Chapter 6

LABOR PRODUCTIVITY: AVERAGE VS. MARGINAL

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1. Introduction

Most headline productivity measures refer to the *average* product of labor, with productivity growth being typically explained by capital deepening and technological progress. One might argue, however, that from an economic perspective a more relevant measure of the productivity of labor is its *marginal* product. This is certainly true if one is interested in the progression of real wages. It turns out, however, that as long as the income share of labor remains essentially constant through time, the two productivity measures give almost identical results. In the case of the United States, the share of labor *has* been fairly steady over the past thirty years.[2] Moreover, the paths of both measures of labor productivity for the United States have been very similar over the 1971-2001 period.

The stability of the labor share also explains why the Cobb-Douglas production function – which restricts the Hicksian elasticity of complementarity between inputs to be unity and thus forces the input shares to be constant – appears to fit U.S. data reasonably well. Any increase in the relative endowment of capital or any technological change, independently of whether it is labor or capital augmenting, necessarily leaves factor shares unchanged with this specification, and thus is measured to impact on the average and marginal products of labor to exactly the same extent. A more thorough look at the evidence, however, reveals that the historical empirical constancy of U.S. factor shares is *not* a law of nature; it is the outcome of opposing forces.

Using a functional form more flexible than the Cobb-Douglas, we find on the one hand that over the past three decades the Hicksian elasticity of complementarity between labor and capital has been significantly greater than one. Thus capital deepening, other things equal, has lead to an increase in the share of labor and thus raised its marginal product by relatively more than its average product. On the other hand, we find that technological change has had an offsetting effect over 1971-2001. It has basically been labor augmenting, and given the large elasticity of complementarity, has tended to reduce the share of labor and thus to raise its average product relative to its marginal product. This paper seeks to analytically disentangle these effects and proposes a measurement methodology which is then applied to produce a multiplicative decomposition of the average and marginal U.S. labor productivity over the past three decades.

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[2] Between 1971 and 2001, the GDP share of labor in the United States fluctuated over the range of .70 to .74, with an apparent mild downward time trend. See the appendix for a description of the data.

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While labor productivity is often the focus of attention, many economists are more interested in *total factor productivity*. Though less intuitive, total factor productivity, as indicated by its name, is more general. It encompasses all factors of production rather than just one of them. It turns out that total factor productivity is an essential component of the average productivity of labor. A third contribution of this paper is to document this important relationship. We present estimates for the United States for the 1971-2001 period that are derived from two different approaches: an econometric approach and one based on index numbers.

A fourth contribution of the paper is to move beyond the rather restrictive two-input, one-output production-function setting. We expand the model by adopting the GDP function framework that allows for many inputs and outputs, including imports and exports. This not only makes it possible to get a better estimate of the elasticity of complementarity between domestic primary inputs, but it also shows that there are additional forces at work including changes in the terms of trade and in the real exchange rate. Complete multiplicative decompositions of both measures of labor productivity and of total factor productivity are provided for this case as well for the United States for the 1971-2001 time period.

2. The Two-Input, One-Output Case

Assume that the aggregate technology can be represented by the following two-input, one-output production function:

$$y_t = y(v_{L,t}, v_{K,t}, t) \ , \tag{1}$$

where y_t measures the quantity of output, $v_{L,t}$ denotes the input of labor services, and $v_{K,t}$ is the input of capital services, with all three quantities being measured at time *t*. Note that the production function itself is allowed to shift over time to account for technological change. We assume that the production function is linearly homogeneous, increasing, and concave with respect to the two input quantities.

Under competitive conditions and profit maximization, the following first order conditions must be met:

$$y_L(v_{L,t}, v_{K,t}, t) \equiv \frac{\partial y(v_{L,t}, v_{K,t}, t)}{\partial v_{L,t}} = \frac{w_{L,t}}{p_t} \tag{2}$$

$$y_K(v_{L,t}, v_{K,t}, t) \equiv \frac{\partial y(v_{L,t}, v_{K,t}, t)}{\partial v_{K,t}} = \frac{w_{K,t}}{p_t} , \tag{3}$$

where $w_{L,t}$ and $w_{K,t}$ represent the rental prices of labor and capital, and p_t is the price of output. The partial derivative $y_L(\cdot)$ on the left-hand side of (2) is the *marginal* product of labor.

The *average* product of labor ($g_{L,t}$), on the other hand, is simply defined as:

$$g_{L,t} \equiv \frac{y_t}{v_{L,t}} . \tag{4}$$

Using production function (1), we can also write the average product of labor as follows:

$$(5) \qquad g_{L,t} = g_L(v_{L,t}, v_{K,t}, t) \equiv \frac{y(v_{L,t}, v_{K,t}, t)}{v_{L,t}}.$$

An index of *average* labor productivity ($A_{t,t-1}$) can be expressed as one plus the rate of increase in the average product of labor between period *t*-1 and period *t*, which is:

$$(6) \qquad A_{t,t-1} \equiv \frac{g_L(v_{L,t}, v_{K,t}, t)}{g_L(v_{L,t-1}, v_{K,t-1}, t-1)}.$$

Similarly, we can define an index of *marginal* labor productivity ($M_{t,t-1}$) as:

$$(7) \qquad M_{t,t-1} \equiv \frac{y_L(v_{L,t}, v_{K,t}, t)}{y_L(v_{L,t-1}, v_{K,t-1}, t-1)}.$$

Note that it follows from the linear homogeneity of the production function that both $g_L(\cdot)$ and $y_L(\cdot)$ are homogeneous of degree zero in $v_{L,t}$ and $v_{K,t}$. The same is therefore true for the two measures of labor productivity, which thus depend only on changes in *relative* factor endowments and on the passage of time.

Next, we define $s_{L,t}$ as the share of labor in total revenues (i.e., GDP):

$$(8) \qquad s_{L,t} \equiv \frac{w_{L,t} v_{L,t}}{p_t y_t}.$$

It follows from (1), (2), (4) and (5) that:

$$(9) \qquad s_{L,t} = s_L(v_{L,t}, v_{K,t}, t) \equiv \frac{y_L(v_{L,t}, v_{K,t}, t)}{g_L(v_{L,t}, v_{K,t}, t)}.$$

Using (9) in (6) and (7), we can rewrite the index of marginal labor productivity growth as

$$(10) \qquad M_{t,t-1} = S_{t,t-1} \cdot A_{t,t-1},$$

where $S_{t,t-1}$ is the labor share index:

$$(11) \qquad S_{t,t-1} \equiv \frac{s_L(v_{L,t}, v_{K,t}, t)}{s_L(v_{L,t-1}, v_{K,t-1}, t-1)}.$$

This index is greater or smaller than one, depending on whether the share of labor has increased or fallen between period *t*-1 and period *t*.

3. The Role of the Hicksian Elasticity of Complementarity

According to (10), the growth of the marginal productivity of labor will be higher (lower) than the growth of the average productivity if technological progress and changes in relative factor endowments lead to an increase (decrease) in labor's share over time. Using (9), we find:

$$\begin{aligned}\frac{\partial s_L(\cdot)}{\partial v_{K,t}} &= \frac{g_L(\cdot)\,\partial y_L(\cdot)\big/\partial v_{K,t} - y_L(\cdot)\,\partial g_L(\cdot)\big/\partial v_{K,t}}{g_L(\cdot)^2} \\ &= \frac{v_{L,t}}{y(\cdot)^2}\left[y_{LK}(\cdot)y(\cdot) - y_L(\cdot)y_K(\cdot)\right] \qquad (12) \\ &= \frac{s_L(\cdot)s_K(\cdot)}{v_{K,t}}(\psi_{LK} - 1),\end{aligned}$$

where $y_{LK}(\cdot) \equiv \partial^2 y(\cdot)/(\partial v_{L,t}\partial v_{K,t})$ and where ψ_{LK} is the Hicksian elasticity of complementarity between labor and capital defined as:[3]

$$(13) \qquad \psi_{LK} \equiv \frac{y_{LK}(\cdot)y(\cdot)}{y_L(\cdot)y_K(\cdot)}.$$

Thus, capital deepening will lead to an increase (decrease) in the share of labor if and only if the elasticity of complementarity is greater (smaller) than one.

Next, to assess the impact of the passage of time, we take the partial derivative of s_L with respect to *t*, which yields:

$$\begin{aligned}\frac{\partial s_L(\cdot)}{\partial t} &= \frac{g_L(\cdot)\,\partial y_L(\cdot)\big/\partial t - y_L(\cdot)\,\partial g_L(\cdot)\big/\partial t}{g_L(\cdot)^2} \\ &= \frac{v_{L,t}}{y(\cdot)^2}\left[y_{LT}(\cdot)y(\cdot) - y_L(\cdot)y_T(\cdot)\right] \qquad (14) \\ &= \frac{s_{L,t}(\cdot)y_T(\cdot)}{y(\cdot)}(\psi_{LT} - 1),\end{aligned}$$

where $y_T(\cdot) \equiv \partial y(\cdot)/\partial t$, $y_{LT}(\cdot) \equiv \partial^2 y(\cdot)/(\partial v_{L,t}\partial t)$, and where ψ_{LT} is defined as follows:

$$(15) \qquad \psi_{LT} \equiv \frac{y_{LT}(\cdot)y(\cdot)}{y_L(\cdot)y_T(\cdot)}.$$

The ratio $y_{LT}(\cdot)/y_L(\cdot)$ is the elasticity of the real wage rate with respect to time. The ratio $y(\cdot)/y_T(\cdot)$, on the other hand, is the inverse of the instantaneous rate of technological change (μ_t). Thus, ψ_{LT} will be greater than one if and only if technological change tends to favor labor relative to capital, in the sense that the wage rate increases by relatively more than the return to capital.[4] In that case the share of labor will increase with the passage of time.

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[3] In the two input case, ψ_{LK} is necessarily positive; that is, the two inputs are necessarily Hicksian complements for each other. Moreover, in the two input case, the Hicksian elasticity of complementarity is then equal to the inverse of the Allen-Uzawa elasticity of substitution (see footnote 12).

[4] In that case, technological change is said to be pro-labor biased. See Kohli (1994) and section 6 below.

4. Disembodied Factor Augmenting Technological Change

To better track the impact of technological change on the share of labor, let us assume for a moment that technological change is disembodied, factor augmenting, and takes place exponentially. We can then rewrite the production function (1) as follows:

$$y(v_{L,t}, v_{K,t}, t) = f(v_{L,t}e^{\mu_L t}, v_{K,t}e^{\mu_K t}) = f(\tilde{v}_{L,t}, \tilde{v}_{K,t}), \tag{16}$$

where μ_L and μ_K are the rates of factor-augmenting technological change for labor and capital ($\mu_L \geq 0, \mu_K \geq 0$), and $\tilde{v}_{L,t}$ and $\tilde{v}_{K,t}$ are the quantities of labor and capital measured in terms of efficiency units ($\tilde{v}_{L,t} \equiv v_{L,t}e^{\mu_L t}$, $\tilde{v}_{K,t} \equiv v_{K,t}e^{\mu_K t}$). The marginal product of labor $y_L(\cdot)$ is:

$$y_L(v_{L,t}, v_{K,t}, t) = \frac{\partial f(v_{L,t}e^{\mu_L t}, v_{K,t}e^{\mu_K t})}{\partial v_{L,t}} = e^{\mu_L t} f_L(\cdot) , \tag{17}$$

where $f_L(\cdot) \equiv \partial f(\cdot)/\partial \tilde{v}_{L,t}$. The average product of labor, on the other hand, is equal to:

$$g_L(v_{L,t}, v_{K,t}, t) = \frac{f(v_{L,t}e^{\mu_L t}, v_{K,t}e^{\mu_K t})}{v_{L,t}}, \tag{18}$$

and the labor share can now be expressed as:

$$s_L(v_{L,t}, v_{K,t}, t) = \frac{v_{L,t}e^{\mu_L t} f_L(v_{L,t}e^{\mu_L t}, v_{K,t}e^{\mu_K t})}{f(v_{L,t}e^{\mu_L t}, v_{K,t}e^{\mu_K t})}. \tag{19}$$

Differentiating expression (19) with respect to time, we get:

$$\begin{aligned} \frac{\partial s_L(\cdot)}{\partial t} &= \frac{\mu_L \tilde{v}_{L,t} f_L(\cdot) + \tilde{v}_{L,t}(f_{LL}\mu_L\tilde{v}_{L,t} + f_{LK}\mu_K\tilde{v}_{K,t})}{f(\cdot)} \\ &\quad - \frac{\tilde{v}_{L,t} f_L(\cdot)(f_L\mu_L\tilde{v}_{L,t} + f_K\mu_K\tilde{v}_{K,t})}{f(\cdot)^2} = s_{L,t}(1 - s_{L,t})(\mu_K - \mu_L)(\psi_{LK} - 1) \end{aligned} \tag{20}$$

where we have taken into account the restrictions $f_{LL}\tilde{v}_{L,t} + f_{LK}\tilde{v}_{K,t} = 0$ and $f_L\tilde{v}_{L,t} + f_K\tilde{v}_{K,t} = f(\cdot)$ that arise from the linear homogeneity of the production function.

Thus, the labor share will increase with the passage of time if $\mu_K > \mu_L$ and $\psi_{LK} > 1$, or, alternatively, if $\mu_K < \mu_L$ and $\psi_{LK} < 1$. If technological change is Harrod-neutral, for instance ($\mu_L > 0$ and $\mu_K = 0$ in that case) and if labor and capital are relatively good complements, then the share of labor will tend to fall over time. The increase in the available amount of labor measured in terms of efficiency units will tend to have a sufficiently large positive impact on the marginal product of capital for the share of capital to increase and the share of labor to fall.

5. The Cobb-Douglas Functional Form

Suppose the production function (1) has the Cobb-Douglas form:

$$(21) \qquad y(v_{L,t}, v_{K,t}, t) = e^{\alpha_0} v_{K,t}^{\beta_K} v_{L,t}^{1-\beta_K} e^{\mu t},$$

where $0 < \beta_K < 1$ and μ is the rate of Hicks-neutral technological change. One would normally expect this rate to be positive. This indeed turns out to be the case as indicated by the estimates of (21) reported in table 1, column 1.[5]

Note that the production function (21) could just as well have been written as:

$$(22) \qquad y(v_{L,t}, v_{K,t}, t) = e^{\alpha_0} (v_{K,t} e^{\mu_K t})^{\beta_K} v_{L,t}^{1-\beta_K},$$

or as

$$(23) \qquad y(v_{L,t}, v_{K,t}, t) = e^{\alpha_0} v_{K,t}^{\beta_K} (v_{L,t} e^{\mu_L t})^{1-\beta_K},$$

where $\mu_K \equiv \mu / \beta_K$ and $\mu_L \equiv \mu /(1-\beta_K)$. What this means is that it is not possible, in the Cobb-Douglas case, to discriminate between the Hicks-neutral, the Solow-neutral, and the Harrod-neutral cases of technological change.

In any case, it is well known that in the Cobb-Douglas case, the marginal product of labor is proportional to its average product:

$$(24) \qquad y_L(v_{L,t}, v_{K,t}, t) = (1-\beta_K) \frac{e^{\alpha_0} v_{K,t}^{\beta_K} v_{L,t}^{1-\beta_K} e^{\mu t}}{v_{L,t}} = (1-\beta_K) g_L(v_{L,t}, v_{K,t}, t).$$

It follows from (9) and (24) that $1-\beta_K$ can be interpreted as the share of labor in total income, which is thus invariant by construction in the Cobb-Douglas case:

$$(25) \qquad s_{L,t} = 1-\beta_K .$$

To sum up, in the Cobb-Douglas case, the two measures of labor productivity defined in (6) and (7) must give exactly the same result because $S_{t,t-1}$ is equal to unity in (10).

6. The Translog Functional Form

The Cobb-Douglas function forces the Hicksian elasticity of complementarity to be unity. A more general representation of the technology is given by the translog functional form.[6] Maintaining for the time being the assumption of disembodied, factor-augmenting technological change, we can represent the translog production function as follows:

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[5] See the appendix for a description of the data. We jointly estimated equations (21) (in logarithmic form) and (25). The estimation method is Zellner's method for seemingly unrelated equations as implemented in TSP, version 4.3A. The value of the logarithm of the likelihood function (LL) is also reported.

[6] See Christensen, Jorgenson and Lau (1973), and Diewert (1974).

$$\ln y_t = \alpha_0 + \beta_K \ln \tilde{v}_{K,t} + (1-\beta_K)\ln \tilde{v}_{L,t} + \frac{1}{2}\phi_{KK}(\ln \tilde{v}_{K,t} - \ln \tilde{v}_{L,t})^2 . \tag{26}$$

Making use of the definitions of $\tilde{v}_{L,t}$ and $\tilde{v}_{K,t}$, we get:

$$\begin{aligned} \ln y_t &= \alpha_0 + \beta_K \ln v_{K,t} + (1-\beta_K)\ln v_{L,t} + \frac{1}{2}\phi_{KK}(\ln v_{K,t} - \ln v_{L,t})^2 \\ &\quad + \left\{\mu_L + (\mu_K - \mu_L)\left[\beta_K + \phi_{KK}(\ln v_{K,t} - \ln v_{L,t})\right]\right\}t \\ &\quad + \frac{1}{2}\phi_{KK}(\mu_K - \mu_L)^2 t^2 . \end{aligned} \tag{27}$$

The labor share is obtained by logarithmic differentiation:

$$s_{L,t} = (1-\beta_K) - \phi_{KK}(\ln v_{K,t} - \ln v_{L,t}) - \phi_{KK}(\mu_K - \mu_L)t . \tag{28}$$

The Hicksian elasticity of complementarity can be obtained as:

$$\psi_{LK} = \frac{-\phi_{KK} + s_{L,t}(1-s_{L,t})}{s_{L,t}(1-s_{L,t})} . \tag{29}$$

ψ_{LK} is greater than one if and only if ϕ_{KK} is negative.[7] In that case the share of labor increases with capital intensity. This matches our result of section 2.

However, it is also apparent from (28) that the form of technological change plays a role. If $\mu_L > \mu_K$ and $\phi_{KK} > 0$, or, alternatively, if $\mu_K > \mu_L$ and $\phi_{KK} < 0$, technological change is pro-labor biased in that the share of labor will increase as the result of the passage of time. In that case, the marginal product of labor will tend to increase more rapidly than the average product.

Parameter estimates for equation (27) are reported in table 1, column 2.[8] These results suggest that $\mu_L > \mu_K$ and $\phi_{KK} < 0$ in the case of the United States. Thus, technological change is labor augmenting, but anti-labor biased.

Function (27) is flexible with respect to the quantities of labor and capital, but not with respect to time.[9] A TP-flexible translog production function formulation is given by:

$$\begin{aligned} \ln y_t &= \alpha_0 + \beta_K \ln v_{K,t} + (1-\beta_K)\ln v_{L,t} + \frac{1}{2}\phi_{KK}(\ln v_{K,t} - \ln v_{L,t})^2 + \\ &\quad \phi_{KT}(\ln v_{K,t} - \ln v_{L,t})t + \beta_T t + \frac{1}{2}\phi_{TT} t^2 . \end{aligned} \tag{30}$$

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[7] Note that concavity of the production function requires ψ_{LK} to be positive; that is, the following constraint must hold: $\phi_{KK} < s_{L,t}(1-s_{L,t})$.

[8] Equation (27) was estimated jointly with (28) by nonlinear iterative Zellner as implemented into TSP, version 4.3A; see Berndt, Hall, Hall, and Hausman (1974). The standard errors are computed from the quadratic form of the analytic first-order derivatives. The estimate of ψ_{LK} is reported in table 1 as well.

[9] Here we are using the terminology of Diewert and Wales (1992). They define as TP flexible functional form a function that not only is flexible (i.e. gives a second-order approximation) with respect to input quantities, but that is also flexible with respect to technological progress (i.e. it is quadratic with respect to time).

Table 1. Parameter Estimates

	Equation:			
	(21)	**(27)**	**(30)**	**(76)**
	Cobb-Douglas production function	**Translog production function**	**TP-flexible translog production function**	**Translog real value added function**
α_0	8.966^a	8.970^a	8.971^a	8.970^a
α_Q				-0.125
α_E				-0.019^a
β_K	0.277^a	0.284^a	0.285^a	0.285^a
γ_{QQ}				0.0109
γ_{QE}				-0.0940^a
γ_{EE}				0.0997^a
φ_{KK}		-0.1360^a	-0.1532^a	-0.2806^a
δ_{QK}				0.0679^a
δ_{EK}				-0.0354
δ_{QT}				-0.0043^a
δ_{ET}				0.0018^a
φ_{KT}			0.0017^a	0.0016^a
β_T			0.0111^a	0.0108^a
φ_{TT}			0.00008	0.00010^a
μ	0.0098^a			
μ_K		0.0018		
μ_L		0.0134^a		
LL	213.63	224.09	226.70	469.78
ψ_{KL}	1.00	1.67	1.75	2.39

Note: A superscript a indicates a coefficient that is significantly different from zero with a 95 percent level of confidence using a two tailed test.

Comparing (27) with (30), one sees that the latter contains one extra parameter. The share of labor is now given by:

(31) $$s_{L,t} = (1-\beta_K) - \phi_{KK}(\ln v_{K,t} - \ln v_{L,t}) - \phi_{KT}\, t\,.$$

It is immediately obvious that technological change is anti-labor biased in the sense that it leads to a reduction in the share of labor if and only if $\phi_{KT} > 0$. This turns out to be the case as shown by the parameter estimates of (30) reported in table 1, column 3.[10]

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[10] These figures were obtained by estimating (30) and (31) jointly, the estimation method again being iterative Zellner.

7. On the Form of Technological Change: a Digression

When it comes to technological progress and the analysis of its impact on labor and capital, one finds many different competing concepts in the literature. The overall picture can therefore become quite confusing. Thus, does technological progress favor labor or capital? Is technological progress labor saving, labor using, labor augmenting, labor rewarding, or labor penalizing? Is it pro- (or anti-) labor biased, or even ultra pro- (or anti-) labor biased? To some extent, these concepts apply to different situations and they are not mutually exclusive. In the production-function context, where the input quantities are taken as exogenous and their marginal products as endogenous, technological change will tend to impact on these marginal products. Technological progress can be said to favor – or reward – labor and/or capital, in so far as it increases the marginal products of labor and/or capital, respectively. Technological progress may favor one more than the other when it favors both. It may also penalize one factor by reducing its marginal product, although, other things equal, a technological improvement must necessarily have a favorable impact on at least one factor.

In the production function context, one can also think of technological change as being factor augmenting; i.e., it can increase the endowment of one or both factors in terms of efficiency units even if the observed quantities have not changed. If technological change is labor augmenting in this sense (i.e. $\mu_L > 0$), it will, other things equal, tend to depress the marginal product of an efficiency unit of labor and enhance the marginal product of capital.[11] Whether the actual marginal product of labor increases or not will ultimately depend on the Hicksian elasticity of complementarity between the two factors. If labor and capital are strong complements, labor might well be penalized and suffer a drop in its wage. Unless labor and capital are indeed rather weak Hicksian complements, the share of labor will tend to decrease. In that sense, technological change can be said to be inherently anti-labor biased. If the share of labor not only falls, but the wage rate declines too, one could think about this as an ultra anti-labor bias.

Appendix table A1 gives an overview of the cases that might occur with just two inputs, and assuming that technological change is disembodied and factor augmenting. For simplicity, we only consider the polar cases of Harrod-, Hicks- and Solow-neutrality, but intermediate situations can obviously arise as well.[12] Based on the estimates of the translog function (27) discussed in section 5 and reported in table 1, column 2, technological change is nearly Harrod-neutral. It is thus labor augmenting. The elasticity of complementarity is greater than one, but less than the inverse of the capital share. The case described in the second column of table A1 is therefore the one that is relevant for the United States over the 1971-2001 period. Although technological change is labor (and capital) rewarding, viewed in a 2-input production function framework it is nevertheless anti-labor (and pro-capital) biased.

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[11] The return of labor per unit of efficiency can be defined as $\tilde{w}_{L,t} \equiv w_{L,t} e^{-\mu_L t}$.

[12] The ε_{jT} 's ($j=L,K$) are the semi-elasticities of factor rewards with respect to time; see Diewert and Wales (1987). The κ_j 's ($\kappa_j \equiv \varepsilon_{jT} - \mu$) measure the bias; see Kohli (1994) for details. The hats indicate relative changes. The changes in factor rental prices are derived under the assumption that the price of output remains constant.

The terms labor (or capital) using and saving are what are relevant when the technology is described by a cost function instead of a production function.[13] In the aggregate, this would be appropriate in a Keynesian setting, where output and factor rental prices can be viewed as predetermined and where the model yields the demand for labor and capital services. For a given level of output, technological progress will lead to a reduction in the demand for one or both inputs. In that sense, technological progress can be labor and/or capital saving, just like it could be labor or capital using (but not both). In this context, factor rental prices are assumed to be given, but the share of labor can change either way depending on how strongly technological change impacts on labor relative to capital.

If the labor share increases, one might say that technological progress is pro-labor biased, although this outcome is possible whether technological progress is labor using or labor saving. If the labor share falls, technological change would necessarily have to be labor saving, but at the same time, it can be either capital using or capital saving. In this context, we can also think of technological change as modifying the effective rental price of one or both inputs. That is, technological progress could lead to the lowering of the rental price of labor per unit of efficiency. Other things equal, this will favor the demand for labor at the expense of capital in terms of efficiency units, but whether or not the measured demand for labor increases or not depends on the size of the Allen-Uzawa elasticity of substitution between labor and capital. If that elasticity is close to zero, the actual demand for labor might well fall. It is easy to see that the share of labor could in general go in either direction.

Appendix table A2 summarizes the possible outcomes in the cost function setting.[14] Given the empirical results to which we alluded earlier, we can conclude that in the U.S. case, technological change is labor- (and capital-) saving, and anti-labor biased.

In the two input case, there is a simple correspondence between the cost function setting and the production function setting, since the elasticity of substitution is then equal to the inverse of the elasticity of complementarity. This is no longer the case if the number of inputs exceeds two, since the transformation of one type of elasticity into the other requires the inversion of a bordered Hessian matrix.[15] It is no longer true, then, that an elasticity of complementarity between a pair of inputs greater than one necessarily implies that the corresponding elasticity of substitution is less than unity. In fact, the two elasticities need not even have the same sign. This makes any characterization of technological progress without reference to the analytical framework at best ambiguous, and at worst useless.

8. Accounting for Labor Productivity

We now turn to the task of accounting for the changes over time in the average and the marginal products of labor. Using (6) as a starting point, we can define the following index that

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[13] See Jorgenson and Fraumeni (1981), for instance.

[14] The function $c(\cdot)$'s (*j=L,K*) is the unit cost function, and σ_{LK} is the Allen-Uzawa elasticity of substitution. The ε_{jT} 's (*j=L,K*) now designate the semi-elasticities of input demands with respect to time. In deriving these results, we have assumed that output remains constant.

[15] See Kohli (1991).

isolates the impact of changes in factor endowments over consecutive periods of time on the average productivity of labor:

(32) $$A^L_{V,t,t-1} \equiv \frac{g_L(v_{L,t}, v_{K,t}, t-1)}{g_L(v_{L,t-1}, v_{K,t-1}, t-1)}.$$

When defining $A^L_{V,t,t-1}$ we have held the technology constant at its initial (period *t*-1) state. $A^L_{V,t,t-1}$ thus has the Laspeyres form, so to speak. Alternatively, we could adopt the technology of period *t* as a reference. We would then get the following Paasche-like index:

(33) $$A^P_{V,t,t-1} \equiv \frac{g_L(v_{L,t}, v_{K,t}, t)}{g_L(v_{L,t-1}, v_{K,t-1}, t)}.$$

Since there is no reason *a priori* to prefer either measure (32) or (33), we follow Diewert and Morrison’s (1986) example and take the geometric mean of the two indexes. We thus get:

(34) $$A_{V,t,t-1} \equiv \left\{\frac{g_L(v_{L,t}, v_{K,t}, t-1)}{g_L(v_{L,t-1}, v_{K,t-1}, t-1)} \cdot \frac{g_L(v_{L,t}, v_{K,t}, t)}{g_L(v_{L,t-1}, v_{K,t-1}, t)}\right\}^{1/2}.$$

Note that if capital deepening takes place, both $A^L_{V,t,t-1}$ and $A^P_{V,t,t-1}$ will be greater than one, in which case $A_{V,t,t-1}$ must exceed one as well.

Similarly, we can define the following index that isolates the impact of technological change. That is, we compute the index of average labor productivity allowing for the passage of time, but holding factor endowments fixed, first at their level of period *t-1*, and then at the level of period *t*:

(35) $$A^L_{T,t,t-1} \equiv \frac{g_L(v_{L,t-1}, v_{K,t-1}, t)}{g_L(v_{L,t-1}, v_{K,t-1}, t-1)}$$

(36) $$A^P_{T,t,t-1} \equiv \frac{g_L(v_{L,t}, v_{K,t}, t)}{g_L(v_{L,t}, v_{K,t}, t-1)}.$$

Taking the geometric mean of these two indexes, we get:

(37) $$A_{T,t,t-1} \equiv \left\{\frac{g_L(v_{L,t-1}, v_{K,t-1}, t)}{g_L(v_{L,t-1}, v_{K,t-1}, t-1)} \cdot \frac{g_L(v_{L,t}, v_{K,t}, t)}{g_L(v_{L,t}, v_{K,t}, t-1)}\right\}^{1/2}.$$

It can easily be seen that $A_{V,t,t-1}$ given in (34) and $A_{T,t,t-1}$ given in (37) together yield a complete decomposition of the index of average labor productivity:

(38) $$A_{t,t-1} = A_{V,t,t-1} \cdot A_{T,t,t-1}.$$

We can proceed along exactly the same lines with the marginal productivity index. We thus get the two following partial indexes:

$$(39)\qquad M_{V,t,t-1} \equiv \left\{ \frac{y_L(v_{L,t}, v_{K,t}, t-1)}{y_L(v_{L,t-1}, v_{K,t-1}, t-1)} \cdot \frac{y_L(v_{L,t}, v_{K,t}, t)}{y_L(v_{L,t-1}, v_{K,t-1}, t)} \right\}^{1/2}$$

$$(40)\qquad M_{T,t,t-1} \equiv \left\{ \frac{y_L(v_{L,t-1}, v_{K,t-1}, t)}{y_L(v_{L,t-1}, v_{K,t-1}, t-1)} \cdot \frac{y_L(v_{L,t}, v_{K,t}, t)}{y_L(v_{L,t}, v_{K,t}, t-1)} \right\}^{1/2} .$$

Together these two partial indexes provide a complete decomposition of $M_{t,t-1}$:

$$(41)\qquad M_{t,t-1} = M_{V,t,t-1} \cdot M_{T,t,t-1} .$$

An alternative way of tackling the decomposition of $M_{t,t-1}$ would be on the basis of (9). Indeed, since $y_L(\cdot) = s_L(\cdot) g_L(\cdot)$, $M_{V,t,t-1}$ could also be expressed as:

$$(42)\qquad M_{V,t,t-1} = S_{V,t,t-1} \cdot A_{V,t,t-1},$$

where

$$(43)\qquad S_{V,t,t-1} \equiv \left\{ \frac{s_L(v_{L,t}, v_{K,t}, t-1)}{s_L(v_{L,t-1}, v_{K,t-1}, t-1)} \cdot \frac{s_L(v_{L,t}, v_{K,t}, t)}{s_L(v_{L,t-1}, v_{K,t-1}, t)} \right\}^{1/2}$$

measures the contribution of changes in factor endowments on the share of labor. Similarly, it can be seen that:

$$(44)\qquad M_{T,t,t-1} = S_{T,t,t-1} \cdot A_{T,t,t-1},$$

where

$$(45)\qquad S_{T,t,t-1} \equiv \left\{ \frac{s_L(v_{L,t-1}, v_{K,t-1}, t)}{s_L(v_{L,t-1}, v_{K,t-1}, t-1)} \cdot \frac{s_L(v_{L,t}, v_{K,t}, t)}{s_L(v_{L,t}, v_{K,t}, t-1)} \right\}^{1/2} .$$

$S_{T,t,t-1}$ measures the contribution of technological progress to changes in the share of labor; it will be greater than one if technological change is pro-labor biased, and less than one otherwise.

Note that (38) and (41) only hold as long as $A_{t,t-1}$ and $M_{t,t-1}$ are indeed given by (6) and (7). If one uses actual data and if the average product of labor is measured as output per unit of labor and its marginal product is measured by its real wage rate, then one cannot expect expressions such as (38) and (41) to hold exactly, since production function (1) itself is only an approximation of reality, and the same is true for first order condition (2).

Let $AA_{t,t-1}$ and $MM_{t,t-1}$ be the *observed* values of the average and marginal productivities of labor, respectively:

(46) $$AA_{t,t-1} \equiv \frac{y_t / v_{L,t}}{y_{t-1} / v_{L,t-1}}$$

(47) $$MM_{t,t-1} \equiv \frac{w_{L,t} / p_t}{w_{L,t-1} / p_{t-1}}.$$

The full decomposition of both indexes is then given by:

(48) $$AA_{t,t-1} = A_{V,t,t-1} \cdot A_{T,t,t-1} \cdot A_{U,t,t-1}$$

(49) $$MM_{t,t-1} = M_{V,t,t-1} \cdot M_{T,t,t-1} \cdot M_{U,t,t-1},$$

where $A_{U,t,t-1}$ and $M_{U,t,t-1}$ are error (or unexplained) components defined by:

(50) $$A_{U,t,t-1} \equiv \frac{AA_{t,t-1}}{A_{t,t-1}}$$

(51) $$M_{U,t,t-1} \equiv \frac{MM_{t,t-1}}{M_{t,t-1}}.$$

9. Labor Productivity vs. Total Factor Productivity

While labor productivity remains the concept of choice when it comes to the public debate, most economists prefer to think in terms of total factor productivity. The measure of total factor productivity treats all inputs symmetrically. In the production function context, it can be defined as the increase in output that is not explained by increases in input quantities. Put differently, it is the increase in output made possible by technological change, holding all inputs constant. One state-of-the art definition of total factor productivity, $Y_{T,t,t-1}$, is drawn from the work of Diewert and Morrison (1986):[16]

(52) $$Y_{T,t,t-1} \equiv \left\{ \frac{y(v_{L,t-1}, v_{K,t-1}, t)}{y(v_{L,t-1}, v_{K,t-1}, t-1)} \cdot \frac{y(v_{L,t}, v_{K,t}, t)}{y(v_{L,t}, v_{K,t}, t-1)} \right\}^{1/2}.$$

In view of the definition of $g_L(\cdot)$, clearly $Y_{T,t,t-1}$ as given by (52) is in fact identical to $A_{T,t,t-1}$ as defined by (37). That is, total factor productivity in this model is equal to the contribution of technological change when explaining the average productivity of labor. The average productivity of labor will exceed total factor productivity to the extent that capital deepening occurs ($A_{V,t,t-1} > 1$).

10. Measurement

Consider first the case of the Cobb-Douglas production function. It is straightforward to show that:

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[16] It too can be thought of as the geometric average of Laspeyres-like and Paasche-like measures.

(53) $$A_{X,t,t-1} = M_{X,t,t-1} = \left(\frac{v_{K,t}}{v_{L,t}} \Big/ \frac{v_{K,t-1}}{v_{L,t-1}} \right)^{\beta_K}$$

(54) $$A_{T,t,t-1} = M_{T,t,t-1} = e^{\mu} .$$

It is interesting to note that, since $e^{\mu} = e^{(1-\beta_K)\mu_L} = e^{\beta_K \mu_K}$, it does not matter for (54) to hold whether technological change is Hicks-neutral, Harrod-neutral, or Solow-neutral, or more general form.[17]

Recall that we report in table 1, first column, the parameter estimates of the Cobb-Douglas production function (21). In table 2 we show annual estimates of the decomposition of the average and marginal productivity of labor. The factor endowments and the technological change components are the same in both tables, but the observed values of average and marginal productivity differ, so that the corresponding error terms differ as well. According to table 2, labor productivity has increased by close to 1.3% per annum over the sample period. Technological progress accounted for the bulk of the increase, with a contribution of about one percentage point. Capital deepening added about a quarter of a percentage point on average.

(56) $$\ln A_{T,t,t-1} = \beta_T + \frac{1}{2}\phi_{KT}\left(\ln\frac{v_{K,t}}{v_{L,t}} + \ln\frac{v_{K,t-1}}{v_{L,t-1}} \right) + \frac{1}{2}\phi_{TT}(2t-1) .$$

For the marginal productivity indexes, we can apply (42) and (44) after having introduced (31) into (43) and (45) to get:

(57) $$S_{V,t,t-1} \equiv \sqrt{ \frac{1-\beta_K - \phi_{KK}\ln\frac{v_{K,t}}{v_{L,t}} - \phi_{KT}(t-1)}{1-\beta_K - \phi_{KK}\ln\frac{v_{K,t-1}}{v_{L,t-1}} - \phi_{KT}(t-1)} \cdot \frac{1-\beta_K - \phi_{KK}\ln\frac{v_{K,t}}{v_{L,t}} - \phi_{KT}t}{1-\beta_K - \phi_{KK}\ln\frac{v_{K,t-1}}{v_{L,t-1}} - \phi_{KT}t} }$$

(58) $$S_{T,t,t-1} \equiv \sqrt{ \frac{1-\beta_K - \phi_{KK}\ln\frac{v_{K,t-1}}{v_{L,t-1}} - \phi_{KT}t}{1-\beta_K - \phi_{KK}\ln\frac{v_{K,t-1}}{v_{L,t-1}} - \phi_{KT}(t-1)} \cdot \frac{1-\beta_K - \phi_{KK}\ln\frac{v_{K,t}}{v_{L,t}} - \phi_{KT}t}{1-\beta_K - \phi_{KK}\ln\frac{v_{K,t}}{v_{L,t}} - \phi_{KT}(t-1)} } .$$

Consider next the translog functional form. Introducing (30) into (34) and (37), we find that:

(55) $$\begin{aligned} \ln A_{V,t,t-1} = {} & \left(\beta_K + \frac{1}{2}\phi_{KT}(2t-1) \right)\left(\ln\frac{v_{K,t}}{v_{L,t}} - \ln\frac{v_{K,t-1}}{v_{L,t-1}} \right) \\ & + \frac{1}{2}\phi_{KK}\left[\left(\ln\frac{v_{K,t}}{v_{L,t}} \right)^2 - \left(\ln\frac{v_{K,t-1}}{v_{L,t-1}} \right)^2 \right] \end{aligned}$$

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[17] See (21)–(23) above.

Table 2. Decompositions of the 2-Input Cobb-Douglas Production Function

	Average productivity of labor				**Marginal productivity of labor**			
	$AA_{t,t-1}$	$A_{V,t,t-1}$	$A_{T,t,t-1}$	$A_{U,t,t-1}$	$MM_{t,t-1}$	$M_{V,t,t-1}$	$M_{T,t,t-1}$	$M_{U,t,t-1}$
Year	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1971	1.0325	1.0082	1.0099	1.0140	1.0210	1.0082	1.0099	1.0028
1972	1.0182	0.9999	1.0099	1.0083	1.0126	0.9999	1.0099	1.0028
1973	1.0186	0.9996	1.0099	1.0091	1.0112	0.9996	1.0099	1.0018
1974	0.9901	1.0071	1.0099	0.9736	1.0108	1.0071	1.0099	0.9939
1975	1.0281	1.0149	1.0099	1.0031	1.0200	1.0149	1.0099	0.9952
1976	1.0214	0.9976	1.0099	1.0138	1.0193	0.9976	1.0099	1.0117
1977	1.0121	0.9989	1.0099	1.0033	1.0042	0.9989	1.0099	0.9955
1978	1.0076	0.9965	1.0099	1.0013	1.0029	0.9965	1.0099	0.9966
1979	1.0020	1.0013	1.0099	0.9910	1.0088	1.0013	1.0099	0.9977
1980	1.0018	1.0087	1.0099	0.9835	1.0203	1.0087	1.0099	1.0016
1981	1.0199	1.0058	1.0099	1.0041	1.0126	1.0058	1.0099	0.9969
1982	1.0064	1.0127	1.0099	0.9841	1.0185	1.0127	1.0099	0.9959
1983	1.0251	1.0010	1.0099	1.0141	1.0120	1.0010	1.0099	1.0011
1984	1.0174	0.9933	1.0099	1.0143	0.9936	0.9933	1.0099	0.9906
1985	1.0107	1.0007	1.0099	1.0001	1.0165	1.0007	1.0099	1.0058
1986	1.0238	1.0051	1.0099	1.0087	1.0405	1.0051	1.0099	1.0251
1987	1.0025	0.9987	1.0099	0.9940	0.9962	0.9987	1.0099	0.9878
1988	1.0125	0.9991	1.0099	1.0036	1.0011	0.9991	1.0099	0.9923
1989	1.0055	0.9987	1.0099	0.9970	1.0070	0.9987	1.0099	0.9985
1990	1.0119	1.0047	1.0099	0.9973	1.0182	1.0047	1.0099	1.0035
1991	1.0156	1.0103	1.0099	0.9954	1.0219	1.0103	1.0099	1.0016
1992	1.0230	1.0027	1.0099	1.0103	1.0278	1.0027	1.0099	1.0150
1993	1.0009	0.9985	1.0099	0.9927	0.9963	0.9985	1.0099	0.9881
1994	1.0050	0.9962	1.0099	0.9990	1.0001	0.9962	1.0099	0.9941
1995	1.0014	0.9992	1.0099	0.9924	0.9908	0.9992	1.0099	0.9819
1996	1.0199	1.0028	1.0099	1.0072	1.0095	1.0028	1.0099	0.9969
1997	1.0112	0.9984	1.0099	1.0029	1.0041	0.9984	1.0099	0.9958
1998	1.0177	1.0014	1.0099	1.0064	1.0261	1.0014	1.0099	1.0147
1999	1.0144	1.0011	1.0099	1.0034	1.0175	1.0011	1.0099	1.0065
2000	1.0236	1.0047	1.0099	1.0089	1.0323	1.0047	1.0099	1.0174
2001	1.0154	1.0104	1.0099	0.9951	1.0246	1.0104	1.0099	1.0042
1971-2001	**1.0134**	**1.0025**	**1.0099**	**1.0010**	**1.0128**	**1.0025**	**1.0099**	**1.0004**

Recall that parameter estimates of the TP-flexible translog production function are reported in table 1, column 3. A decomposition of the average and marginal productivity indexes based on the translog functional form is provided in columns 1-4 and 5-8 of table 3. Remember that $A_{T,t,t-1}$ in table 3 can also be interpreted as a model-based measure of total factor productivity. The decomposition of the average productivity index is similar to the one obtained with the Cobb-Douglas, with total factor productivity accounting for about four fifths of the increase in average labor productivity. The decomposition of the marginal productivity index, on the other hand, shows a somewhat different picture: technological progress accounts for less than two thirds of real wage increases with capital deepening now playing a larger role. The reason has to do with the estimate of the elasticity of complementarity, which is significantly larger than one. We find that by restricting this elasticity to be unity, the Cobb-Douglas functional form leads to an underestimation of the impact of capital deepening on the marginal product of labor.

11. The Average Productivity of Labor: An Index Number Approach

To make the decomposition (55)–(58) operational one needs econometric estimates of the parameters of the translog production function. This is indeed how we were able to construct the figures reported in columns 1-4 and 5-8 of table 3. It turns out, however, that, as long as the true production function is translog, the decomposition of the average productivity of labor can also be obtained on the basis of knowledge of the data alone; that is, without needing to know the individual parameters of the production function.

Table 3. Decompositions for a 2-Input Translog Production Function

	Average productivity of labor				**Marginal productivity of labor**				**Average productivity of labor: index number approach**		
Year	$AA_{t,t-1}$ (1)	$A_{V,t,t-1}$ (2)	$A_{T,t,t-1}$ (3)	$A_{U,t,t-1}$ (4)	$MM_{t,t-1}$ (5)	$M_{V,t,t-1}$ (6)	$M_{T,t,t-1}$ (7)	$M_{U,t,t-1}$ (8)	$AA_{t,t-1}$ (9)	$A_{V,t,t-1}$ (10)	$A_{T,t,t-1}$ (11)
1971	1.0325	1.0081	1.0088	1.0152	1.0210	1.0144	1.0065	1.0000	1.0325	1.0081	1.0243
1972	1.0182	0.9999	1.0089	1.0093	1.0126	0.9999	1.0066	1.0061	1.0182	0.9999	1.0183
1973	1.0186	0.9996	1.0090	1.0100	1.0112	0.9993	1.0066	1.0053	1.0186	0.9996	1.0191
1974	0.9901	1.0070	1.0091	0.9744	1.0108	1.0125	1.0067	0.9917	0.9901	1.0070	0.9833
1975	1.0281	1.0145	1.0092	1.0041	1.0200	1.0260	1.0069	0.9874	1.0281	1.0145	1.0135
1976	1.0214	0.9977	1.0094	1.0142	1.0193	0.9959	1.0070	1.0163	1.0214	0.9977	1.0238
1977	1.0121	0.9989	1.0094	1.0037	1.0042	0.9981	1.0071	0.9991	1.0121	0.9989	1.0132
1978	1.0076	0.9966	1.0095	1.0016	1.0029	0.9939	1.0071	1.0019	1.0076	0.9966	1.0112
1979	1.0020	1.0013	1.0096	0.9913	1.0088	1.0023	1.0072	0.9993	1.0020	1.0013	1.0007
1980	1.0018	1.0087	1.0097	0.9837	1.0203	1.0154	1.0073	0.9975	1.0018	1.0087	0.9934
1981	1.0199	1.0057	1.0098	1.0043	1.0126	1.0102	1.0074	0.9950	1.0199	1.0057	1.0142
1982	1.0064	1.0123	1.0099	0.9843	1.0185	1.0221	1.0076	0.9890	1.0064	1.0123	0.9943
1983	1.0251	1.0010	1.0100	1.0139	1.0120	1.0017	1.0077	1.0026	1.0251	1.0010	1.0241
1984	1.0174	0.9934	1.0101	1.0139	0.9936	0.9884	1.0077	0.9976	1.0174	0.9934	1.0243
1985	1.0107	1.0007	1.0102	0.9998	1.0165	1.0013	1.0078	1.0073	1.0107	1.0007	1.0099
1986	1.0238	1.0051	1.0103	1.0083	1.0405	1.0090	1.0079	1.0232	1.0238	1.0051	1.0186
1987	1.0025	0.9987	1.0104	0.9935	0.9962	0.9977	1.0080	0.9906	1.0025	0.9987	1.0038
1988	1.0125	0.9991	1.0104	1.0030	1.0011	0.9984	1.0080	0.9948	1.0125	0.9991	1.0135
1989	1.0055	0.9987	1.0105	0.9964	1.0070	0.9977	1.0081	1.0012	1.0055	0.9987	1.0069
1990	1.0119	1.0047	1.0106	0.9966	1.0182	1.0084	1.0082	1.0015	1.0119	1.0047	1.0071
1991	1.0156	1.0104	1.0107	0.9946	1.0219	1.0184	1.0083	0.9952	1.0156	1.0104	1.0053
1992	1.0230	1.0027	1.0108	1.0094	1.0278	1.0048	1.0084	1.0143	1.0230	1.0027	1.0203
1993	1.0009	0.9985	1.0109	0.9917	0.9963	0.9973	1.0085	0.9906	1.0009	0.9984	1.0024
1994	1.0050	0.9962	1.0110	0.9979	1.0001	0.9933	1.0086	0.9983	1.0050	0.9962	1.0088
1995	1.0014	0.9992	1.0110	0.9912	0.9908	0.9986	1.0086	0.9837	1.0014	0.9992	1.0021
1996	1.0199	1.0028	1.0111	1.0059	1.0095	1.0050	1.0087	0.9958	1.0199	1.0028	1.0170
1997	1.0112	0.9984	1.0112	1.0016	1.0041	0.9972	1.0088	0.9982	1.0112	0.9984	1.0129
1998	1.0177	1.0014	1.0113	1.0049	1.0261	1.0025	1.0089	1.0146	1.0177	1.0014	1.0162
1999	1.0144	1.0012	1.0114	1.0019	1.0175	1.0020	1.0089	1.0065	1.0144	1.0012	1.0132
2000	1.0236	1.0049	1.0115	1.0071	1.0323	1.0086	1.0090	1.0143	1.0236	1.0049	1.0187
2001	1.0154	1.0107	1.0116	0.9931	1.0246	1.0188	1.0092	0.9965	1.0153	1.0107	1.0048
1971-2001	**1.0134**	**1.0025**	**1.0102**	**1.0006**	**1.0128**	**1.0044**	**1.0078**	**1.0005**	**1.0134**	**1.0025**	**1.0109**

Following Diewert and Morrison (1986), one can show that, as long as the true production function is given by (30), $A_{T,t,t-1}$ defined by (37) – or, equivalently, $Y_{T,t,t-1}$ defined by (52) – can be computed as:

$$(59) \qquad A_{T,t,t-1} = \frac{Y_{t,t-1}}{V_{t,t-1}},$$

where $Y_{t,t-1}$ is the index of real GDP:

$$(60) \qquad Y_{t,t-1} \equiv \frac{y_t}{y_{t-1}},$$

and $V_{t,t-1}$ is a Törnqvist index of input quantities:

$$(61) \qquad V_{t,t-1} \equiv \exp\left[\sum_{i\in\{L,K\}} \frac{1}{2}(s_{i,t} + s_{i,t-1}) \ln \frac{v_{i,t}}{v_{i,t-1}}\right],$$

where $s_{K,t}$ $(= 1 - s_{L,t})$ is the income share of capital. Hence the following gives a complete decomposition of real GDP growth:

$$(62) \qquad Y_{t,t-1} = Y_{L,t,t-1} \cdot Y_{K,t,t-1} \cdot A_{T,t,t-1},$$

where

$$(63) \qquad Y_{L,t,t-1} \equiv \exp\left[\frac{1}{2}(s_{L,t} + s_{L,t-1}) \ln \frac{v_{L,t}}{v_{L,t-1}}\right]$$

$$(64) \qquad Y_{K,t,t-1} \equiv \exp\left[\frac{1}{2}(s_{K,t} + s_{K,t-1}) \ln \frac{v_{K,t}}{v_{K,t-1}}\right].$$

$Y_{L,t,t-1}$ and $Y_{K,t,t-1}$ can be interpreted as the contributions of labor and capital to real GDP growth.

Next, let $L_{t,t-1}$ be the labor input index:

$$(65) \qquad L_{t,t-1} \equiv \frac{v_{L,t}}{v_{L,t-1}}.$$

It follows from (46) that:

$$(66) \qquad AA_{t,t-1} \equiv \frac{Y_{t,t-1}}{L_{t,t-1}}.$$

Making use of (62) – (64), we get:

$$(67) \qquad AA_{t,t-1} = A_{V,t,t-1} \cdot A_{T,t,t-1},$$

where

(68) $$A_{V,t,t-1} \equiv \exp\left[\frac{1}{2}(s_{K,t} + s_{K,t-1})\left(\ln\frac{v_{K,t}}{v_{L,t}} - \ln\frac{v_{K,t-1}}{v_{L,t-1}}\right)\right].$$

We show in columns 9-11 of table 3 the decomposition of the average productivity of labor based on (67). This decomposition does not require knowledge of the parameters of the translog function.[18] This is obviously very convenient. On the other hand, as indicated by (59), the total factor productivity term ($A_{T,t,t-1}$) is obtained as a Solow residual. Hence, unlike what is done in (48), it is not possible to split it up into a secular component and an error term.[19] Note that the estimates shown in columns 9-11 of table 3 are very similar to those shown in columns 1-4 of the table, except obviously for the total factor productivity term that now incorporates the unexplained component.

12. Domestic Real Value Added

A production function framework is limiting since it requires the number of outputs to be one.[20] Moreover, the production function approach makes it impossible to take into account imports and exports. In what follows, we therefore opt for the description of the aggregate technology by a real value added (or real income) function, such as the one proposed by Kohli (2004a) that is based on the GDP function approach to modeling the production sector of an open economy.[21] We assume that the technology has two outputs, domestic (nontraded) goods (*D*) and exports (*X*) and three inputs: labor (*L*), capital (*K*), and imports (*M*). Treating imports as a variable input is equivalent to treating imports as a negative output.

We denote the output quantities (including imports) by y_i and their prices by p_i, $i \in \{D, X, M\}$. Furthermore, we denote the inverse of the terms of trade by q ($q \equiv p_M / p_X$) and the relative price of tradables vs. nontradables by e ($e \equiv p_X / p_D$). Note that for given terms of trade, a change in e can be interpreted as a change in the real exchange rate, an increase in e being equivalent to a real depreciation of the home currency. Let π_t be nominal GDP:

(69) $$\pi_t \equiv p_{D,t} y_{D,t} + p_{X,t} y_{X,t} - p_{M,t} y_{M,t} = p_t y_t \ .$$

Domestic real value added (z_t) – or real gross domestic income (GDI) – is defined as nominal GDP deflated by the price of domestic output:

(70) $$z_t \equiv \frac{\pi_t}{p_{D,t}} = y_{D,t} + e_t y_{X,t} - e_t q_t y_{M,t} \ .$$

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[18] This index number approach essentially boils down to using the observed share of labor (8) instead of the fitted one as given by (31). See Kohli (1990) for a further discussion of the differences between the two approaches.

[19] An index number approach is not feasible for the marginal productivity index because, even if the true production function is translog, the first-order condition is not; as shown by (31), it is linear in logarithms, rather than quadratic.

[20] Alternatively, one must assume that outputs are globally separable from domestic inputs.

[21] See Kohli (1978) and Woodland (1982).

Let T_t be the production possibilities set at time *t*. We assume that T_t is a convex cone. The aggregate technology can be described by a real valued added function defined as follows:

$$(71)\qquad z(q_t, e_t, v_{K,t}, v_{L,t}, t) \equiv \max_{y_D, y_X, y_M} \left\{ \begin{aligned} & y_{D,t} + e_t y_{X,t} - e_t q_t y_{M,t} : \\ & \qquad (y_{D,t}, y_{X,t}, y_{M,t}, v_{K,t}, v_{L,t}) \in T_t \end{aligned} \right\}.$$

In this context, the *average* real value added of labor ($h_{L,t}$) can be expressed as:

$$(72)\qquad h_{L,t} = h_L(q_t, e_t, v_{K,t}, v_{L,t}, t) \equiv \frac{z(q_t, e_t, v_{K,t}, v_{L,t}, t)}{v_{L,t}},$$

whereas as the *marginal* real value added of labor ($z_{L,t}$) is given by:

$$(73)\qquad z_{L,t} = z_L(q_t, e_t, v_{K,t}, v_{L,t}, t) \equiv \frac{\partial z(q_t, e_t, v_{K,t}, v_{L,t}, t)}{\partial v_{L,t}}.$$

The average and marginal productivity indexes are now as follows:

$$(74)\qquad A_{t,t-1} \equiv \frac{h_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{h_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)}$$

$$(75)\qquad M_{t,t-1} \equiv \frac{z_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{z_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)}.$$

The translog representation of the real value added function is as follows:

$$(76)\qquad \begin{aligned} \ln z_t = {} & \alpha_0 + \alpha_Q \ln q_t + \alpha_E \ln e_t + \beta_K \ln v_{K,t} + (1-\beta_K) \ln v_{L,t} \\ & + \frac{1}{2}\gamma_{QQ} (\ln q_t)^2 + \gamma_{QE} \ln q_t \ln e_t + \frac{1}{2}\gamma_{EE} (\ln e_t)^2 \\ & + \frac{1}{2}\phi_{KK} (\ln v_{K,t} - \ln v_{L,t})^2 + (\delta_{QK} \ln q_t + \delta_{EK} \ln e_t)(\ln v_{K,t} - \ln v_{L,t}) \\ & + (\delta_{QT} \ln q_t + \delta_{ET} \ln e_t) t + \phi_{KT} (\ln v_{K,t} - \ln v_{L,t}) t + \beta_T t + \frac{1}{2}\phi_{TT} t^2. \end{aligned}$$

Logarithmic differentiation yields the following system of equations:[22]

$$(77)\qquad \frac{\partial \ln z(\cdot)}{\partial \ln q_t} = -s_{M,t} = \alpha_Q + \gamma_{QQ} \ln q_t + \gamma_{QE} \ln e_t + \delta_{QK} (\ln v_{K,t} - \ln v_{L,t}) + \delta_{QT}\, t$$

$$(78)\qquad \frac{\partial \ln z(\cdot)}{\partial \ln e_t} = s_{B,t} = \alpha_E + \gamma_{QE} \ln q_t + \gamma_{EE} \ln e_t + \delta_{EK} (\ln v_{K,t} - \ln v_{L,t}) + \delta_{ET}\, t$$

$$(79)\qquad \frac{\partial \ln z(\cdot)}{\partial \ln v_{L,t}} = s_{L,t} = 1 - \beta_K - \delta_{QK} \ln q_t - \delta_{EK} \ln e_t - \phi_{KK} (\ln v_{K,t} - \ln v_{L,t}) - \phi_{KT}\, t$$

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[22] See Kohli (2004a).

(80) $$\frac{\partial \ln z(\cdot)}{\partial t} \equiv \mu_t = \beta_T + \delta_{QT} \ln q_t + \delta_{ET} \ln e_t + \phi_{KT} (\ln v_{K,t} - \ln v_{L,t}) + \phi_{TT}\, t\,,$$

where s_M is the GDP share of imports ($s_M \equiv p_M y_M / \pi$), s_B is the trade balance relative to GDP ($s_B \equiv (p_X y_X - p_M y_M)/\pi$), s_L is, as before, the GDP share of labor, and μ is again the instantaneous rate of technological change.

Parameter estimates, obtained from the joint estimation of equations (76)–(80), are reported in the last column of table 1.[23] It is noteworthy that the labor share now depends on four items. Besides relative factor endowments and the passage of time, the terms of trade and the real exchange rate may influence the share of labor as well now. A deterioration in the terms of trade (an increase in *q*) will tend to lower the share of labor as indicated by the positive estimate of δ_{QK}. Similarly, a real appreciation of the home currency (a fall in *e*) will tend to reduce s_L in view of the negative estimate of δ_{EK}. In both these cases, the marginal product of labor would, *ceteris paribus*, increase less rapidly than its average product.

13. Average Productivity in the Open Economy

Proceeding along the same lines as in section 8, we can define the following index to capture the contribution of changes in the terms of trade to the average productivity of labor:

(81) $$A_{Q,t,t-1} \equiv \left\{ \frac{h_L(q_t, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)}{h_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{h_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{h_L(q_{t-1}, e_t, v_{K,t}, v_{L,t}, t)} \right\}^{1/2} .$$

Similarly, we can identify the contribution of changes in the real exchange rate as:

(82) $$A_{E,t,t-1} \equiv \left\{ \frac{h_L(q_{t-1}, e_t, v_{K,t-1}, v_{L,t-1}, t-1)}{h_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{h_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{h_L(q_t, e_{t-1}, v_{K,t}, v_{L,t}, t)} \right\}^{1/2} ,$$

the contribution of changes in domestic factor endowments:

(83) $$A_{V,t,t-1} \equiv \left\{ \frac{h_L(q_{t-1}, e_{t-1}, v_{K,t}, v_{L,t}, t-1)}{h_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{h_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{h_L(q_t, e_t, v_{K,t-1}, v_{L,t-1}, t)} \right\}^{1/2} ,$$

and, finally, the contribution of technological progress:

(84) $$A_{T,t,t-1} \equiv \left\{ \frac{h_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t)}{h_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{h_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{h_L(q_t, e_t, v_{K,t}, v_{L,t}, t-1)} \right\}^{1/2} .$$

Assuming that the real value added function is given by (76) and that its parameters are known, it is straightforward to compute the values of (81)–(84). Moreover, it can easily be

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[23] The estimation method again is the non-linear iterative algorithm for estimating Zellner's seemingly unrelated equations as implemented in TSP.

shown that these four effects together give a complete decomposition of the average productivity of labor as defined by (74):

$$A_{t,t-1} = A_{Q,t,t-1} \cdot A_{E,t,t-1} \cdot A_{V,t,t-1} \cdot A_{T,t,t-1}. \tag{85}$$

Moreover, if we seek to explain the *observed* increase in average labor productivity, we get:

$$AA_{t,t-1} = A_{Q,t,t-1} \cdot A_{E,t,t-1} \cdot A_{V,t,t-1} \cdot A_{T,t,t-1} \cdot A_{U,t,t-1}, \tag{86}$$

where $AA_{t,t-1}$ is now defined as:

$$AA_{t,t-1} \equiv \frac{z_t / v_{L,t}}{z_{t-1} / v_{L,t-1}}, \tag{87}$$

and $A_{U,t,t-1}$ is the unexplained component of $AA_{t,t-1}$:

$$A_{U,t,t-1} = \frac{AA_{t,t-1}}{A_{t,t-1}}. \tag{88}$$

If the true real value added function is translog it is possible to compute (81)–(84) based on the data alone, without knowledge of the parameters of (76). Indeed, one can show that:[24]

$$A_{Q,t,t-1} = \exp\left[\frac{1}{2}(-s_{M,t} - s_{M,t-1}) \ln \frac{q_t}{q_{t-1}}\right] \tag{89}$$

$$A_{E,t,t-1} = \exp\left[\frac{1}{2}(s_{B,t} + s_{B,t-1}) \ln \frac{e_t}{e_{t-1}}\right] \tag{90}$$

$$A_{V,t,t-1} \equiv \exp\left[\frac{1}{2}(s_{K,t} + s_{K,t-1})\left(\ln \frac{v_{K,t}}{v_{L,t}} - \ln \frac{v_{K,t-1}}{v_{L,t-1}}\right)\right] \tag{91}$$

$$A_{T,t,t-1} \equiv \frac{Y_{t,t-1}}{V_{t,t-1}}, \tag{92}$$

so that:

$$AA_{t,t-1} = A_{Q,t,t-1} \cdot A_{E,t,t-1} \cdot A_{V,t,t-1} \cdot A_{T,t,t-1}. \tag{93}$$

A decomposition of the average productivity of labor according to (86) and (93) is reported in columns 1-6 and 7-11 of table 4, respectively.

14. Accounting for Changes in the Share of Labor

In the next section, we will focus on the decomposition of the *marginal* productivity index, but first we will briefly turn our attention to the behavior of the labor share. Indeed, we

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[24] See Kohli (2004a).

will follow here essentially the same route as in the production function context; that is, we will exploit the link between the marginal and average productivity measures via a labor share index.

There is one key difference compared to the treatment in section 8, however. A decomposition such as (41), which is exact independently of the underlying functional form, only holds if the number of elements on the right-hand side is two. However, if the underlying functional form is translog, the decomposition is exact even if the number of components is larger; see (85), for instance. But even if the underlying function is translog, as here, the first-order conditions are not. As shown by (77)–(80), the share equations are linear in logarithms. Hence the best we can hope for is a linear approximation of the decomposition of the marginal productivity and labor share indices. With this in mind, we will proceed as in sections 8 and 9.[25]

In the context of the real value-added function, the labor share index can be defined as:

$$(94)\qquad S_{t,t-1} \equiv \frac{s_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{s_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)},$$

where $s_L(q_t, e_t, v_{K,t}, v_{L,t}, t)$ is given by the right-hand side of (79). This index can easily be calculated once the parameters of the real value-added function are known. The same holds true for the following four indices that identify the contributions of the terms of trade, the real exchange rate, the relative factor endowments and the passage of time:

$$(95)\qquad S_{Q,t,t-1} \equiv \left\{ \frac{s_L(q_t, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)}{s_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{s_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{s_L(q_{t-1}, e_t, v_{K,t}, v_{L,t}, t)} \right\}^{1/2}$$

$$(96)\qquad S_{E,t,t-1} \equiv \left\{ \frac{s_L(q_{t-1}, e_t, v_{K,t-1}, v_{L,t-1}, t-1)}{s_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{s_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{s_L(q_t, e_{t-1}, v_{K,t}, v_{L,t}, t)} \right\}^{1/2}$$

$$(97)\qquad S_{V,t,t-1} \equiv \left\{ \frac{s_L(q_{t-1}, e_{t-1}, v_{K,t}, v_{L,t}, t-1)}{s_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{s_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{s_L(q_t, e_t, v_{K,t-1}, v_{L,t-1}, t)} \right\}^{1/2}$$

$$(98)\qquad S_{T,t,t-1} \equiv \left\{ \frac{s_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t)}{s_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{s_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{s_L(q_t, e_t, v_{K,t}, v_{L,t}, t-1)} \right\}^{1/2}.$$

An approximation to $S_{t,t-1}$ is given by the following:[26]

$$(99)\qquad S_{t,t-1} \cong S_{Q,t,t-1} \cdot S_{E,t,t-1} \cdot S_{V,t,t-1} \cdot S_{T,t,t-1}.$$

We next define the *observed* labor share index:

$$(100)\qquad SS_{t,t-1} \equiv \frac{x_{L,t} w_{L,t} / \pi_t}{x_{L,t-1} w_{L,t-1} / \pi_{t-1}}.$$

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[25] See Sfreddo (2001) for a further discussion and for three alternative decompositions of the first-order conditions.

[26] See Sfreddo (2001); we have verified that the residual is almost zero.

A complete decomposition of the change in the labor share is hence given by:

$$(101) \quad SS_{t,t-1} \cong S_{Q,t,t-1} \cdot S_{E,t,t-1} \cdot S_{V,t,t-1} \cdot S_{T,t,t-1} \cdot S_{U,t,t-1},$$

where $S_{U,t,t-1}$ is the unexplained component which can be represented as:

$$(102) \quad S_{U,t,t-1} = \frac{SS_{t,t-1}}{S_{t,t-1}}.$$

Table 4. Decompositions for a 2-Input, 3-Output Translog Real Domestic Value Added Function

	Average productivity of labor						Average productivity of labor: index number approach				
Year	***$AA_{t,t-1}$*** (1)	***$A_{Q,t,t-1}$*** (2)	***$A_{E,t,t-1}$*** (3)	***$A_{V,t,t-1}$*** (4)	***$A_{T,t,t-1}$*** (5)	***$A_{U,t,t-1}$*** (6)	***$AA_{t,t-1}$*** (7)	***$A_{Q,t,t-1}$*** (8)	***$A_{E,t,t-1}$*** (9)	***$A_{V,t,t-1}$*** (10)	***$A_{T,t,t-1}$*** (11)
1971	1.0310	0.9986	1.0000	1.0081	1.0098	1.0143	1.0310	0.9986	1.0000	1.0079	1.0229
1972	1.0161	0.9979	1.0001	0.9999	1.0097	1.0084	1.0161	0.9979	1.0001	0.9999	1.0162
1973	1.0164	0.9980	0.9997	0.9996	1.0097	1.0093	1.0164	0.9980	0.9998	0.9996	1.0168
1974	0.9792	0.9892	0.9998	1.0071	1.0096	0.9737	0.9792	0.9890	0.9999	1.0070	0.9724
1975	1.0298	1.0016	1.0000	1.0144	1.0097	1.0038	1.0298	1.0016	1.0000	1.0144	1.0151
1976	1.0214	1.0001	1.0000	0.9978	1.0098	1.0136	1.0214	1.0001	0.9999	0.9977	1.0238
1977	1.0083	0.9962	1.0001	0.9989	1.0098	1.0034	1.0083	0.9961	1.0002	0.9989	1.0094
1978	1.0068	0.9991	1.0001	0.9966	1.0097	1.0014	1.0068	0.9991	1.0001	0.9965	1.0104
1979	0.9976	0.9959	0.9999	1.0013	1.0097	0.9908	0.9976	0.9959	0.9997	1.0013	0.9963
1980	0.9894	0.9882	1.0000	1.0089	1.0095	0.9831	0.9894	0.9876	1.0000	1.0085	0.9810
1981	1.0219	1.0018	1.0002	1.0059	1.0094	1.0044	1.0219	1.0019	1.0001	1.0056	1.0163
1982	1.0105	1.0038	1.0008	1.0124	1.0096	0.9841	1.0105	1.0038	1.0003	1.0121	0.9984
1983	1.0295	1.0042	1.0004	1.0009	1.0099	1.0139	1.0295	1.0040	1.0003	1.0009	1.0286
1984	1.0198	1.0018	1.0003	0.9935	1.0101	1.0141	1.0197	1.0018	1.0005	0.9933	1.0267
1985	1.0128	1.0006	1.0007	1.0007	1.0101	1.0007	1.0128	1.0006	1.0015	1.0008	1.0121
1986	1.0233	0.9985	1.0006	1.0050	1.0101	1.0089	1.0233	0.9984	1.0011	1.0051	1.0181
1987	0.9995	0.9968	1.0001	0.9987	1.0101	0.9938	0.9995	0.9967	1.0002	0.9987	1.0007
1988	1.0124	1.0004	0.9997	0.9991	1.0102	1.0031	1.0124	1.0004	0.9995	0.9991	1.0134
1989	1.0052	0.9993	1.0003	0.9987	1.0103	0.9967	1.0052	0.9993	1.0003	0.9987	1.0065
1990	1.0102	0.9978	1.0006	1.0048	1.0103	0.9968	1.0102	0.9979	1.0004	1.0047	1.0054
1991	1.0178	1.0021	1.0004	1.0104	1.0104	0.9945	1.0178	1.0020	1.0002	1.0103	1.0074
1992	1.0227	0.9995	1.0006	1.0027	1.0105	1.0093	1.0227	0.9995	1.0001	1.0027	1.0200
1993	1.0020	1.0010	1.0005	0.9985	1.0106	0.9916	1.0020	1.0010	1.0002	0.9985	1.0036
1994	1.0053	1.0003	1.0002	0.9962	1.0107	0.9981	1.0053	1.0002	1.0001	0.9962	1.0091
1995	1.0009	0.9996	1.0000	0.9992	1.0107	0.9915	1.0009	0.9996	1.0000	0.9992	1.0017
1996	1.0210	1.0007	1.0006	1.0028	1.0108	1.0059	1.0210	1.0007	1.0004	1.0029	1.0180
1997	1.0142	1.0026	1.0006	0.9984	1.0109	1.0016	1.0142	1.0026	1.0004	0.9983	1.0159
1998	1.0223	1.0042	1.0005	1.0014	1.0111	1.0050	1.0223	1.0041	1.0004	1.0014	1.0208
1999	1.0137	0.9988	1.0004	1.0012	1.0112	1.0021	1.0137	0.9988	1.0005	1.0012	1.0126
2000	1.0196	0.9960	1.0002	1.0049	1.0112	1.0072	1.0196	0.9957	1.0003	1.0049	1.0147
2001	1.0195	1.0029	1.0006	1.0106	1.0113	0.9941	1.0195	1.0032	1.0010	1.0105	1.0090
1971-2001	**1.0128**	**0.9993**	**1.0003**	**1.0025**	**1.0102**	**1.0006**	**1.0128**	**0.9992**	**1.0002**	**1.0025**	**1.0104**

Table 5. Decompositions for a 2-Input, 3-Output Translog Real Domestic Value Added Function

	Share of labor						**Marginal productivity of labor**					
Year	$SS_{t,t-1}$ (1)	$S_{Q,t,t-1}$ (2)	$S_{E,t,t-1}$ (3)	$S_{V,t,t-1}$ (4)	$S_{T,t,t-1}$ (5)	$S_{U,t,t-1}$ (6)	$MM_{t,t-1}$ (7)	$M_{Q,t,t-1}$ (8)	$M_{E,t,t-1}$ (9)	$M_{V,t,t-1}$ (10)	$M_{T,t,t-1}$ (11)	$M_{U,t,t-1}$ (12)
1971	0.9889	0.9976	0.9991	1.0115	0.9978	0.9831	1.0196	0.9962	0.9992	1.0196	1.0075	0.9971
1972	0.9945	0.9965	0.9994	0.9999	0.9978	1.0008	1.0105	0.9945	0.9995	0.9998	1.0075	1.0092
1973	0.9927	0.9970	1.0036	0.9994	0.9978	0.9949	1.0090	0.9951	1.0033	0.9990	1.0075	1.0042
1974	1.0209	0.9864	1.0056	1.0099	0.9978	1.0213	0.9996	0.9758	1.0055	1.0171	1.0074	0.9945
1975	0.9921	1.0018	1.0006	1.0207	0.9978	0.9719	1.0217	1.0034	1.0006	1.0354	1.0074	0.9756
1976	0.9979	1.0001	0.9988	0.9967	0.9978	1.0044	1.0193	1.0003	0.9988	0.9945	1.0076	1.0181
1977	0.9922	0.9958	0.9987	0.9984	0.9978	1.0014	1.0005	0.9920	0.9988	0.9974	1.0075	1.0049
1978	0.9953	0.9991	0.9995	0.9951	0.9978	1.0039	1.0021	0.9982	0.9995	0.9917	1.0075	1.0053
1979	1.0067	0.9959	1.0015	1.0018	0.9978	1.0098	1.0043	0.9919	1.0014	1.0031	1.0074	1.0006
1980	1.0184	0.9885	0.9998	1.0123	0.9977	1.0202	1.0076	0.9768	0.9999	1.0213	1.0072	1.0030
1981	0.9928	1.0017	0.9992	1.0082	0.9978	0.9861	1.0146	1.0036	0.9994	1.0141	1.0071	0.9904
1982	1.0121	1.0036	0.9975	1.0177	0.9978	0.9956	1.0226	1.0074	0.9982	1.0303	1.0074	0.9798
1983	0.9872	1.0040	0.9986	1.0014	0.9978	0.9855	1.0164	1.0082	0.9990	1.0023	1.0077	0.9992
1984	0.9766	1.0017	0.9988	0.9907	0.9978	0.9875	0.9959	1.0035	0.9991	0.9843	1.0078	1.0013
1985	1.0057	1.0006	0.9973	1.0010	0.9978	1.0091	1.0186	1.0012	0.9980	1.0018	1.0079	1.0098
1986	1.0163	0.9985	0.9982	1.0071	0.9978	1.0147	1.0400	0.9970	0.9988	1.0122	1.0079	1.0238
1987	0.9937	0.9971	0.9997	0.9982	0.9978	1.0010	0.9931	0.9939	0.9998	0.9969	1.0079	0.9947
1988	0.9888	1.0003	1.0009	0.9987	0.9978	0.9911	1.0011	1.0007	1.0006	0.9977	1.0079	0.9942
1989	1.0014	0.9994	0.9990	0.9981	0.9977	1.0072	1.0066	0.9987	0.9994	0.9968	1.0080	1.0038
1990	1.0062	0.9982	0.9984	1.0066	0.9977	1.0053	1.0165	0.9960	0.9990	1.0115	1.0080	1.0021
1991	1.0062	1.0017	0.9990	1.0145	0.9978	0.9932	1.0241	1.0038	0.9994	1.0251	1.0081	0.9878
1992	1.0046	0.9996	0.9987	1.0038	0.9978	1.0048	1.0274	0.9991	0.9992	1.0065	1.0082	1.0141
1993	0.9953	1.0008	0.9989	0.9978	0.9978	1.0000	0.9974	1.0019	0.9994	0.9963	1.0083	0.9916
1994	0.9951	1.0002	0.9996	0.9947	0.9978	1.0029	1.0004	1.0005	0.9997	0.9908	1.0084	1.0011
1995	0.9894	0.9996	1.0001	0.9989	0.9977	0.9930	0.9903	0.9992	1.0000	0.9981	1.0084	0.9846
1996	0.9898	1.0005	0.9985	1.0039	0.9977	0.9892	1.0105	1.0012	0.9990	1.0068	1.0085	0.9950
1997	0.9930	1.0020	0.9984	0.9978	0.9977	0.9971	1.0070	1.0046	0.9990	0.9961	1.0086	0.9987
1998	1.0083	1.0031	0.9985	1.0019	0.9977	1.0070	1.0308	1.0073	0.9990	1.0033	1.0088	1.0120
1999	1.0031	0.9991	0.9988	1.0016	0.9977	1.0058	1.0169	0.9980	0.9993	1.0027	1.0089	1.0079
2000	1.0085	0.9971	0.9995	1.0067	0.9977	1.0075	1.0282	0.9932	0.9997	1.0117	1.0089	1.0147
2001	1.0091	1.0021	0.9987	1.0147	0.9977	0.9960	1.0288	1.0050	0.9992	1.0254	1.0090	0.9901
1971-2001	**0.9994**	**0.9990**	**0.9994**	**1.0035**	**0.9978**	**0.9997**	**1.0122**	**0.9983**	**0.9997**	**1.0060**	**1.0079**	**1.0002**

15. Accounting for Changes in Real Wages

We are now in a position to account for the marginal productivity index and the changes in real wages. Recall that the marginal productivity index is defined by (75). We can also identify the following partial effects:

$$M_{Q,t,t-1} \equiv \left\{ \frac{z_L(q_t, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)}{z_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{z_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{z_L(q_{t-1}, e_t, v_{K,t}, v_{L,t}, t)} \right\}^{1/2} \qquad (103)$$

(104) $$M_{E,t,t-1} \equiv \left\{ \frac{z_L(q_{t-1}, e_t, v_{K,t-1}, v_{L,t-1}, t-1)}{z_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{z_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{z_L(q_t, e_{t-1}, v_{K,t}, v_{L,t}, t)} \right\}^{1/2}$$

(105) $$M_{V,t,t-1} \equiv \left\{ \frac{z_L(q_{t-1}, e_{t-1}, v_{K,t}, v_{L,t}, t-1)}{z_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{z_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{z_L(q_t, e_t, v_{K,t-1}, v_{L,t-1}, t)} \right\}^{1/2}$$

(106) $$M_{T,t,t-1} \equiv \left\{ \frac{z_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t)}{z_L(q_{t-1}, e_{t-1}, v_{K,t-1}, v_{L,t-1}, t-1)} \cdot \frac{z_L(q_t, e_t, v_{K,t}, v_{L,t}, t)}{z_L(q_t, e_t, v_{K,t}, v_{L,t}, t-1)} \right\}^{1/2} .$$

Since $z_L(\cdot) = h_L(\cdot) \cdot s_L(\cdot)$ under profit maximization (as long as the timing of the arguments is the same in all three functions), it immediately follows that:

(107) $$M_{t,t-1} = A_{t,t-1} \cdot S_{t,t-1}$$

(108) $$M_{Q,t,t-1} = A_{Q,t,t-1} \cdot S_{Q,t,t-1}$$

(109) $$M_{E,t,t-1} = A_{E,t,t-1} \cdot S_{E,t,t-1}$$

(110) $$M_{V,t,t-1} = A_{V,t,t-1} \cdot S_{V,t,t-1}$$

(111) $$M_{T,t,t-1} = A_{T,t,t-1} \cdot S_{T,t,t-1} .$$

Furthermore, it follows from (85) and (99) that:

(112) $$M_{t,t-1} \cong M_{Q,t,t-1} \cdot M_{E,t,t-1} \cdot M_{V,t,t-1} \cdot M_{T,t,t-1}.$$

We finally consider the *observed* marginal productivity of labor index. It is now as follows:

(113) $$MM_{t,t-1} \equiv \frac{w_{L,t} / p_{D,t}}{w_{L,t-1} / p_{D,t-1}} .$$

A complete decomposition of the progression in real wages is therefore given by:

(114) $$MM_{t,t-1} \cong M_{Q,t,t-1} \cdot M_{E,t,t-1} \cdot M_{V,t,t-1} \cdot M_{T,t,t-1} \cdot M_{U,t,t-1},$$

where $M_{U,t,t-1}$ is as usual the unexplained component which can be represented as

(115) $$M_{U,t,t-1} = \frac{MM_{t,t-1}}{M_{t,t-1}} .$$

We show in columns 7-12 of table 5 the decomposition of the marginal productivity of labor based on (114). Real wages increased by just over 1.2% per year over the sample period. This increase is dominated by technological progress, though capital deepening played a role too. In fact, comparing these results with those in columns 1-6 of table 4, we again find that capital deepening has a relatively larger impact on marginal productivity than on average productivity. Terms-of-trade changes have reduced real wages by approximately 0.1% per annum on average.

Changes in the real exchange rate have had a negligible effect on average, although the impact has been noticed in some years such as 1974 when it added about 0.6% to real wages.

16. About Unit Labor Costs

Many economic analysts attach much importance to the development of unit labor costs. An increase in unit labor costs – that is, an increase in nominal wages that is not matched by an increase in average productivity – is often viewed as being a threat to price stability. This concern as to the inflationary consequences of an increase in unit labor costs can be understood if one considers that, in most industries and for the economy as a whole, labor costs are the largest component of total costs. This might explain why increases in unit labor costs are sometimes thought of as being the prime source of inflation, even though a theory of inflation that leaves no place for money may sound suspect. In any case, it may be useful to investigate what role unit labor costs play in the analysis in this paper.

Unit labor costs ($\omega_{L,t}$) can be defined as follows:

$$\text{(116)} \quad \omega_{L,t} \equiv \frac{w_{L,t}}{z_t / v_{L,t}} .$$

In view of our earlier definitions, unit labor costs can also be expressed as:

$$\text{(117)} \quad \omega_{L,t} = \frac{w_{L,t}}{h_{L,t}} = \frac{z_{L,t} \cdot p_{D,t}}{h_{L,t}} = s_{L,t} \cdot p_{D,t} .$$

In terms of change factors we get:

$$\text{(118)} \quad \Omega_{L,t,t-1} = SS_{L,t,t-1} \cdot P_{D,t,t-1} ,$$

where $\Omega_{L,t,t-1}$ is the unit labor cost index and $P_{D,t,t-1}$ is (one plus) the domestic inflation rate:

$$\text{(119)} \quad \Omega_{L,t,t-1} \equiv \frac{\omega_{L,t}}{\omega_{L,t-1}} \text{ and}$$

$$\text{(120)} \quad P_{D,t,t-1} \equiv \frac{p_{D,t}}{p_{D,t-1}} .$$

Looking at (118), the link between increases in unit labor costs and inflation is evident. In fact, if the share of labor is constant ($SS_{t,t-1} = 1$ in that case), the correlation is perfect. An increase in unit labor costs, be it as the result of an increase in nominal wages or a reduction in average productivity, would necessarily go hand in hand with an increase in the price of output. Correlation is not causation, however. Nominal wages need not be exogenous, no more than average productivity. It is reasonable to assume both are endogenous for the economy as a whole, and this is how they have been treated in the model developed in this paper. Similarly, as stressed throughout the paper, the share of labor is endogenous too. Rather than viewing changes in unit labor costs as an exogenous factor impacting on prices, it might be more useful to explain the changes in unit labor costs as a function of the factors that we have identified earlier on.

In the context of our model, it is clear from (118) that changes in unit labor costs reflect changes in (i) the share of labor and (ii) the price of output. As to the second item, it could be argued that unit labor costs mirror changes in the general price level, rather than cause them. Regarding the impact of changes in the labor share, we refer the reader to section 13 as summarized in columns 1-6 of table 5. Thus, in the U.S. case, a worsening in the terms of trade and/or a real appreciation of the currency, other things equal, reduce unit labor costs. The same is true for technological change, whereas capital deepening acts to increase unit labor costs. Some of these results may sound counter-intuitive. Thus, an increase in the stock of capital, which, for a given labor endowment, must unambiguously increase output and average labor productivity, might yet increase unit labor costs if the marginal product of labor (i.e. real wages) increases by relatively more. If the Hicksian elasticity of complementarity is greater than one, this will be so.

17. Conclusions

In this paper we try to sort out some ideas linked to productivity and to identify the main components of labor productivity. A distinction is drawn between the marginal and the average productivity of labor. This leads to a focus on the GDP share of labor. This in turn helps to illuminate the main forces at work: technological progress, capital deepening, terms-of-trade changes, and changes in the real exchange rate. These last two factors, though statistically significant, were found to play minor roles. This may be because the United States is a relatively closed economy. It is very possible that changes in the terms of trade and in the real exchange rate play a more important role for labor productivity in more open economies.

Our analysis leads to an emphasis on the role played by the Hicksian elasticity of complementarity. This elasticity is significantly greater than unity. This explains to a large extent why the share of labor has been fairly steady over time, and thus why the marginal and average measures of labor productivity have moved in unison. Capital deepening tends to increase the marginal product of labor, and given the large elasticity of complementarity this tends to increase the share of labor. Technological progress, on the other hand, by being mainly labor augmenting can be thought of as anti-labor biased (although not ultra anti-labor biased). This tends to reduce the share of labor, largely offsetting the impact of capital deepening. The slight deterioration in the terms of trade and the small real appreciation of the U.S. dollar that took place over the sample period have further contributed to containing the increase in the labor share.

This paper documents the relationship between total factor and labor productivity. Even if total factor productivity is the main driving force in the increase in output and average productivity, expression (93) shows that there are other forces at work as well. The growth in U.S. labor productivity since the mid-1990s is often considered as a tribute and testimony to the performance of American workers. However, the headline figures typically relate to the nonfarm business sector only. The farming sector, the government sector and the household sector – close to half the economy – are left out of the calculation. Also, productivity growth can be the outcome of a conjunction of favorable events. Thus, capital deepening will unavoidably increase the average and the marginal productivity of labor. And technological progress will necessarily increase average productivity too, but it may impact either way on real wages, although in the U.S. case, the effect is positive. An improvement in the terms of trade and a depreciation of the home currency also lead to increases in average labor productivity, and, in the U.S. case, the impact on real wages is magnified through the increase in the GDP share of labor.

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Appendix

A1. Description of the Data

All data are annual for the period 1970 to 2001. We require the prices and quantities of all inputs and outputs. The data for GDP and its components, in nominal and in real terms, are taken from the *Bureau of Economic Analysis* website. Prices are then obtained by deflation. Data on the capital stock, labor compensation, and national income are also retrieved from the *BEA* website. The quantity of capital services is assumed to be proportional to the stock. Capital income is defined as national income minus labor compensation. The quantity of labor services is computed by multiplying the total number of employees on nonfarm payrolls by an index of the average number of weekly hours worked in the nonfarm business sector. Both these series are taken from the *Bureau of Labor Statistics* website. The user costs of labor and capital are then obtained by dividing labor and capital income by the corresponding quantity series. For the purpose of sections 9 and 10, output is expressed as an implicit Törnqvist index of real GDP; see Kohli (2004b) for details. In sections 11 to 15, the price of nontraded goods is computed as a Törnqvist price index of the deflators of consumption, investment and government purchases.

A2. Neutral, Disembodied and Factor-Augmenting Technological Change

The first of the two tables that follow (A1) gives an overview, in a production function setting, of the cases that might occur with just two inputs, and assuming that technological change is disembodied and factor augmenting. For simplicity, we only consider the polar cases of Harrod-, Hicks- and Solow-neutrality, but intermediate situations can obviously arise as well.

The second of the following tables (A2) summarizes the possible outcomes in a cost function setting.

Table A1. Neutral, Disembodied, Factor-Augmenting Technological Change in the 2-Input Case: Production-Function Setting

$$y_t = f(v_{L,t}e^{\mu_L t}, v_{K,t}e^{\mu_K t})$$

$\mu_L > 0, \mu_K = 0$ Harrod-neutral ***(L-augmenting)***				$\mu_L = \mu_K (\equiv \mu) > 0$ Hicks-neutral ***(L & K-augmenting)***	$\mu_L = 0, \mu_K > 0$ Solow-neutral ***(K-augmenting)***			
$\hat{\hat{w}}_L < 0, \hat{\hat{w}}_K > 0$				$\hat{\hat{w}}_L = \hat{\hat{w}}_K = 0$	$\hat{\hat{w}}_L > 0, \hat{\hat{w}}_K < 0$			
$\psi_{LK} > 1$		$\psi_{LK} = 1$	$\psi_{LK} < 1$		$\psi_{LK} > 1$		$\psi_{LK} = 1$	$\psi_{LK} < 1$
$\psi_{LK} > 1/s_K$	$1 < \psi_{LK} < 1/s_K$				$\psi_{LK} > 1/s_L$	$1 < \psi_{LK} < 1/s_L$		
$\hat{w}_L < 0, \hat{w}_K > 0$ ***L-penalizing*** ***K-rewarding***	$0 < \hat{w}_L < \hat{w}_K$ *L- & K-rewarding*	$\hat{w}_L = \hat{w}_K > 0$ *L- & K-rewarding*	$\hat{w}_L > \hat{w}_K > 0$ *L- & K-rewarding*	$\hat{w}_L = \hat{w}_K = \mu > 0$ *L- & K-rewarding*	$\hat{w}_L > 0, \hat{w}_K < 0$ *L-rewarding* *K-penalizing*	$\hat{w}_L > \hat{w}_K > 0$ *L- & K-rewarding*	$\hat{w}_L = \hat{w}_K > 0$ *L- & K-rewarding*	$0 < \hat{w}_L < \hat{w}_K$ *L- & K-rewarding*
$\varepsilon_{LT} < 0, \varepsilon_{KT} > 0$ $\kappa_L < 0, \kappa_K > 0$ $\hat{s}_L < 0, \hat{s}_K > 0$ ***ultra anti-labor,*** ***pro-capital biased***	$0 < \varepsilon_{LT} < \varepsilon_{KT}$ $\kappa_L < 0, \kappa_K > 0$ $\hat{s}_L < 0, \hat{s}_K > 0$ *anti-labor,* *pro-capital biased*	$\varepsilon_{LT} = \varepsilon_{KT} > 0$ $\kappa_L = \kappa_K = 0$ $\hat{s}_L = \hat{s}_K = 0$ *unbiased*	$\varepsilon_{LT} > \varepsilon_{KT} > 0$ $\kappa_L > 0, \kappa_K < 0$ $\hat{s}_L > 0, \hat{s}_K < 0$ *pro-labor,* *anti-capital biased*	$\varepsilon_{LT} = \varepsilon_{KT} = \mu > 0$ $\kappa_L = \kappa_K = 0$ $\hat{s}_L = \hat{s}_K = 0$ *unbiased*	$\varepsilon_{LT} > 0, \varepsilon_{KT} < 0$ $\kappa_L > 0, \kappa_K < 0$ $\hat{s}_L > 0, \hat{s}_K < 0$ *pro-labor,* *ultra anti-capital biased*	$\varepsilon_{LT} > \varepsilon_{KT} > 0$ $\kappa_L > 0, \kappa_K < 0$ $\hat{s}_L > 0, \hat{s}_K < 0$ *pro-labor,* *anti-capital biased*	$\varepsilon_{LT} = \varepsilon_{KT} > 0$ $\kappa_L = \kappa_K = 0$ $\hat{s}_L = \hat{s}_K = 0$ *unbiased*	$0 < \varepsilon_{LT} < \varepsilon_{KT}$ $\kappa_L < 0, \kappa_K > 0$ $\hat{s}_L < 0, \hat{s}_K > 0$ *anti-labor,* *pro-capital biased*

Table A2. Neutral, Disembodied, Factor-Augmenting Technological Change in the 2-Input Case: Cost-Function Setting

$$p_t = c(w_{L,t}e^{-\mu_L t}, w_{K,t}e^{-\mu_K t})$$

$\mu_L > 0, \mu_K = 0$ Harrod-neutral ***(L-augmenting)***				$\mu_L = \mu_K (\equiv \mu) > 0$ Hicks-neutral ***(L & K-augmenting)***	$\mu_L = 0, \mu_K > 0$ Solow-neutral ***(K-augmenting)***			
$\hat{\hat{v}}_L > 0, \hat{\hat{v}}_K < 0$				$\hat{\hat{v}}_L = \hat{\hat{v}}_K = 0$	$\hat{\hat{v}}_L < 0, \hat{\hat{v}}_K > 0$			
$\sigma_{LK} > 1$		$\sigma_{LK} = 1$	$\sigma_{LK} < 1$		$\sigma_{LK} > 1$		$\sigma_{LK} = 1$	$\sigma_{LK} < 1$
$\sigma_{LK} > 1/s_K$	$1 < \sigma_{LK} < 1/s_K$				$\sigma_{LK} > 1/s_L$	$1 < \sigma_{LK} < 1/s_L$		
$\hat{v}_L > 0, \hat{v}_K < 0$ ***L-using*** ***K-saving***	$0 > \hat{v}_L > \hat{v}_K$ *L- & K-saving*	$\hat{v}_L = \hat{v}_K < 0$ *L- & K-saving*	$\hat{v}_L < \hat{v}_K < 0$ *L- & K-saving*	$\hat{v}_L = \hat{v}_K = -\mu < 0$ *L- & K-saving*	$\hat{v}_L < 0, \hat{v}_K > 0$ *L-saving* *K-using*	$\hat{v}_L < \hat{v}_K < 0$ *L- & K-saving*	$\hat{v}_L = \hat{v}_K < 0$ *L- & K-saving*	$0 > \hat{v}_L > \hat{v}_K$ *L- & K-saving*
$\varepsilon_{LT} > 0, \varepsilon_{KT} < 0$ $\kappa_L > 0, \kappa_K < 0$ $\hat{s}_L > 0, \hat{s}_K < 0$ ***ultra pro-labor, anti-capital biased***	$0 > \varepsilon_{LT} > \varepsilon_{KT}$ $\kappa_L > 0, \kappa_K < 0$ $\hat{s}_L > 0, \hat{s}_K < 0$ *pro-labor, anti-capital biased*	$\varepsilon_{LT} = \varepsilon_{KT} < 0$ $\kappa_L = \kappa_K = 0$ $\hat{s}_L = \hat{s}_K = 0$ *unbiased*	$\varepsilon_{LT} < \varepsilon_{KT} < 0$ $\kappa_L < 0, \kappa_K > 0$ $\hat{s}_L < 0, \hat{s}_K > 0$ *anti-labor, pro-capital biased*	$\varepsilon_{LT} = \varepsilon_{KT} < 0$ $\kappa_L = \kappa_K = 0$ $\hat{s}_L = \hat{s}_K = 0$ *unbiased*	$\varepsilon_{LT} < 0, \varepsilon_{KT} > 0$ $\kappa_L < 0, \kappa_K > 0$ $\hat{s}_L < 0, \hat{s}_K > 0$ *anti-labor, ultra pro-capital biased*	$\varepsilon_{LT} < \varepsilon_{KT} < 0$ $\kappa_L < 0, \kappa_K > 0$ $\hat{s}_L < 0, \hat{s}_K > 0$ *anti-labor, pro-capital biased*	$\varepsilon_{LT} = \varepsilon_{KT} < 0$ $\kappa_L = \kappa_K = 0$ $\hat{s}_L = \hat{s}_K = 0$ *unbiased*	$0 > \varepsilon_{LT} > \varepsilon_{KT}$ $\kappa_L > 0, \kappa_K < 0$ $\hat{s}_L > 0, \hat{s}_K < 0$ *pro-labor, anti-capital biased*