Chapter 3

ON MEASURING THE CONTRIBUTION OF ENTERING AND EXITING FIRMS TO AGGREGATE PRODUCTIVITY GROWTH

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1. Introduction

A recent development in productivity analysis is the increased focus on the impact of firm entry and exit on aggregate levels of productivity growth. Haltiwanger, and Bartelsman and Doms, in their survey papers make the following observations:[2]

> "There are large and persistent differences in productivity across establishments in the same industry... – for total factor productivity [TFP] the ratio of the productivity level for the plant at the 75th percentile to the plant at the 5th percentile in the same industry is 2.4 (this is the average across industries) – the equivalent ratio for labour productivity is 3.5." John Haltiwanger (2000; 9)

> "The ratio of average TFP for plants in the ninth decile of the productivity distribution relative to the average in the second decile was about 2 to 1 in 1972 and about 2.75 to 1 in 1987." Eric J. Bartelsman and Mark Doms (2000; 579)

Thus the recent productivity literature has demonstrated empirically that increases in the productivity of the economy can be obtained by reallocating resources[3] away from low

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[2] See their papers for many additional references to the literature. Some of the more important references are Baldwin and Gorecki (1991), Baily, Hulten and Campbell (1992), Griliches and Regev (1995), Baldwin (1995), Haltiwanger (1997), Ahn (2001), Foster, Haltiwanger and Krizan (2001), Aw, Chen and Roberts (2001), Fox (2002), Baldwin and Gu (2002), Balk (2003), Bartelsman, Haltiwanger and Scarpetta (2004) and Foster, Haltiwanger and Syverson (2008).

[3] A more precise meaning for the term "reallocating resources" is "changing input shares".

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productivity firms in an industry to the higher productivity firms.[4] However, different investigators have chosen different methods for measuring the contributions to industry productivity growth of entering and exiting firms and the issue remains open as to which method is "best". We propose yet another method for accomplishing this decomposition. It differs from existing methods in that it treats time in a symmetric fashion so that the industry productivity difference in levels between two periods reverses sign when the periods are interchanged, as do the various contribution terms.[5] Our proposed productivity decomposition is explained in sections 2 and 3 below, assuming that each firm in the industry produces only one homogeneous output and uses only one homogeneous input. In the literature, it is often assumed there is only one output and one input that each firm produces and uses.

With multiple outputs and inputs, so long as the list of outputs being produced and inputs being used by each firm is constant across firms, then there is no problem in using normal index number theory to construct output and input aggregates for each continuing firm that is present for the two periods under consideration.[6] We turn our attention to the multiple input, multiple output case beginning with section 4. However, this approach does not work with entering and exiting firms, since there is no natural base observation for comparing the single period data for these firms. This problem does not seem to have been widely recognized in the literature, though there are notable exceptions.[7] Hence in the remainder of the paper, we focus on this problem. Our proposed approach is to use *multilateral index number theory* so that each firm's data in each time period is treated as if it were pertaining to a "country." There are many multilateral methods that could be used, and we compare our new method with some of the alternatives.

In section 5 below, we construct an artificial data set involving three continuing firms, one entering firm and one exiting firm. In the remaining sections of the paper, we use various multilateral aggregation methods in order to construct firm output and input aggregates, which we then use in our suggested productivity growth decomposition formula. The multilateral aggregation methods we consider are: the star system (section 6); the GEKS system (section 7); the own share system (section 8); the "spatial" linking method due to Robert Hill (section 9), and a simple deflation of value aggregates method (section 10). Section 11 concludes.

2. Aggregate Productivity Level Measurement in the One Output One Input Case

We begin with a very simple case where firms produce one output with one input so it is very straightforward to measure the productivity of each firm by dividing its output by its input.[8]

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[4] This conclusion has also emerged from the extensive literatures on benchmarking and on data envelopment analysis; e.g., see Coelli, Prasada Rao and Battese (1998).

[5] Balk (2003; 29) also emphasized the importance of a symmetric treatment of time. A symmetric decomposition was proposed by Griliches and Regev (1995) and a modification of it was used by Aw, Chen and Roberts (2001).

[6] An economic justification for using a superlative index to accomplish this aggregation can be supplied under some separability assumptions; see Diewert (1976).

[7] Aw, Chen and Roberts (2001) and Aw, Chung and Roberts (2003) recognized the importance of this problem and used a modification of a multilateral method proposed by Caves, Christensen and Diewert (1982). The modification is due to Good (1985) and explained in Good, Nadiri and Sickles (1997). The Caves, Christensen and Diewert method was designed for a single cross section and is not suitable for use in a panel data context with inflation.

[8] We will consider the case of many outputs and many inputs in sections 4-10 below.

We assume that these firms are all in the same industry, producing the same output and using the same input, so that it is very straightforward to measure industry productivity for each period by dividing aggregate industry output by aggregate industry input. *Our measurement problem is to account for the contributions to industry productivity growth of entering and exiting firms*.

In what follows, C denotes the set of continuing production units that are present in periods 0 and 1, X denotes the set of exiting firms that are present in only period 0, and N denotes the set of new firms present in only period 1. Let $y_{Ci}^{t} > 0$ and $x_{Ci}^{t} > 0$ denote, respectively, the output and input for continuing firm $i \in C$ during periods t = 0, 1. Let $y_{Xi}^{0} > 0$ and $x_{Xi}^{0} > 0$ denote the output and input of exiting firm $i \in X$ in period 0. Finally, let $y_{Ni}^{1} > 0$ and $x_{Ni}^{1} > 0$ denote the output and input of the new firm $i \in N$ in period 1.

The *productivity level* Π_{Ci}^{t} of a continuing firm $i \in C$ in each period t can be defined as:

(1) $$\Pi_{Ci}^{t} \equiv y_{Ci}^{t} / x_{Ci}^{t}, \quad i \in C, \quad t = 0,1.$$

The productivity levels of each *exiting* firm in period 0 and each *entering* firm in period 1 are defined in a similar fashion, as follows:

(2) $$\Pi_{Xi}^{0} \equiv y_{Xi}^{0} / x_{Xi}^{0}, \quad i \in X;$$

(3) $$\Pi_{Ni}^{1} \equiv y_{Ni}^{1} / x_{Ni}^{1}, \quad i \in N.$$

Assuming for now that the output and input products are the same for all firms, a natural definition for period 0 *industry productivity* Π^{0} is aggregate output divided by aggregate input:[9]

(4) $$\begin{aligned}\Pi^{0} &\equiv [\textstyle\sum_{i \in C} y_{Ci}^{0} + \sum_{i \in X} y_{Xi}^{0}]/[\sum_{i \in C} x_{Ci}^{0} + \sum_{i \in X} x_{Xi}^{0}] \\ &= S_{C}^{0} \textstyle\sum_{i \in C} s_{Ci}^{0} \Pi_{Ci}^{0} + S_{X}^{0} \sum_{i \in X} s_{Xi}^{0} \Pi_{Xi}^{0},\end{aligned}$$

where the *aggregate input shares* of the continuing and exiting firms in period 0 are:

(5) $$S_{C}^{0} \equiv \textstyle\sum_{i \in C} x_{Ci}^{0} / [\sum_{i \in C} x_{Ci}^{0} + \sum_{i \in X} x_{Xi}^{0}];$$

(6) $$S_{X}^{0} \equiv \textstyle\sum_{i \in X} x_{Xi}^{0} / [\sum_{i \in C} x_{Ci}^{0} + \sum_{i \in X} x_{Xi}^{0}].$$

The period 0 *micro input share*, s_{Ci}^{0}, for a *continuing* firm $i \in C$ is defined as follows:

(7) $$s_{Ci}^{0} \equiv x_{Ci}^{0} / \textstyle\sum_{k \in C} x_{Ck}^{0}, \quad i \in C.$$

Similarly, the period 0 *micro input share* for *exiting* firm $i \in X$ is:

[9] It is possible to rework our analysis by reversing the role of inputs and outputs so that output shares replace input shares in the decomposition formulae. Then at the end, we can take the reciprocal of the aggregate inverse productivity measure and obtain an alternative productivity decomposition. We owe this suggestion to Bert Balk.

(8) $s_{Xi}^0 \equiv x_{Xi}^0 / \sum_{k\in X} x_{Xk}^0$, $i\in X$.

The period 0 *aggregate productivities for continuing and exiting firms*, Π_C^0, and Π_X^0, can be defined in a similar manner to the definition in (4) of Π^0 for the entire industry as:

(9) $\Pi_C^0 \equiv \sum_{i\in C} y_{Ci}^0 / \sum_{i\in C} x_{Ci}^0 = \sum_{i\in C} s_{Ci}^0 \Pi_{Ci}^0$;

(10) $\Pi_X^0 \equiv \sum_{i\in X} y_{Xi}^0 / \sum_{i\in X} x_{Xi}^0 = \sum_{i\in X} s_{Xi}^0 \Pi_{Xi}^0$.

Substituting of (9) and (10) back into (4) for the aggregate period 0 level of productivity of the industry and using $S_C^0 = 1 - S_X^0$ leads to the following decomposition of period 0 productivity:

(11) $\Pi^0 = S_C^0 \Pi_C^0 + S_X^0 \Pi_X^0$

(12) $\quad = \Pi_C^0 + S_X^0 (\Pi_X^0 - \Pi_C^0)$.

In expression (12), the first term, Π_C^0, represents the productivity contribution of continuing firms while the second term, $S_X^0(\Pi_X^0 - \Pi_C^0)$, represents the contribution of exiting firms, relative to continuing firms, to the overall period 0 productivity level. Usually the exiting firm will have lower productivity levels than the continuing firms so that Π_X^0 will be less than Π_C^0. Thus, under normal conditions, the second term on the right-hand side of (12) will make a *negative contribution* to the overall level of period 0 productivity.[10] Substituting (9) and (10) into (12) leads to the following decomposition of the period 0 productivity into the contributions of firms grouped by whether they are continuing or exiting:

(13) $\Pi^0 = \sum_{i\in C} s_{Ci}^0 \Pi_{Ci}^0 + S_X^0 \sum_{i\in X} s_{Xi}^0 (\Pi_{Xi}^0 - \Pi_C^0)$,

where we have also used the fact that $\sum_{i\in X} s_{Xi}^0$ sums to unity.

The above material can be repeated with minor modifications to provide a decomposition of the industry period 1 productivity level Π^1 into its components. Thus, Π^1 is defined as:

(14)
$$\begin{aligned}\Pi^1 &\equiv [\textstyle\sum_{i\in C} y_{Ci}^1 + \sum_{i\in N} y_{Ni}^1]/[\sum_{i\in C} x_{Ci}^1 + \sum_{i\in N} x_{Ni}^1] \\ &= S_C^1 \textstyle\sum_{i\in C} s_{Ci}^1 \Pi_{Ci}^1 + S_N^1 \sum_{i\in N} s_{Ni}^1 \Pi_{Ni}^1,\end{aligned}$$

where the period 1 aggregate input shares of continuing and new firms, S_C^1 and S_N^1, and individual continuing and new firm shares, s_{Ci}^1 and s_{Ni}^1, are defined as follows:

[10] Olley and Pakes (1996; 1290) have an alternative covariance type decomposition of the overall level of productivity in a given period into firm effects but it is not suitable for our purpose, which is to highlight the differential effects on overall period 0 productivity of the exiting firms compared to the continuing firms.

(15) $S_C^1 \equiv \sum_{i\in C} x_{Ci}^1 / [\sum_{i\in C} x_{Ci}^1 + \sum_{i\in N} x_{Ni}^1]$;

(16) $S_N^1 \equiv \sum_{i\in N} x_{Ni}^1 / [\sum_{i\in C} x_{Ci}^1 + \sum_{i\in N} x_{Ni}^1]$;

(17) $s_{Ci}^1 \equiv x_{Ci}^1 / \sum_{k\in C} x_{Ck}^1$, $i\in C$;

(18) $s_{Ni}^1 \equiv x_{Ni}^1 / \sum_{k\in N} x_{Nk}^1$, $i\in N$.

The period 1 counterparts to Π_C^0 and Π_X^0 in (9) and (10) are the *aggregate period one productivity levels of continuing firms* Π_C^1 and *entering firms* Π_N^1, defined as follows:

(19) $\Pi_C^1 \equiv \sum_{i\in C} y_{Ci}^1 / \sum_{i\in C} x_{Ci}^1 = \sum_{i\in C} s_{Ci}^1 \Pi_{Ci}^1$;

(20) $\Pi_N^1 \equiv \sum_{i\in N} y_{Ni}^1 / \sum_{i\in N} x_{Ni}^1 = \sum_{i\in N} s_{Ni}^1 \Pi_{Ni}^1$.

Substituting (19) and (20) back into definition (14) for the aggregate period 1 level of productivity leads to the following decomposition counterparts of (11) and (12) -- a decomposition of aggregate period 1 productivity into its continuing and new components:

(21) $\Pi^1 = S_C^1 \Pi_C^1 + S_N^1 \Pi_N^1$

(22) $\quad = \Pi_C^1 + S_N^1 (\Pi_N^1 - \Pi_C^1)$,

where (22) follows from (21) using $S_C^1 = 1 - S_N^1$. Thus the aggregate period 1 productivity level Π^1 is equal to the aggregate period 1 productivity level of continuing firms, Π_C^1, plus a second term, $S_N^1(\Pi_N^1 - \Pi_C^1)$, which represents the contribution of the new entrants' productivity levels, Π_N^1, relative to that of the continuing firms, Π_C^1.[11] Substituting (19) and (20) into (22) leads to the following decomposition of the aggregate period 1 productivity level Π^1 into its individual firm contributions:

(23) $\Pi^1 = \sum_{i\in C} s_{Ci}^1 \Pi_{Ci}^1 + S_N^1 \sum_{i\in N} s_{Ni}^1 (\Pi_{Ni}^1 - \Pi_C^1)$.

This completes our discussion of how the levels of productivity in periods 0 and 1 can be decomposed into individual firm contribution effects. In the following section, we study the more difficult problem of decomposing the aggregate productivity change, Π^1 / Π^0, into individual firm growth effects, taking into account that not all firms are present in both periods.

[11] Baldwin (1995) in his study of the Canadian manufacturing sector showed that on average, the productivity level of new entrants is below that of continuing firms. However, he found that for new entrants that survive, they reach the average productivity level of continuing firms in about a decade. For additional empirical evidence on the relative productivity levels of entering and exiting firms, see Bartelsman and Doms (2000; 581). See also Aw, Chen and Roberts (2001) (who also find that the productivity level of new entrants is below that of incumbents) and section 5 of Bartelsman, Haltiwanger and Scarpetta (2004).

3. The Measurement of Productivity Change between Two Periods

It is traditional to define productivity change from period 0 to period 1 as a ratio of the productivity levels in the two periods rather than as a difference. This is because the ratio measure will be independent of the units of measurement while the difference measure will not (unless some normalization is performed). However, in the present context, as we are attempting to calculate the contribution of new and disappearing production units to overall productivity change, it is more convenient to work with the difference concept, at least initially.

Using formula (13) for the period 0 productivity level Π^0 and (23) for the period 1 productivity level Π^1, we obtain the following decomposition of the *productivity difference*:

$$(24)\quad \Pi^1 - \Pi^0 = \sum_{i\in C} s_{Ci}^1 \Pi_{Ci}^1 - \sum_{i\in C} s_{Ci}^0 \Pi_{Ci}^0 + S_N^1 \sum_{i\in N} s_{Ni}^1 (\Pi_{Ni}^1 - \Pi_C^1) - S_X^0 \sum_{i\in X} s_{Xi}^0 (\Pi_{Xi}^0 - \Pi_C^0)$$

$$(25)\quad = \Pi_C^1 - \Pi_C^0 + S_N^1(\Pi_N^1 - \Pi_C^1) - S_X^0(\Pi_X^0 - \Pi_C^0),$$

where (25) follows from (24) using (12) and (22). Thus the overall industry productivity change, $\Pi^1 - \Pi^0$, is equal to the productivity change of the continuing firms, $\Pi_C^1 - \Pi_C^0$, plus a term that reflects the contribution to overall productivity change of new entrants, $S_N^1(\Pi_N^1 - \Pi_C^1)$,[12] and a term that reflects the contribution to overall productivity change of exiting firms, $-S_X^0(\Pi_X^0 - \Pi_C^0)$.[13] Note that the reference productivity levels that the productivity levels of the entering and exiting firms are compared with, Π_C^1 and Π_C^0 respectively, are *different* in general, so even if the average productivity levels of entering and exiting firms are the *same* (so that Π_N^1 equals Π_X^0), the contributions to overall industry productivity *growth* of entering and exiting firms can still be nonzero if $\Pi_N^1 \neq \Pi_C^1$ or $\Pi_X^0 \neq \Pi_C^0$.[14]

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[12] This term is positive if and only if the average level of productivity of the new entrants in period 1, Π_N^1, is *greater* than the average productivity level of continuing firms in period 1, Π_C^1.

[13] This term is positive if and only if the average level of productivity of the firms who exit in period 0, Π_X^0, is *less* than the average productivity level of continuing firms in period 0, Π_C^0.

[14] Haltiwanger (1997) (2000; 10) argues that if the productivity levels of entering and exiting firms or establishments are exactly the same, then the sum of the contribution terms of entering and exiting firms should be zero. However, our perspective is different: we want to measure the differential effects on productivity *growth* of entering and exiting firms and so what counts in our framework are the productivity levels of entering firms relative to continuing firms in period 1 and the productivity levels of exiting firms relative to continuing firms in period 0. Thus if continuing firms show productivity growth over the two periods, then if the entering and exiting firms have the same productivity levels, the effects of entry and exit will be to decrease productivity growth compared to the continuing firms. Balk (2003; 28) follows the example of Haltiwanger (1997) in choosing a common reference level of productivity to compare the productivity levels of entering and exiting firms but Balk chooses the arithmetic average

The first two terms on the right-hand side of (24) give the aggregate effects of the changes in productivity levels of the continuing firms. It is useful to further decompose this aggregate change in the productivity levels of continuing firms into two sets of components: the first set of terms measures the productivity change of each continuing production unit, $\Pi_{Ci}^{1} - \Pi_{Ci}^{0}$, and the second set reflects the shifts in the share of resources used by each continuing production unit, $s_{Ci}^{1} - s_{Ci}^{0}$. As Balk (2003; 26) notes, there are two natural decompositions for the difference in the productivity levels of the continuing firms, (27) and (29) below, that are the difference counterparts to the decomposition of a value ratio into the product of a Laspeyres (or Paasche) price index times a Paasche (or Laspeyres) quantity index:

(26) $\Pi_{C}^{1} - \Pi_{C}^{0} = \sum_{i\in C} s_{Ci}^{1}\Pi_{Ci}^{1} - \sum_{i\in C} s_{Ci}^{0}\Pi_{Ci}^{0}$

(27) $= \sum_{i\in C} s_{Ci}^{0}(\Pi_{Ci}^{1} - \Pi_{Ci}^{0}) + \sum_{i\in C} \Pi_{Ci}^{1}(s_{Ci}^{1} - s_{Ci}^{0})$;

(28) $\Pi_{C}^{1} - \Pi_{C}^{0} = \sum_{i\in C} s_{Ci}^{1}\Pi_{Ci}^{1} - \sum_{i\in C} s_{Ci}^{0}\Pi_{Ci}^{0}$

(29) $= \sum_{i\in C} s_{Ci}^{1}(\Pi_{Ci}^{1} - \Pi_{Ci}^{0}) + \sum_{i\in C} \Pi_{Ci}^{0}(s_{Ci}^{1} - s_{Ci}^{0})$.

We now note a severe disadvantage associated with either (27)[15] or (29): *these decompositions are not invariant with respect to the treatment of time*. If we reverse the roles of periods 0 and 1, we would like the decomposition of the aggregate productivity difference for continuing firms, $\Pi_{C}^{0} - \Pi_{C}^{1} = \sum_{i\in C} s_{Ci}^{0}\Pi_{Ci}^{0} - \sum_{i\in C} s_{Ci}^{1}\Pi_{Ci}^{1}$ (an aggregate productivity difference that involves the individual productivity differences $\Pi_{Ci}^{0} - \Pi_{Ci}^{1}$ and share differences $s_{Ci}^{0} - s_{Ci}^{1}$) to satisfy a symmetry or invariance property, but unfortunately it does not.[16] A solution to this problem is to take the arithmetic average of (26) and (28), leading to a *Bennet* (1920) *type decomposition of the productivity change of continuing firms*:

(30) $\Pi_{C}^{1} - \Pi_{C}^{0} = \sum_{i\in C} (1/2)(s_{Ci}^{0} + s_{Ci}^{1})(\Pi_{Ci}^{1} - \Pi_{Ci}^{0}) + \sum_{i\in C} (1/2)(\Pi_{Ci}^{0} + \Pi_{Ci}^{1})(s_{Ci}^{1} - s_{Ci}^{0})$.

The use of this decomposition for continuing firms dates back to Griliches and Regev (1995; 185).[17] Balk (2003; 29) also endorsed the use of this symmetric decomposition.[18] We endorse it since it is symmetric and can also be given a strong axiomatic justification.[19]

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of the industry productivity levels in periods 0 and 1 (which is at least a symmetric choice) whereas Haltiwanger chooses the industry productivity level of period 0 (which is not a symmetric choice). In any case, our approach seems to be different from other approaches suggested in the literature.

[15] The decomposition defined by (26) is the one used by Baily, Hulten and Campbell (1992; 193) for continuing firms except that they used *logs* of the TFP levels Π_{Ci}^{t} instead of the levels themselves.

[16] We want the difference decomposition to satisfy a differences counterpart to the index number time reversal test.

[17] Griliches and Regev (1995; 185) also have a symmetric treatment of the industry difference in total factor productivity (TFP) levels, but firms that exit and enter during the two periods being compared are treated as one firm and they make a direct comparison of the change in productivity of all entering and exiting firms on this basis. There are problems in interpretation if there happen to be no entering (or exiting) firms in the sample or more generally, if there are big differences in the shares of entering and exiting firms. Aw, Chen and Roberts (2001; 73) also use this symmetric methodology, except they work with logs of TFP.

Substitution of (30) into (24) gives our final "best" decomposition of the aggregate productivity difference $\Pi^1 - \Pi^0$ into micro firm effects:

$$(31) \quad \Pi^1 - \Pi^0 = \sum_{i\in C}(1/2)(s_{Ci}^0 + s_{Ci}^1)(\Pi_{Ci}^1 - \Pi_{Ci}^0) + \sum_{i\in C}(1/2)(\Pi_{Ci}^0 + \Pi_{Ci}^1)(s_{Ci}^1 - s_{Ci}^0) + S_N^1 \sum_{i\in N} s_{Ni}^1(\Pi_{Ni}^1 - \Pi_C^1) - S_X^0 \sum_{i\in X} s_{Xi}^0(\Pi_{Xi}^0 - \Pi_C^0).$$

The first set of terms on the right hand side of (31), $\sum_{i\in C}(1/2)(s_{Ci}^0 + s_{Ci}^1)(\Pi_{Ci}^1 - \Pi_{Ci}^0)$, gives the contribution of the productivity growth of each continuing firm to the aggregate productivity difference between periods 0 and 1, $\Pi^1 - \Pi^0$. The second set of terms, $\sum_{i\in C}(1/2)(\Pi_{Ci}^0 + \Pi_{Ci}^1)(s_{Ci}^1 - s_{Ci}^0)$, gives the contribution of the effects of the reallocation of resources between continuing firms. The third set of terms, $S_N^1 \sum_{i\in N} s_{Ni}^1(\Pi_{Ni}^1 - \Pi_C^1)$, gives the contribution of each entering firm to productivity growth. The final set of terms, $-S_X^0 \sum_{i\in X} s_{Xi}^0(\Pi_{Xi}^0 - \Pi_C^0)$, gives the contribution of each exiting firm.

Note that the decomposition in (31) is symmetric: if we reverse the role of periods 0 and 1, then the new aggregate productivity difference will equal the negative of the original productivity difference and each individual firm contribution term on the new right hand side will equal the negative of the original firm contribution effect. The only decomposition we are aware of in the literature that has this time reversal property is due to Balk (2003; 28). His decomposition differs from what we propose in that he compares the productivity levels of entering and exiting firms to the arithmetic average of the industry productivity levels in periods 0 and 1 instead of to the average productivity level of continuing firms in period 1 for entering firms, and continuing firms in period 0 for exiting firms as we do.

We now make a final adjustment to (31) in order to achieve invariance to changes in the units of measurement of output and input: we divide both sides of (31) by the base period productivity level Π^0.[20] With this adjustment, (31) becomes the following TFPG expression:

$$(32) \quad [\Pi^1/\Pi^0] - 1 = [\sum_{i\in C}(1/2)(s_{Ci}^0 + s_{Ci}^1)(\Pi_{Ci}^1 - \Pi_{Ci}^0) + \sum_{i\in C}(1/2)(\Pi_{Ci}^0 + \Pi_{Ci}^1)(s_{Ci}^1 - s_{Ci}^0) + S_N^1 \sum_{i\in N} s_{Ni}^1(\Pi_{Ni}^1 - \Pi_C^1) - S_X^0 \sum_{i\in X} s_{Xi}^0(\Pi_{Xi}^0 - \Pi_C^0)]/\Pi^0.$$

In the following sections, we illustrate the aggregate productivity decomposition (32) using an artificial data set. *Note that (32) is only valid for an industry that produces a single output and*

[18] "In view of its symmetry it should be the preferred one." Bert M. Balk (2003; 29).

[19] Diewert (2005) showed that the Bennet decomposition of a difference of the form $\sum_i p_i^1 q_i^1 - \sum_i p_i^0 q_i^0$ into a sum of terms reflecting price change and a sum of terms reflecting quantity change can be given an axiomatic justification that is analogous to the axiomatic justification for the use of the Fisher (1922) ideal index in index number theory. The adaptation of this axiomatic theory to provide a decomposition of $\sum_i p_i^1 s_i^1 - \sum_i p_i^0 s_i^0$ is straightforward.

[20] Instead of dividing by Π^0, we could divide by the logarithmic mean of Π^0 and Π^1. The left hand side of the resulting counterpart to (32) reduces to $\ln(\Pi^0/\Pi^1)$, which is completely symmetric in the data whereas the left hand side of (32) is not. We owe this suggestion to Bert Balk.

uses a single input. However, in practice, firms in an industry produce many outputs and use many inputs. Hence, before decomposition (32) can be implemented, it is necessary to aggregate the many outputs produced and inputs used by each firm into aggregate firm output and input. This problem is not straightforward because of firms entering and exiting. In the following section, we address this unconventional aggregation problem.[21]

4. How Can the Inputs and Outputs of Entering and Exiting Firms Be Aggregated?

The aggregate productivity decomposition defined by (32) above assumes that each firm produces only one output and uses only one input. However, firms in the same industry typically produce many outputs and utilize many inputs. Thus in order to apply (32), we have to somehow aggregate all of the outputs produced by each firm into an aggregate output that is comparable across firms and across time periods and aggregate all of the inputs utilized by each firm into an aggregate input that is comparable across firms and across time periods. It can be seen that these two aggregation problems are in fact *multilateral aggregation problems*;[22] i.e., the output, or input, vector of each firm in each period must be compared with the corresponding output, or input, vectors of all other firms in the industry over the two time periods involved in the aggregate productivity comparison.[23] In the following sections of this paper, we will illustrate how these firm output and input aggregates can be formed using several methods that have been suggested in the multilateral aggregation literature.

In order to make the comparison of alternative multilateral methods of aggregation more concrete, we will utilize an artificial data set. In the following section, we table our data set and calculate the aggregate productivity of the industry using normal index number methods.

5. Industry Productivity Aggregates Using an Artificial Data Set

We consider an industry over two periods, 0 and 1, that consists of five firms. Each firm f produces varying amounts of the same two outputs and uses varying amounts of the same two inputs. The output vector of firm f in period t is defined as $y_f^t \equiv [y_{f1}^t, y_{f2}^t]$ and the corresponding input vector is defined as $x_f^t \equiv [x_{f1}^t, x_{f2}^t]$ for t = 0,1 and f = 1,2,…,5. Firms 1,2 and 3 are continuing firms, firm 4 is present in period 0 but not 1 (and hence is an exiting firm) and firm 5 is not present in period 0 but is present in period 1 (and hence is an entering firm). Firm 1 is medium sized, firm 2 is tiny and firm 3 is very large. The output price vector of firm f in period t is $p_f^t \equiv [p_{f1}^t, p_{f2}^t]$ and the corresponding input price vector is $w_f^t \equiv [w_{f1}^t, w_{f2}^t]$ for t = 0,1 and f = 1,2,…,5. The firm price and quantity data are listed in table 1.

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[21] As noted earlier, Aw, Chen and Roberts (2001) (2003) also addressed this aggregation problem.

[22] Bilateral index number theory compares the price and quantity vectors pertaining to two situations whereas multilateral index number theory attempts to construct price and quantity aggregates when there are more than two situations to be compared. See Balk (1996) (2001) and Diewert (1999) for recent surveys of multilateral methods.

[23] Fox (2002) seems to have been the first to notice that aggregating firm outputs and inputs into aggregate outputs and inputs should be treated as a multilateral aggregation problem in order to avoid paradoxical results.

Table 1. Firm Price and Quantity Data for Periods 0 and 1

	Firm 1		Firm 2		Firm 3		Firm 4		Firm 5	
	Output prices									
	p_{11}^t	p_{12}^t	p_{21}^t	p_{22}^t	p_{31}^t	p_{32}^t	p_{41}^t	p_{42}^t	p_{51}^t	p_{52}^t
t=0	1	1	0.8	1.2	0.9	0.8	1.2	1.1	---	---
t=1	15	7	13	8	14	7	---	---	16	8
	Output quantities									
	y_{11}^t	y_{12}^t	y_{21}^t	y_{22}^t	y_{31}^t	y_{32}^t	y_{41}^t	y_{42}^t	y_{51}^t	y_{52}^t
t=0	12	8	1	1	50	50	7	9	---	---
t=1	15	8	3	2	60	45	---	---	16	8
	Input prices									
	w_{11}^t	w_{12}^t	w_{21}^t	w_{22}^t	w_{31}^t	w_{32}^t	w_{41}^t	w_{42}^t	w_{51}^t	w_{52}^t
t=0	1	1	0.7	0.8	0.9	1.1	1.2	1	---	---
t=1	10	23	13	16	8	26	---	---	14	20
	Input quantities									
	x_{11}^t	x_{12}^t	x_{21}^t	x_{22}^t	x_{31}^t	x_{32}^t	x_{41}^t	x_{42}^t	x_{51}^t	x_{52}^t
t=0	10	10	1	1	45	35	13	12	---	---
t=1	8	6	2	2	35	30	---	---	7	6

Thus the period 0 output price vector for firm 1 is $p_1^0 = [1,1]$, the period 1 output price vector for firm 1 is $p_1^1 = [15,7]$ and so on. Note that there has been a great deal of general price level change going from period 0 to 1.[24]

In the following sections, we will look at various methods for forming output and input aggregates for each firm and each period but before we do this, it is useful to compute *total industry supplies* of the two outputs, $y^t \equiv [y_{\bullet 1}^t, y_{\bullet 2}^t]$ for each period t and *total industry demands* for each of the two inputs $x^t \equiv [x_{\bullet 1}^t, x_{\bullet 2}^t]$ for each period t as well as the corresponding *unit value prices*, $p^t \equiv [p_{\bullet 1}^t, p_{\bullet 2}^t]$ and $w^t \equiv [w_{\bullet 1}^t, w_{\bullet 2}^t]$.[25] This information is listed in (33) below:

(33) $p^0 = [0.946, 0.869]$; $p^1 = [14.468, 7.159]$; $w^0 = [0.968, 1.057]$; $w^1 = [9.308, 24.318]$;

$y^0 = [70, 68]$; $y^1 = [94, 63]$; $x^0 = [69, 58]$; $x^1 = [52, 44]$.

[24] In some applications of the literature on the contribution of entry and exit to aggregate productivity growth, the comparison periods are a decade apart and so the period 0 and 1 price levels can differ considerably.

[25] The unit value price of output n in period t is defined as $p_{\bullet n}^t \equiv \sum_{f=1}^5 p_{fn}^t y_{fn}^t / \sum_{f=1}^5 y_{fn}^t$ for n = 1,2 and t = 0,1. The unit value price of input n in period t is defined as $w_{\bullet n}^t \equiv \sum_{f=1}^5 w_{fn}^t x_{fn}^t / \sum_{f=1}^5 x_{fn}^t$ for n = 1,2 and t = 0,1.

Note that industry output 1 has increased from 70 to 94 but industry output 2 decreased slightly from 68 to 63. However, both industry input demands dropped markedly; input 1 decreased from 69 to 52 and input 2 decreased from 58 to 45. Thus overall, industry productivity improved markedly going from period 0 to 1.

In order to benchmark the reasonableness of the various productivity decompositions given by (32) above for different multilateral methods to be discussed in the following four sections, it is useful to use the industry data in (33) to construct normal index number estimates of industry total factor productivity growth (TFPG). Following Jorgenson and Griliches (1967) (1972),[26] TFPG can be defined as a quantity index of output growth, $Q(p^0,p^1,q^0,q^1)$, divided by a quantity index of input growth, $Q^*(w^0,w^1,x^0,x^1)$:

(34) $\quad \text{TFPG} \equiv Q(p^0,p^1,q^0,q^1)/Q^*(w^0,w^1,x^0,x^1)$.

In order to implement (34), one needs to choose an index number formula for Q and Q^*. From an axiomatic perspective, the "best" choices seem to be the Fisher (1922) ideal formula[27] or the Törnqvist (1936) Theil (1967) formula.[28] With these two choices of index number formula, the resulting TFP growth rates[29] for the data listed in (33) are as follows:

(35) $\quad \text{TFPG}_F = 1.5553;\ \text{TFPG}_T = 1.5573$.

If we subtract 1 from the above TFPG rates, we obtain industry aggregate counterparts to the left hand side of (32), $[\Pi^1/\Pi^0]-1$. Thus using the Fisher formula, industry productivity improved 55.53% and using the Törnqvist Theil formula, industry productivity improved 55.73%. These productivity growth rates should be kept in mind as we look at alternative multilateral methods for constructing output and input aggregates for each firm in each period so that we can implement the decomposition formula (32). In other words, a multilateral method that leads to an aggregate productivity growth rate $[\Pi^1/\Pi^0]-1$ that is different from the range of .5553 to .5573 is probably not very reliable. We now turn to our first multilateral method.

6. The Star System for Making Multilateral Comparisons

Recall that in the previous section, we defined the firm f and period t output and input vectors as $y_f^t \equiv [y_{f1}^t, y_{f2}^t]$ and $x_f^t \equiv [x_{f1}^t, x_{f2}^t]$ for t = 0,1 and f = 1,2,…,5. However, for t = 0 and f = 5 and also for t = 1 and f = 4, there are no data, since these two firms are entering and exiting respectively. Thus there are actually a total of 8 output and input quantity vectors instead of 10. It will prove to be more convenient to relabel our data so that there are only 8 distinct

[26] For recent surveys on how to measure TFPG, see Balk (2003) and Diewert and Nakamura (2003).

[27] See Diewert (1992). The Fisher output quantity index is defined as $Q_F(p^0,p^1,q^0,q^1) \equiv [p^0q^1p^1q^1/p^0q^0p^1q^0]^{1/2}$ where p·q denotes the inner product of the vectors p and q.

[28] See Diewert (2004). Both of these formulae can be given economic justifications as well; see Diewert (1976).

[29] Actually these rates are 1 plus the total factor productivity growth rates.

output and input quantity vectors. Thus define the output quantity vectors y^1, y^2, y^3 and y^4 as the previously defined vectors y_1^0, y_2^0, y_3^0 and y_4^0 respectively (these are the nonzero period 0 output quantity vectors) and define the vectors y^5, y^6, y^7 and y^8 as the previously defined vectors y_1^1, y_2^1, y_3^1 and y_5^1 respectively (these are the nonzero period 1 output quantity vectors). Similarly, define the output price vectors p^1, p^2, p^3 and p^4 as the previously defined vectors p_1^0, p_2^0, p_3^0 and p_4^0 respectively and define the vectors p^5, p^6, p^7 and p^8 as the previously defined vectors p_1^1, p_2^1, p_3^1 and p_5^1, respectively. Undertake the same reordering of the data for inputs. Now we can apply multilateral methods. In effect, we treat each of the 8 output (or input) price and quantity vectors as if they corresponded to the data for a country.[30]

The first multilateral method we consider is the star system.[31] To implement this, we choose a bilateral index number formula, say the Fisher formula Q_F, and choose one observation as the base (or star), say observation k, and then compute the Fisher quantity aggregate of each observation relative to the base k, $Q_F(p^k, p^1, y^k, y^1)$, $Q_F(p^k, p^2, y^k, y^2), \ldots, Q_F(p^k, p^8, y^k, y^8)$. The resulting sequence of 8 numbers can serve as output aggregates for our 8 observations.

Of course, the problem with the star system aggregates is that it is necessary to asymmetrically choose one observation as the "star" and usually, it is not clear which observation should be chosen.[32] Thus in tables 2 and 3 below, we list each of the 8 output and input aggregates respectively, choosing each observation as the base in turn. In order to make these output and input aggregates comparable, we divide each set of parities by the parity for the first observation. Thus the output and input parities listed in tables 2 and 3 for are the following normalized parities for outputs and inputs, for k = 1,…,8, respectively:[33]

(36) $1,\ Q_F(p^k, p^2, y^k, y^2)/Q_F(p^k, p^1, y^k, y^1),\ \ldots,\ Q_F(p^k, p^8, y^k, y^8)/Q_F(p^k, p^1, y^k, y^1)$,

(37) $1,\ Q_F(w^k, w^2, x^k, x^2)/Q_F(w^k, w^1, x^k, x^1),\ \ldots,\ Q_F(w^k, w^8, x^k, x^8)/Q_F(w^k, w^1, x^k, x^1)$.

The input aggregates for observations 1 and 2 are the same regardless of the base. This is due to the use of the Fisher formula and the fact the input vectors for observations 1 and 2 are proportional.[34] If the quantity vectors for the observations being compared are proportional, then the Fisher quantity index will reflect this factor of proportionality.[35] In general, however, the choice of the base observation affects the output and input parities.

[30] Note that we need to make two multilateral comparisons: one for outputs and one for inputs.
[31] This terminology is due to Kravis (1984; 10).
[32] In our particular example, a case could be made for choosing either observation 3 or 7; i.e., the observations that correspond to the very large firm. So, there are two choices and again, it is not clear which of these two is better.
[33] Recall that our final decomposition of the industry productivity change defined by (32) does not depend on our rather arbitrary units of measurement for aggregate firm outputs and inputs.
[34] The input vector for firm 1 in period 0 is $x^1 = [10,10]$ and for firm 2 in period 0 is $x^2 = [1,1]$.
[35] Similarly, if the two price vectors are proportional, then the Fisher price index between the two observations will reflect this factor of proportionality. The Fisher formula seems to be the only superlative formula that is consistent with both Hicks' and Leontief's aggregation theorems; see Allen and Diewert (1981).

Table 2. Fisher Star Output Aggregates

Outputs	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
Base=1	1	0.102	4.971	0.794	1.170	0.250	5.203	1.225
Base=2	1	0.102	5.103	0.824	1.199	0.256	5.405	1.247
Base=3	1	0.099	4.971	0.79	1.216	0.256	5.365	1.270
Base=4	1	0.098	4.997	0.794	1.243	0.26	5.482	1.296
Base=5	1	0.100	4.785	0.748	1.17	0.247	5.070	1.232
Base=6	1	0.100	4.857	0.764	1.184	0.25	5.169	1.243
Base=7	1	0.098	4.821	0.754	1.201	0.252	5.203	1.261
Base=8	1	0.100	4.794	0.751	1.163	0.246	5.052	1.225

Table 3. Fisher Star Input Aggregates

Inputs	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
Base=1	1	0.100	3.975	1.252	0.680	0.200	3.183	0.646
Base=2	1	0.100	3.958	1.251	0.677	0.200	3.175	0.644
Base=3	1	0.100	3.975	1.243	0.692	0.201	3.281	0.650
Base=4	1	0.100	4.005	1.252	0.690	0.200	3.229	0.650
Base=5	1	0.100	3.904	1.234	0.680	0.201	3.260	0.644
Base=6	1	0.100	3.949	1.250	0.675	0.200	3.170	0.643
Base=7	1	0.100	3.856	1.235	0.664	0.201	3.183	0.637
Base=8	1	0.100	3.946	1.244	0.682	0.201	3.228	0.646

Now go along each row of table 2 and divide by the corresponding input aggregate listed in the corresponding row of table 3. This determines the productivity level of each observation. These star productivity levels are listed in table 4. There can be considerable variation in the productivity levels for each observation, depending on which observation is chosen as the base in the star system comparison. Thus if we choose the base to equal 1 (firm 1 in period 0), the productivity level of firm 2 in period 0 is 1.021 whereas if we choose the base to equal 7 (firm 3 in period 1), the productivity level of firm 2 in period 0 is 0.980: a 4% variation in productivity.[36]

Aggregate output prices that correspond to the 8 output aggregates that are listed in table 2 for each choice of base observation can be obtained by dividing the value of output produced by each firm in each period by the corresponding output listed for that observation in table 2. Similarly, aggregate input prices that correspond to the 8 input aggregates that are listed in table 3 for each choice of base observation can be obtained by dividing the value of inputs used by each firm in each period by the corresponding input listed for that observation in table 3. Once these aggregate output and input prices have been constructed, then we are in a position to apply the decomposition analysis that was discussed in sections 2 and 3 above.

[36] Ideally, we would like all the entries in each column of table 4 to be identical so that the productivity levels of each firm observation do not depend on the choice of index number base.

Table 4. Fisher Star Productivity Levels

Prod Levels	y_1/x_1	y_2/x_2	y_3/x_3	y_4/x_4	y_5/x_5	y_6/x_6	y_7/x_7	y_8/x_8
Base=1	1	1.021	1.251	0.634	1.721	1.250	1.635	1.897
Base=2	1	1.021	1.289	0.659	1.771	1.279	1.703	1.937
Base=3	1	0.990	1.251	0.636	1.756	1.271	1.635	1.953
Base=4	1	0.982	1.247	0.634	1.802	1.296	1.698	1.993
Base=5	1	0.992	1.226	0.606	1.721	1.226	1.555	1.914
Base=6	1	0.997	1.230	0.612	1.754	1.250	1.630	1.933
Base=7	1	0.980	1.250	0.611	1.809	1.253	1.635	1.981
Base=8	1	1	1.215	0.604	1.706	1.227	1.565	1.897

We define the various terms that occur on the right and left hand sides of the aggregate productivity growth decomposition (32) as follows:

(38) $\Gamma \equiv [\Pi^1/\Pi^0]-1$ (aggregate industry productivity growth);

(39) $\Gamma_{CD} \equiv \sum_{i\in C}(1/2)(s_{Ci}^0+s_{Ci}^1)(\Pi_{Ci}^1-\Pi_{Ci}^0)/\Pi^0$

(the direct productivity growth contribution of continuing firms);

(40) $\Gamma_{CR} \equiv \sum_{i\in C}(1/2)(\Pi_{Ci}^0+\Pi_{Ci}^1)(s_{Ci}^1-s_{Ci}^0)/\Pi^0$ (continuing firm reallocation contribution);

(41) $\Gamma_N \equiv S_N^1\sum_{i\in N}s_{Ni}^1(\Pi_{Ni}^1-\Pi_C^1)/\Pi^0$ (the contribution of entering firms to TFPG);

(42) $\Gamma_X \equiv S_X^0\sum_{i\in X}s_{Xi}^0(\Pi_{Xi}^0-\Pi_C^0)/\Pi^0$ (the contribution of exiting firms to TFPG).

The terms defined by (38)-(42) are listed in table 5 for each choice of base; i.e., we use the data in tables 2-4 above (along with the corresponding prices) in order to construct an aggregate industry productivity growth decomposition for each of the 8 bases.

Table 5. Aggregate Productivity Growth Decompositions for Each Choice of Base

	Γ	Γ_{CD}	Γ_{CR}	Γ_N	Γ_X
Base=1	0.5356	0.4054	-0.0061	0.0337	0.1025
Base=2	0.5496	0.4247	-0.0062	0.0300	0.1010
Base=3	0.5471	0.4128	-0.0063	0.0391	0.1015
Base=4	0.6025	0.4704	-0.0071	0.0374	0.1017
Base=5	0.5174	0.3739	-0.0066	0.0440	0.1061
Base=6	0.5684	0.4311	-0.0070	0.0387	0.1056
Base=7	0.5678	0.4249	-0.0075	0.0425	0.1080
Base=8	0.5296	0.3887	-0.0065	0.0418	0.1056

The choice of base matters. Aggregate productivity growth using observation 5 (data of firm 1 in period 1) as the base leads to estimating industry productivity growth as 51.74% whereas if observation 4 (data of the disappearing firm 4 in period 0) is used as the base, then industry productivity growth is estimated as much larger at 60.25%.[37] In the last 4 columns in table 5, the direct productivity growth of continuing firms accounts for most of the industry productivity growth (between 37.39% and 47.04%), the contribution of the exiting firm is between 10% and 11%, the contribution of the entering firm is between 3.0% and 4.4%, and the reallocation of resources between continuing firms sums to a negligible contribution.

Now, define the three continuing firm terms on the right hand side of (39) as Γ_{CD1}, Γ_{CD2} and Γ_{CD3}: the direct productivity growth contributions of continuing firms 1, 2 and 3 respectively. Define the three terms on the right hand side of (40) as Γ_{CR1}, Γ_{CR2} and Γ_{CR3}: the reallocation contributions of continuing firms 1, 2 and 3 respectively. These terms are listed in table 6. We see that the largest contribution to industry TFP growth is the direct TFP growth of firm 3 (the large firm); it contributes between 24.29% (the index base 5 estimate) and 32.61% (the index base 4 estimate). The next largest contribution comes from the medium sized firm 1. The other contribution terms are all less than 5%.

Some form of averaging of the star decompositions is called for. Our next method takes the geometric averages of the output and input aggregates in tables 2 and 3 and implements (32).

Table 6. Direct and Reallocation Contributions to Aggregate Productivity Growth for Each Continuing Firm and for Each Choice of Base

	Γ_{CD1}	Γ_{CD2}	Γ_{CD3}	Γ_{CR1}	Γ_{CR2}	Γ_{CR3}
Base=1	0.1210	0.0073	0.2771	-0.0372	0.0309	0.0002
Base=2	0.1262	0.0080	0.2905	-0.0381	0.0305	0.0014
Base=3	0.1263	0.0088	0.2777	-0.0396	0.0296	0.0037
Base=4	0.1345	0.0099	0.3261	-0.0367	0.0305	-0.0009
Base=5	0.1234	0.0076	0.2429	-0.0456	0.0298	0.0093
Base=6	0.1289	0.0082	0.2939	-0.0403	0.0312	0.0021
Base=7	0.1372	0.0089	0.2788	-0.0492	0.0304	0.0112
Base=8	0.1216	0.0074	0.2597	-0.0414	0.0305	0.0044

7. The GEKS Method for Making Multilateral Comparisons

The GEKS method dates back to Gini (1931), Eltetö and Köves (1964) and Szulc (1964). As already indicated, this method takes the geometric mean of the star output and input

[37] The choice of observations 2, 3, 6 and 7 as the index number base gives rise to industry TFP growth rates that are closest to our target rates of around 55.53% and 55.73%; recall (35) above. Note that the average of the industry productivity growth rates for the large firm observations (3 and 7) is 55.74%.

parities.[38] The GEKS relative output and input aggregates are listed in table 7. Once the output and input aggregates have been constructed, then the GEKS productivity levels can be constructed by dividing each output aggregate by the corresponding input aggregate, as in table 7.

Table 7. GEKS Output and Input Aggregates and Productivity Levels

Outputs	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
	1	0.100	4.911	0.777	1.193	0.252	5.242	1.250
Inputs	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
	1	0.100	3.946	1.245	0.680	0.201	3.213	0.645
Prod Levels	y_1/x_1	y_2/x_2	y_3/x_3	y_4/x_4	y_5/x_5	y_6/x_6	y_7/x_7	y_8/x_8
	1	0.998	1.245	0.624	1.755	1.256	1.631	1.938

Aggregate output prices that correspond to the 8 output aggregates that are listed in table 7 can be obtained by dividing the value of output produced by each firm in each period by the corresponding output listed for that observation in table 7. Similarly, aggregate input prices that correspond to the 8 input aggregates that are listed in table 7 can be obtained by dividing the value of inputs used by each firm in each period by the corresponding input listed for that observation in table 7. Once these aggregate output and input prices have been constructed, then we can repeat the decomposition analysis that was implemented in the previous section.

The productivity growth decomposition terms defined by (38)-(42) are listed in table 8 below. We also list the direct and reallocation contribution terms defined by the individual terms in (39) and (40) for each continuing firm in table 8.

Table 8. The GEKS Aggregate Productivity Growth Decomposition

Γ	Γ_{CD}	Γ_{CR}	Γ_N	Γ_X	
0.5521	0.4162	-0.0066	0.0384	0.1040	
Γ_{CD1}	Γ_{CD2}	Γ_{CD3}	Γ_{CR1}	Γ_{CR2}	Γ_{CR3}
0.1274	0.0083	0.2806	-0.041	0.0304	0.0039

From Table 8, the GEKS aggregate productivity growth Γ is 55.21%, which is reasonably close to our target rates of around 55.53% to 55.73%; recall (35) above. Thus we conclude that the GEKS method for constructing relative output and input levels for each firm in each period is satisfactory, at least for our particular numerical example.

One problem with the (unweighted) GEKS method is that each firm observation is given equal weighting. For example, for small firms, their star parities could be quite different than for large firms. Hence it may not be wise to give these small firms equal weighting in the construction of the output and input aggregates. In the following section, we look at a multilateral method that gives large firms more weight.

[38] The GEKS aggregates can be defined in a number of equivalent ways; see for example, Diewert (1999; 31-37).

8. The Own Share Method for Making Multilateral Comparisons

Recall our discussion in section 6 when we described how the star output aggregates could be constructed using observation k as the base. We noted that the sequence of 8 numbers, $Q_F(p^k,p^1,y^k,y^1)$, $Q_F(p^k,p^2,y^k,y^2)$, … , $Q_F(p^k,p^8,y^k,y^8)$, could serve as comparable output aggregates for our 8 observations. Hence, using observation k as the base, the *share of total output of observation k* is:

$$(43)\quad \begin{aligned} s_k^* &\equiv Q_F(p^k,p^k,y^k,y^k)/[Q_F(p^k,p^1,y^k,y^1)+Q_F(p^k,p^2,y^k,y^2)+\ldots+Q_F(p^k,p^8,y^k,y^8)] \\ &= 1/[Q_F(p^k,p^1,y^k,y^1)+Q_F(p^k,p^2,y^k,y^2)+\ldots+Q_F(p^k,p^8,y^k,y^8)]; \qquad k=1,\ldots,8. \end{aligned}$$

The last equation in (43) follows from the fact that the Fisher ideal quantity index satisfies an identity test and hence $Q_F(p^k,p^k,y^k,y^k)$ equals 1. Thus, using the metric of observation k to make the index number comparisons, the share of observation k in "world" output, s_k^*, is defined by (43) for k = 1,2, …, 8. Each observation's *own share* of "world" output is defined by (43). Put another way, if we look at the entries in table 2 above, the numbers listed in the Base=1 row determine the share of observation 1 in total output over the two periods, s_1^*; the numbers listed in the Base=2 row determine the share of observation 2 in total output over the two periods, s_2^*; …; and the numbers listed in the Base=8 row determine the share of observation 8 in total output over the two periods, s_8^*. Thus each row in table 2 determines only one share of "world" output and so the rows that correspond to smaller shares of world output get a smaller influence in the overall multilateral comparison. This means that the own share system does weight the individual star parities according to their economic importance as opposed to the more democratic GEKS method where each star parity has the same importance.

Unfortunately, the own shares s_k^* defined by (43) do not sum up to unity and so we renormalize these "shares" to sum up to unity as follows:[39]

$$(44)\quad y_k \equiv s_k^*/[\textstyle\sum_{j=1}^{8} s_j^*], \quad k = 1,\ldots,8.$$

The output aggregates y_k defined by (44) are the *own share output aggregates*.[40] The same procedure can be used to define own share input aggregates. The own share relative output and input aggregates as well as the output to input productivity ratios are listed in table 9. Once these aggregates have been constructed, then the own share productivity levels can be constructed by dividing each output aggregate by the corresponding input aggregate. The resulting 8 productivity levels are listed in the bottom row of table 9.[41]

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[39] In our empirical example, the s_k^* summed up to 0.99996 so that the differences between the y_k and the s_k^* were negligible. The corresponding input shares summed up to 0.99997.

[40] The own share system was proposed by Diewert (1988; 69). For axiomatic properties see Diewert (1999; 37-39).

[41] Note that the units of measurement for the output and input aggregates are quite different in Tables 7 and 9. This illustrates the importance of providing a productivity growth decomposition that is independent of the units.

Table 9. Own Share Output and Input Aggregates and Productivity Levels

Outputs	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
	0.068	0.007	0.332	0.052	0.082	0.017	0.357	0.085
Inputs	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
	0.091	0.009	0.357	0.113	0.062	0.018	0.293	0.058
Prod Levels	y_1/x_1	y_2/x_2	y_3/x_3	y_4/x_4	y_5/x_5	y_6/x_6	y_7/x_7	y_8/x_8
	0.75	0.742	0.931	0.465	1.322	0.943	1.218	1.462

Aggregate output prices that correspond to the 8 output aggregates listed in table 9 can be obtained by dividing the value of output produced by each firm in each period by the corresponding output listed for that observation in table 9. Similarly, aggregate input prices that correspond to the 8 input aggregates that are listed in table 9 can be obtained by dividing the value of inputs used by each firm in each period by the corresponding input listed for that observation in table 9. Once these aggregate output and input prices have been constructed, then we can repeat the decomposition analysis that was implemented in the previous sections.

The productivity growth decomposition terms defined by (38)-(42) are listed in table 10. Here we also list the direct and reallocation contribution terms, defined by the individual terms in (39) and (40) for each continuing firm. From table 10, the own share aggregate productivity growth Γ is 55.45%, which is close to our target rate of 55.53% to 55.73%; recall (35) above.[42] Thus we conclude that the own share method is very satisfactory, at least for our example.

Table 10. The Own Share Aggregate Productivity Growth Decomposition

Γ	Γ_{CD}	Γ_{CR}	Γ_N	Γ_X	
0.5545	0.4165	-0.0067	0.0403	0.1044	
Γ_{CD1}	Γ_{CD2}	Γ_{CD3}	Γ_{CR1}	Γ_{CR2}	Γ_{CR3}
0.1290	0.0086	0.2789	-0.0423	0.0302	0.0054

9. Hill's Method for Making Multilateral Comparisons

Another method for finding output and input aggregates is based on the following idea: observations which are most similar in their price structures (i.e., their output prices are closest to being proportional across items) should be linked using a bilateral index number formula first. Then the observation outside of the first two observations that has the most similar relative prices to the first two observations should be added, etc. This basic idea has been successfully exploited by Robert Hill at higher levels of aggregation[43] with complete price and expenditure data.

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[42] The own share decomposition is very close to the GEKS decomposition in table 8. Diewert (1988; 69) (1999; 38) showed that the own share aggregates and the GEKS aggregates will usually approximate each other closely.

[43] See Robert Hill (1999a) (1999b) (2001) (2004). The basic idea of spatially linking countries with the most similar price and quantity structures dates back to Fisher (1922; 271-272). We apply the idea here to firm observations.

To apply this idea, it is necessary to choose a measure of the degree of *dissimilarity* for the (relative) output prices corresponding to any two observations. There are many measures of relative price dissimilarity that could be chosen.[44] Our pick is the following one that measures the degree of dissimilarity between the output prices of observations j and k ($j,k = 1,\ldots,8$):

(45) $$D(p^j,p^k) \equiv \{\ln[p_1^k / P_F(p^k,p^j,q^k,q^j)p_1^j]\}^2 + \{\ln[p_2^k / P_F(p^k,p^j,q^k,q^j)p_2^j]\}^2 ,$$

where $P_F(p^k,p^j,q^k,q^j)$ is the Fisher output price index of observation j relative to k.[45] Thus instead of comparing the price of output 1 for observation k, p_1^k, with the price of output 1 for observation j, p_1^j, we multiply p_1^j by the Fisher price index for observation k relative to j, $P_F(p^k,p^j,q^k,q^j)$, which inflates the base prices j by a general inflation factor that makes the prices of k comparable to the inflated j prices. In particular, if the j prices are equal to λ times the k prices, so that $p^j = \lambda p^k$, then the Fisher index that compares the j prices to the k prices will pick up this proportionality factor so that $P_F(p^k,p^j,q^k,q^j) = \lambda$. In this case, the dissimilarity measure defined by (45) will be zero; i.e., we will have $D(p^j,p^k) = 0$. It can also be verified that the dissimilarity measure defined by (45) satisfies the following *symmetry property*:

(46) $$D(p^k,p^j) = D(p^j,p^k) \qquad j,k = 1,\ldots,8 .$$

Table 11 lists the Fisher output price indexes $P_F(p^k,p^j,q^k,q^j)$ between pairs of observations.[46]

Table 11. Fisher Output Price Indexes Between Each Pair of Observations

Base k=1	1	0.980	0.855	1.152	12.007	11.000	11.100	13.064
Base k=2	1.021	1	0.850	1.133	11.963	10.969	10.904	13.093
Base k=3	1.170	1.176	1	1.354	13.518	12.570	12.591	14.738
Base k=4	0.868	0.883	0.738	1	9.811	9.190	9.145	10.720
Base k=5	0.083	0.084	0.074	0.102	1	0.927	0.949	1.081
Base k=6	0.091	0.091	0.080	0.109	1.079	1	1.016	1.170
Base k=7	0.090	0.092	0.079	0.109	1.054	0.985	1	1.143
Base k=8	0.077	0.076	0.068	0.093	0.925	0.855	0.875	1

[44] See Diewert (2009) for an axiomatic treatment of the topic.

[45] This dissimilarity measure is essentially equal to that used by Allen and Diewert (1981) except that they used the Törnqvist index $P_T(p^k,p^j,q^k,q^j)$ to adjust for general price level change in place of the Fisher index $P_F(p^k,p^j,q^k,q^j)$ in (45). Diewert (2009) defined a weighted counterpart to (45) which he called the weighted log quadratic index of relative price dissimilarity.

[46] The Fisher (1922) output price index is defined as $P_F(p^0,p^1,q^0,q^1) \equiv [p^1q^0p^1q^1 / p^0q^0p^0q^1]^{1/2}$. Row k of table 11 is equal to $P_F(p^k,p^1,q^k,q^1)$, $P_F(p^k,p^2,q^k,q^2)$, … , $P_F(p^k,p^8,q^k,q^8)$.

Note that Fisher output price levels for firms present in period 1 are 9.145 to 14.738 times the levels of prices for firms present in period 0 (see the entries in the northeast corner of Table 11). Table 12 lists the dissimilarity measures $D(p^j, p^k)$ defined by (45): a symmetric matrix.

Table 12. Log Quadratic Output Price Dissimilarity Measures

0	0.08220	0.00705	0.00380	0.34067	0.12932	0.26641	0.28161
0.08220	0	0.13759	0.12365	0.71777	0.40242	0.61510	0.63505
0.00705	0.13759	0	0.00047	0.23304	0.07164	0.17715	0.18557
0.00380	0.12365	0.00047	0	0.24608	0.08182	0.19079	0.19811
0.34067	0.71777	0.23304	0.24608	0	0.04837	0.00305	0.00324
0.12932	0.40242	0.07164	0.08182	0.04837	0	0.02565	0.02722
0.26641	0.61510	0.17715	0.19079	0.00305	0.02565	0	0
0.28161	0.63505	0.18557	0.19811	0.00324	0.02722	0	0

Note that the dissimilarity measure between observations 7 and 8 is 0; this is due to the fact that the output price vectors for these two observations are proportional.

Inspection of table 12 shows that the lowest dissimilarity measures that link the data are: 7-8; 7-5; 7-6; 3-4; 1-4; 1-2 and 3-5. This set of links will enable us to construct output aggregates, $y_1,\ldots,y_8$, which are listed in table 15 below. The same strategy that was used to construct Hill output aggregates can be used to construct input aggregates. The input counterparts to tables 11 and 12 are tables 13 and 14.

Table 13. Fisher Input Price Indexes Between Each Pair of Observations

Base k=1	1	0.750	0.994	1.102	16.029	14.500	16.650	16.884
Base k=2	1.333	1	1.331	1.471	21.474	19.333	22.257	22.570
Base k=3	1.006	0.752	1	1.117	15.842	14.497	16.254	16.867
Base k=4	0.907	0.680	0.895	1	14.338	13.131	14.896	15.214
Base k=5	0.062	0.047	0.063	0.070	1	0.898	1.014	1.056
Base k=6	0.069	0.052	0.069	0.076	1.114	1	1.153	1.169
Base k=7	0.060	0.045	0.062	0.067	0.986	0.867	1	1.028
Base k=8	0.059	0.044	0.059	0.066	0.947	0.855	0.972	1

Fisher input price levels for firms present in period 1 are 13.131 to 22.570 times the levels of prices for firms present in period 0 meaning that input prices grew faster than output prices.

Inspection of table 14 shows that the lowest dissimilarity measures that link the data are: 3-6; 2-3; 1-2; 1-4; 6-8; 5-7 and 5-8. This set of links will enable us to construct Hill input aggregates, $x_1,\ldots,x_8$, which are listed in table 15. The eight Hill productivity levels, $y_1/x_1,\ldots,y_8/x_8$, are also listed in table 15.

Table 14. Log Quadratic Input Price Dissimilarity Measures

0	0.00893	0.02014	0.01669	0.35300	0.02161	0.73589	0.06377
0.00893	0	0.00225	0.04993	0.25127	0.00277	0.58761	0.02507
0.02014	0.00225	0	0.07378	0.20284	0.00002	0.50447	0.01219
0.01669	0.04993	0.07378	0	0.51780	0.07605	0.95664	0.14529
0.35300	0.25127	0.20284	0.5178	0	0.20208	0.06805	0.11723
0.02161	0.00277	0.00002	0.07605	0.20208	0	0.51192	0.01122
0.73589	0.58761	0.50447	0.95664	0.06805	0.51192	0	0.36697
0.06377	0.02507	0.01219	0.14529	0.11723	0.01122	0.36697	0

Table 15. Hill Output and Input Aggregates and Productivity Levels

Outputs	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
	1	0.102	4.997	0.794	1.222	0.256	5.295	1.284
Inputs	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
	1	0.100	3.958	1.252	0.681	0.200	3.263	0.644
Prod Levels	y_1/x_1	y_2/x_2	y_3/x_3	y_4/x_4	y_5/x_5	y_6/x_6	y_7/x_7	y_8/x_8
	1	1.021	1.262	0.634	1.795	1.277	1.623	1.992

Comparing the entries in table 15 with the corresponding GEKS entries in table 7, it can be seen that with the exceptions of observations 1 and 7, the Hill productivity levels tend to be greater than the corresponding GEKS productivity levels.

Aggregate output prices that correspond to the 8 output aggregates that are listed in table 15 can be obtained by dividing the value of output produced by each firm in each period by the corresponding output listed for that observation in table 15. Similarly, aggregate input prices that correspond to the 8 input aggregates that are listed in table 15 can be obtained by dividing the value of inputs used by each firm in each period by the corresponding input listed for that observation in table 15. Once these aggregate output and input prices have been constructed, then we can repeat the decomposition analysis that was implemented in the previous sections.

The productivity growth decomposition terms defined by (38)-(41) are listed in table 16 below. We also list the direct and reallocation contribution terms defined by the individual terms in (39) and (40) for each continuing firm in table 16.

Table 16. The Hill Aggregate Productivity Growth Decomposition

Γ	Γ_{CD}	Γ_{CR}	Γ_N	Γ_X	
0.5401	0.3986	-0.0063	0.044	0.1038	
Γ_{CD1}	Γ_{CD2}	Γ_{CD3}	Γ_{CR1}	Γ_{CR2}	Γ_{CR3}
0.1318	0.008	0.2588	-0.0428	0.0301	0.0064

From table 16, the Hill aggregate productivity growth Γ is 54.01%, which is not as close to our target rates of around 55.53% to 55.73% compared to the GEKS and own share decompositions of productivity growth. Thus for this particular numerical example, we conclude that the Hill method for constructing relative output and input levels for each firm in each period is satisfactory but not as good at the GEKS and own share estimates.

10. An Approximate Method for Constructing Output and Input Aggregates

The multilateral methods for constructing output and input aggregates that have been discussed in the previous 3 sections are theoretically satisfactory methods. However, they suffer from two major disadvantages:

- They may not be practical for very large data sets; i.e., they are computation intensive.
- Detailed price and quantity information may not be available for each firm; i.e., only information on output revenues and input costs by unit may be available.

Thus in the present section, we assume that we have only information on firm revenues and costs by period and that we also have aggregate intertemporal price indexes for both outputs and inputs available. In particular, we assume that we have the aggregate Fisher output and input price indexes at our disposal. Using the aggregate period 0 and 1 information on the industry's two outputs and inputs listed in section 5 above,[47] the Fisher and Törnqvist output price index numbers for period 1 are 12.283 and 12.239 respectively[48] while the Fisher and Törnqvist input price index numbers for period 1 are 16.035 and 15.998 respectively. We will use the Fisher industry price index values for outputs and inputs for period 1, $P_F(p^0,p^1,q^0,q^1)$ and $P_F{}^*(w^0,w^1,x^0,x^1)$ respectively, to deflate all of the period 1 firm revenues and costs in order to make them at least approximately comparable to the period 0 firm revenues and costs. Recalling the notation that was introduced at the beginning of section 6, for observations 1-4, we define firm aggregate outputs and inputs as firm revenues and costs respectively; i.e., define the *output and input aggregates,* y_1, y_2, y_3, y_4 and x_1, x_2, x_3, x_4 respectively, as follows:

(47) $\quad y_k \equiv p^k y^k$; k = 1,2,3,4 ; $\quad x_k \equiv w^k x^k$; k = 1,2,3,4 .

For observations 5-8 (the period 1 observations), we define firm aggregate outputs and inputs as Fisher index deflated firm revenues and costs respectively; i.e., define the *output and input aggregates,* y_5, y_6, y_7, y_8 and x_5, x_6, x_7, x_8 respectively, as follows:

(48) $\quad y_k \equiv p^k y^k / P_F(p^0, p^1, q^0, q^1)$ and $x_k \equiv w^k x^k / P_F^*(w^0, w^1, x^0, x^1)$ for k=5,6,7,8.

Obviously, the output and input aggregates defined by (47) and (48) are not going to be as accurate as the output and input aggregates defined in the previous 3 sections. However, it is still of some interest to see how close these approximate aggregates are to the previously defined multilateral aggregates. The approximate output and input aggregates are listed in table 17 along with the corresponding plant productivity levels.

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[47] See the listing of the industry data in (33) above.
[48] The corresponding index values are 1 in period 0.

Table 17. Approximate Output and Input Aggregates and Productivity Levels

Outputs	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8
	20.000	2.000	85.000	18.300	22.877	4.478	94.031	26.052
Inputs	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8
	20.000	1.500	79.000	27.600	13.595	3.617	66.105	13.595
Prod Levels	y_1/x_1	y_2/x_2	y_3/x_3	y_4/x_4	y_5/x_5	y_6/x_6	y_7/x_7	y_8/x_8
	1	1.333	1.076	0.663	1.683	1.238	1.422	1.916
GEKS	1	0.998	1.245	0.624	1.755	1.256	1.631	1.938

In order to make the units of measurement for outputs and inputs listed in table 17 comparable to the units listed in the corresponding GEKS table 7, it is necessary to divide the outputs row by 20 and the inputs row by 20. The productivity levels row in table 17 is comparable to the corresponding row in table 7. For easy reference, the GEKS productivity levels are listed as the last row in table 17. It can be seen that there are some rather substantial differences in the GEKS productivity levels compared to the corresponding approximate ones.

As usual, aggregate output prices that correspond to the 8 output aggregates listed in table 17 can be obtained by dividing the value of output produced by each firm in each period by the corresponding output listed for that observation in table 17. Similarly, aggregate input prices that correspond to the 8 input aggregates listed in table 17 can be obtained by dividing the value of inputs used by each firm in each period by the corresponding input listed for that observation in table 17. Once these aggregate output and input prices have been constructed, then we can repeat the decomposition analysis that was implemented in the previous sections.

The productivity growth decomposition terms defined by (38)-(41) are listed in table 18 below. We also list the direct and reallocation contribution terms defined by the individual terms in (39) and (40) for each continuing firm in table 18. For ease of comparison, we list the decompositions for the GEKS, own share and Hill methods in table 18 as well.

Table 18. The Approximate Method Aggregate Productivity Growth Decomposition

	Γ	Γ_{CD}	Γ_{CR}	Γ_N	Γ_X	
Approx Method	0.5553	0.4033	-0.0023	0.0659	0.0885	
GEKS	0.5521	0.4162	-0.0066	0.0384	0.104	
Own Share	0.5545	0.4165	-0.0067	0.0403	0.1044	
Hill	0.5401	0.3986	-0.0063	0.044	0.1038	
	Γ_{CD1}	Γ_{CD2}	Γ_{CD3}	Γ_{CR1}	Γ_{CR2}	Γ_{CR3}
Approx Method	0.1264	-0.0028	0.2798	-0.0491	0.0374	0.0094
GEKS	0.1274	0.0083	0.2806	-0.041	0.0304	0.0039
Own Share	0.129	0.0086	0.2789	-0.0423	0.0302	0.0054
Hill	0.1318	0.008	0.2588	-0.0428	0.0301	0.0064

From table 18, the approximate method aggregate productivity growth Γ is 55.53%, which is exactly equal to our target Fisher rate of 55.53%. This exact equality is not a statistical fluke but is a consequence of the fact that we have used the industry Fisher price indexes to deflate the period 1 value data. Thus our approximate method works extremely well in terms of replicating the industry's aggregate productivity growth. However, the other terms on the right hand side of (32) are not always well predicted by the approximate method. In particular, it leads to a contribution of entry term Γ_N equal to 6.59% whereas the other methods lead to contribution terms in the 3.84 to 4.40% range. Also, the approximate method leads to a contribution of exit term Γ_X equal to 8.85% whereas the other methods lead to contribution terms in the 10.38 to 10.44% range. However, considering the simplicity of the approximate method, we conclude that at least for this example, this method was suitable for constructing output and input aggregates to be used in a productivity growth decomposition such as (32), though, of course, it was not as good as the GEKS and own share methods.

11. Conclusion

This paper proposes a new formula (32) for decomposing industry productivity growth into terms that reflect the productivity growth of individual production units that operate in both the base and comparison periods, and also the reallocation of resources among continuing firms from lower productivity to higher productivity units, as well as entry and exit contribution terms. Unfortunately, this formula and the other formulae presented in the literature are derived under the assumption that each production unit produces a single homogeneous output and uses a single homogeneous input. Most of the paper (sections 4-10) is concerned with the problems involved in aggregating many outputs and many inputs into output and input aggregates. In order to accomplish this aggregation, we suggested the use of multilateral methods and we implemented four multilateral methods on a test data set that is described in section 5 above. For our test data set, we found that the own share method worked best but the GEKS method was very close. The Hill methods and an approximate method that used value aggregates in the base period and deflated value aggregates in the comparison period also worked reasonably well for our data set. The fact that the approximate method worked so well is very encouraging for empirical work in this area, since variants of it are what have been used in empirical applications of productivity decompositions that involve entry and exit.[49]

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[49] In our test example, we used the actual "industry" Fisher output and input price indexes as the deflators. In empirical work, the deflators that are available are unlikely to be the exact industry deflators. Also, in real life, it is unlikely that all of the production units in a given industry are producing positive amounts of a common list of outputs and using positive amounts of a common list of inputs, as for our example.

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