Chapter 9

LOWE INDICES

Peter Hill[1]

1. Introduction

The Lowe price index is a type of index in which the quantities are fixed and predetermined. The Lowe quantity index is a type of index in which the prices are fixed and predetermined. Many of the indices produced by statistical agencies turn out to be Lowe indices. They range from Consumer Price Indices to the Geary-Khamis quantity indices used in the first three phases of the International Comparisons Project of the United Nations and the World Bank. Lowe indices have certain characteristic features that throw light on their underlying properties.

The name "Lowe Index" was introduced in the international *Consumer Price Index Manual: Theory and Practice* (2004) -- the 2004 CPI Manual hereafter -- and in a paper by Balk and Diewert (2003, 2009).[2] However, it is not a new index number formula. It makes its appearance in paragraph 1.17 of Chapter 1 of the 2004 CPI Manual where it is described as follows:

> "One very wide, and popular, class of price indices is obtained by defining the index as the percentage change, between the periods compared, in the total cost of purchasing a given set of quantities generally described as a 'basket'... This class of index is called a Lowe index after the index number pioneer who first proposed it in 1823" (see Chapter 15).

Such indices are often described loosely as Laspeyres indices or Laspeyres type indices. However, a true Laspeyres price index is one in which the quantities that make up the basket are the actual quantities of the price reference period. This is the earlier of the two periods compared, assuming that the price changes are being measured forwards in time. Consumer Price Indices, or CPIs, are not Laspeyres indices as just defined, even though they may officially be described as Laspeyres type indices. The expenditures and quantities used as weights for CPIs typically come from household budget surveys undertaken some years before the price reference period for the CPI. For practical reasons, the quantities always refer to a period which pre-dates the price reference period, possibly by a considerable length of time.

---

[1] The author would like to thank Bert Balk and Erwin Diewert for a number of helpful and constructive comments on an earlier draft of this paper.

[2] The 2004 international *CPI Manual* was produced under the auspices of a group of international agencies - the ILO, IMF, OECD, EU (Eurostat), UNECE, and World Bank - advised by an international group of experts that included Bert Balk, Erwin Diewert and the author. Erwin Diewert wrote the entire sequence of chapters on index number theory from Chapter 15 onwards.

Citation: **Peter Hill (2010), "Lowe Indices," chapter 9, pp. 197-216 in**
W.E. Diewert, B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura (2010),
***PRICE AND PRODUCTIVITY MEASUREMENT: Volume 6 -- Index Number Theory*. Trafford Press.**
Also available as a free e-publication at www.vancouvervolumes.com and www.indexmeasures.com.

© Alice Nakamura, 2009. Permission to link to, or copy or reprint, these materials is granted without restriction, including for use in commercial textbooks, with due credit to the authors and editors.

In the interest of greater accuracy and precision, and to avoid confusion about the actual status of CPIs, it was decided to introduce the concept of a 'Lowe index' in the 2004 CPI Manual. The term 'Lowe index' will not be found in the index number literature before 2003. In a Lowe price index the quantities are not restricted to those in one or another of the periods compared. Any set of quantities may be used. They could even be hypothetical quantities that do not refer to any actual period of time.

This paper is not just concerned with CPIs. Lowe indices are used extensively throughout the entire field of economic statistics. This paper introduces the concept of the Lowe quantity index which is defined as the ratio of total costs, or values, of two different baskets of goods and services valued at the same set of prices. Any set of prices may be used and they do not have to be those observed in either of the two periods compared. Lowe quantity indices are used extensively by statistical agencies. They are commonly used in national accounts.

Moreover, Lowe indices are not confined to inter-temporal comparisons. As they are transitive, Lowe indices have been widely used in multilateral comparisons of real product between countries. For example, the Geary-Khamis method used in the first three phases of the International Comparisons Project of the United Nations and World Bank uses a Lowe quantity index in which the prices are the average prices for the group of countries as a whole[3]. Other types of Lowe indices have also been used for international comparisons.

The first section of the paper focuses on the use of Lowe price indices as CPIs drawing upon material contained in the 2004 CPI Manual. Later sections focus mainly on Lowe quantity indices, particularly as used in international comparisons.

2. CPIs as Lowe Price Indices

An inter-temporal Lowe price index compares the total value of a given set, or basket, of quantities in two different time periods. The quantities that make up the basket are described as the reference quantities and are denoted by q_i^r $(i = 1,2,\dots,n)$. The period with which prices in other periods are compared is described as the price reference period. The Lowe index for period t with period 0 as the price reference period, $P_{LO}^{0,t}$, is defined as follows:

(1) $$P_{LO}^{0,t} \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^r}{\sum_{i=1}^{n} p_i^0 q_i^r}$$

In a Lowe index, any set of quantities could serve as the reference quantities. They do not have to be the quantities purchased in one or other of the two periods compared, or indeed in any other period of time. They could, for example, be arithmetic or geometric averages of the quantities in the two periods compared or purely hypothetical quantities.

[3] See Kravis, Heston and Summers (1982, pp. 89-94) and Hill (1997, pp. 57, 58).

In CPIs, the quantities selected to serve as the reference quantities are generally those actually purchased by households over the course of a year or possibly over a longer period. The data source is typically a survey of household consumption expenditures conducted well in advance of the period which is to serve as the price reference period. For example, if Jan. 2000 is chosen as the price reference period for a monthly CPI, the quantities may be derived from an annual expenditure survey carried out in 1997 or 1998, or perhaps spanning both years. As it takes a long time to collect and process expenditure data, there is usually a considerable time lag before such data can be introduced into the calculation of CPIs. The basket may also refer to a year, whereas the periodicity of the index may be a month or quarter.

When the reference quantities in a CPI belong to an actual time period it is described as the quantity reference period. It will be denoted as period b. As just noted, the quantity reference period b is likely to precede price reference period 0 and it will be assumed throughout this section that the order of the three time periods is $b < 0 < t$. The Lowe index for period t with period b as the quantity reference period and period 0 as the price reference period is written as follows:

$$(2) \qquad P_{LO}^{0,t} = \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \sum_{i=1}^{n} \left(\frac{p_i^t}{p_i^0} \right) s_i^{0b} \qquad \text{where} \quad s_i^{0b} = \frac{p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}.$$

The index can be written, and calculated, in two ways: either as the ratio of two value aggregates, or as an arithmetic weighted average of the price ratios, or price relatives, p_i^t / p_i^0, for the individual products using the hybrid expenditures shares s_i^{0b} as weights. They are described as hybrid because the prices and quantities belong to two different time periods, 0 and b respectively. The hybrid weights may be obtained by updating the actual expenditure shares in period b, namely $p_i^b q_i^b / \sum p_i^b q_i^b$, for the price changes occurring between periods b and 0 by multiplying them by the price relatives p_i^0 / p_i^b and then normalising them to sum to unity.

3. Laspeyres and Paasche Indices

Laspeyres and Paasche indices are special cases of the Lowe index. The Laspeyres price index is the Lowe index in which the reference quantities are those of the price reference period 0 -- that is, period b coincides with period 0 in equation (2).[4] The Paasche price index is the Lowe index in which the reference quantities are those of period t -- that is, period b coincides

[4] When the quantity reference period is not the same as the price reference period, the term 'base period' can be ambiguous as it could mean either period. The term 'base period' is therefore avoided here where possible. In a Laspeyres index the price and quantity reference periods are the same so that it can unambiguously be described as the base period.

with t. Assuming that $0 < t$, the Laspeyres index uses the basket of the earlier of the two periods while the Paasche uses that of the later period.

The properties of Laspeyres and Paasche indices are well known and discussed extensively in the index number literature. When the price and quantity relatives for period t based on period 0 are negatively correlated, which happens when consumers substitute goods that are becoming relatively cheaper for goods that are becoming relatively dearer, the Laspeyres index exceeds the Paasche.[5] This almost invariably happens in practice, at least with CPIs. For a more detailed and rigorous discussion of the inter-relationships between Laspeyres and Paasche indicies, see paragraphs 15.11 to 15.17 and Appendix 15.1 by Erwin Diewert in the 2004 CPI Manual. In the present context, it is necessary to consider the relationships beween Lowe, Laspeyres and Paasche indices.

A Lowe price index can be expressed as the ratio of two Laspeyres prices indices based on the quantity reference period b. For example, the Lowe index for period t with price reference period 0 is equal to the Laspeyres index for period t based on period b divided by the Laspeyres index for period 0 also based on period b. Thus,

$$(3) \qquad P_{LO}^{0,t} = \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \left[\frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b} \Bigg/ \frac{\sum_{i=1}^{n} p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b} \right] = \frac{P_{LA}^{b,t}}{P_{LA}^{b,0}},$$

where the subscript $_{LA}$ denotes Laspeyres.

Equation (3) also implies that the Laspeyres index for period t based on period b can be factored into the product of two Lowe indices, namely for period 0 on period b multiplied by that for period t on period 0. Re-arranging (3) we have

$$(4) \qquad P_{LA}^{b,t} = P_{LA}^{b,0} \cdot P_{LO}^{0.t} = P_{LO}^{b,0} \cdot P_{LO}^{0,t} = P_{LO}^{b,t},$$

since $P_{LO}^{b,0}$ is identical to $P_{LA}^{b,0}$ and $P_{LO}^{b,t}$ is identical to $P_{LA}^{b,t}$. Equation (4) illustrates an important property of Lowe price indices, namely that they are transitive. The Lowe (=Laspeyres) index for period t based on the quantity reference period b can be viewed as a chain Lowe index in which periods b and t are linked through the intermediate period 0.

It is more interesting and important to consider the case where the link is through a period that does not lie between the two periods compared. The Lowe index for period for period t on period 0 can be factored as follows by rearranging (3):

[5] This result was first derived by von Bortkiewicz (1923). The proof is given in Appendix 15.9 by Erwin Diewert in the 2004 CPI Manual. See also Hill (2006), pp. 315- 323.

$$(5)\qquad P_{LO}^{0,t} \equiv \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \frac{\sum_{i=1}^{n} p_i^b q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} \cdot \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b} = P_{LO}^{0,b} \cdot P_{LO}^{b,t} = P_{PA}^{0,b} \cdot P_{LA}^{b,t}.$$

This shows that the direct Lowe index for t on 0 is identical to the chain Lowe index that links t with 0 via period b. This reflects the fact that Lowe indices are transitive. However, as just noted, $P_{LO}^{b,0}$ is identical to $P_{LA}^{b,0}$ and $P_{LO}^{b,t}$ is identical to $P_{LA}^{b,t}$. Thus, the direct Lowe index is also identical to the Paasche index for b based on 0 multiplied by the Laspeyres index for t based on b. Given that the order of the three periods is $b < 0 < t$, the Paasche index for b with period 0 as the price reference period measures the price change *backwards* from 0 to b. Thus, (5) can be interpreted as showing that the Lowe index for t on 0 is a chain index in which the first link is the backwards Paasche[6] from 0 to b while the second link is the forwards Laspeyres from b to t.

This roundabout way of measuring the change between 0 and t via period b becomes increasingly arbitrary and unsatisfactory the further back in time the quantity reference period b is from the price reference period 0.

4. Short Term Price Movements

Most users of CPIs are more interested in short term price movements in the recent past than in the total price change between the possibly remote price reference period 0 and period t. Consider the index for period $t+1$ on period t with price reference period 0 and quantity reference period b. The order of the periods remains $b < 0 < t < t+1$. The change between t and t+1 is obtained indirectly by dividing the index of t+1 by the index for t, as follows:

$$(6)\qquad P_{LO}^{t,t+1} = \left[\frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} \Bigg/ \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} \right] = \frac{\sum_{i=1}^{n} p_i^{t+1} q_i^b}{\sum_{i=1}^{n} p_i^t q_i^b}.$$

In general, the ratio of two Lowe indices is also a Lowe index. Here, the index for t+1 on t is a Lowe index with period b as the quantity reference period. It does not depend on the quantities in the original price reference period 0.

As just shown above, this index can also be viewed as a chain index in which the first link is $P_{PA}^{t,b}$, the *backwards* Paasche index that measures the price change from period t back to period b, while the second link is $P_{PA}^{t,b+1}$, the *forwards* Laspeyres from b to t+1. Linking two

[6] The backwards Paasche index is equal to $1/P_{LA}^{0,b}$. It is the reciprocal of the Laspeyres index for period 0 that uses period b as the price (and quantity) reference period.

consecutive time periods in this roundabout way through some third period in the past is inherently arbitrary and unreasonable. There can be no economic rationale for such a procedure. With the passage of time, the relative quantities in periods t and t+1 are likely to diverge increasingly from the relative quantities in period b. In this case, the quantities of period b become increasingly irrelevant to a price comparison between t and t+1 the longer the lapse of time between period b and period t.[7]

In order to have short term Lowe indices whose reference quantities are of some relevance to the two periods compared, the gap between the quantity reference period b and period t should be kept to a minimum. This implies that the quantity reference period itself should be updated as frequently as possible. The Lowe indices themselves need to be chained.

5. Lowe, Laspeyres and Cost of Living Indices

A cost of living index, or COLI, may be defined as the ratio of minimum expenditures needed to attain the same level of utility in two time periods. Assuming that the actual expenditures in the first period are minimal, the COLI measures the minimum amount by which expenditures need to change in order to maintain the level of utility in the first period.

COLIs cannot be calculated exactly because the second set of expenditures cannot be observed. However, a COLI may be approximated by means of a superlative index. The concept of a superlative index was introduced by Erwin Diewert (1976). Superlative indices treat both periods symmetrically, the two most widely used examples of superlative indices being the Fisher index and the Törnqvist index. These indices and their properties are explained in some detail in Chapters 1, 15, 16 and 17 of the 2004 CPI Manual.

A well known result in index number theory is that the Laspeyres price index places an upper bound on the COLI based on the first period, while the Paasche index places a lower bound on the COLI based on the second period.[8] It useful therefore to establish how a Lowe index that uses as reference quantities the quantities of period b may be expected to relate to the Laspeyres based on period 0 where, as usual, b is earlier than 0.

This relationship is examined in paragraphs 15.44 to 15.48 and Appendix 15.2 of the 2004 CPI Manual. As it depends on the behaviour of prices and quantities over time, no unconditional generalizations can be made. However, it is possible to make generalizations that are conditional on particular types of behaviour, just as it can predicted that a Laspeyres index will exceed the corresponding Paasche index if there is a negative correlation between the price and quantity relatives. The conclusion reached in paragraph 15.45 of the 2004 CPI Manual is that "under the assumptions that there are long-term trends in prices and normal consumer substitution responses, the Lowe index will normally be greater than the corresponding Laspeyres index."

---

[7] In a paper on the relative merits of direct and chained indices included in the present volume, Balk (2009) also concludes that when measuring the change between consecutive time periods "it is not at all clear why period *0* price and/or quantity data should play a role in the comparison of periods τ and $\tau-1$ ($\tau = 2,\ldots,t$)."

[8] The proof is given in paragraphs 17.9 to 17.17 of the 2004 CPI Manual. The proof is attributable to Konüs (1924).

It is reasonable to conclude that, in most cases, the Lowe index will exceed the corresponding Laspeyres index, and that the gap between them is likely to increase the further back in time period b for the Lowe reference quantities is compared with period 0, the base period for the Laspeyres index.

Given that period b precedes period 0, the ranking of the indices for period t on period 0 under the assumed conditions will be:

Lowe ≥ Laspeyres ≥ Fisher ≥ Paasche.

As the Fisher is a superlative index it may be expected to approximate a COLI.

Statistical offices need to take these relationships into consideration. There may be practical advantages and financial savings from continuing to make repeated use over many years of the same fixed set of quantities to calculate a CPI. However, the amount by which such a CPI exceeds some conceptually preferred target index, such as a COLI, is likely to get steadily larger the longer the same set of reference quantities is used. Many users are likely to interpret the difference as upward bias, which may eventually undermine the credibility and acceptability of the index.

Assuming long term trends in prices and normal consumer substitution, Balk and Diewert (2003) conclude that, the difference between a Lowe index and a COLI may be “reduced to a negligible amount if:

- the lag in obtaining the base year quantity weights is minimized, and

- the base year is changed as frequently as possible.”

Essentially the same recommendation was made at the end of the previous section but on slightly different grounds.

6. Lowe Price Indices as Deflators and Their Associated Implicit Quantity Indices

Lowe price indices may be used to deflate time series of consumption expenditures at current prices in order to obtain the implicit quantity indices. The two implicit quantity indices of main interest are the index for period t on period 0 and for period t+1 on period t. Deflating the change in current expenditures between period 0 and period t by the Lowe index for period t, we have:

$$(7) \qquad \left[\frac{\sum_{i=1}^{n} p_i^t q_i^t}{\sum_{i=1}^{n} p_i^0 q_i^0} \middle/ \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} \right] = \left[\frac{\sum_{i=1}^{n} p_i^t q_i^t}{\sum_{i=1}^{n} p_i^t q_i^b} \middle/ \frac{\sum_{i=1}^{n} p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_l^b} \right] = \frac{Q_{PA}^{b,t}}{Q_{PA}^{b,0}}$$

where Q_{PA} denotes a Paasche quantity index. The implicit quantity index is therefore equal to the ratio of the Paasche quantity index for t on b divided by that for 0 on b.

In the likely case in which the Lowe price index for t on 0 exceeds the Laspeyres index for t on 0, then the implicit quantity index for t on 0 will be less than the Paasche index for t on 0.

The implicit quantity index between period t and period t+1 is as follows.

(8) $$\left[\frac{\sum_{i=1}^{n} p_i^{t+1} q_i^{t+1}}{\sum_{i=1}^{n} p_i^{t} q_i^{t}} \bigg/ \frac{\sum_{i=1}^{n} p_i^{t+1} q_i^{b}}{\sum_{i=1}^{n} p_i^{t} q_i^{b}}\right] = \left[\frac{\sum_{i=1}^{n} p_i^{t+1} q_i^{t+1}}{\sum_{i=1}^{n} p_i^{t+1} q_i^{b}} \bigg/ \frac{\sum_{i=1}^{n} p_i^{t} q_i^{t}}{\sum_{i=1}^{n} p_i^{t} q_l^{b}}\right] = \frac{Q_{PA}^{b,t+1}}{Q_{PA}^{b,t}}$$

Thus, it equals the ratio of the Paasche quantity index for t+1 on b to the Paasche index of t on b. It does not depend on the prices or quantities in the price reference period 0.

The ratio of two Paasche quantity indices is a conceptually complex measure whose meaning is not intuitively obvious. Such indices are not common and Lowe price indices do not seem to be widely used as deflators.[9]

6. Inter-temporal Lowe Quantity Indices

Consider a set of n products with quantities q_i ($i = 1,2,\ldots,n$). A Lowe quantity index is defined as the ratio of the total values of the quantities in two different time periods valued at the same set of reference prices. Any set of prices may be chosen as the reference prices. They do not have to be those observed in some actual period.

The inter-temporal Lowe quantity index, $Q_{LO}^{0,t}$, for period t with period 0 as the quantity reference period is defined as follows:

(9) $$Q_{LO}^{0,t} = \frac{\sum_{i=1}^{n} p_i^{r} q_i^{t}}{\sum_{i=1}^{n} p_i^{r} q_i^{0}}$$

where the p_i^r denote the reference prices. When the reference prices are those observed in some actual time period b it will be described as the price reference period.

The Lowe quantity index for period t with period 0 as the quantity reference period and period b as the price reference period is defined as follows:

[9] Although aggregate Lowe price indices, such as the overall CPI, may not be widely used to deflate expenditure aggregates, the detailed disaggregated price indices of which they are composed are commonly used to deflate individual components of final expenditures or output in national accounts. The disaggregated component indices may be reweighted as required or appropriate.

$$(10) \qquad Q_{LO}^{0,t} = \frac{\sum_{i=1}^{n} p_i^b q_i^t}{\sum_{i=1}^{n} p_i^b q_i^0} = \sum_{i=1}^{n} \left(\frac{q_i^t}{q_i^0} \right) s_i^{b0}, \qquad \text{where} \quad s_i^{b0} = \frac{p_i^b q_i^0}{\sum_{i=1}^{n} p_i^b q_i^0}.$$

Like the Lowe price index, the Lowe quantity index can be written, and calculated, in two ways: either as the ratio of two value aggregates, or as an arithmetic weighted average of the quantity relatives, q_i^t / q_i^0, using the hybrid expenditures shares s_i^{0b} as weights. The hybrid weights may be obtained by updating the actual expenditure shares in period b, namely $p_i^b q_i^b / \sum p_i^b q_i^b$, by multiplying them by the quantity relatives q_i^0 / q_i^b and then normalising them to sum to unity.

7. Lowe, Laspeyres and Paasche Quantity Indices

The properties and behaviour of Lowe quantity indices match those of the corresponding price indices and will therefore only be summarized here.

First, the Laspeyres and Paasche quantity indices are special cases of the Lowe quantity index. Second, any Lowe quantity index can be expressed as the ratio of two Laspeyres quantity indices based on the price reference period b.

Third, the Lowe quantity index is transitive. Consider a pair of Lowe quantity indices using the same set of reference prices such as those of period b. The Lowe quantity index for period k with period j as the quantity reference period multiplied by the index for period l with period k as the quantity reference period is identical with the Lowe quantity index for period l with period j as the quantity reference period:

$$(10) \qquad \frac{\sum_{i=1}^{n} p_i^b q_i^k}{\sum_{i=1}^{n} p_i^b q_i^j} \cdot \frac{\sum_{i=1}^{n} p_i^b q_i^l}{\sum_{i=1}^{n} p_i^b q_i^k} \equiv \frac{\sum_{i=1}^{n} p_i^b q_i^l}{\sum_{i=1}^{n} p_i^b q_i^j}, \qquad \text{or} \qquad Q_{LO}^{j,k} \cdot Q_{LO}^{k,l} \equiv Q_{LO}^{j,l}.$$

8. Time series at Constant Prices

It is common in national accounts to publish time series for an aggregate such as Household Consumption Expenditures at constant prices. A convenient year such as 2,000 is chosen as the base year and the values of the aggregate in subsequent years are given by revaluing the quantities at the reference prices of year 2,000. The constant price series is usually obtained by deflating the values of the aggregate at current prices by Paasche price indices based on 2,000. The movements in the resulting constant prices series are, of course, identical with those of a Laspeyres quantity index based on 2,000. The base year 2,000 will be denoted here simply as year 0.

The proportionate change in the constant price series between any pair of consecutive years, such as t and t+1, that do not include the base year is a Lowe quantity index: namely,

$$(11) \qquad Q_{LO}^{t,t+1} = \frac{\sum_{i=1}^{n} p_i^0 q_i^{t+1}}{\sum_{i=1}^{n} p_i^0 q_i^t}$$

where p_i^0 is the price of product i in 2,000. It equals the Laspeyres quantity index for year t+1 divided by that for year t. In practice, the change between t and t+1 may be of greater interest to users and of more relevance for policy purposes than the total change between the price reference year 2,000 and year t+1.

As in the corresponding case of a Lowe price index, this Lowe quantity index can be viewed as a chain index that links t and t+1 via the base year 0:

$$(12) \qquad Q_{LO}^{t,t+1} = \frac{\sum_{i=1}^{n} p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^t} \cdot \frac{\sum_{i=1}^{n} p_i^0 q_i^{t+1}}{\sum_{i=1}^{n} p_i^0 q_i^0} = \frac{Q_{LA}^{0,t+1}}{Q_{LA}^{0,t}}$$

The first term on the right of (12) is the (backwards) Paasche quantity index for year 0 with year t as the quantity reference period. It measures the quantity change from t back to 0. The second term in (12), the Laspeyres for t+1 based on 0, then measures the forward change from 0 up to t+1. As in the case of the corresponding Lowe price index, this roundabout way of measuring the change between t and t+1 is inherently arbitrary and unsatisfactory. The reference prices for year 0 are likely to become increasingly inappropriate for a comparison between t and t+1 with the passage of time.

For this reason, it is generally accepted that, despite the convenience of constant price series for many uses, it not desirable to permit the series to continue for more than a few years before moving the price reference year forwards. Each new price reference year then acts as the link between the previous series and the new series[10].

9. Lowe Indices in International Comparisons

As Lowe indices are transitive, they are widely used for purposes of multilateral comparisons within groups of countries. There are many ways in which the reference quantities

[10] The *1993 SNA Manual* remarks in paragraph 16.77 that "… the underlying issue is not whether to chain or not but how often to rebase. Sooner or later the base year for fixed weight Laspeyres volume indices and their associated constant price series has to be updated because the prices in the base year become increasingly irrelevant."

or prices might be specified. Two important classes of multilateral price and quantity indices actually used in international comparisons are the average quantity methods and the average price methods[11].

The average quantity methods use as reference quantities a basket whose quantities consist of some kind of average of the quantities in all the countries in the group. The purchasing power parity, or PPP, for a pair of countries is then defined as the ratio of the values of the reference basket in the two countries valued at their own prices in their own national currencies. Either country may serve as the reference country. The average price methods use a set of average prices for the group as a whole as the reference prices to construct international quantity indices.

The average quantity methods generate Lowe PPPs while the average price methods generate international Lowe quantity indices. However, they are not described as 'Lowe' PPPs or indices in the existing literature on PPPs and International Comparisons as the term 'Lowe' index was only introduced in 2003, as already mentioned.

10. Lowe PPPs

Consider first the PPP between a single pair of countries, j and k. A basket of n reference quantities is specified, the quantities being denoted by q_i^r. The prices in each country denoted by p_i^j, are denominated in the national currency of the country. The Lowe PPP, or PPP_{LO}, for country k with country j as the reference country is defined as follows:

$$(13) \qquad PPP_{LO}^{j,k} = \frac{\sum_{i=1}^{n} p_i^k q_i^r}{\sum_{i=1}^{n} p_i^j q_i^r}$$

Any set of quantities could serve as the reference quantities. They do not have to be the quantities purchased in one or other of the two countries compared, or indeed in any actual country. They could be arithmetic or geometric averages of the quantities in the two countries compared or averages over a larger group of countries for which multilateral PPPs are required.

If the reference quantities are specified to be those of the reference country j the PPP becomes a Laspeyres PPP. If the reference quantities are those of country k, it becomes a Paasche PPP.[12] As in inter-temporal indices, Laspeyres and Paasche indices are special cases of Lowe indices.

[11] See Kravis, Heston and Summers (1982) pp. 77-79 and Hill (1997, pp. 54-62).

[12] See, for example, Table 7.2 of Kravis, Heston and Summers (1982) which lists all the Laspeyres and Paasche indices between 34 countries.

11. Multilateral PPPs Using the Star method

The attraction of a Lowe PPP in the context of a set of multilateral comparisons for a group of countries is that the Lowe index is transitive. There are many possible sets of reference quantities to choose from. One possibility to select the quantities in one of the countries in the group, say country b, and to use them as the reference quantities for the PPPs between every pair of countries in the group. In this case, country b acts as the base country for the multilateral comparisons. The reference quantities q_i^r become the actual quantities in country b or q_i^b. The Lowe PPP between country k and country j is then equal to the ratio of their Laspeyres PPPs based on country b:

$$(14) \qquad PPP_{LO}^{j,k} = \frac{\sum_{i=1}^{n} p_i^k q_i^b}{\sum_{i=1}^{n} p_i^j q_i^b} = \left[\frac{\sum_{i=1}^{n} p_i^k q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b} \Bigg/ \frac{\sum_{i=1}^{n} p_i^j q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b} \right] = \frac{PPP_{LA}^{r,k}}{PPP_{LA}^{r,j}}.$$

In practice, Laspeyres PPPs are calculated between every country and the base country b. The various Lowe PPPs between pairs of countries that do not include the base country are all derived indirectly by dividing one Laspeyres by another.

This arrangement can be portrayed graphically by a star in which the base country is placed at the centre and every other country is placed in a ring around the centre. This kind of method is therefore described as a star method.

Star methods in which an individual country is chosen to be at the centre of the star have been used in the past.[13] However, the results obtained obviously vary according to the subjective choice of country to act as the base country. For this reason, star methods that place an actual country at the centre of the star are generally considered to be unacceptable. A less arbitrary method is needed.

As quantities of the same product can be summed across countries, another obvious possibility is to choose the total quantities of each product over the group of countries concerned as the reference quantities. This makes the reference quantities characteristic of the group of countries as a whole, which may be considered a desirable property for a set of multilateral comparisons. Alternatively, the total quantities may be replaced by the average quantities obtained by dividing the total quantities by the number of countries. Dividing by a constant does not change the relative quantities of different kinds of product and it is immaterial whether the average or the total quantities are used in Lowe PPPs.

The use of such average or total quantities as reference quantities for international Lowe PPPs was first proposed by Walsh (1901) and also considered as a possibility by Van Ijzeren (1956)[14]. Both Walsh and Van Ijzeren also examined the possibility of using other kinds of

[13] For example, a type of star method was used to calculate PPPs among the so-called Group II countries of Eastern Europe in the 1980's with Austria at the centre of the star.

[14] See Diewert (1993) and Hill (1997, p. 55).

averages such as geometric averages. Lowe PPPs that use arithmetic average quantities as reference quantities have been calculated by the United Nations Economic Commission for Latin America and the Caribbean.

When average quantities are used as the reference quantities, the method still remains a star method, but one in which an ‘average country’ is placed at the centre of the star instead of an actual country.[15] This is explained more fully below.

12. Lowe PPPs as deflators

Using Lowe PPPs as deflators produces derived or implicit measures of relative real expenditures that are conceptually complex. As illustrated in equation (15), if the ratio of the expenditures in national currencies for countries k and j is divided by a Lowe PPP with reference quantities q^b, the result is the ratio of two Paasche quantity indices based on country b. Country b may be an average country or an actual country:

$$(15) \qquad \left[\frac{\sum_{i=1}^{n} p_i^k q_i^k}{\sum_{i=1}^{n} p_i^j q_i^j} \Bigg/ \frac{\sum_{i=1}^{n} p_i^k q_i^b}{\sum_{i=1}^{n} p_i^j q_i^b} \right] = \left[\frac{\sum_{i=1}^{n} p_i^k q_i^k}{\sum_{i=1}^{n} p_i^k q_i^b} \Bigg/ \frac{\sum_{i=1}^{n} p_i^j q_i^j}{\sum_{i=1}^{n} p_i^j q_l^b} \right] = \frac{Q_{PA}^{b,k}}{Q_{PA}^{b,j}}.$$

In international comparisons, there is typically more interest in the quantity comparisons than in the PPPs and the international agencies tend to give quantity comparisons priority over PPPs. Lowe quantity indices which provide conceptually simple and meaningful comparisons of real expenditures have therefore been preferred to the kinds of complex implicit quantity measures given in (14).

13. International Lowe Quantity Indices

Let the two countries compared be j and k and let the selected reference prices be denoted by p_i^r. The Lowe quantity index for country k based on country j, or $Q_{LO}^{j,k}$, is defined as follows:

$$(16) \qquad Q_{LO}^{j,k} = \frac{\sum_{i=1}^{n} p_i^r q_i^k}{\sum_{i=1}^{n} p_i^r q_i^j}.$$

[15] The fact that all average quantity and average price methods are examples of the star method was pointed out by Hill (1997, pp. 54-60).

Any set of prices could be selected as the reference prices, p_i^r.

In a set of multilateral comparisons, if the prices of one of the countries, say country b, are selected as the reference quantities, the method becomes a star method in which that country is placed at the centre of the star. As shown above in the corresponding case of Lowe PPPs, the Lowe quantity index between countries j and k can then be obtained as the ratio of the Laspeyres quantity indices for countries k and j based on b.

Each Lowe quantity index can also be interpreted as a chain index in which country j is compared indirectly with country k via country b at the centre of the star:

$$(17) \qquad Q_{LO}^{j,k} \equiv \frac{\sum_{i=1}^{n} p_i^b q_i^k}{\sum_{i=1}^{n} p_i^b q_i^j} = \frac{\sum_{i=1}^{n} p_i^b q_i^b}{\sum_{i=1}^{n} p_i^b q_i^j} \cdot \frac{\sum_{i=1}^{n} p_i^b q_i^k}{\sum_{i=1}^{n} p_i^b q_i^b} = Q_{PA}^{j,b} \cdot Q_{LA}^{b,k}.$$

The Lowe index in (17) is the product of the Paasche quantity index for country b based on country j multiplied by the Laspeyres index for country k based on country b. If countries j and k are very different from each other while country b is intermediate between them, the chain index may provide a satisfactory quantity measure. However, if countries j and k are very similar to each other while country b is very different from both of them, the chain index is not likely to provide a satisfactory quantity measure. In any case, the arbitrary selection of the prices of one country to act as the reference prices is not generally consider to be an acceptable method. In practice, some kind of average prices for the group are preferred.

14. The Geary Khamis quantity index

The Geary Khamis, or GK, quantity index is a Lowe index that uses average international prices as the reference prices. The GK index has been widely used. It was used in the first three phases of the International Comparisons Project, or ICP, of the United Nations and World Bank that started in 1970. It has also been used by the OECD as one of the methods for making comparisons among OECD countries. The GK method may be described as follows.

Assume there are C countries in the group. The GK quantity index for country k on country j, Q_{GK}^{jk}, is defined as follows.

$$(18) \qquad Q_{GK}^{j,k} = \frac{\sum_{i=1}^{n} \tilde{p}_i^G q_i^k}{\sum_{i=1}^{n} \tilde{p}_i^G q_i^j} \quad \text{where} \quad \tilde{p}_i^G = \sum_{c=1}^{C} \frac{p_i^c}{PPP^c} \frac{q_i^c}{Q_i^G} \quad \text{and} \quad Q_i^G = \sum_{c=1}^{C} q_i^c.$$

The aggregate purchasing power parity for country c, PPP^c, is defined as follows:

$$(19) \qquad PPP^c = \frac{\sum_{i=1}^{n} p_i^c q_i^c}{\sum_{i=1}^{n} \bar{p}_i^G q_i^c}.$$

The average prices and the PPPs are determined simultaneously in the GK method[16]. The average prices are denominated in the numeraire currency for the group. The method is invariant to the choice of numeraire currency,

The GK average international price has a simple interpretation because it is defined in the same way as the national average price for a single country. A national average price is defined as the total value of the transactions in the product divided by the total quantity of the product. It is a quantity weighted arithmetic average of the prices at which the product is sold within the country. Similarly, a GK average international price as defined in (18) is a quantity weighted average of the prices at which the product is sold across the entire group of countries after the prices have all been converted into the designated numeraire currency. The group can therefore be regarded as if it were a super country with average prices p_i^G and total quantities Q_i^G, the p_i^G s serving as the reference prices for the Lowe quantity index.

The PPP for country c in (19) is a Paasche price index for country c based on the group G. This means that the PPP between any two countries, such as countries j and k, is the ratio of the two Paasche indices based on the group G:

$$(20) \qquad PPP_{GK}^{j,k} = \frac{\sum_{i-1}^{n} p_i^k q_i^k}{\sum_{i=1}^{n} \bar{p}_i^G q_i^k} \Bigg/ \frac{\sum_{i=1}^{n} p_i^j q_i^j}{\sum_{i=1}^{n} \bar{p}_i^G q_i^j}$$

This expression does not simplify. The properties of GK PPPs are not so simple and transparent as those of GK quantity indices.

As it is a Lowe index, the GK quantity index can be expressed as the ratio of two Laspeyres quantity indices based on the group G:

$$(21) \qquad Q_{GK}^{j,k} \equiv \frac{\sum_{i=1}^{n} \bar{p}_i^G q_i^k}{\sum_{i=1}^{n} \bar{p}_i^G q_i^j} = \left[\frac{\sum_{i=1}^{n} \bar{p}_i^G q_i^k}{\sum_{i=1}^{n} \bar{p}_i^G Q_i^G} \Bigg/ \frac{\sum_{i=1}^{n} \bar{p}_i^G q_i^j}{\sum_{i=1}^{n} \bar{p}_i^G Q_i^G} \right] = \frac{Q_{LA}^{G,k}}{Q_{LA}^{G,j}}.$$

The GK method can be viewed as a star method which places the group itself at the centre of the star.[17] The denominator in each of the two Laspeyres indices in (21) is the total value of all

[16] See Kravis, Heston and Summers (1982), pp. 89-94 for a full explanation of the GK method and its properties.

[17] See Hill (1997) pp. 54-60.

transactions in all products in all countries of the group valued at the average prices for the group expressed in the numeraire currency. The resulting GK indices do not actually depend on the total quantities Q_i^G, however, as the two denominators in (21) cancel each other out.[18]

Alternatively, the GK quantity index can be viewed as chain index as follows.

$$(22)\qquad Q_{GK}^{j,k} = \frac{\sum_{i=1}^{n} \bar{p}_i^G Q_i^G}{\sum_{i=1}^{n} \bar{p}_i^G q_i^j} \cdot \frac{\sum_{i=1}^{n} \bar{p}_i^G q_i^k}{\sum_{i=1}^{n} \bar{p}_i^G Q_i^G} = Q_{PA}^{j,G} \cdot Q_{LA}^{G,k}.$$

Q_{GK}^{jk} is a chain index in which k is linked to j via the group G. It equals the Paasche quantity index for G on j multiplied by the Laspeyres quantity index for k on G.

15. Missing Products

Linking countries through the group G can have advantages over direct comparisons between the countries concerned. One of the main problems encountered in constructing international price and quantity indices is the fact that not all the products that can be found in the group as a whole are to be found in every country. On the contrary, in any one country, many products are likely to be missing, especially if the group of countries is large and economically diverse and patterns of consumption vary considerably among the countries.

There are obviously no prices to be observed for products whose quantities are zero. As the number products available varies from country to country, the sets of country prices that might potentially be used as reference prices for Lowe quantity indices also vary in size from country to country. The largest and most comprehensive set of prices consists of the average prices for the group as a whole. This set must include every product in every country.

A direct binary comparison between two countries carried out independently of other countries uses only the prices and quantities of those two countries. However, it may not be possible to use all the quantity information if there are some products that are found in only one of the two countries. For example, it is not possible to include products that are found in country k but not in country j in the direct Laspeyres quantity index for k based on j because there are no prices for them in j. Similarly, it is not possible to include products found in j but not in k in the Paasche quantity index for k on j as there are no prices for them in k. In these circumstances, the Laspeyres and Paasche quantity indices may be regarded as being subject to bias.

[18] In the UN / World Bank *International Comparisons Project*, the denominator in each of the two Laspeyres indices in (21) could be interpreted as the total GDP of the group of countries as a whole expressed in the numeraire currency. If the group included all countries in the world, the denominator would be World GDP. Implicitly, in the GK method the Laspeyres index for the GDP of each individual country is calculated based on the total GDP for the group. The GDP quantity indices for pairs of countries are then obtained indirectly as the ratios of the corresponding Laspeyres indices based on the group as a whole.

However, all the products in both countries can be included in the Lowe quantity index between them that uses the average international prices for the group of countries to which they belong as the reference prices. As already noted, there must be an average international price for every product that is found in any country in the group. Thus, a Lowe quantity index that uses international prices is able to utilize all the quantities in both countries. For this reason it might provide a better measure of the relative quantities in the two countries than a direct binary index that has to ignore certain quantities. .

Much depends on how appropriate or relevant the average international prices are considered to be for a comparison between two countries. The GK average international prices have a clear economic interpretation and must be relevant for comparisons between countries within the group. As already explained, the GK index can be viewed as the ratio of the two Laspeyres quantity indices based on the group as a whole. Each index is able to include all the quantities in that country irrespectively of whether they are found in the other country. As shown in (22), the GK quantity index can also be viewed as a chain index that links the two countries though the group. Each link covers all the products in the country in question even though the coverage is not the same in the two links. However, this is an advantage. The main reason for chaining is that this approach is able to deal with situations, whether over time periods or countries, in which the set of products covered is variable.

If the two countries are very different with a relatively small overlap of products between them, chaining through the group as a whole is likely to produce a better quantity index than a direct comparison between them that is restricted to using only the price and quantities in the two countries concerned. On the other hand, if the sets of products available in the two countries largely coincide, the direct quantity index between them may be preferable to a chain index through the group. The argument is similar to that used earlier to argue that consecutive time periods for the same country, which are likely to have almost identical sets of products, should not be linked through some earlier time period, and especially not through some period in the remote past. In some circumstances chain indices are superior to direct indices while in other circumstances, direct indices are superior. The choice of preferred index depends on the circumstances.

16. Other Average Price Methods

Any set of prices can serve as the reference prices in a Lowe quantity index. Although a GK average price, being the international equivalent of a national average price, has a meaningful economic interpretation, there are other ways in which an average international price might be defined. Different types of averages might be used instead of an arithmetic average, and different kinds of weighting may be used.

For example, one possibility would be to use unweighted geometric means of the national average prices as the reference prices in an international Lowe quantity index. This method was advocated by Gerardi (1982), but it was first proposed by Walsh (1901).[19] An intriguing feature

[19] See Diewert (1993).

of this method is that it makes no difference to the quantity indices whether the national average prices are converted into a common numeraire currency or not.

When considering the relative merits of the different methods, the key issue is what effect the different kinds of average prices may be expected to have on the international quantity indices. This depends on how closely the various sets of national prices are correlated with the average international prices. Whatever international prices are used, they must be closer to some sets of national prices than others. Because of ordinary substitution effects, for a given set of quantities, the Lowe quantity index for a country may be expected to be lower the more closely the pattern of the reference prices resembles the pattern for the prices of that country.[20]

Consider the example of two countries. Suppose the initial vector of average international prices used as the reference prices for the Lowe quantity index is roughly equidistant from the price vectors for countries k and j. Next, suppose the vector of reference prices is changed to bring it closer to country k's vector and further away from j's vector, the quantities in both countries remaining unchanged. As the vector of reference prices approaches the actual pattern of relative prices in k that was responsible for generating the actual quantities in k, the Laspeyres quantity index for k based on the group G will tend to fall. Conversely, as the vector of reference prices moves away from the actual prices in j, the Laspeyres index for j will tend to rise. Thus, the Lowe quantity index for k on j will tend to fall.

The effects on the Lowe quantity indices of defining the average international prices in different ways are sometimes predictable for reasons just given. Suppose the average prices are weighted according to the economic size of the country as in the GK method. The vector of average international prices will tend to be closer to the vectors of actual prices for the richest countries than if the average prices are unweighted, as in the Gerardi method. Thus, the GK quantity indices for the richest countries will tend to be lower than if unweighted Gerardi prices are used.

This is an illustration of the Gerschenkron effect. It does not demonstrate which of the two types of index is biased. If unweighted average prices are preferred on the grounds that each country should be given equal weight as a matter of principle, then the Lowe quantity indices for the richest countries using GK prices as reference prices may be regarded as having a downward bias, thereby understating the gap between rich and poor countries. On the other hand, in reality countries are not all the same size and some countries account for much larger shares of total world income and output than others. From this perspective, Lowe quantity indices that use unweighted average international prices may be regarded as having an upward bias for the largest and richest countries. These are issues that cannot be decided on technical grounds alone. Value judgments are inevitably involved.

[20] This phenomenon is described in the literature as the 'Gerschenkron effect.' It occurs with all multilateral indices that use some kind of average group prices as the reference prices.

17. Conclusions

Lowe indices are popular for several reasons. They are conceptually simple and meaningful. They enable statistical agencies to economize by continuing to make use of the same set of reference prices of quantities over many years. They are transitive and additive. These two properties are particularly attractive to users of both inter-temporal and international Lowe quantity indices.

Very many of the price and quantity indices produced by statistical agencies turn out to be Lowe indices although their generic similarity has not been so obvious until recently because of the lack of a common name.

Lowe indices have two closely interrelated characteristics. They can be expressed as ratios of Laspeyres indices and they can be viewed as chain indices that link through some other period, country or group of countries. The quality of a Lowe index depends on the relevance or suitability of the link. In the case of temporal price or quantity indices where the link is some past period, its relevance must diminish as it recedes into the past. In these circumstances, a Lowe price index is also likely to be subject to increasing upward bias as compared with a cost of living index. In the case of international Lowe quantity indices, the situation is complicated. Linking a pair of countries through the group of countries to which they belong may be regarded as strengthening or weakening the comparison between them depending on circumstances.

References

Balk, B.M. (2006), "Direct and Chained Indices: A Review of Two Paradigms", chapter , in W.E. Diewert, B.M. Balk, D. Fixler, K.J. Fox, and A.O. Nakamura (eds.), *Price and Productivity Measurement: Volume 6 -- Index Number Theory*, Trafford Press.

Balk, B.M. and W.E. Diewert (2003), "The Lowe Consumer Price Index and its Substitution Bias", Discussion Paper No. 04-07, Department of Economics, University of British Columbia, Vancouver. Published as chapter 7 in W.E. Diewert, B.M. Balk, D. Fixler, K.J. Fox, and A.O. Nakamura (eds.) (2009), *Price and Productivity Measurement: Volume 6 -- Index Number Theory*, Trafford Press.

Bortkiewicz, L. von (1923), "Zweck und Struktur einer Preisindexzahl", *Nordic Statistical Journal*, 2.

Diewert, W.E. (1976), "Exact and Superlative Index Numbers", *Journal of Econometrics*, Vol. 4, pp. 224-245.

Diewert, W.E. (1993), "The Early History of Price Index Research", in *Essays in Index Number Theory, Volume 1*, edited by W.E. Diewert and A.O. Nakamura, North Holland.

EU, IMF, OECD, UN, World Bank, 1993, *System of National Accounts, 1993*, Brussels/Luxembourg, New York, Paris, Washington D.C., 1993 chapters can be downloaded for free at http://unstats.un.org/unsd/sna1993/toctop.asp?L1=1

Gerardi, D. (1982), "Selected Problems of Inter-County Comparisons on the Basis of the Experience of the EEC", *Review of Income and Wealth*, Series 28, December 1092, pp. 381-405.

Hill, R.J. (1997), "A Taxonomy of Multilateral Methods for Making International Comparisons of Prices and Quantities", *Review of Income and Wealth*, Series 43 (1) March, 49-69.

Hill, R.J. (2006), "When Does Chaining Reduce the Paasche-Laspeyres Spread? An Application to Scanner Data", *Review of Income and Wealth*, Series 52, No 2, June, 309-325.

ILO, IMF, OECD, Eurostat, UNECE, and the World Bank (2004), *Consumer Price Index Manual: Theory and Practice*, International Labour Office.

Konüs, A.A. (1924), "The Problem of the True Index of the Cost of Living", *The Economic Bulletin of the Institute of Economic Conjuncture*, No. 9-10 reprinted in English in *Econometrica*, volume 7, 1939.

Kravis, I.B, A.W. Heston and R. Summers (1982), *World Product and Income: International Comparisons of Real Gross Product*, Johns Hopkins University Press.

Van Ijzeren, J. (1956), "Three Methods of Comparing the Purchasing Power of Currencies," *Statistical Studies* No.7, The Netherlands Central Bureau of Statistics.

Walsh, C.M. (1901), *The Measurement of the General Exchange Value*, Macmillan and Co.