observed inputs, and the latter depend in turn on prices of inputs and outputs as well as on mark-up size.

The contribution of productivity change to output growth is given by AMFP. Clearly, the interpretation of AMFP has to be kept in mind: it reflects the combined effects of technical change, of non-observed inputs, of non-constant returns to scale and of deviations from perfect competition in product markets. In other words, AMFP is a true ‘residual’ or a non-theoretic productivity measure. But for many practical purposes, it will fulfil its role as a multi-faceted measure of productivity growth.[9] We note in passing that AMFP could also serve as a useful measure of productivity growth when technical change is of a more general nature, and not necessarily Hicks-neutral.

If one wants to extend the analysis, an additional analytical step could be taken to decompose AMFP into its technical change component and other effects. However, this requires invoking parametric methods of estimation if one does not want to impose competitive behaviour on output product markets.

[9] As can be seen from the list of assets in our empirical implementation, one important asset that is left out is land, which is not considered a produced asset by the national accounts. This asset presumably does not grow much so the last term in (17c) is likely to be negative in OECD countries, pointing to a downward bias of AMFP as a measure of technical change.

5.2 Invoking Additional Assumptions

This subsection follows the approach (i) outlined at the start of the section: additional hypotheses are invoked to deal with the possible presence of unobserved inputs, non-constant returns to scale and mark-ups. Each set of hypotheses is designed so as to lead to a 'correct' measure of MFP in the sense that it reflects Hicks neutral technical change *if the hypotheses hold*. In addition, consideration is given to how, under the assumed circumstances, the pragmatic AMFP measure would fare. It is of interest, for example, whether its use would imply an upward or downward bias for measuring technical change and when the values would coincide with those for the conventional MFP measure.

5.2.1 Assuming no unobserved input, common mark-up factors and CRS

If one assumes that there are no unobserved inputs (D = 0), and a common mark-up factor in the different output markets ($\mu_i = \mu$), the only possibility to explain a difference between total costs of observed measures and GOS are the combined effects of a positive mark-up and non-constant returns. In this case, the mark-up/returns to scale ratio is given by $\frac{\mu}{\varepsilon} = 1 - \frac{M}{P \cdot Q}$, so it is determined by the ratio $M / P \cdot Q$ where M corresponds to the difference between non-labour income (GOS) and the sum of observed capital costs and where $P \cdot Q$ is the sum of revenues. If empirical information exists on the average mark-up factor, μ, it can be used to determine ε. Alternatively, information may exist on the average degree of returns to scale in the economy. If not, an additional assumption has to be invoked – typically that of constant returns to scale ($\varepsilon = 1$). Having defined away D, one finds that total costs equal observed costs, or TC = C.

In this case, the growth accounting equation (15b) can be expressed as

(18a) $$\sum_{i=1}^{M} \left(\frac{P_i Q_i}{P \cdot Q} \right) \frac{d \ln Q_i}{dt} - \left(\frac{wL}{C} \frac{d \ln L}{dt} + \sum_{i=1}^{N} \frac{u_i K_i}{C} \frac{d \ln K_i}{dt} \right) = MFP1 \text{, with}$$

(18b) $$MFP1 = -\frac{\partial \ln TC1}{\partial t}.$$

Here, TC1 is the modified cost function that applies under the conditions D = 0, $\mu_i = \mu$ and $\varepsilon = 1$. Expression (18b) indicates that MFP1 correctly traces technical change provided the assumptions D = 0, $\mu_i = \mu$ and $\varepsilon = 1$ are accurate. Under the stated assumptions, it is easy to see that $MFP1 = \frac{d \ln Q}{dt} - \frac{d \ln X}{dt}$, where this is the definition of AMFP given in (17a). Thus, if the assumptions above hold, the true productivity measure MFP1 given in (18b) coincides with the result obtained from applying an AMFP measure.

5.2.2 Assuming proportionality of D and K, absence of mark-up factors and CRS

A second possibility is to allow for an unobserved factor (D > 0) but to impose marginal cost pricing (hence $\mu_i = 1$ for $i = 1, \ldots, M$) and constant returns to scale. This is equivalent to assuming fully competitive output markets, and means that (10) implies that $P \cdot Q = TC$. With this setup, it follows that the entire difference between GOS and the sum of observed payments to capital is identified with payments to the unobserved input: $M = \phi D$. To evaluate (15b) for this situation, an additional assumption is needed. One possibility would be to assume that the rate of change of the unobserved input D equals that of the observed capital inputs: $\frac{d\ln D}{dt}=\frac{d\ln K}{dt}$, where $\frac{d\ln K}{dt}\equiv\sum_{i=1}^{N}\frac{u_i K_i}{u\cdot K}\frac{d\ln K_i}{dt}$ is a Divisia quantity index of observed fixed assets. Under these conditions, the growth accounting equation (15b) can be written as

(19a)
$$\sum_{i=1}^{M}\frac{P_iQ_i}{P\cdot Q}\frac{d\ln Q_i}{dt}=\frac{wL}{P\cdot Q}\frac{d\ln L}{dt}+\frac{u\cdot K}{P\cdot Q}\frac{d\ln K}{dt}+\frac{M}{P\cdot Q}\frac{d\ln D}{dt}+MFP2$$
$$=\frac{wL}{P\cdot Q}\frac{d\ln L}{dt}+\left(\frac{u\cdot K}{P\cdot Q}+\frac{M}{P\cdot Q}\right)\frac{d\ln K}{dt}+MFP2,\text{ with}$$

(19b)
$$MFP2=-\frac{\partial\ln TC2}{\partial t}.$$

Now MFP2 given in (19b) traces the shift of the cost function TC2 correctly as long as the assumptions hold. The measured growth contribution of the observed capital inputs merits further discussion. It is easily verified that $\left(\frac{wL}{P\cdot Q}+\frac{u\cdot K}{P\cdot Q}+\frac{M}{P\cdot Q}\right)=1$ so that the weight that now attaches to the observed capital inputs, $\left(\frac{u\cdot K}{P\cdot Q}+\frac{M}{P\cdot Q}\right)=\frac{GOS}{P\cdot Q}$, equals the share of GOS in total production, which in turn is the complement to the labour share in total income. Thus, the income of the unobserved factor D is distributed across the observed capital inputs, and (19a) can be rewritten as:

(19c)
$$\frac{d\ln Q}{dt}=\left(\frac{wL}{P\cdot Q}\frac{d\ln L}{dt}+\frac{GOS}{P\cdot Q}\frac{d\ln K}{dt}\right)+MFP2,$$

using the expression for $(d\ln Q/dt)$ in the paragraph that follows equation (17a). Equation (19c) bears a strong resemblance to a model with endogenous net rates of return as described below in section 5.2.3. In both cases, the overall rate of growth of capital services, $d\ln K/dt$, enters with the same weight – one minus the labour share in total income. Of course, in the endogenous model, the growth rate of observed capital services, $d\ln K^*/dt$, will in general be different from $d\ln K/dt$ in the present case since each asset's user cost term is based on an endogenous rather than an exogenous rate of return. Nonetheless, as will be apparent from the empirical section, the two MFP measures trace each other quite closely, at least for the four countries examined and for the time period used in the empirical part of this paper.

Suppose the above assumptions are true but an AMFP measure is applied. What would be the resulting bias with regard to the 'true' MFP2 measure? After some manipulations, it can be shown that

(20)
$$AMFP=MFP2+\frac{M}{PQ}\left(\frac{d\ln K}{dt}-\frac{d\ln X}{dt}\right).$$

Thus, AMFP will overstate MFP2 if the growth rate of observed capital assets – and by assumption the growth rate of the unobserved asset – exceeds the growth rate of all observed inputs. In the empirical examples presented in section 6, this is the case and AMFP turns out to be consistently higher than MFP2.

5.2.3 Defining away mark-ups and unobserved inputs and assuming CRS

These are the assumptions invoked when MFP computations rely on endogenous rates of return: output markets are taken as competitive ($\mu_i = 1;\ i = 1,\ldots,M$), there are no unobserved factors (D = 0) and there are constant returns to scale. The endogenous approach goes back to Christensen and Jorgenson (1969), and has been applied in many subsequent studies of productivity growth, including many carried out by national statistical offices (e.g., BLS 2003). This is the most widely-used methodology but also the one that requires the most restrictive set of assumptions: the assumptions needed to justify the use of endogenous rates of return[10]. In addition to the above assumptions, there must be perfect anticipation of asset price changes and depreciation. This implies that $P \cdot Q = wL + u^* \cdot K$. The growth accounting model (15b) becomes

(21a) $$\sum_{i=1}^{M}\left(\frac{P_iQ_i}{P \cdot Q}\right)\frac{d\ln Q_i}{dt} = \left(\frac{wL}{P \cdot Q}\frac{d\ln L}{dt} + \sum_{i=1}^{N}\frac{u_i^* K_i}{P \cdot Q}\frac{d\ln K_i}{dt}\right) + \text{MFP3, where}$$

(21b) $$\text{MFP3} = -\frac{\partial \ln TC3}{\partial t},$$

and where TC3 denotes a cost function with observed inputs only. If the additional restrictions hold, measured productivity change corresponds to the shift of a cost function TC3 with CRS and only observed inputs. As alluded to above, there is similarity with (19c) because (21a) can be re-written as

(22) $$\frac{d\ln Q}{dt} = \left(\frac{wL}{P \cdot Q}\frac{d\ln L}{dt} + \frac{GOS}{P \cdot Q}\frac{d\ln K^*}{dt}\right) + \text{MFP3 where } \frac{d\ln K^*}{dt} \equiv \sum_{i=1}^{N}\frac{u_i^* K_i}{u^* \cdot K}\frac{d\ln K_i}{dt},$$

so that the contribution of capital assets is the product of the rate of growth of observed capital services and the share of GOS in total output or cost.

If the above assumptions are correct, and if an endogenous rate of return is used, the evaluation of AMFP* would yield the correct result since $\text{AMFP*} = -\frac{\partial \ln TC3}{\partial t} = \text{MFP3}$ in this case. We have marked AMFP* with an asterisk here to draw attention to the fact that AMFP is based on a capital measure that reflects endogenous rates of return. If AMFP is computed on the basis of exogenous rates, it would clearly differ from MFP3. This is also borne out in the empirical example below. However, no a-priori statement can be made as to the sign of this difference.

5.2.4 Assuming no mark-ups, no unobserved input and decreasing returns to scale: an input-based measure

This constitutes yet another possibility for dealing with the difference between revenues and observed factor payments: the unobserved factor is defined away (D = 0) as well as mark-ups of prices over marginal costs ($\mu_i = 1$; i = 1,…M), but the production technology is assumed

[10] The endogenous rate of return is computed by choosing that net rate of return that just equalizes the sum of user costs of observed assets with non-labour income (GOS for simplicity). Using the same notation for user costs as in footnote 2, this means that $GOS = \sum_{i=1}^{N} q^i (r^* + \delta^i - d\ln q^i / dt) K^i$.

to exhibit decreasing returns to scale ($\varepsilon > 1$). Then, the entire difference between GOS and observed asset rental payments is ascribed to the effects of marginal cost pricing under decreasing returns to scale: $M = (\varepsilon - 1)TC$ and TC = C. Under these circumstances, the returns to scale parameter can be computed as $\varepsilon = M / TC + 1$. Given a value for ε, (15b) can be rewritten as

(23a) $$\varepsilon \sum_{i=1}^{M} \left(\frac{P_i Q_i}{P \cdot Q} \right) \frac{d \ln Q_i}{dt} = \left(\frac{wL}{TC} \frac{d \ln L}{dt} + \sum_{i=1}^{N} \frac{u_i K_i}{TC} \frac{d \ln K_i}{dt} \right) + MFP4, \text{where}$$

(23b) $$MFP4 = -\frac{\partial \ln TC4}{\partial t}$$

and where TC4 is a cost function with decreasing returns to scale and with observed factor inputs only. Some more discussion is useful here. First, because all costs are observed, TC = C and (23a) can be written as

(23c) $$\varepsilon \frac{d \ln Q}{dt} = \frac{d \ln X}{dt} + MFP4 .$$

We note in passing that the same growth accounting equation and/or productivity measure MFP4 could have been derived from a model with a constant returns to scale cost function for observed and unobserved inputs, but with the added assumption that the quantity of the unobserved input is positive and fixed.[11] The unobserved input then acts as the additional cost factor that is equivalent to a decreasing returns to scale technology.

If the assumptions above are correct, how does MFP4 relate to AMFP? It is easily established that under these circumstances, $AMFP = MFP4 - (\varepsilon - 1)\frac{d \ln Q}{dt}$. If returns to scale are decreasing ($\varepsilon > 1$), and if the quantity of output increases ($d \ln Q / dt > 0$), AMFP will be smaller than MFP4, since AMFP captures both the effects of pure technical change and non-constant returns. This is borne out in the empirical section 6.

5.2.5 Assuming no mark-ups, no unobserved input and decreasing returns to scale: an output-based measure

It is well known that a production technology with non-constant returns to scale gives rise to several productivity measures (see, for example, Balk 1998). In particular there are differences between output-based measures of technology such as the shift of a production function or of a revenue function over time and input-based measures of technology such as the shift of a cost function or of an input distance function over time. In the sections above, the analysis has been based on a cost function, i.e., an input-based measure. To introduce an alternative and output-based measure of technical change, we shall consider a revenue function and its shift over time. As in section 5.2.4, we assume that there is no unobserved input and that there are no mark-ups. As a consequence, the value of M is entirely determined by the decreasing returns to scale and total costs equal observed costs (since $M = (\varepsilon - 1)TC$ where TC = C).

[11] The idea is based on Diewert and Nakamura (2007) who introduce an unknown variable into a cost function to deal with decreasing returns to scale.

To derive the output-based productivity measure, consider the revenue function[12] R, defined so as to show maximum revenues given a vector of inputs and a vector of output prices:

(24) $$R(P,L,K,t) = \max_{Q}\{P \cdot Q : (Q,L,K) \text{ belongs to } Z(t)\}.$$

Diewert (1983) first used a revenue function to define a theoretical productivity index, albeit in discrete time. We follow his approach and define the continuous-time equivalent as the partial derivative of the revenue function with respect to time: total differentiation of R yields the following output-based measure of technical change:

(25) $$\frac{\partial \ln R}{\partial t} = \frac{d \ln R}{dt} - \sum_{i=1}^{M} \frac{\partial \ln R}{\partial \ln P_i} \frac{d \ln P_i}{dt} - \frac{\partial \ln R}{\partial \ln L} \frac{d \ln L}{dt} - \sum_{i=1}^{N} \frac{\partial \ln R}{\partial \ln K_i} \frac{d \ln K_i}{dt}.$$

To derive a computable measure of the output-based productivity measure, an additional assumption has to be introduced: revenue-maximising behaviour on the part of producers. Then, observed revenues equal maximum revenues: $P \cdot Q = R$. If in addition firms are price takers, one gets $Q_i = \partial R / \partial P_i$. It is then straightforward to obtain a computable expression for the elasticity of revenues with respect to output prices:

(26) $$\frac{\partial \ln R}{\partial \ln P_i} = \frac{P_i Q_i}{R} = \frac{P_i Q_i}{P \cdot Q}.$$

Note that the assumptions of revenue maximisation and price taking on output markets were not necessary for the derivation of the input-based measure in section 5.2.4. Thus, the output-based productivity statistic requires different assumptions than the input-based statistic.

Now define the Divisia decomposition of total revenues into a price and a quantity index:

(27) $$\frac{d \ln R}{dt} = \sum_{i=1}^{M} \frac{P_i Q_i}{P \cdot Q} \frac{d \ln P_i}{dt} + \sum_{i=1}^{M} \frac{P_i Q_i}{P \cdot Q} \frac{d \ln Q_i}{dt}.$$

The first two expressions on the right hand side of (25) are equivalent to a Divisia quantity index of outputs: $\frac{d \ln R}{dt} - \sum_{i=1}^{M} \frac{P_i Q_i}{P \cdot Q} \frac{d \ln P_i}{dt} = \sum_{i=1}^{M} \frac{P_i Q_i}{P \cdot Q} \frac{d \ln Q_i}{dt} \equiv \frac{d \ln Q}{dt}$.

To find computable expressions for the input elasticities of the revenue function, we invoke the profit-maximising behaviour of producers. This implies that they solve a maximisation problem of the kind $\max_{L,K}\{R(P,L,K,t) - wL - u \cdot K\}$. The first order conditions for a maximum are $\partial R / \partial L = w$ and $\partial R / \partial K_i = u_i$ (i=1,…N). Consequently, $\partial \ln R / \partial \ln L = wL / R$ and $\partial \ln R / \partial \ln K_i = u_i K_i / R$ (i=1,…N). Then, the third and fourth expression on the right-hand side of (25) can be rewritten as

[12] The concept of a revenue function is due to Samuelson (1953-54).

(28)
$$\frac{\partial \ln R}{\partial \ln L}\frac{d\ln L}{dt}+\sum_{i=1}^{N}\frac{\partial \ln R}{\partial \ln K_i}\frac{d\ln K_i}{dt}=\frac{wL}{R}\frac{d\ln L}{dt}+\sum_{i=1}^{N}\frac{u_iK_i}{R}\frac{d\ln K_i}{dt}$$
$$=\frac{C}{R}\left(\frac{wL}{C}\frac{d\ln L}{dt}+\sum_{i=1}^{N}\frac{u_iK_i}{C}\frac{d\ln K_i}{dt}\right).$$
$$=\frac{C}{R}\frac{d\ln X}{dt}.$$

But $C/R = TC/P\cdot Q = 1/\varepsilon$ and the final computable expression for the output side-based productivity measure in (25) is

(29)
$$\frac{\partial \ln R}{\partial t}=\frac{d\ln Q}{dt}-\frac{1}{\varepsilon}\frac{d\ln X}{dt}=MFP5.$$

The link to AMFP is readily established: it can be shown that

(30)
$$MFP5=\frac{d\ln Q}{dt}-\frac{d\ln X}{dt}-\left(\frac{1}{\varepsilon}-1\right)\frac{d\ln X}{dt}=AMFP+\left(1-\frac{1}{\varepsilon}\right)\frac{d\ln X}{dt}.$$

Thus, MFP5 will exceed AMFP if the quantity index of inputs grows at a positive rate as can be observed in the country examples in section 6.

5.2.6 A note on increasing returns to scale

There is no reason to believe that returns to scale may not be locally increasing; hence this case must be treated as well. Suppose that $\varepsilon < 1$. Unless the case of M < 0 is allowed, implying continuing losses for producers, increasing returns to scale must go together with positive mark-ups over marginal costs. Thus, in order to have M > 0 under increasing returns to scale, μ_i must be positive, but also less than unity for at least one product.

Then, $M = P\cdot Q(1-\mu/\varepsilon)$ if we assume that there is no unobserved input (D = 0). Under these assumptions, the growth accounting and productivity equation (15b) takes the form:

(31)
$$\frac{\varepsilon}{\mu}\sum_{i=1}^{M}\frac{P_iQ_i}{P\cdot Q}\mu_i\frac{d\ln Q_i}{dt}=\frac{d\ln X}{dt}+MFP6.$$

While (31) is a valid measure for the shift in a cost function, TC6, given increasing returns to scale and without unobserved inputs, it is apparent that with the observable information on prices, quantities and factor remuneration (31) still cannot be computed. Although ε/μ is known, there is not enough information to deduce values for product-specific mark-up factors μ_i. Extraneous information about mark-ups is required to compute MFP6. While such information sometimes is available, this cannot be expected on an ongoing, timely and comprehensive basis. For example, Oliveira-Martins et al. (1996) estimated mark-up ratios for 14 OECD countries by industry and report estimates of the typically positive mark-ups. But one-off studies are quickly outdated. Also, industry-level mark-up estimates are frequently confined to manufacturing industries, leaving uncovered important areas of the service sector. Overall, it would not seem practical for a statistical office to rely on mark-up estimates for purposes of productivity statistics. For the same reason, we are not in a position to compute empirical results for MFP6.

6. Empirical Implementation

After the theoretical derivations in section 5, we shall now move on to empirical considerations. Several questions arise. One concerns index numbers: how should the continuous-time formulae be translated into discrete index number formulae to accommodate the fact that data observations come in discrete form? A second question relates to how exactly some of the variables should be measured, in particular capital services and user costs of capital. Finally, we wish to compare the various productivity measures to get a sense of the importance of choices of assumptions.

6.1 Choice of Index Number Formulae

Concerning the index number issue, our approach has been one of approximating the continuous-time Divisia indices in the theoretical part of the paper by Törnqvist-type indices for the present empirical part. We are aware of the methodological shortcomings of this procedure: this discrete approximation is essentially an arbitrary choice,[13] not rigorously backed up by theory. A more thorough procedure would have been to start out with discrete formulations for the cost and revenue functions and then derive the appropriate index number formulae together with the productivity measure.[14] However, we feel that the theoretical advantages of a full derivation in discrete time are outweighed by the algebraic complications that such an approach brings along with including all the interaction terms which would add little to the message delivered in the present paper while making the exposition much less readable.

For the purpose at hand then, we chose the following Törnqvist-type approximations to the above Divisia-type formulations of the various productivity indices:

(32a) $$\frac{d\ln Q}{dt} \approx \sum_{i=1}^{M} \frac{1}{2}\left(\frac{P_i^t Q_i^t}{P^t \cdot Q^t} + \frac{P_i^{t-1} Q_i^{t-1}}{P^{t-1} \cdot Q^{t-1}}\right) \ln\left(\frac{Q_i^t}{Q_i^{t-1}}\right) \equiv \ln\left(\frac{Q^t}{Q^{t-1}}\right)$$

(32b)
$$\frac{d\ln X}{dt} \approx \frac{1}{2}\left(\frac{w^t L^t}{C^t} + \frac{w^{t-1} L^{t-1}}{C^{t-1}}\right) \ln\left(\frac{L^t}{L^{t-1}}\right) + \sum_{i=1}^{N} \frac{1}{2}\left(\frac{u_i^t K_i^t}{C^t} + \frac{u_i^{t-1} K_i^{t-1}}{C^{t-1}}\right) \ln\left(\frac{K_i^t}{K_i^{t-1}}\right) \equiv \ln\left(\frac{X^t}{X^{t-1}}\right)$$

(32c) $$AMFP^{t/t-1} = \ln\left(\frac{Q^t}{Q^{t-1}}\right) - \ln\left(\frac{X^t}{X^{t-1}}\right)$$

(32d) $$MFP1^{t/t-1} = AMFP^{t/t-1}$$

[13] That nearly all common index number formulae can be considered as discrete approximations to the Divisia index has already been shown by Frisch (1936). For a more recent statement, see Diewert (1980) or Balk (2005).

[14] Examples are provided by Balk (1998, section 3.7).

(32e)
$$MFP2^{t/t-1} = \ln\left(\frac{Q^t}{Q^{t-1}}\right) - \frac{1}{2}\left(\frac{w^t L^t}{P^t \cdot Q^t} + \frac{w^{t-1}L^{t-1}}{P^{t-1} \cdot Q^{t-1}}\right)\ln\left(\frac{L^t}{L^{t-1}}\right) - \sum_{i=1}^{N}\frac{1}{2}\left(\frac{u_i^t K_i^t + M^t}{P^t \cdot Q^t} + \frac{u_i^{t-1}K_i^{t-1} + M^{t-1}}{P^{t-1} \cdot Q^{t-1}}\right)\ln\left(\frac{K_i^t}{K_i^{t-1}}\right)$$

(32f)
$$MFP3^{t/t-1} = \ln\left(\frac{Q^t}{Q^{t-1}}\right) - \frac{1}{2}\left(\frac{w^t L^t}{P^t \cdot Q^t} + \frac{w^{t-1}L^{t-1}}{P^{t-1} \cdot Q^{t-1}}\right)\ln\left(\frac{L^t}{L^{t-1}}\right) - \sum_{i=1}^{N}\frac{1}{2}\left(\frac{u_i^{*t} K_i^t}{P^t \cdot Q^t} + \frac{u_i^{*t-1}K_i^{t-1}}{P^{t-1} \cdot Q^{t-1}}\right)\ln\left(\frac{K_i^t}{K_i^{t-1}}\right)$$

(32g)
$$MFP4^{t/t-1} = \frac{1}{2}\left(\varepsilon^t + \varepsilon^{t-1}\right)\ln\left(\frac{Q^t}{Q^{t-1}}\right) - \ln\left(\frac{X^t}{X^{t-1}}\right)$$

(32h)
$$MFP5^{t/t-1} = \ln\left(\frac{Q^t}{Q^{t-1}}\right) - \frac{1}{2}\left(\frac{1}{\varepsilon^t} + \frac{1}{\varepsilon^{t-1}}\right)\ln\left(\frac{X^t}{X^{t-1}}\right).$$

6.2 Measuring Outputs and Inputs

The empirical productivity measures developed in the present paper all relate to the total economy. This reflects data constraints rather than a preferred choice which would have been to limit computations to the corporate or business sector. Neither capital input measures nor hours worked are easily available in such a sectoral breakdown and calculations remain at the aggregate level, in line with the data available from the *OECD Productivity Database*.[15]

6.2.1 Outputs

Value-added has been measured at basic prices, i.e., excluding taxes on products but including product subsidies, because this valuation constitutes the economically relevant variable from a producer perspective. Time series on value-added and net indirect taxes were taken from the *OECD Annual National Accounts*.

A second adjustment to aggregate value-added is also required to maintain consistency between input and output data: capital input in the *OECD Productivity Database* is limited to non-residential, fixed assets in scope and consequently, the value-added produced with residential assets should be excluded from productivity calculations. Thus, total value-added is corrected for the production of owner-occupiers.[16] Note that both adjustments (valuation of output at basic prices and exclusion of the production of owner-occupiers of dwellings) have immediate consequence for the size of the endogenous rate of return as computed in MFP3 and for the weights that attach to capital and labour in MFP2. AMFP, on the other hand, is influenced

[15] www.oecd.org/statistics/productivity.

[16] The need for this exclusion and possible consequences for the measurement of the endogenous rates of return were pointed out to me by Mathilde Mas (University of Valencia).

by these adjustments only to the extent that they bear on the volume growth rate of output. Moreover, current-price value-added does not enter the AMFP computation because labour and capital weights are determined independently of the output measure. This is a distinct advantage in the presence of the AMFP approach.[17]

6.2.2 Inputs

Labour input is measured as total hours worked in the economy – a difficult task, especially at the international level. Even so, this remains an imperfect measure: no account is taken of differences in the value of hours of persons with different skill and experience levels. A more appropriate index of labour input would weight different types of hours worked by their corresponding shares in overall compensation. The most important measurement issues are described in a note available on the site of the *OECD Productivity Database*.

Capital inputs are derived with the perpetual inventory method. The estimation of capital service flows starts with identifying those assets that correspond to the breakdown currently available from the OECD/Eurostat National Accounts questionnaire, augmented by information on information and communication technology assets. Only non-residential gross fixed capital formation is considered for seven types of assets or products: Products of agriculture, metal products and machinery (IT hardware; communications equipment; other); transport equipment; non-residential construction; other products (software; other).

Investment. For each type of asset, a time series of current-price investment expenditure and the corresponding price indices are assembled starting with 1960. For many countries, this involves a certain amount of estimation, in particular for the period 1960-80. Such estimates are typically based on national accounts data prior to the introduction of SNA93, or on relationships between different types of assets that are established for recent periods and projected backwards. For purposes of exposition of the methodology, the current price investment series for asset type i in year t are denoted by IN_t^i (i=1,2, ..., 7) and the corresponding price index is denoted by $q_{t,0}^i$. Price indices are normalised to the reference year 1995 where $q_{t,0}^i = 1$.

Price indices should be constant quality deflators that reflect price changes for a given investment good. This is particularly important for those items that have seen rapid quality change such as information and communication technology (ICT) assets. For instance, observed price changes of 'computer boxes' had to be quality-adjusted to permit comparison of different vintages. Schreyer (2000) used a set of 'harmonised' ICT deflators to control for some of the differences in methodology.[18] We follow this approach and assume that the ratios between ICT and non-ICT asset prices evolve in a similar manner across countries, using the United States as the benchmark. Although no claim is made that the 'harmonised' deflator is necessarily the

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[17] For example, the available OECD national accounts data do not allow us to single out the production of the owner-occupied dwellings industry – only the parent aggregate with real estate, renting and business activities is available. For purposes of the present computations, an assumption had to be made that the production of owner-occupiers accounts for one third of the entire industry. Obviously, this introduces a potential bias in those computations that depend on this adjustment.

[18] Wyckoff (1995) was one of the first to point out that the large differences that could be observed between computer price indices in OECD countries were likely much more a reflection of differences in statistical methodology than true differences in price changes. In particular, those countries that employ hedonic methods to construct ICT deflators tend to register a larger drop in ICT prices than countries that do not.

correct price index for a given country, the possible error due to using a harmonised price index is smaller than the bias arising from comparing capital services based on national deflators[19].

Productive stocks. Given price and volume series for investment goods, for each of the (supposedly) homogenous asset types, a productive stock S_t^i has been constructed as follows:

$$S_t^i = \sum_{\tau=1}^{T^i} (IN_{t-\tau}^i / q_{t-\tau,0}^i) h_\tau^i F_\tau^i \,, \quad i=1, \ldots, 7. \tag{33}$$

In this expression, the productive stock of asset i at the beginning of period t is the sum over all past investments for this asset, where current price investment in past periods, $IN_{t-\tau}^i$ has been deflated with the purchase price index of new capital goods, $q_{t-\tau,0}^i$. T^i represents the maximum service life of asset type i. Because past vintages of capital goods are less efficient than new ones, an age efficiency function h_τ^i has been applied. It describes the efficiency time profile of an asset, conditional on its survival and is defined as a hyperbolic function of the form used by the United States Bureau of Labor Statistics (BLS 1983), $h_\tau^i = (T^i - \tau)/(T^i - \beta\tau)$.

Capital goods of the same type purchased in the same year do not generally retire at the same moment. More likely, there is a retirement distribution around a mean service life. In the present calculations, a normal distribution with a standard deviation of 25 percent of the average service life is chosen to represent the probability of retirement. The distribution was truncated at an assumed maximum service life of 1.5 times the average service life. The parameter F_τ^i is the cumulative value of this distribution, describing the probability of survival over a cohort's life span. The following average service lives are assumed for the different assets: 7 years for IT equipment; 15 years for communications equipment, other equipment and transport equipment; 60 years for non-residential structures; 3 years for software; and 7 years for the remaining products. The parameter β in the age-efficiency function was set to 0.8. Service lives and parameter values were specified following BLS practices.

User costs of capital. In a fully functioning asset market, the purchase price of an asset will equal the discounted flow of the value of services that the asset is expected to generate in the future. This equilibrium condition is used to derive the rental price or user cost expression for assets. Let $q_{t,0}^i$ denote the purchase price in year t of a new (zero-year old) asset of type i, and let $u_{t+\tau,\tau}^i$ be the rental price that this asset is expected to fetch in period $t+\tau$ (first subscript to the right) when the asset will be of age τ (second subscript to the right). With r as the nominal discount rate valid at time t, the asset market equilibrium condition for a new asset (age zero) is:

$$q_{t,0}^i = \sum_{\tau=0}^{\infty} u_{t+\tau+1,\tau}^i (1+r)^{-(\tau+1)} \,. \tag{34a}$$

[19] See Schreyer et al. (2003) for details. There is a difficulty with the harmonised deflator that should be noted. From an accounting perspective, adjusting the price index for investment goods for any country implies an adjustment of the volume index of output. In most cases, such an adjustment would increase the measured rate of volume output change. At the same time, effects on the economy-wide rate of GDP growth appear to be relatively small (see Schreyer (2002) for a discussion).

This formulation implies that rentals are paid at the end of each period. To solve this expression for the rental price, the price for a one year old asset in period t+1 is computed as $q^i_{t+1,1} = \sum_{\tau=0}^{\infty} u^i_{t+\tau+2,\tau+1}(1+r)^{-(\tau+1)}$ and then subtracted from the expression above to obtain $u^i_{t+1,0} = q^i_{t,0}(1+r) - q^i_{t+1,1}$ or $u^i_{t,0} = q^i_{t-1,0}(1+r) - q^i_{t,1}$ which can be transformed into

(34b) $$u^i_{t,0} = q^i_{t-1,0}(r + d^i_{t,0} - \zeta^i_{t,0} + d^i_{t,0}\zeta^i_{t,0}).$$

This is the user cost formulation[20] applied in the present paper, where the rate of depreciation of asset i has been defined as $d^i_{t,s} \equiv 1 - q^i_{t,s+1}/q^i_{t,s}$ and the rate of price change of the same asset is given by $\zeta^i_t \equiv q^i_{t,s}/q^i_{t-1,s} - 1$. Note that the different variables in the user cost equation are expectations because they invoke knowledge about price changes in future periods.

These expectations govern the rental price. The System of National Accounts that capital stock data should tie into is based on ex-post prices, observed in the context of actual transactions. Would the use of user cost expressions such as those discussed above be in contradiction with the principles of national accounts?

The answer is 'no'. The presence of expectations does not make the user cost term less 'real': transactions are concluded at this price, even if with hindsight (ex post) the expectations underlying the transactions may turn out to be wrong. This is most apparent when one thinks of a case where capital goods are rented: the observed rental price characterises the transaction and is the relevant market price, typically dependent on expectations on the side of the lessor and the lessee. Nobody would challenge using such observed prices in the national accounts. If rental prices are not observable, values have to be imputed, and the expression above indicates how this can be done on the basis of economic theory. Imputations are numerous in the national accounts, and in this sense, the imputation of user costs would not constitute an exception.

Thus, it is not the presence of an expected variable as such that is at issue. The real issue from a capital and productivity measurement viewpoint is whether the realised, but unobserved, marginal productivity of fixed assets is better approximated by an ex-ante or an ex-post measure of user costs.[21] On this matter, Berndt (1990) points out that: "…if one wants to use a measure of capital to calculate actual multifactor productivity growth, then theory tells us quite clearly that we should weight the various traditionally measured capital inputs by their realised marginal products, not their expected marginal products. This means that in choosing capital service price weights, one should employ shadow values or ex post rates of return, and not the ex ante rates of return that are appropriate in the investment context."

While we concur with Berndt's statement that for purposes of productivity measurement, realised marginal products are the appropriate weights, this does not mean that ex post rates of return are always the preferred approximation to realised marginal productivity. Suppose that a

[20] Jorgenson and Yun (2001) show how tax considerations enter the user cost of capital and how they affect measured economic performance. This is one of the projects for expansion of the OECD Productivity Database. At present, however, these parameters are not considered in our set of user costs and capital measures.

[21] The distinction between ex-ante and ex-post user costs has been discussed by Berndt and Fuss (1986), Harper et al. (1989), Diewert (2001), Berndt (1990) in his discussion of Hulten (1990) and Hill and Hill (2003).

capital asset is rented by a producer at a given, pre-agreed rental price to be paid by the end of the period. The lessee of the asset will use it in his production process as planned regardless of the ex-post rental price. Therefore, the marginal productivity of the asset in the production process would best be approximated by the ex-ante rental price that is the price at which the rental transaction actually took place.

Take another case of an owner/producer and suppose that there has been investment at the beginning of the period in line with the ex-ante user cost. Now let there be a change in market conditions that leads to a modification of expectations and of user costs. If capital is fully flexible and can be adjusted continuously, it will be adjusted in line with the new user cost term. But the user cost term is governed by expectations, even though the expectations may have changed. It is only when capital cannot be adjusted that the ex-post user cost term would furnish the preferred approximation to the realised marginal productivity of an asset. This is the case that Berndt (1990) and Berndt and Fuss (1986) have in mind and it relies on quasi-fixity of capital in the production process. Thus, there is no general conclusion that ex-post user cost measures should always be preferred to ex-ante ones for measuring and aggregating capital input.

There is another conceptual difficulty with ex post user costs: the computation of the realised rates of return is commonly done by choosing a rate of return so that the ensuing user cost and total value of capital services just exhausts the measured gross operating surplus available from the national accounts. This computation relies, however, on the assumption that there is only one ex-post rate of return that applies to all assets. While equalisation of rates of return across assets is a natural assumption in an ex-ante context, it is much harder to justify in an ex-post context, especially given states of disequilibrium. Essentially it amounts to imposing an equilibrium condition to implement an (ex-post) measure that was specifically chosen on the grounds that it captures the nature of a situation of disequilibrium.

Diewert (2001) also points out that while the ex-post measure (of the nominal rate of return) is widely used in empirical research, it is subject to measurement error and it may not reflect the economic conditions facing producers at the beginning of the period.

A practical argument against the use of an ex-post rate is that its calculation requires information on the level of the productive capital stock at current prices (or alternatively on the wealth stock at current prices). However, levels of capital stocks tend to be less reliable statistics than their rates of change, especially when long historical investment series have to be estimated. This problem does not arise when user costs and nominal rates of return are of an ex-ante nature and therefore are exogenous variables. On the other hand, ex-post rates of return are of interest as such, and straightforward to compute. In sum then, there is no clear conclusion on this matter. In the present work, preference is given to an ex-ante approach, mainly because it allows us to develop capital service measures independently from measures of labour compensation, gross operating surplus and mixed income in the national accounts.

Exogenous net rate of return. To compute the net rate of return, following a suggestion of Diewert (2001), the starting point is the constant value for the expected real interest rate rr. The constant real rate is computed by taking a series of annual observed nominal rates (an unweighted average of interest rates with different maturities[22]) and deflating them by the

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[22] These are the average bank rate, the bank rate on prime loans, long-term government bond yields, short-term government bond yields, the interest rate on a 90 day bank fixed deposit, and the treasury bill rate.

consumer price index. The resulting series of real interest rates is averaged over the period (1980-2000) to yield a constant value for rr. The expected nominal interest rate for every year is then computed as $r_t = (1+rr)(1+p_t)-1$ where p is the expected value of an overall deflator, the consumer price index.

To obtain a measure for p, the expected overall inflation, we construct a 5-year centered moving average of the rate of change of the consumer price index $p_t = \sum_{s=-2}^{+2} CPI_{t-s}$, where CPI_t is the annual percentage change of the consumer price index. This equals the expected rate of overall price change and, by implication, the nominal net rate of return.

Expected asset price changes, another element in the user cost equation, are derived as a smoothed series of actual asset price changes: a 5-year centered moving average filter is used.

Depreciation rates have been computed using the definition given above, $d_{t,0}^i \equiv 1 - q_{t,1}^i / q_{t,0}^i$. So, the rate of depreciation for a new asset equals one minus the ratio of the market price for a year old asset over the market price for a new asset. The market price for a new asset can be observed directly, but the price for a one-year old asset must be computed using the asset market equilibrium condition (34), the age-efficiency function h and the discount rate.

6.3 Results

Tables 1-4 summarise empirical results for Canada, France, the United States and Japan. They show the rates of change of output (GDP) and labour input as well as the volume changes of capital services alternatively based on exogenous and endogenous rates of return as well as the various MFP measures. The first observation is that moving from an endogenous to an exogenous rate of return leads to a rise in the observed measure of capital input – at least in the case of the countries considered and for the period at hand. Also, labour and capital shares turn out to be quite different when based on total costs rather than total revenue.

The second panel in each table reviews results for the five alternative MFP measures presented in the text above. It is immediately apparent that the different options – each associated with a particular set of assumptions about market structures or production technology – can lead to considerable variation in the resulting MFP measures, France being a noticeable exception. Unless a-priori knowledge about technology and market structure are available, it will be difficult to choose between the different options. Also, every different MFP measure implies a different message about the relative contribution of capital services to output growth. For all four countries examined, measured productivity growth turns out to be slowest when based on endogenous rate models (MFP3) or when assuming proportionality between capital input and an unobserved factor (MFP2). The output-based productivity measure that allows for decreasing returns to scale (MFP5) is generally the fastest-growing item in each country, followed by the input-based productivity measure with decreasing returns to scale (MFP4).

However, a simple geometric average of the five specific MFP measures yields a time series that is very close to the simple AMFP measure. In the absence of a-priori information on mark-ups, returns to scale, or unobserved assets, the choice of a measure that is close to the average of the different options may be a reasonable one. This is one of our conclusions.

Table 1. Canada

Basic series

	Output	Hours worked		Capital services			
				Exogenous RoR		Endogenous RoR	
	Volume index	Cost share	Volume index	Cost share	Volume index	Revenue share	Volume index
1985	100.0	71.70%	100.0	28.30%	100.0	33.30%	100.0
1986	102.4	72.69%	102.9	27.31%	105.0	32.28%	105.8
1987	106.7	73.54%	106.4	26.46%	110.6	32.41%	112.1
1988	112.0	73.69%	110.7	26.31%	116.9	32.07%	119.2
1989	114.9	71.39%	112.9	28.61%	123.1	31.36%	126.5
1990	115.1	70.34%	112.6	29.66%	128.7	30.49%	133.1
1991	112.6	69.61%	109.2	30.39%	133.7	28.71%	139.1
1992	113.6	73.23%	108.0	26.77%	138.3	28.19%	144.5
1993	116.3	72.76%	110.1	27.24%	142.7	28.52%	149.7
1994	121.8	75.35%	113.3	24.65%	147.6	30.91%	155.4
1995	125.2	74.38%	114.6	25.62%	152.9	32.32%	161.2
1996	127.3	76.10%	116.8	23.90%	158.6	32.16%	167.3
1997	132.7	74.31%	118.5	25.69%	166.5	32.65%	175.5
1998	138.1	74.22%	121.5	25.78%	175.3	31.48%	184.6
1999	145.8	73.00%	125.4	27.00%	184.9	32.94%	194.5
2000	153.5	74.31%	128.4	25.69%	194.3	34.56%	204.3
2001	156.4	72.66%	128.4	27.34%	202.9	..	204.3
2002	161.6	72.29%	130.3	27.71%	209.7	..	204.3
85-90	2.81%	72.23%	2.37%	27.77%	5.05%	31.98%	5.71%
90-95	1.70%	72.61%	0.35%	27.39%	3.45%	29.86%	3.84%
95-00	4.07%	74.38%	2.28%	25.62%	4.80%	32.69%	4.73%
95-02	3.64%	73.91%	1.84%	26.09%	4.51%	32.69%	3.38%

Productivity measures

	MFP1 =AMFP	MFP2	MFP3	MFP4	MFP5		Average
1985	100.0	100.0	100.0	100.0	100.0		100.0
1986	99.0	98.9	98.6	99.2	99.2		99.0
1987	99.2	99.1	98.6	99.8	99.8		99.3
1988	99.7	99.4	98.8	100.6	100.6		99.8
1989	99.3	98.9	98.1	100.4	100.4		99.4
1990	98.4	97.9	96.9	99.5	99.5		98.5
1991	97.3	96.9	95.7	98.4	98.4		97.3
1992	98.0	97.5	96.2	99.1	99.0		98.0
1993	98.1	97.6	96.2	99.2	99.2		98.0
1994	99.7	99.2	97.7	101.1	101.0		99.7
1995	100.7	100.1	98.4	102.4	102.2		100.8
1996	100.0	99.2	97.6	101.9	101.7		100.0
1997	101.9	100.8	99.2	104.3	103.8		102.0
1998	102.7	101.5	99.9	105.5	105.0		102.9
1999	104.5	103.1	101.5	107.9	107.1		104.8
2000	106.7	105.1	103.5	110.7	109.7		107.1
2001	107.5	..	..	110.7	..		109.1
2002	108.8	..	..	108.5	..		108.7
85-90	-0.31%	-0.42%	-0.63%	-0.09%	-0.10%		-0.31%
90-95	0.46%	0.43%	0.32%	0.58%	0.53%		0.46%
95-00	1.15%	0.97%	0.99%	1.56%	1.42%		1.22%
95-02	1.10%	..	..	0.82%	..		1.08%

Source: OECD Productivity Database May 2004.

Table 2. France

Basic series

	Output	Hours worked		Capital services			
				Exogenous RoR		Endogenous RoR	
	Volume index	Cost share	Volume index	Cost share	Volume index	Revenue share	Volume index
1985	100.0	69.53%	100.0	30.47%	100.0	24.32%	100.0
1986	102.4	70.42%	99.8	29.58%	102.7	26.56%	102.8
1987	105.0	71.36%	100.3	28.64%	105.7	27.46%	105.9
1988	109.8	71.48%	101.3	28.52%	109.4	28.88%	109.6
1989	114.4	70.06%	101.9	29.94%	113.7	30.06%	113.9
1990	117.4	69.81%	102.8	30.19%	118.4	29.80%	118.6
1991	118.6	69.11%	102.4	30.89%	122.7	29.94%	123.0
1992	120.4	68.10%	101.8	31.90%	126.5	30.39%	126.9
1993	119.3	68.26%	99.7	31.74%	129.4	30.59%	129.9
1994	121.8	67.91%	99.5	32.09%	132.3	31.64%	132.7
1995	123.8	68.67%	98.8	31.33%	135.0	31.89%	135.4
1996	125.2	68.86%	99.5	31.14%	137.7	31.71%	138.2
1997	127.5	69.50%	99.5	30.50%	140.5	32.30%	141.0
1998	131.9	70.50%	100.3	29.50%	144.0	33.09%	144.4
1999	136.1	71.89%	101.9	28.11%	148.5	32.87%	148.6
2000	141.3	71.12%	101.3	28.88%	153.6	33.21%	153.5
2001	144.2	68.83%	101.5	31.17%	158.5	33.00%	158.3
2002	146.0	69.40%	101.1	30.60%	161.7	32.38%	161.5
85-90	3.21%	70.44%	0.56%	29.56%	3.37%	27.85%	3.42%
90-95	1.06%	68.64%	-0.79%	31.36%	2.62%	30.71%	2.64%
95-00	2.64%	70.09%	0.50%	29.91%	2.59%	32.51%	2.50%
95-02	2.36%	69.85%	0.33%	30.15%	2.59%	32.56%	2.52%

Productivity measures

	MFP1 =AMFP	MFP2	MFP3	MFP4	MFP5	Average
1985	100.0	100.0	100.0	100.0	100.0	100.0
1986	101.8	101.9	101.9	101.6	101.7	101.8
1987	103.1	103.3	103.2	102.9	103.0	103.1
1988	106.0	106.2	106.2	105.7	105.9	106.0
1989	108.8	109.0	108.9	108.5	108.7	108.8
1990	109.6	109.8	109.7	109.3	109.5	109.6
1991	109.8	110.0	109.9	109.5	109.7	109.8
1992	110.8	111.1	111.0	110.5	110.7	110.8
1993	110.5	110.9	110.8	110.3	110.4	110.6
1994	112.2	112.6	112.5	111.9	112.1	112.3
1995	113.9	114.3	114.1	113.6	113.8	113.9
1996	113.9	114.3	114.2	113.6	113.8	114.0
1997	115.3	115.7	115.6	115.0	115.2	115.4
1998	117.7	118.1	118.0	117.6	117.7	117.8
1999	119.1	119.4	119.3	119.2	119.2	119.2
2000	122.9	122.9	123.0	123.3	123.1	123.1
2001	124.2	124.1	124.1	124.7	124.4	124.3
2002	125.2	125.1	125.1	125.8	125.4	125.3
85-90	1.83%	1.86%	1.85%	1.78%	1.81%	1.83%
90-95	0.77%	0.80%	0.80%	0.77%	0.77%	0.78%
95-00	1.53%	1.46%	1.49%	1.65%	1.57%	1.54%
95-02	1.35%	1.29%	1.31%	1.46%	1.39%	1.36%

Source: OECD Productivity Database May 2004.

Table 3: Japan

Basic series

	Output	Hours worked		Capital services			
				Exogenous RoR		Endogenous RoR	
	Volume index	Cost share	Volume index	Cost share	Volume index	Revenue share	Volume index
1985	100.0	73.61%	100.0	26.39%	100.0	29.35%	100.0
1986	103.0	75.51%	100.7	24.49%	104.4	29.98%	104.4
1987	106.9	77.02%	101.1	22.98%	109.3	30.28%	109.2
1988	114.1	77.74%	102.0	22.26%	115.2	31.72%	114.8
1989	120.1	77.24%	102.4	22.76%	122.2	32.42%	121.3
1990	126.4	73.80%	102.2	26.20%	129.1	32.60%	128.0
1991	130.6	73.72%	102.6	26.28%	136.4	32.19%	135.1
1992	131.9	72.24%	102.0	27.76%	143.2	32.16%	141.8
1993	132.2	72.20%	99.3	27.80%	149.2	31.81%	147.7
1994	133.7	72.84%	99.0	27.16%	154.4	31.49%	152.9
1995	136.2	72.40%	98.4	27.60%	160.2	31.23%	158.6
1996	140.9	71.80%	99.3	28.20%	167.2	32.28%	165.2
1997	143.5	70.92%	98.8	29.08%	174.4	32.22%	172.0
1998	141.9	69.20%	97.0	30.80%	181.0	31.91%	178.6
1999	142.0	71.00%	94.5	29.00%	187.3	32.11%	184.5
2000	146.0	71.88%	95.0	28.12%	192.9	32.60%	189.5
2001	146.7	69.41%	93.8	30.59%	197.2	32.61%	194.0
2002	146.1	71.57%	92.0	28.43%	198.9	32.80%	195.5
85-90	4.68%	75.82%	0.43%	24.18%	5.11%	31.06%	4.94%
90-95	1.50%	72.87%	-0.75%	27.13%	4.32%	31.91%	4.29%
95-00	1.38%	71.20%	-0.71%	28.80%	3.71%	32.06%	3.56%
95-02	1.00%	71.02%	-0.96%	28.98%	3.08%	32.22%	2.99%

Productivity measures

	MFP1 =AMFP	MFP2	MFP3	MFP4	MFP5	Average
1985	100.0	100.0	100.0	100.0	100.0	100.0
1986	101.3	101.1	101.2	101.5	101.4	101.3
1987	103.7	103.3	103.4	104.3	103.9	103.7
1988	108.6	107.8	108.2	110.1	109.1	108.7
1989	112.5	111.1	111.8	114.8	113.2	112.7
1990	117.0	115.0	115.7	120.1	117.9	117.1
1991	118.9	116.4	117.2	122.4	119.9	118.9
1992	118.9	116.1	116.7	122.6	120.1	118.9
1993	120.2	117.1	117.4	123.9	121.3	119.9
1994	120.6	117.3	117.3	124.4	121.7	120.2
1995	122.3	118.7	118.8	126.2	123.4	121.8
1996	124.2	120.4	120.8	128.5	125.5	123.8
1997	125.3	121.3	121.7	129.8	126.7	124.9
1998	124.2	120.0	120.3	128.5	125.6	123.7
1999	125.2	120.9	121.1	129.6	126.6	124.6
2000	127.3	122.7	122.8	132.0	128.7	126.7
2001	128.1	123.4	123.3	132.9	129.6	127.4
2002	129.1	124.3	123.5	133.8	130.5	128.2
85-90	3.14%	2.79%	2.91%	3.67%	3.29%	3.16%
90-95	0.88%	0.63%	0.54%	0.99%	0.92%	0.79%
95-00	0.80%	0.68%	0.66%	0.89%	0.84%	0.77%
95-02	0.78%	0.66%	0.56%	0.84%	0.79%	0.72%

Source: OECD Productivity Database May 2004.

Table 4: United States

Basic series

	Output	Hours worked		Capital services			
				Exogenous RoR		Endogenous RoR	
	Volume index	Cost share	Volume index	Cost share	Volume index	Revenue share	Volume index
1985	100.0	73.26%	100.0	26.74%	100.0	27.77%	100.0
1986	103.4	73.67%	101.1	26.33%	103.7	27.54%	103.7
1987	106.8	74.09%	104.0	25.91%	107.3	27.42%	107.1
1988	111.3	74.58%	107.1	25.42%	110.7	27.06%	110.5
1989	115.2	73.18%	110.0	26.82%	114.5	28.07%	114.2
1990	117.2	72.79%	110.7	27.21%	118.0	27.51%	117.7
1991	116.6	72.21%	108.9	27.79%	121.0	27.12%	120.7
1992	120.2	73.41%	109.2	26.59%	124.2	27.33%	123.9
1993	123.4	75.19%	112.0	24.81%	127.6	27.55%	127.1
1994	128.4	75.47%	114.8	24.53%	131.4	28.19%	130.7
1995	131.9	76.20%	117.4	23.80%	136.3	28.50%	135.2
1996	136.7	76.16%	119.0	23.84%	142.4	29.37%	140.6
1997	142.8	76.71%	122.3	23.29%	150.2	29.91%	147.3
1998	148.9	76.08%	125.3	23.92%	159.4	28.95%	155.1
1999	155.1	76.76%	127.7	23.24%	169.6	28.59%	163.8
2000	161.0	76.51%	129.4	23.49%	179.5	27.52%	172.4
2001	161.4	74.35%	128.4	25.65%	186.9	27.56%	179.1
2002	165.3	73.32%	127.8	26.68%	192.6	..	..
85-90	3.17%	73.60%	2.03%	26.40%	3.32%	27.56%	3.25%
90-95	2.36%	74.21%	1.18%	25.79%	2.88%	27.70%	2.78%
95-00	3.98%	76.40%	1.95%	23.60%	5.51%	28.81%	4.86%
95-02	3.22%	75.76%	1.21%	24.24%	4.94%	28.63%	..

Productivity measures

	MFP1 =AMFP	MFP2	MFP3	MFP4	MFP5	Average
1985	100.0	100.0	100.0	100.0	100.0	100.0
1986	101.6	101.6	101.5	101.6	101.6	101.6
1987	101.9	101.8	101.7	102.0	102.0	101.9
1988	103.0	103.0	102.8	103.2	103.2	103.0
1989	103.6	103.6	103.5	103.9	103.9	103.7
1990	104.1	104.0	103.8	104.4	104.3	104.1
1991	104.1	104.1	103.7	104.4	104.3	104.1
1992	106.3	106.2	105.9	106.6	106.5	106.3
1993	106.4	106.3	106.0	106.7	106.7	106.4
1994	107.9	107.8	107.6	108.4	108.3	108.0
1995	108.0	107.8	107.9	108.7	108.5	108.2
1996	109.6	109.3	109.6	110.6	110.3	109.9
1997	110.7	110.2	111.0	112.1	111.7	111.1
1998	111.8	111.1	112.4	113.7	113.1	112.4
1999	113.1	112.1	113.7	115.3	114.6	113.7
2000	114.6	113.4	115.1	117.2	116.4	115.3
2001	114.4	113.0	114.4	117.0	116.2	115.0
2002	116.7	..	..	..	..	116.7
85-90	0.80%	0.79%	0.75%	0.86%	0.84%	0.81%
90-95	0.73%	0.72%	0.76%	0.81%	0.79%	0.76%
95-00	1.19%	1.00%	1.30%	1.50%	1.39%	1.28%
95-02	1.11%	..	..	..	..	1.08%

Source: OECD Productivity Database May 2004.

7. Conclusions

This paper examined productivity and growth accounting measures when rates of return to capital inputs are exogenously determined. Several hypotheses about competition on output markets and about technology are invoked, each of which is compatible with exogenous rates of return. The following conclusions emerge.

The endogenous case – widely used in empirical research – imposes quite stringent assumptions: constant returns to scale and fully competitive output markets, no unobserved capital inputs and perfect foresight for producers regarding the expected changes in prices and rates of return and depreciation.

Different hypotheses are consistent with different MFP measures. In the absence of further a-priori information or recourse to parametric techniques, there is no obvious way of discriminating among the different hypotheses and hence making an informed choice among the alternative productivity and growth accounting measures.

This is evidenced for the four countries. In total, five different productivity measures were computed, each consistent with a set of assumptions. Empirically, the differences matter.

The paper suggests a pragmatic way forward: using an 'Apparent' MFP (AMFP) measure that is simply the ratio between a volume index of output and a volume index of the observed inputs. AMFP is not a pure measure of technical change defined as a path-independent shift in the production or cost function. However, AMFP is shown to have the following properties:

- The measure is close to the average of other measures.
- It is easy to explain.
- It can be applied when assumptions about the nature of 'pure' technical change are relaxed allowing, for example, formulations that encompass neutral and biased technical change.
- It relies on input measures that are independent from output measures and whose quality therefore does not vary with the quality and available detail of production or value-added data.

Clearly, the interpretation of AMFP has to be kept in mind: it reflects the combined effects of technical change, of unobserved inputs, of non-constant returns to scale and, indirectly, of deviations from perfect competition in product markets. In other words, AMFP is a true 'residual'. But for many practical purposes, it should prove useful as a multi-faceted measure of productivity growth.

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