Chapter 5

EXACT INDUSTRY CONTRIBUTIONS TO LABOR PRODUCTIVITY CHANGE

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1. Introduction

Industry contributions to aggregate productivity growth have been a topic of great interest in recent years. One reason for this is a desire for insight into the sources of the remarkable speedup of productivity growth in the late 1990s. The U.S. Bureau of Labor Statistics (BLS) estimates that output per hour in the nonfarm business sector grew at an average rate of around 3 percent per year from 1995 to 2003, compared with 1.5 percent per year between 1987 and 1995. Interest in investigating industry sources of productivity change has been further heightened by the availability of new and improved data on industry gross output, intermediate inputs and value added, resulting from the integration of the GDP-by-industry accounts and the annual I-O accounts in data sets released in June 2004. Evidence on industry contributions to productivity change has been used to resolve controversies concerning the economic gains from information technology (IT), the causes of the post-1995 speedup in productivity growth, possible measurement errors in prices or output, and other important questions.[2]

As Nordhaus (2002, p. 213) observes, the use of chain-weighted output measures makes disentangling the contributions of individual components to aggregate productivity growth a complex problem. To account for substitution effects, non-linear chain-weighted index number formulas such as the Fisher index or the Törnqvist index must be used to measure aggregate real output growth. Although in nominal terms, aggregate output is the sum of every industry's value added, with the chain-weighted index number formulas, aggregate real output fails to equal the sum over all industries of each industry's real value added. The lack of an additive formula for industry contributions to real output growth implies that formulas for industry contributions to

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[2] Some recent studies of industry contributions to productivity change are Bosworth and Triplett (2004), Klein et al. (2003), Basu and Fernald (2002), Gullickson and Harper (2002), Nordhaus (2002), Stiroh (2002), Jorgenson (2001), Mc Kinsey Global Institute (2001), ten Raa and Wolff (2001), Jorgenson and Stiroh (2000a, 2000b), Oliner and Sichel (2000), and Corrado and Slifman (1999). The present paper is part of a collaborative project on this topic between the U.S. Bureau of Economic Analysis (BEA) and the Office of Productivity and Technology of the U.S. Bureau of Labor Statistics (BLS).

Citation: **Marshall Reinsdorf and Robert Yuskavage (2010), "Exact Industry Contributions to Labor Productivity Change," chapter 5, pp. 77-102 in**
W.E. Diewert, B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura (2010),
***PRICE AND PRODUCTIVITY MEASUREMENT: Volume 6 -- Index Number Theory*. Trafford Press.**
Also available as a free e-publication at www.vancouvervolumes.com and www.indexmeasures.com.

aggregate productivity growth also generally add up to incorrect totals, because aggregate productivity is measured as the difference between the log-change in aggregate real output and the log-change in aggregate inputs.

The unavailability of exact formulas for industry contributions to aggregate productivity growth has led to reliance on approximate decomposition formulas. In the appendix we derive the change in real GDP implied by a commonly used Törnqvist approach to industry contributions to productivity change. The resulting expression in equation (A-4) is different enough from ordinary approximations for the change in real GDP to raise questions about the accuracy of those approximations: questions for which we nevertheless produce a reassuring answer in this paper.

In this paper we derive an exactly additive decomposition of aggregate labor productivity growth into industry sources using results from the literature on index number formulas. Included in our decomposition are contributions to aggregate productivity growth due to changes at the industry level in real gross output per hour and in the relative use of intermediate inputs. The sum of the first two of these effects equals the contribution to aggregate productivity of changes in an industry's real value added per hour. A third effect comes from changes in the allocation of labor between industries with different productivity levels. In the productivity literature, this effect has been variously referred to as a "shift effect," a "Denison effect," or a "labor reallocation effect." Bosworth and Triplett (2004) point out that ignoring the labor allocation effect may lead to misleading inferences concerning the proportion of aggregate productivity change attributable to a particular group of industries, such as ones that produce information technology (IT) products. Previous authors have treated the labor reallocation term as a kind of residual that cannot be included in the additive decomposition, but we show how it can be included.

In addition to its methodological contributions, this paper makes an empirical contribution to the literature on the industry sources of the post-1995 rebound in productivity growth. Among its empirical findings are a modest direct contribution of the IT-producing industries to the productivity speedup, large contributions for Wholesale trade and Retail trade, and a negative contribution for the Electric, gas and sanitary services industry, reflecting the increased use of intermediate inputs.

2. Exactly Additive Contributions of Commodities to Change in Fisher Indexes

The two widely used chain-weighted index number formulas are the Fisher index and the Törnqvist index.[3] Here we take the Fisher measure of aggregate productivity growth as the object of investigation. Although the Törnqvist index is easily decomposed into *commodity* contributions to the log-change in the aggregate, for the problem of finding *industry* contributions to change, the Fisher index is actually more tractable. Another advantage of our Fisher approach is that the results can be used to obtain decompositions of productivity growth that are precisely consistent with official measures of real output, which the BEA constructs

[3] Diewert and Nakamura (2003) survey some of the reasons for this.

from Fisher indexes. Furthermore, the Fisher contributions formula has an economic justification that other formulas lack. Finally, we note that the Fisher index has an appealing justification as a measure of aggregate welfare change for a society as a whole. In particular, Pollak (1981) shows that the aggregate Laspeyres price index is an upper bound for the Scitovsky-Laspeyres social cost of living index, which measures the change in the aggregate income that would be required for a social planner to keep every household in a society on its original indifference curve.[4] Diewert (2001, pp. 172-173) observes that the Paasche index is a lower bound for the analogously defined Scitovsky-Paasche social cost of living index and that the Fisher index can therefore be justified as an average of lower and upper bounds for social cost of living indexes based on a pair of relevant Scitovsky contours.[5]

To solve the problem of identifying industry sources of productivity change, we use the formula for additive contributions to the change in a Fisher quantity index that underlies the tables of contributions to change reported in the U.S. National Income and Product Accounts (NIPAs). This formula was discovered by van IJzeren (1952) as part of an argument that the Fisher index had a unique property that could justify its use.[6] It was then forgotten, until its independent rediscovery by Dikhanov (1997).

Van IJzeren considered the problem of finding an average basket for a price index that would be unaffected by an equiproportional change in all quantities and an average price vector for a quantity index that would be unaffected by an equiproportional change in all prices. In doing this, he effectively posited the desirability of the decomposition formula now used by BEA and Statistics Canada, and then showed this property implies the Fisher formula for the index.[7]

Index number formulas that use simple averages of prices or baskets are known as Edgeworth (or Edgeworth-Marshall) indexes. The Edgeworth quantity index, E^Q , uses an average of initial and final prices to value quantity changes:

$$E^Q(\mathbf{p}_t, \mathbf{p}_{t+1}, \mathbf{q}_t, \mathbf{q}_{t+1}) \equiv \frac{\mathbf{q}_{t+1}'(\mathbf{p}_t + \mathbf{p}_{t+1})/2}{\mathbf{q}_t'(\mathbf{p}_t + \mathbf{p}_{t+1})/2}. \tag{1}$$

Similarly, the Edgeworth price index uses as its basket an average of the baskets from the initial and final periods:

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[4] This theory concerns commodities that are consumed directly, but, under certain assumptions, it can be extended to the measurement of output that includes investment goods used to produce commodities for consumption in future time periods. In particular, we can treat investments that raise future consumption possibilities as part of consumption for welfare measurement purposes; see Basu and Fernald (2002) and Weitzman (1976).

[5] To justify a Törnqvist index as a measure of aggregate welfare change requires stronger assumptions. Assuming that households have preferences that are homothetic — but not necessarily identical — and they have total expenditures that are constant shares of aggregate total expenditures, the aggregate log Törnqvist index is a weighted average of individual log Törnqvist indexes, which are themselves superlative measures of individual consumers' welfare change. Exactly additive industry contributions to a Törnqvist measure of aggregate productivity growth are available from the authors upon request. For more on the properties of Fisher indexes, see also Diewert (1992).

[6] See Reinsdorf, Diewert and Ehemann (2002), and Balk (2004).

[7] See van IJzeren (1987) for more background. The use of this formula in BEA's National Income and Product Accounts (NIPAs) is discussed in Moulton and Seskin (1999). A related multiplicative decomposition of the change in the Fisher index is presented in Kohli (2010).

(2) $$E^P(\mathbf{p}_t,\mathbf{p}_{t+1},\mathbf{q}_t,\mathbf{q}_{t+1}) \equiv \frac{\mathbf{p}_{t+1}'(\mathbf{q}_t+\mathbf{q}_{t+1})/2}{\mathbf{p}_t'(\mathbf{q}_t+\mathbf{q}_{t+1})/2}.$$

A high rate of inflation (or the multiplication of all final period prices by any scalar other than 1) will arbitrarily change the weights in the Edgeworth quantity index, and similarly a high rate of real growth will arbitrarily change the weights in the Edgeworth price index. To correct the Edgeworth indexes so that they always give equal weight to relative prices and quantities in both periods, period t prices must be rescaled by a price index I^P before averaging them with prices from period t+1, and period t quantities must be rescaled by a quantity index I^Q before they can be averaged with quantities from period t+1. This gives the pair of simultaneous equations:

(3) $$I^Q(\mathbf{p}_t,\mathbf{p}_{t+1},\mathbf{q}_t,\mathbf{q}_{t+1};I^p) \equiv \frac{\mathbf{q}_{t+1}'(\mathbf{p}_t I^P+\mathbf{p}_{t+1})/2}{\mathbf{q}_t'(\mathbf{p}_t I^P+\mathbf{p}_{t+1})/2}, \text{ and}$$

(4) $$I^P(\mathbf{p}_t,\mathbf{p}_{t+1},\mathbf{q}_t,\mathbf{q}_{t+1};I^Q) \equiv \frac{\mathbf{p}_{t+1}'(\mathbf{q}_t I^Q+\mathbf{q}_{t+1})/2}{\mathbf{p}_t'(\mathbf{q}_t I^Q+\mathbf{q}_{t+1})/2}.$$

Van IJzeren shows that the solution to these equations sets I^Q equal the Fisher quantity index, F^Q, and I^P equal the Fisher price index, F^P, where a Fisher index is defined as the geometric mean of a Paasche index and a Laspeyres index.

In addition to van IJzeren's axiomatic justification for the decomposition formula for Fisher indexes given by the right side of equation (3) or equation (4), it has an economic justification. Reinsdorf, Diewert and Ehemann (2002) show that this formula is a second order approximation to a decomposition formula that measures the contribution of each item i to the change in a flexible production function of the form $[\sum_i \sum_j a_{ij} q_i q_j]^{1/2}$, where the coefficients satisfy $a_{ij} = a_{ji}$ The van IJzeren decomposition can, therefore, be expected to provide a good measure of the economic contributions of the various inputs to the change in output.

In the NIPAs, commodity contributions to the change in F_t^Q, the Fisher index for real GDP, are calculated by expressing F_t^Q in the form given by equation (3). In this index, the quantities from period t and the quantities from period t+1 are both valued at a constant set of prices. These constant prices equal inflation-corrected averages of the prices from the periods being compared. Hence, the constant price for the arbitrary commodity c, denoted by p_{ct}^*, equals $(p_{ct}F_t^P + p_{c,t+1})/2$, where F_t^P denotes the Fisher price index calculated from final expenditures on commodities, e_{ct} and $e_{c,t+1}$, and the corresponding price indexes. To adjust the expenditure on commodity c, denoted by e_{ct}, from current-year dollars to the constant price p_{ct}^*, it is multiplied by an average of F_t^P and the price relative for commodity c, $p_{c,t+1}/p_{ct}$; hence,

(5) $$E_{ct} = e_{ct}\frac{p_{ct}F_t^P + p_{c,t+1}}{2p_{ct}}.$$

Similarly, to adjust the final expenditure $e_{c,t+1}$ to equal the value it would have had at price p^*_{ct}, it is multiplied by an average of the ratio of F^P to commodity c's price relative and 1:

(6) $$E_{c,t+1} = e_{c,t+1} \frac{p_{ct} F_t^P + p_{c,t+1}}{2 p_{c,t+1}}.$$

The Fisher quantity index for GDP then tracks the change in GDP measured using the constant prices p^*_{ct}:

(7) $$F_t^Q = \frac{\sum_c E_{c,t+1}}{\sum_c E_{ct}}.$$

The contribution to the change in F_t^Q of the arbitrary commodity γ is, then, given by:

$$\frac{E_{\gamma,t+1} - E_{\gamma t}}{\sum_c E_{ct}}.$$

3. Exactly Additive Contributions of Industries to Change in Fisher Indexes

The production approach estimate of GDP is calculated as the sum over all industries of current-year dollar value added v_t. If y_{it} is the gross output of industry i and m_{it} is its use of intermediate inputs, then $v_t = \sum_i v_{it} = \sum_i (y_{it} - m_{it})$. Given consistent data, the production approach estimate of current-year dollar GDP equals the expenditure approach estimate of GDP, defined as $\sum_c e_{ct}$ where e_{ct} is the final demand for commodity c. BEA calculates the Fisher index for the total value added of all industries — the production approach estimate of real GDP — in a way that makes it theoretically equal to the expenditure approach estimate of real GDP.[8]

The same estimate of real GDP can be obtained if the adjustment factors on the right side of equations (5) and (6) are used to convert current-year dollar values of gross output and intermediate inputs into constant dollar values. To convert to constant dollars for decomposing real GDP change between period t and period t+1, measures of gross output and intermediate inputs based on prices from period t are multiplied by the same factor as e_{ct} in equation (5), and measures in prices from period t+1 are multiplied by the same factor as $e_{c,t+1}$ in equation (6). If

[8] See Moyer, Reinsdorf and Yuskavage (2003) and also Yuskavage (1996). These authors used a consistent set of data from the GDP by Industry Accounts, so their estimate of real GDP was the same using either the production approach or the expenditure approach. However, aggregate real output for all industries from the GDP by Industry accounts usually differs from real GDP from the NIPAs because of inconsistencies between deflators in the two sets of accounts. Also, before June 2004, the sum of value added from the GDP-by-Industry Accounts equaled the income side estimate of the GDP, not the expenditure approach estimate. For related productivity measurement issues, see Eldridge (1999).

L_{it}^{YP} represents the Laspeyres price index for the gross output of industry i and P_{it}^{YP} represents the Paasche price index, the constant-price measure of this industry's gross output of industry i in year t, denoted by Y_{it}, is:

(8) $$Y_{it} = y_{it}[F_t^P + L_{it}^{YP}]/2,$$

and, using this same set of prices to value its output in year t+1 gives a constant-price measure of:

(9) $$Y_{i,t+1} = y_{i,t+1}[F_t^P / P_{it}^{YP} + 1]/2.$$

The equations for constant-price intermediate inputs, denoted by M_{it} and $M_{i,t+1}$, are analogous to those for constant-price gross output.

Constant-price value added in industry i, V_{it}, is defined as $Y_{it} - M_{it}$, and constant-price GDP, denoted by V_t, is defined as $\sum_i V_{it}$. Moyer, Reinsdorf and Yuskavage (2004, proposition 1) show that V_{t+1}/V_t equals the Fisher index for real GDP. That is,

(10) $$F_t^Q = \frac{Y_{t+1} - M_{t+1}}{Y_t - M_t}.$$

Industry i's additive contribution to the change in real GDP, C_{it}, can then be calculated as:

(11) $$C_{it} = \frac{V_{i,t+1} - V_{it}}{V_t},$$

where $\sum_i C_{it} = F_t^Q - 1$.

4. Exactly Additive Industry Contributions to Change in Labor Productivity

4.1 Contributions to the Change in Aggregate Real Value Added per Hour

Some simple measures of industry contributions to productivity change are decompositions of the change in the production approach estimate of real GDP per hour. In a Laspeyres framework, these decompositions provide industry contributions that sum exactly to the change in aggregate productivity because the sum of Laspeyres real value added over all industries equals Laspeyres real GDP. However, the existing methods for calculating industry contributions to real GDP per hour provide only approximate decompositions for Fisher or Törnqvist measures of real GDP and real value added.

Balk (2003, p. 28, equation 51) provides an appealing formula for industry contributions to real GDP per hour based on the Bennet decomposition.[9] Using Fisher indexes for real value

[9] Diewert (2000; 2005) shows that the quantity components of the Bennet decomposition of nominal output change, defined as $\bar{p}_i \Delta q_i + \bar{q}_i \Delta p_i$, have an economic interpretation as an approximation to the contributions to change in production function that implies the Fisher index formula.

added in this formula, which preserves the symmetry properties of the Bennet decomposition, creates a discrepancy between the total over all industries of real value added and the Fisher measure of real GDP. As a result, the industry contributions fail to sum to the total change in GDP per hour.

Adopting the Fisher index for measurement of real output, and letting H_t denote aggregate hours or full-time equivalents (FTEs), the objective of the Bennet decomposition is to calculate industry contributions to aggregate productivity change, measured in dollars of year t per hour, as $F_t^Q v_t / H_{t+1} - v_t / H_t$. Using the constant-price measures of value added in the Bennet decomposition corrects its non-additivity because the industry contributions based on V_{it} and the $V_{i,t+1}$ add up to the Fisher measure of real GDP per hour.[10] Let $\bar{h}_i$ denote the average of H_{it} / H_t and $H_{i,t+1} / H_{t+1}$ and let Δh_i equal $H_{i,t+1} / H_{t+1} - H_{it} / H_t$, where H_{it} / H_t is industry i's share of aggregate labor input in year t. Then industry i has an additive Bennet contribution C_{it}^* to arithmetic change in aggregate labor productivity equal to:

$$\begin{aligned} C_{it}^* &= \frac{v_t}{V_t} \bar{h}_i \left[\frac{V_{i,t+1}}{H_{i,t+1}} - \frac{V_{it}}{H_{it}} \right] \\ &+ \frac{v_t}{V_t} \Delta h_i \left\{ \frac{1}{2} \left[\frac{V_{i,t+1}}{H_{i,t+1}} + \frac{V_{it}}{H_{it}} \right] - \frac{1}{2} \left[\frac{V_{t+1}}{H_{t+1}} + \frac{V_t}{H_t} \right] \right\} \end{aligned} \quad (12)$$

The term on the first line of equation (12) represents the direct effect from productivity growth in industry i. The term on the second line of equation (12) represents a labor allocation effect, or shift effect. An increase in the share of aggregate labor allocated to an industry with above-average productivity will raise productivity by an amount that is measured by the expression on the second line of equation (12).

4.2 Contributions to Log-Change in Output

The use in equation (12) of differences in real valued added per hour to measure productivity change has the advantage of simplicity, but it also has some disadvantages. First, this measure can be distorted by substitution induced by changes in the relative price of intermediate inputs; for example, it will tend to rise if the price of intermediate inputs falls even in the absence of any genuine productivity gain.[11] Second, often researchers are interested in comparing multi-year periods of high productivity growth with multi-year periods of low productivity growth but, unlike logarithmic measures of productivity change, the C_{it}^* cannot be averaged over years.

[10] This method also offers the advantage of a unified approach to statistical agencies that publish contributions to change in Fisher indexes as well as contributions to productivity change, such as Statistics Canada.
[11] Capital deepening can cause a similar rise in any kind of measure of labor productivity, but in this case measures of labor productivity that include gains from capital deepening are still of interest.

To avoid such problems, researchers generally use the log-change in gross output per hour as the measure of an industry's labor productivity.[12] Since BEA measures real output growth by a Fisher quantity index, let the log-change in real GDP be the log of the Fisher index F_t^Q, which may be calculated using the price indexes for commodities and the final expenditures on commodities, e_{ct} and $e_{c,t+1}$. The aggregate labor productivity change between year t and year $t+1$ is then:

(13) $$ALP_t = \log F_t^Q - d\log H_t$$

where $d\log H_t \equiv \log(H_{t+1}/H_t)$ is the log-change in hours of labor input.[13]

Identification of the industry sources of aggregate labor productivity change as measured by equation (13) requires formulas for contributions to the log-change in real GDP and in aggregate hours. Equation (12) describes a contribution to a difference; not to a log-change. Yet, as Balk (2003, pp. 41-2) points out, logarithmic means can be used to convert difference measures to log-change measures. If $s_{it} \neq s_{i,t+1}$, the logarithmic mean $m(s_{it}, s_{i,t+1})$ is defined as:

(14) $$m(s_{it}, s_{i,t+1}) \equiv (s_{i,t+1} - s_{it})/\log(s_{i,t+1}/s_{it}).$$

The main index formula that uses logarithmic means is the Sato-Vartia index (see Sato, 1976 and Vartia, 1976).[14] The log Sato-Vartia quantity index is defined as a weighted average of log-changes in quantities, where the weights are normalized to sum to 1 and are proportional to logarithmic means of the expenditure shares s_{it} and $s_{i,t+1}$.

To decompose the log-change in GDP into industry contributions, let w_{it}^Y denote the weight for the gross output of industry i and let w_{it}^M denote the weight for its intermediate inputs. In this case, the Sato-Vartia weights are normalized so that the sum of the gross output weights less the sum of the intermediate input weights w_{it}^M equals 1. The Sato-Vartia weight w_{it}^Y for the log change in industry i's gross output is:

(15) $$w_{it}^Y = \frac{m(Y_{it}/V_t, Y_{i,t+1}/V_{t+1})}{\sum_j [m(Y_{jt}/V_t, Y_{j,t+1}/V_{t+1}) - m(M_{jt}/V_t, M_{j,t+1}/V_{t+1})]}.$$

Similarly, the Sato-Vartia weight w_{it}^M for the log change in industry i's intermediate inputs is:

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[12] Hulten (1978) shows that use of the log-change in industry gross output to calculate industry contributions to total factor productivity growth results in estimates with an economic interpretation as measures of technological change.
[13] A Fisher index of various types of labor input would provide valuable additional information on the effects of changes in the composition of industry labor forces. Unfortunately, data to compute such input indexes are lacking.
[14] Balk (1995) discusses the axiomatic properties of the Sato-Vartia index, including the basket test, and finds that its axiomatic properties are on a par with the Fisher index. Its economic interpretation is discussed in Lau (1979).

(16) $$w_{it}^{M} = \frac{m(M_{it}/V_t, M_{i,t+1}/V_{t+1})}{\sum_j [m(Y_{jt}/V_t, Y_{j,t+1}/V_{t+1}) - m(M_{jt}/V_t, M_{j,t+1}/V_{t+1})]}.$$

Proposition 1 shows that the weights defined in (15) and (16) furnish exactly additive contributions by industry to the log change in real GDP.

PROPOSITION 1: Let $\hat{C}_{it} = w_{it}^{Y} d\log Y_{it} - w_{it}^{M} d\log M_{it}$. Then $\sum_i \hat{C}_{it} = \log F_t^{Q}$.

PROOF:

(17)
$$\begin{aligned}
&\sum_i w_{it}^{Y}(d\log Y_{it} - \log F^{Q}) - \sum_i w_{it}^{M}(d\log M_{it} - \log F^{Q}) \\
&= \frac{\sum_i (Y_{i,t+1}/V_{t+1} - Y_{it}/V_t) - \sum_i (M_{i,t+1}/V_{t+1} - M_{it}/V_t)}{\sum_j [m(Y_{jt}/V_t, Y_{j,t+1}/V_{t+1}) - m(M_{jt}/V_t, M_{j,t+1}/V_{t+1})]} \\
&= \frac{\sum_i (Y_{i,t+1} - M_{i,t+1})/V_{t+1} - \sum_i (Y_{it} - M_{it})/V_t}{\sum_j [m(Y_{jt}/V_t, Y_{j,t+1}/V_{t+1}) - m(M_{jt}/V_t, M_{j,t+1}/V_{t+1})]} \\
&= 0
\end{aligned}$$

Consequently, $\sum_i w_{it}^{Y} d\log Y_{it} - \sum_i w_{it}^{M} d\log M_{it} = [\sum_i w_{it}^{Y} - \sum_i w_{it}^{M}](\log F_t^{Q}) = \log F_t^{Q}$.

4.3 Exact Industry Contributions to Aggregate Productivity

Following the approach of Stiroh (2002, p. 1572, equation (6)), the weights that permit a decomposition of the log-change in real output can be used to show how industry productivity changes contribute to aggregate productivity change. Let LP_{it}^{Y} denote labor productivity in industry i, defined as the log-change in gross output per hour, or $d\log(Y_{it}/H_{it})$. In addition, define the value-added shares w_{it}^{V} as $w_{it}^{Y} - w_{it}^{M}$. Then a partial decomposition of the log-change in aggregate labor productivity is:

(18)
$$\begin{aligned}
ALP_t^{V} &= [\sum_i w_{it}^{V} LP_{it}^{Y}] - [\sum_i w_{it}^{M}(d\log M_{it} - d\log Y_{it})] \\
&\quad + [\sum_i w_{it}^{V}(d\log H_{it}) - d\log H_t].
\end{aligned}$$

The first term in equation (18) shows that an industry's direct contribution to aggregate labor productivity is its productivity in producing gross output times its average share of value added $w_{it}^{Y} - w_{it}^{M}$. The second term adjusts the industry's direct contribution to aggregate productivity for the effect of the change in the intermediate inputs required to produce a given amount of gross output. Combined, these terms provide the contribution of an industry's log-change in real value added per employee hour to the log-change in aggregate productivity.

Equation (18) is an incomplete decomposition of aggregate productivity growth because in the last term d log H_t is not expressed as a sum of industry contributions. This term represents an effect from changes in the allocation of hours between industries with different levels of average output per hour. For example, suppose that a high-productivity industry begins to contract out some average-productivity activity it had performed in-house to a low-productivity industry, with a concomitant movement of employees. Aggregate productivity is, of course, unchanged, but productivity (as measured by real value added per hour) rises in both of the affected industries. The negative allocation effect offsets the positive contributions of the rising productivity within the two industries to hold aggregate productivity constant.

We can add an expression that exactly accounts for the contributions of the labor allocation effect to equation (18) using an approach similar to Nordhaus' (2002, pp. 214-5) "Denison effect." Under this approach, the difference between an industry's value added share in the economy and its labor input share in the economy is used to measure the contribution to aggregate output of changes in the relative size of its labor force. For our exact decomposition of the labor reallocation effect, we use labor shares w_{it}^{H} that resemble Sato-Vartia weights:

(19) $$w_{it}^{H} = \frac{m(H_{it} / H_t, H_{i,t+1} / H_{t+1})}{\sum_j m(H_{jt} / H_t, H_{j,t+1} / H_{t+1})} .$$

Equation (19) makes an exact decomposition of the labor reallocation effect possible because the weights w_{it}^{H} add up to 1 and the weighted average $\sum_i w_{it}^{H}(d\log H_{it})$ equals $d\log H_t$, which is the only term in equation (19) not decomposed by industry. The relative amount of labor that is reallocated into industry i equals $d\log H_{it} - d\log H_t$. We assume that reallocated labor always has an opportunity cost equal to the economy's average level of productivity; that is, the labor that is released by an industry has the average level of real value added per hour in the industries where it is redeployed, and the extra labor that is absorbed by an industry would have had the average level of real valued added per hour in its alternative use.[15] Then the marginal effect on aggregate real output of labor reallocation into industry i is $w_{it}^{V*} - w_{it}^{H}$, where w_{it}^{V*} is the log-change in aggregate real output from a 1 log point change in hours in industry i and w_{it}^{H} is the log-change in aggregate real output per hour when the amount of labor representing a 1 log point change in industry i is added to an industry with the average productivity level. Thus, the contribution to the log-change in real output due to reallocation is $(w_{it}^{V*} - w_{it}^{H})(d\log H_{it} - d\log H_t)$, which essentially equals $(w_{it}^{V} - w_{it}^{H})(d\log H_{it} - d\log H_t)$.

Substituting $\sum_i w_{it}^{H}(d\log H_{it})$ for $d\log H_t$ in equation (18) and then subtracting $\sum_i (w_{it}^{V} - w_{it}^{H})(d\log H_t)$, which equals 0, gives:

[15] Note that when the aggregate under investigation excludes important industries, the average level of productivity in the aggregate may differ significantly from the average level of productivity in the economy as a whole. The decomposition of the labor reallocation effect must reflect the average level of productivity in the aggregate under investigation, because the labor reallocation effect for any aggregate reflects only reallocation within that aggregate.

(20)
$$ALP_t^V = \left[\sum_i w_{it}^V LP_{it}^Y\right] - \left[\sum_i w_{it}^M (d\log M_{it} - d\log Y_{it})\right] + \left[\sum_i (w_{it}^V - w_{it}^H)(d\log H_{it} - d\log H_t)\right].$$

In the last term in (20), an industry's contribution to the labor reallocation effect depends on its relative efficiency at using labor, measured by the difference between its share of GDP and its share of labor input, and the growth of its labor input share. An inefficient industry—one with a value added share w_{it}^V below its labor share w_{it}^H —has a positive labor reallocation effect if it releases labor for use in other industries, and a relatively efficient industry has a positive reallocation effect if it absorbs labor released by other industries.

4.4 Comparison with a Decomposition that Uses Real Value Added per Hour

The exact decomposition in equation (20) closely approximates a decomposition that, like the Bennet decomposition in equation (12), uses real value added per hour, albeit in log-change form. In the decomposition of the log-change in real value added per hour, the weights $\bar{h}_i$ in equation (12) are replaced by Sato-Vartia weights based not on industry hours but on industry constant-price value added. Let $w_{it}^{V*} \equiv m(V_{it}/V_t, V_{i,t+1}/V_{t+1}) / \sum_j m(V_{jt}/V_t, V_{j,t+1}/V_{t+1})$, and let the direct measure of the change in real value added per hour in industry i be $LP_{it}^V = d\log V_{it} - d\log H_{it}$. Then the logarithmic decomposition based on industries' value added productivity is:

(20′)
$$ALP_t^V = \left[\sum_i w_{it}^H LP_{it}^V\right] + \left[\sum_i (w_{it}^{V*} - w_{it}^H)[d\log(V_{it}/V_t)]\right] = \left[\sum_i w_{it}^{V*} LP_{it}^V\right] + \left[\sum_i (w_{it}^{V*} - w_{it}^H)[(d\log H_{it}/H_t)]\right].$$

In the first line of equation (20′) the weights on industry productivity gains are similar to the $\bar{h}_i$ weights in equation (12), but in the shift effect term on that line, the measure of changes in industry relative size, $d\log(V_{it}/V_t)$, differs from the measure given by Δh_i in equation (12) because it uses industry output; not labor input. An input-based measure of relative size would be more consistent with the intuition that the shift effect comes from changes in allocation of labor from low-productivity to high-productivity industries. Such a measure appears in the second line of equation (20′), but with this version of the shift effect, the weights on the industry productivity changes, w_{it}^{V*}, differ from the $\bar{h}_i$ weights in equation (12) because they are based on industry output. Nevertheless, the pattern of direct contributions implied by the $w_{it}^{V*} LP_{it}^V$ in equation (20′) can be expected to resemble the pattern implied by the first term in equation (12).

The measure of the contribution of industry i's value added productivity to aggregate productivity given by the second line of equation (20′) closely approximates the measure of the contribution of industry i's gross output productivity adjusted for its use of intermediate inputs that appears in equation (20). The value added contribution measure in $(20')$ equals the gross output contribution measure in (20) times a slope coefficient that approximately equals 1 plus an intercept that approximately equals 0. The slope λ_t equals the ratio of the normalization factors for the Sato-Vartia weights under the two approaches:

$$\lambda_t \equiv \frac{\sum_j [m(Y_{it}/V_t, Y_{j,t+1}/V) - m(M_{jt}/V_t, M_{j,t+1}/V_{t+1})]}{\sum_j m(V_{jt}/V_t, V_{j,t+1}/V_{t+1})}. \tag{21}$$

The intercept equals $(w_{it}^{V*} - \lambda_t w_{it}^{V})(d \log F_t^Q)$.

4.5 Consistency with Decompositions that Use Domar Weights

Readers familiar with the literature on industry contributions to productivity change may wonder whether the decomposition given by equation (20) is consistent with well-known decompositions that use Domar weights. Domar weights are ratios, such as w_{it}^{Y}, of industry gross output to aggregate value added. Domar weights are required to decompose multifactor productivity growth (see Gullikson and Harper, 1999, p. 51.) The use of w_{it}^{V} as a weight on gross output productivity in the first term of equation (20) may appear inconsistent with the need to use Domar weights. Equation (20) is, however, easily reconciled with the Domar weighting scheme. For this reconciliation, the third term in equation (20) can be disregarded because a reallocation effect is not part of the original Domar (1961) framework.[16] The Domar contribution of an industry to productivity change can be described as the sum of an output change contribution and an input change contribution. The output change contribution $w_{it}^{Y}(d \log Y_{it})$ is the sum of the $w_{it}^{M}(d \log Y_{it})$ part of the second term in (20) and the $w_{it}^{V}(d \log Y_{it})$ part of $w_{it}^{V} LP_{it}^{Y}$ in the first term. The only inputs explicitly considered in this paper are M_{it} and H_{it}, which is consistent with a production model in which V_{it} is identified with the cost of labor inputs in period t. The sum of $w_{it}^{V}(d \log H_{it})$ implicitly included in the first term of equation (20) and $w_{it}^{M}(d \log M_{it})$ from the second term effectively equals the Domar weight times the measure of combined labor and material inputs.

5. Comparison with Törnqvist Contributions to Productivity Change

In contrast to the exact approach to industry contributions to productivity change, an approximate approach based on industry-level Törnqvist indexes has been used for important studies of the sources of productivity change. The Törnqvist contributions to aggregate

[16] A Domar weighted decomposition of translog aggregate multi-factor productivity growth that includes reallocation effects was developed by Jorgenson, Gollop and Fraumeni (1987, p. 66) and used in modified form by Jorgenson, Ho and Stiroh (2002, p. 9.)

productivity change solve neither the problem of decomposing a Törnqvist measure of aggregate productivity change nor the problem of decomposing an aggregate Fisher measure. They fail to solve the Törnqvist decomposition problem because aggregation of Törnqvist measures of industry value added does not yield the measure of real GDP calculated from a Törnqvist index of final uses of commodities.

Let $\bar{v}_i$ denote a simple average of the current-year dollar shares of value added in periods t and t+1 in industry i, let $\bar{m}_i$ denote the average ratio of current-year dollar intermediate inputs to value added in industry i, let F_{it}^{QM} denote the Fisher quantity index for intermediate inputs to industry i, let F_{it}^{QY} denote the Fisher quantity index for gross output in industry i, and let $LP_{it}^{\tilde{Y}}$ denote labor productivity in industry i measured as the difference between $\log F_{it}^{QY}$ and the log-change in hours. (We use Fisher quantity indexes rather than Törnqvist indexes at the industry level because the available industry level indexes are Fisher indexes.) The Törnqvist decomposition formula from Stiroh (2002) is:

$$\text{(22)} \quad \begin{aligned} ALP_t^{\tilde{V}} &= [\textstyle\sum_i \bar{v}_{it} LP_{it}^{\tilde{Y}}] - [\textstyle\sum_i \bar{m}_{it}(d\log F_{it}^{QM} - d\log F_{it}^{QY})] \\ &\quad + [\textstyle\sum_i \bar{v}_{it} d\log H_{it} - d\log H_t]. \end{aligned}$$

The Törnqvist index weights in equation (22) differ from the Sato-Vartia index weights in equation (18) because they use simple averages rather then normalized logarithmic means and because they are based on current-year dollar measures of value added and intermediate inputs. An analysis of these differences suggests that their effect will often be small.

To explore the effect of the functional form difference, assume that the industry shares of aggregate value added (i.e. of GDP) are the same in current-year dollars as in constant dollars. Let $\gamma_{it} = |v_{i,t+1} - \bar{v}_{it}| / \bar{v}_{it}$, which is the two-period coefficient of variation of the industry i's value added share. Finally, note that a Taylor series for $\log(1+\gamma_{it})$ minus a Taylor series for $\log(1-\gamma_{it})$ equals $\frac{2}{3}\gamma_{it} + \frac{2}{5}\gamma_{it}^3 + \frac{2}{7}\gamma_{it}^5 + \cdots$. Then,

$$\text{(23)} \quad \begin{aligned} m(v_{it}, v_{i,t+1}) &= \bar{v}_{it} 2\gamma_{it} / \log[(1+\gamma_{it})/(1-\gamma_{it})] \\ &= \bar{v}_{it} / (1 + \tfrac{1}{3}\gamma_{it}^2 + \tfrac{1}{5}\gamma_{it}^4 + \tfrac{1}{7}\gamma_{it}^6 + \cdots) \end{aligned}$$

Let $\overline{\gamma_t^2} = \sum_i \bar{v}_{it}\gamma_{it}^2$. Since γ_{it}^4, γ_{it}^6, etc. are very small, a reduction in their coefficients will have almost no effect on the value of the expansion in equation (23). In particular,

$$\bar{v}_{it}(1 - \tfrac{1}{3}\gamma_{it}^2) = \bar{v}_{it} / (1 + \tfrac{1}{3}\gamma_{it}^2 + \tfrac{1}{9}\gamma_{it}^4 + \tfrac{1}{27}\gamma_{it}^6 + \cdots) \approx \bar{v}_{it} / (1 + \tfrac{1}{3}\gamma_{it}^2 + \tfrac{1}{5}\gamma_{it}^4 + \tfrac{1}{7}\gamma_{it}^6 + \cdots) = m(v_{it}, v_{i,t+1}).$$

Therefore, the Sato-Vartia index weight for industry i based on current-year dollar shares, denoted by $\bar{w}_{it}$, approximately equals $\bar{v}_{it}(1 - \frac{1}{3}\gamma_{it}^2)/(1 - \frac{1}{3}\overline{\gamma^2})$. Another version of this approximation simply adjusts the Törnqvist weights by amounts proportional to deviations in the squared coefficients of variation:

$$\bar{w}_{it} \approx \bar{v}_{it}[1-\frac{1}{3}(\gamma_{it}^{2}-\overline{\gamma_{t}^{2}})]. \qquad (24)$$

Equation (24) reveals that the Sato-Vartia index formula differs from the Törnqvist index formula only by incorporating an adjustment to each item weight that is inversely proportional to the excess volatility of its expenditure share. Consequently, industries with volatile shares tend to receive slightly lower weights in the contributions formula based on the Sato-Vartia index than they do in equation (22). However, equation (24) also implies that any differences in weights caused by the use of logarithmic means instead of the simple averages of the Törnqvist index are likely to be small.

Another difference between the exactly additive contributions to productivity change in equation (20) and the Törnqvist contributions in equation (22) is the use of constant-price measures of real change in equation (20) and the use of industry-level Fisher indexes in equation (22). However, $d\log Y_{it}$, the log-change in the constant-price index for gross output in equation (20), can be expected to differ only slightly from $d\log F_{it}^{QY}$ in equation (22), and similarly $d\log F_{it}^{QM} \approx d\log M_{it}$. For example, substituting into equation (3) and simplifying shows that F_{it}^{QY} can be expressed as an average of the Laspeyres and Paasche quantity indexes for gross output in industry i, L_{it}^{QY} and P_{it}^{QY}, with weights proportional to the Laspeyres and Fisher price indexes, L_{it}^{PY} and F_{it}^{PY}.[17] The constant-price measure of gross output change, $d\log Y_{it}$, differs from the Fisher measure only by giving the Laspeyres quantity index a weight proportional to the overall Fisher price index for GDP, F_{t}^{P}, instead of the industry-specific index, F_{it}^{PY}. Thus,

$$\begin{aligned} d\log Y_{it} - d\log F_{it}^{QY} &= \log\left[\frac{F_{t}^{P}}{F_{t}^{P}+L_{it}^{PY}}L_{it}^{QY} + \frac{L_{it}^{PY}}{F_{t}^{P}+L_{it}^{PY}}P_{it}^{QY}\right] \\ &\quad - \log\left[\frac{F_{it}^{PY}}{F_{it}^{PY}+L_{it}^{PY}}L_{it}^{QY} + \frac{L_{it}^{PY}}{F_{it}^{PY}+L_{it}^{PY}}P_{it}^{QY}\right]. \end{aligned} \qquad (25)$$

The difference in weights between the terms of equation (25) generally has a very small effect.

The resemblance of the terms in equation (22) to their counterparts in equation (20) means that the Törnqvist contributions can be expected to approximate the exactly additive contributions closely. Furthermore, it implies that Törnqvist weights can be substituted for the Sato-Vartia weights in the labor reallocation term of equation (20) to obtain approximate contributions to the labor reallocation effect. On the other hand, the formula for the log change in total real GDP implicit in equation (22), which is derived in appendix A as equation (A-4), differs considerably from the direct Fisher measure of this change. This suggests that the total of

[17] Dumagan (2002) discusses this expression for contributions to change in the Fisher index; see his equation (9).

the contributions calculated using the Törnqvist approach could differ from the aggregate change in productivity by a non-trivial amount. We investigate the question of how well (A-4) approximates the log-change in the Fisher quantity index empirically in the next section.

6. Empirical Results

6.1 Differences between Exact Contributions and Törnqvist Contributions

To investigate the differences between exactly additive industry contributions to productivity change and Törnqvist contributions, we use 2003 vintage data for the years from 1987 to 2001 from BEA's GDP-by-Industry accounts. In these accounts in 2003, industry nominal value added was estimated from income data, so that the sum over all industries of value added equals the income-side estimate of GDP. This sum is, therefore, less than the expenditure-side estimate of GDP from the 2003 vintage NIPA data by an amount equal to the statistical discrepancy. For years from 1987 to 1995, the statistical discrepancy averaged 0.36 percent of GDP, but from 1996 to 2001 it averaged about –0.41 percent of GDP. Other things being equal, therefore, output measures based on income side data can be expected to imply larger gains between these periods than output measures based on expenditure data.

We include in our analysis only the industries in the nonfarm private business sector.[18] Some of these industries include nonprofit institutions, many of which are measured in a way that assumes no productivity change; these institutions are important in the health services, educational services, and social services industries, and in membership organizations. We exclude holding and investment companies because of measurement problems and the owner-occupied housing portion of the real estate industry because it has no labor input.[19] These exclusions leave 58 industries in the data set, which account for about 92 percent of the value added of the nonfarm private business sector.

To construct the constant-price measures required for the exact decomposition of industry sources of productivity growth (the V_{it}, Y_{it}, M_{it} and the $V_{i,t+1}$, $Y_{i,t+1}$, $M_{i,t+1}$) we use unpublished data on the Laspeyres and Paasche components of the Fisher indexes in the GDP-by-Industry accounts. In addition, to measure labor inputs we use published data on full-time equivalent employees (FTE's) by detailed industry from the NIPAs.

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[18] Although the theoretical discussion treated all of GDP as the aggregate of interest, studies of industry sources of productivity change generally exclude some industries whose productivity is not well measured.

[19] Owner-occupied housing is removed by subtracting its nominal and deflated gross output and intermediate inputs based on data from NIPA tables 8.12 and 8.13. In data released after the 2003 Comprehensive Revision of the NIPAs (after the research for this paper was done), owner-occupied housing was longer part of the real estate sector.

Table 1. Exact and Törnqvist Contributions to Value Added Productivity of FTEs: Average Growth Rates in Percentage Points

Industry	Exactly Additive Value Added Productivity 1987-1995	Törnqvist Value Added Productivity 1987-1995	Exactly Additive Value Added Productivity 1995-2001	Törnqvist Value Added Productivity 1995-2001
Agricultural, forestry and fishing services	-0.0124	-0.0124	-0.0009	-0.0009
Metal mining	0.0052	0.0052	0.0119	0.0120
Coal mining	0.0252	0.0252	0.0149	0.0150
Oil and gas extraction	0.0475	0.0484	-0.0518	-0.0528
Nonmetallic minerals, except fuels	0.0000	0.0000	0.0068	0.0068
Construction	0.0067	0.0067	-0.0554	-0.0553
Lumber and wood products	-0.0208	-0.0208	-0.0089	-0.0089
Furniture and fixtures	0.0029	0.0029	0.0034	0.0034
Stone, clay, and glass products	0.0164	0.0164	-0.0013	-0.0013
Primary metal industries	0.0212	0.0212	0.0197	0.0198
Fabricated metal products	0.0226	0.0226	0.0021	0.0022
Industrial machinery and equipment	**0.1550**	**0.1545**	**0.2283**	**0.2255**
Electronic and other electric equipment	**0.2914**	**0.2878**	**0.3732**	**0.3639**
Motor vehicles and equipment	0.0154	0.0154	0.0187	0.0190
Other transportation equipment	-0.0098	-0.0097	0.0330	0.0331
Instruments and related products	0.0016	0.0018	-0.0176	-0.0172
Miscellaneous manufacturing industries	0.0065	0.0065	0.0151	0.0151
Food and kindred products	0.0493	0.0494	-0.0708	-0.0701
Tobacco products	-0.0048	-0.0049	-0.0369	-0.0369
Textile mill products	0.0153	0.0153	0.0063	0.0063
Apparel and other textile products	0.0145	0.0145	0.0178	0.0178
Paper and allied products	-0.0010	-0.0010	0.0094	0.0095
Printing and publishing	-0.0375	-0.0375	-0.0163	-0.0163
Chemicals and allied products	0.0558	0.0558	0.0357	0.0357
Petroleum and coal products	-0.0039	-0.0035	0.0022	0.0015
Rubber and miscellaneous plastics products	0.0321	0.0321	0.0279	0.0279
Leather and leather products	0.0041	0.0041	-0.0004	-0.0004
Railroad transportation	0.0265	0.0265	0.0127	0.0127
Local and interurban passenger trans	-0.0070	-0.0070	0.0032	0.0032
Trucking and warehousing	0.0439	0.0438	-0.0018	-0.0019
Water transportation	0.0083	0.0083	0.0032	0.0032
Transportation by air	0.0106	0.0105	0.0074	0.0073
Pipelines, except natural gas	-0.0045	-0.0047	0.0041	0.0041
Transportation services	-0.0017	-0.0018	0.0109	0.0109
Telephone and telegraph	0.1362	0.1362	0.1372	0.1367
Radio and television	0.0429	0.0430	-0.0133	-0.0134
Electric, gas, and sanitary services	0.1053	0.1054	-0.0009	-0.0012

Table 1. Continued

Industry	Exactly Additive Value Added Productivity 1987-1995	Törnqvist Value Added Productivity, 1987-1995	Exactly Additive Value Added Productivity, 1995-2001	Törnqvist Value Added Productivity, 1995-2001
Wholesale trade	0.2419	0.2416	0.5484	0.5482
Retail trade	0.1022	0.1022	0.5180	0.5178
Depository institutions	0.0953	0.0952	0.1424	0.1425
Nondepository institutions	0.0105	0.0101	0.0796	0.0797
Security and commodity brokers	0.0636	0.0617	0.2333	0.2375
Insurance carriers	0.0217	0.0217	0.0218	0.0220
Insurance agents, brokers, and services	-0.0430	-0.0429	-0.0021	-0.0021
Real estate w/o owner occ	0.1227	0.1225	0.1156	0.1155
Hotels and other lodging places	0.0070	0.0070	-0.0149	-0.0149
Personal services	-0.0016	-0.0016	-0.0010	0.0010
Business services	0.0153	0.0153	0.0323	0.0330
Auto repair, services, and parking	-0.0107	-0.0107	0.0097	0.0097
Miscellaneous repair services	-0.0025	-0.0025	-0.0166	-0.0165
Motion pictures	-0.0092	-0.0092	0.0031	0.0031
Amusement and recreation services	-0.0029	-0.0029	-0.0123	-0.0123
Health services	-0.1526	-0.1526	-0.0125	-0.0125
Legal services	-0.0085	-0.0085	0.0035	0.0032
Educational services	-0.0028	-0.0028	-0.0141	-0.0141
Social services	-0.0019	-0.0019	-0.0159	-0.0159
Membership organizations	0.0018	0.0018	-0.0445	-0.0445
Other services	-0.0052	-0.0052	0.0865	0.0865
TOTAL	**1.500**	**1.494**	**2.391**	**2.391**
Addendum:				
Total excluding industrial machinery and electronic equipment industries	**1.054**	**1.052**	**1.790**	**1.794**
Total excluding productivity change in 1987-88 from average for the pre-1995 period	**1.215**	**N/A**	**N/A**	**N/A**

Note: Excludes government, farms, owner-occupied housing, investment and holding company offices, and private households. FTE–Full time equivalent employment.

Table 1 shows the contributions of industries' log-changes in real value added per employee hour to the log-change in aggregate real value added per employee hour net of the labor reallocation effect, which is excluded. The exactly additive contributions in table 1 are calculated as the sum of the first two terms in equation (20), and the Törnqvist contributions are calculated as the sum of the corresponding terms in equation (22). Averages for two periods are shown, one from 1987 to 1995, and another for the period from 1995 to 2001. A speedup in productivity growth seems to start in 1995,[20] so a comparison of these two periods provides important evidence on industry contributions to the productivity speedup.

The exactly additive contributions to productivity change in table 1 generally differ from their Törnqvist counterparts by less than 0.001, but a few important industries have more appreciable discrepancies. Most notably, the "industrial machinery and equipment" industry, which contains computers, and the "electronic and other electric equipment" industry, which contains semiconductors, both have slightly higher contributions to productivity growth based on the exact method than they do based on the Törnqvist method. The combined contribution of these two industries in the pre-1995 period is 0.446 percentage points using the exact method and 0.442 using the Törnqvist method; in the post-1995 period their exact contribution is 0.602 and their Törnqvist contribution is 0.589. Their exact contribution to the productivity speedup is therefore 0.156, compared with a Törnqvist contribution of 0.147.

The tendency of the Törnqvist method to imply smaller estimates is evident in the aggregate, as well. For the pre-1995 period, the total over all industries of value added contributions is 1.500 percentage points using the exact method and 1.494 using the Törnqvist method. Since the goal is to decompose the direct measure of aggregate productivity change, the differences between the total of the Törnqvist contributions and the exact total may be interpreted as indicative of downward bias in the Törnqvist contribution formula. The two approaches give the same total for the post-1995 period, so the speedup in aggregate real value added per FTE net of the reallocation effect is lower using the exact method than using the Törnqvist method: 0.891 compared with 0.897. Aggregate output grew sharply between 1987 and 1988, and a negative statistical discrepancy in 1988 made the growth of the income-based measure particularly strong. As a result, the productivity speedup appears larger when the starting point is 1988 rather than when it is 1987; in particular, the value in the bottom row of table 1 implies a speedup of 1.176 percent per year between the period from 1988 to 1995 and the period from 1995 to 2001.

6.2 Estimates of Industry Contributions to the Productivity Speedup

Table 2 shows contributions of important groups of industries — including those that have negative contributions — to the productivity speedup based on comparisons of the period from 1988 to 1995 in table 1 to the period from 1995 to 2001. One of the advantages of the exact industry contributions is that they can be combined into analytically interesting groups of industries, such as IT or ICT industries, with no loss of precision. Furthermore, combining the exact contributions of the individual industries in a group yields a result virtually identical to the one that could be calculated by aggregating these industries in the I-O tables and then calculating the exact contribution of the aggregate.[21]

[20] Output per hour for nonfarm business from BLS grows at an average rate of about 1.5 percent per year from 1987 to 1995, and at 2.4 percent per year from 1995 to 2001. BLS data used to construct multifactor productivity growth in nonfarm business up to 2001 implies that 0.3 percentage points of the speedup in output per hour growth come from faster multifactor productivity growth, and 0.6 percentage points come from growth of ordinary capital and human capital as measured in the "labor composition" adjustment. See http://www.bls.gov/lpc/home.htm#data.

[21] The two results may not be precisely identical because the factor needed to scale the Sato-Vartia weights to add up to 1, which is itself quite near 1, may change when industries are aggregated in the I-O tables.

In the aggregate, the contribution to the productivity speedup of changes in gross output per hour (0.40 percent per year) is smaller than the indirect effect of declining intermediate input utilization (0.49 percent per year). The labor reallocation effect is negative in both periods, but it contributes to the productivity speedup by rising from –0.46 percent per year before 1995 to – 0.33 percent per year after 1995.

The largest contribution to the productivity speedup in table 2, of 0.74 percentage points, comes from the combined wholesale and retail trade industries. Productivity gains from improvements in business processes (e.g. a "Wal-Mart effect") facilitated by increased IT use, as well as the substitution of capital for labor, are probably both important reasons for the surge in productivity in the distribution industries. In addition, Bosworth and Triplett (2004) observe that productivity in these industries may benefit in another way from rising quality levels of IT goods; in particular, if the amount of real resources required to sell a box to a retail customer is constant, but we count the box as containing twice as much "computing power" as before, measured productivity in retailing will rise. A preliminary analysis suggests, however, that this effect — which is sometimes viewed as a spurious increase in productivity — is small.

Another industry with a large contribution to the productivity speedup in table 2 is securities and commodity brokers. This industry makes intensive use of IT capital goods, so falling prices for these goods may have enabled it to substitute more capital for labor and intermediate inputs in the later period and to realize gains in multifactor productivity.

The health services industry made an important contribution to the overall speedup because its value added productivity went from a negative growth rate to around 0 in the post-1995 period. Most of this improvement resulted from a large improvement in the growth rate of gross output per hour, which became positive after 1995. Although the pickup in productivity in this industry may be real, its relatively poor performance in the pre-1995 period could partly be due to measurement error, perhaps as a result of quality improvements not captured by its output price index. (In addition, health services contains many nonprofit institutions whose real output is partly measured as a deflated cost of inputs including compensation of employees, resulting in a questionable measure of productivity change.) The productivity speedup may, therefore, partly reflect improvements in measurement techniques in the late 1990s.

The industries that contain computers and semiconductors (industrial machinery and electrical equipment) make relatively large contributions to productivity growth in both periods. This qualitative result is consistent with what previous researchers have found, but table 2 shows a slightly smaller pickup in this contribution than others have found; indeed, less than one-sixth of the productivity speedup (or 0.156 percentage points) is directly attributable to these industries.[22] Within these industries, most of the productivity speedup comes from declining relative use of intermediate inputs; not rising gross output per hour. The relative decline in the real intermediate inputs after 1995 partly reflects a relatively large pickup in the rate of decline of the price deflator for these industries' gross output.

[22] Note that contributions to the level or to the speedup of aggregate productivity must be interpreted carefully because some negative contributions are present. In table 2, as a group the industries with positive contributions can "explain" about 180 percent of the total speedup.

Table 2. Contributions to Aggregate Growth in Real Output per FTE for Nonfarm Private Business: Selected Groups of Industries

Group of Industries	Gross Output Labor Productivity	LESS: Intermediate Input Intensity Effect	EQUALS: Value Added Labor Productivity	PLUS: Labor Reallocation Effect	EQUALS: Contribution to Agg. Labor Productivity
Nonfarm Private Business					
Average, 1995-2001	2.290	-0.101	2.391	-0.326	2.065
LESS: Average, 1988-1995	1.891	0.391	1.500	-0.460	1.040
EQUALS: Speedup	0.399	-0.492	0.891	0.134	1.025
Industries with Positive Contributions to Speedup in Aggregate Productivity Growth					
All 34 Industries with Positive Total Contributions					
Average contribution, 1995-2001	2.225	-0.418	2.643	-0.029	2.614
LESS: Average Contribution, 1988-1995	1.525	0.522	1.003	-0.225	0.778
EQUALS: Contribution to Speedup	0.700	-0.940	1.640	0.196	1.836
Wholesale and Retail Trade					
Average contribution, 1995-2001	0.650	-0.417	1.066	-0.008	1.059
LESS: Average Contribution, 1988-1995	0.384	0.040	0.344	-0.022	0.322
EQUALS: Contribution to Speedup	0.266	-0.456	0.722	0.014	0.736
Security and Commodity Brokers					
Average contribution, 1995-2001	0.132	-0.102	0.233	0.047	0.280
LESS: Average Contribution, 1988-1995	0.080	0.016	0.064	0.008	0.071
EQUALS: Contribution to Speedup	0.052	-0.118	0.170	0.039	0.209
Health Services					
Average contribution, 1995-2001	0.050	0.062	-0.012	0.000	-0.012
LESS: Average Contribution, 1988-1995	-0.044	0.108	-0.153	-0.034	-0.187
EQUALS: Contribution to Speedup	0.094	-0.046	0.140	0.034	0.175
Electronic and other electric equipment					
Average contribution, 1995-2001	0.276	-0.097	0.373	-0.011	0.362
LESS: Average Contribution, 1988-1995	0.250	-0.041	0.291	-0.012	0.280
EQUALS: Contribution to Speedup	0.026	-0.056	0.082	0.000	0.082
Telephone and Telegraph					
Average contribution, 1995-2001	0.204	0.067	0.137	0.039	0.176
LESS: Average Contribution, 1988-1995	0.155	0.019	0.136	-0.035	0.101
EQUALS: Contribution to Speedup	0.049	0.048	0.001	0.074	0.075
Industrial Machinery					
Average contribution, 1995-2001	0.139	-0.090	0.228	-0.001	0.227
LESS: Average Contribution, 1988-1995	0.159	0.004	0.155	-0.002	0.153
EQUALS: Contribution to Speedup	-0.021	-0.094	0.073	0.001	0.074

Table 2. Continued

Group of Industries	Gross Output Labor Productivity	LESS: Intermediate Input Intensity Effect	EQUALS: Value Added Labor Productivity	PLUS: Labor Reallocation Effect	EQUALS: Contribution to Agg. Labor Productivity
All Services except health services					
Average contribution, 1995-2001	0.322	0.304	0.018	-0.201	-0.183
LESS: Average Contribution, 1988-1995	0.148	0.169	-0.071	-0.155	-0.226
EQUALS: Contribution to Speedup	0.174	0.135	0.089	-0.046	0.043
Business Services					
Average contribution, 1995-2001	0.205	0.173	0.032	-0.069	-0.036
LESS: Average Contribution, 1988-1995	0.095	0.079	0.015	-0.057	-0.042
EQUALS: Contribution to Speedup	0.111	0.094	0.017	-0.012	0.005
Industries with Negative Contributions to the Speedup in Aggregate Productivity Growth					
All 24 Industries with Negative Total Contributions					
Average contribution, 1995-2001	0.065	0.317	-0.252	-0.297	-0.549
LESS: Average Contribution, 1988-1995	0.366	-0.131	0.498	-0.236	0.262
EQUALS: Contribution to Speedup	-0.301	0.448	-0.749	-0.062	-0.811
Electric, Gas, and Sanitary Services					
Average contribution, 1995-2001	0.060	0.061	-0.001	-0.068	-0.069
LESS: Average Contribution, 1988-1995	0.094	-0.011	0.105	-0.047	0.058
EQUALS: Contribution to Speedup	-0.033	0.073	-0.106	-0.021	-0.128
Food and Kindred Products					
Average contribution, 1995-2001	0.016	0.087	-0.071	-0.003	-0.073
LESS: Average Contribution, 1988-1995	0.026	-0.023	0.049	-0.002	0.048
EQUALS: Contribution to Speedup	-0.010	0.110	-0.120	-0.001	-0.121
Nondurable Manufacturing					
Average contribution, 1995-2001	0.137	0.162	-0.025	0.002	-0.023
LESS: Average Contribution, 1988-1995	0.137	0.013	0.111	-0.017	0.094
EQUALS: Contribution to Speedup	0.000	0.149	-0.136	0.019	-0.117
Membership organizations					
Average contribution, 1995-2001	-0.031	0.014	-0.045	-0.071	-0.116
LESS: Average Contribution, 1988-1995	0.000	-0.002	0.002	-0.029	-0.028
EQUALS: Contribution to Speedup	-0.030	0.016	-0.046	-0.042	-0.088
Construction					
Average contribution, 1995-2001	-0.097	-0.042	-0.055	-0.018	-0.073
LESS: Average Contribution, 1988-1995	-0.049	-0.056	0.007	0.004	0.011
EQUALS: Contribution to Speedup	-0.048	0.014	-0.062	-0.022	-0.084

Two industries in the service sector round out our group of positive contributors to the productivity speedup. These are telephone and telegraph, and business services. In the high-productivity telephone industry, rapid growth of hours boosted its contribution from the labor reallocation effect from –0.035 in the pre-1995 period to 0.039 in the post-1995 period, and value added per hour also accelerated. In business services, gross output per FTE grew much faster in the post-1995 period, but a rise in use of intermediate inputs appeared to account for most of this gain, leaving only a small contribution to aggregate value added per FTE. Also, a pickup in employment in the low-productivity business services industry reduced its labor reallocation contribution from –0.057 to –0.069. This may reflect an increased tendency for high-productivity industries to contract out activities that have low value added per hour worked. Since industries engaged in such contracting out would show a gain in their value added productivity without making any real improvement in production technology, some of the negative labor reallocation contribution of business services could arguably be attributable to other industries that showed large productivity gains.

Three of the four detailed industries in table 2 with noteworthy negative contributions to the productivity speedup include negative components from the labor reallocation effect. Decelerating growth of employment in the capital-intensive electric, gas and sanitary services industry depressed its labor reallocation contribution from –0.047 to –0.068.

Finally, table 2 shows that increased utilization of intermediate inputs is an important cause of a productivity slowdown in two industries: food and kindred products, and electric, gas and sanitary services.[23] Business services such as payroll processing may be increasing their use of computer power more rapidly than their growth of real output. Estimates of the growth of the intermediate inputs in the food product manufacturing industry may be affected by difficulties in estimating the portion of the output of vertically integrated producers of food products attributable to the farm industry. The growing consumption of organic foods, which are more expensive to farm, may also have contributed to the relatively rapid growth of intermediate inputs. Finally, the electric services may have substituted to cleaner, more expensive fuels, such as lower sulfur coal or natural gas, to comply with environmental standards. Such substitution would likely register as growing use of intermediate inputs. Also, since gas-burning electric plants are less capital intensive than most other kinds of powered plants, it may also result in the substitution of intermediate inputs for capital services.

7. Conclusion

This paper has derived exactly additive formulas for the decomposition of industry sources of a Fisher measure of aggregate labor productivity growth. The Törnqvist formulas for industry contributions to labor productivity change developed by Basu and Fernald (1995 and 1997) and by Stiroh (2002) theoretically approximate the exact contributions. In empirical tests, the Törnqvist formulas exhibit a slight downward bias in measuring aggregate productivity growth, and in measuring the contributions of the IT producing industries to aggregate

[23] Basu (1995) was among those who urged a closer look at the role of intermediate inputs more than a decade ago.

productivity growth. Nevertheless, on the whole, the agreement between the Törnqvist formulas and the exact formulas is remarkably close. The results therefore show that the Törnqvist approximations provide acceptable measures of industry contributions to aggregate labor productivity change.

This paper also provides new empirical evidence on industry contributions to labor productivity growth. The IT producing industries directly account for a quarter to two-fifths of aggregate productivity growth, but their direct contribution to the post-1995 productivity speedup in productivity growth is only around one-sixth of the total speedup. In contrast, the wholesale and retail trade industries account for more than half of the speedup.

Appendix: Derivation of Törnqvist Measure of Aggregate Growth

Let $d\log \hat{V}_t d$ denote the change in real GDP implied by the Törnqvist formulas for industry contributions to productivity change. To solve for $d\log \hat{V}_t$, let y_{it} denote current-dollar gross output in industry i, let m_{it} denote current-dollar intermediate inputs, and let v_t denote current-dollar GDP, or the sum over all industries of current-dollar value added. Then, substituting v_{it} for $y_{it} - m_{it}$, the change in aggregate output implied by equation (22) is

(A-1)

$$\begin{aligned} d\log \hat{V}_t &= \sum_i 0.5\left[\frac{y_{i,t+1}}{v_{t+1}} + \frac{y_{it}}{v_t}\right](d\log F_{it}^{QY}) - \sum_i 0.5\left[\frac{m_{i,t+1}}{v_{t+1}} + \frac{m_{it}}{v_t}\right](d\log F_{it}^{QY}) \\ &= \sum_i 0.5\left[\frac{y_{i,t+1}}{v_{t+1}} + \frac{y_{it}}{v_t}\right]\left[d\log F_{it}^{QY} - \frac{m_{i,t+1}/v_{t+1} + m_{it}/v_t}{y_{i,t+1}/v_{t+1} + y_{it}/v_t}(d\log F_{it}^{QM})\right] \\ &= \sum_i 0.5\left[\frac{v_{i,t+1}}{v_{t+1}} + \frac{v_{it}}{v_t}\right]\left[\frac{y_{i,t+1}/v_{t+1} + y_{it}/v_t}{v_{i,t+1}/v_{t+1} + v_{it}/v_t}\right] \\ &\quad \times\left[d\log F_{it}^{QY} - \frac{m_{i,t+1}/v_{t+1} + m_{it}/v_t}{y_{i,t+1}/v_{t+1} + y_{it}/v_t}(d\log F_{it}^{QM})\right]. \end{aligned}$$

Using the notation of Stiroh's (2002) equation (5), define $\bar{s}_{Mit}$ as the ratio of average deflated intermediate inputs to average deflated gross outputs:

(A-2) $$\bar{s}_{Mit} \equiv \frac{m_{i,t+1}/v_{t+1} + m_{it}/v_t}{y_{i,t+1}/v_{t+1} + y_{it}/v_t}.$$

Also, following Basu and Fernald (1997), define the log change in the arbitrary industry i's real value added as:

(A-3) $$d\log \hat{V}_{it} \equiv \frac{d\log F_{it}^{QY} - \bar{s}_{Mit}(d\log F_{it}^{QM})}{1 - \bar{s}_{Mit}}.$$

Then the log change in aggregate output implied by the Törnqvist decomposition is:

(A-4) $$d\log \hat{V}_t = \sum_i \bar{v}_{it}(d\log \hat{V}_{it}).$$

The functional form in equation (A-4) is quite different from the formula for the Fisher quantity index for GDP. This makes an analytical analysis of how well $d\log \hat{V}_t$ approximates $d\log V_t$ difficult, leaving the question to be addressed with empirical evidence.

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