An Index Number Formula Problem: the Aggregation of Broadly Comparable Items

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Abstract

Index number theory informs us that if data on matched prices and quantities are available, a superlative index number formula is best to aggregate *heterogeneous* items, and a unit value index to aggregate *homogeneous* ones. The formulas can give very different results. Neglected is the practical case of broadly comparable items, for which price dispersion can be decomposed into a quality component, say due to product differentiation, and a component that is stochastic or due to price discrimination. This paper analyses why such formulas differ and proposes a solution to this index number problem.

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I. Introduction

There is a consensus as to which price index number formula is best when price and quantity/value information are available for each item aggregated. The economic theoretic approach to index number formulas supports superlative index numbers, primarily the Fisher, Törnqvist, and Walsh indexes, all of which give similar answers. The axiomatic approach supports the Fisher index. Such findings are part of the internationally-accepted manuals on consumer, producer, and (forthcoming)[2] trade price indexes—ILO *et al*. (2004a and 2004b). What is less well known is that for the aggregation of homogeneous items, the unit value index is the best formula and superlative price index numbers are biased, and for the aggregation of heterogeneous items, superlative price index numbers are best and the unit value index is biased.

The bias in superlative index numbers for homogeneous items is a neglected and important index number issue. If, for example, the prices of goods A and B were 10 and 12 in both the reference and current periods, but there was a shift in quantities from say 6, for both A and B in the reference period, to 8 for A and 4 for B in the current period, the superlative, or any other index number formula for heterogeneous goods and services, would give an answer of unity, no overall price change. However, the correct answer for homogeneous items would be a unit value *fall* of 3 per cent appropriately reflecting the shift in the quantity basket in the current period from the higher price *level* of 12 for A to the lower price *level* of 10 for B. The item is, on average, now cheaper. The *CPI Manual* (ILO *et al*. 2004a, Chapter 20) and *1993 SNA*[3] advocate the use of unit value indexes for homogeneous goods and services:

> "Suppose, for example, that a certain quantity of a particular good or service is sold at a lower price to a particular category of purchaser without any difference whatsoever in the nature of the good or service offered, location, timing or conditions of sale, or other factors. A subsequent decrease in the proportion sold at the lower price raises the average price paid by purchasers for quantities of a good or service whose quality is the same and remains unchanged, by assumption. It also raises the average price received by the seller without any change in quality. This must be recorded as a price and not a volume increase." United Nations (1993, paragraph 16.116).

Index number theory recognizes that the appropriateness of each formula depends on whether the items aggregated are homogeneous or otherwise—Diewert (1995) and (Balk, 1998 and 2005). As matters stand the advice is to simply determine whether or not items are homogeneous and apply the appropriate formula. But what if the goods and services are nearly homogeneous? A superlative price index that take account of substitution between items due to different relative price *changes*, but ignore shifts of quantities towards higher or lower price *levels*, would misrepresent price changes. Or what if the items aggregated are comparable, but of different qualities such that some of the price dispersion is due to product

[2] Available on the IMF web site: http://www.imf.org/external/np/sta/tegeipi/index.htm

[3] The same stance is taken in the update *SNA 2008* available at: http://unstats.un.org/unsd/sna1993/draftingphase/pubChapterDetail.asp?ch=16.

differentiation and some due to price discrimination? A superlative index would ignore any shift to higher or lower average (quality-adjusted) price levels, but a unit value index would wrongly treat changes in compositional mix of items of different quality as price changes, the familiar unit value bias. Given that these formulas will generally give quite different answers it is important to determine why they differ, the conditions under which each is suitable, and what to do when, as is likely to be the case, neither is.

The rationale for unit value and superlative indexes are outlined in Section II along with some discussion of the circumstances under which they are appropriate. Section III provides a formal analysis of how unit value and Fisher price index differ. In Section IV a solution is proposed: a hedonic regression equation is used to decompose the price dispersion into heterogeneity-adjusted and heterogeneity-based components and a formula derived based on an average of unit value and Fisher indexes respectively based on these components, with the weights for the respective indexes derived from the explanatory power of the regression. An application using scanner data is provided in Section V with conclusions in VI.

The application of the results of this paper is in the determination of price and volume measures at the national and micro level for economic aggregates. It applies to consumer, commodity, producer (input and output), import, and export price indexes, as well as price indexes of capital goods, such as house price indexes. Since price indexes are used as deflators thee is a concomitant application to volume indexes. The concern is with aggregation where price and quantity/value information is available for broadly comparable items, for example, for measuring the aggregate price and volume change of different qualities of automobiles, but not over automobiles and beef.

II. The Use of Unit Value and Superlative Indexes in Index Number Construction

A. Superlative index numbers

The Fisher, P_F and Törnqvist, P_T, index number formulas are both commonly used superlative indexes.[4] The Fisher price index is a geometric mean of Laspeyres, P_L, and Paasche, P_P, price indexes and is defined for a price comparison between the current period t and a reference period 0, over m=1, ...,M matched items whose respective prices and quantities are given by p_m^t and q_m^t for period t, and p_m^0 and q_m^0 for period 0, by:

$$P_F \equiv \sqrt{\frac{\sum_{m=1}^{M} p_m^t q_m^0}{\sum_{m=1}^{M} p_m^0 q_m^0} \times \frac{\sum_{m=1}^{M} p_m^t q_m^t}{\sum_{m=1}^{M} p_m^0 q_m^t}} = \sqrt{P_L \times P_P} \tag{1}$$

[4] The Walsh price index is a less commonly used superlative index that is similar to a Laspeyres or Paasche price index, but uses a geometric mean of period 0 and t quantities as the fixed basket quantities (ILO et al., 2004a, Chapter 15, paragraphs 15.24-32).

The Törnqvist price index is defined as:

$$P_T = \prod_{m=1}^{M} \left(\frac{p_m^t}{p_m^0} \right)^{\left(s_m^0 + s_m^t\right)/2} \quad \text{where } s_m^t = p_m^t q_m^t / \sum_m p_m^t q_m^t \text{ and } s_m^0 = p_m^0 q_m^0 / \sum_m p_m^0 q_m^0 \tag{2}$$

Both P_F and P_T make symmetric use of each period's price and quantity information. Diewert (1976 and 1978), from an approach based on economic theory, demonstrated that both Fisher and Törnqvist indexes belong to a class of *superlative indexes*[5] that have the desirable property of incorporating substitution effects, that is the effect of say consumers substituting their basket of goods towards those with relatively low price increases, thus lowering the cost of living. Laspeyres and Paasche indexes are fixed (quantity) basket price indexes and allow for no such substitution.

In the test or axiomatic approach desirable properties for an index number are chosen and different formula evaluated against them. Fisher described his index as "ideal" because it satisfied the tests proposed including the "time reversal" and "factor reversal" tests.[6] The Fisher index has also been justified from a fixed basket approach. It is apparent from the Laspeyres and Paasche price indexes that constitute equation (1) that both indexes hold fixed, the basket of quantities. The formulas differ in that Laspeyres holds the basket fixed in the reference period and Paasche in the current period. Neither formula can be judged superior to the other, yet they can both yield different results. A compromise solution for the price index is to use a formula that makes symmetric use of the base and current period information on quantities. The Fisher index can be shown to be the most suitable in this regard (ILO *et al.*, 2004a, Chapter 15.

Thus all three approaches favor the Fisher index in particular and superlative indexes more generally, since they produce very similar results to the Fisher index. In practice of course Laspeyres-type indexes are often calculated because data on current period information are not available.[7] The arguments presented in this paper apply as much to the use of unit value indexes against Laspeyres-type price indexes, as the Fisher price index.

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[5] Aggregator functions underlie the definition of indexes in economic theory, for example, a utility function to define a constant utility cost of living index. Different index number formulas can be shown to correspond with different functional forms of the aggregator function. Laspeyres, for example, corresponds to a highly restrictive Leontief form. The underlying functional forms for superlative indexes, including Fisher and Törnqvist, are flexible: they are second-order approximations to other (twice-differentiable) forms around the same point. It is the generality of functional forms that superlative indexes represent that allows then to accommodate substitution behavior and be desirable indexes.

[6] The time reversal test requires that the index for period *t* compared with period 0, should be the reciprocal of that for period 0 compared with *t*. The factor reversal test requires that the product of the price index and the volume index should be equal to the proportionate change in the current values.

[7] In practice, especially for CPIs where timeliness is of the essence, the price reference period 0 differs from the earlier weight reference period, say *b*, since it takes time to compile the results from the survey of households, establishments and other sources for the weights to use in the index. The Laspeyres index given by the second

(continued)

B. Unit value indexes

A unit value index, P_U, is given by:

$$P_U \equiv \left(\frac{\sum_{m=1}^{M} p_m^t q_m^t}{\sum_{m=1}^{M} q_m^t} \right) \Bigg/ \left(\frac{\sum_{m=1}^{M} p_m^0 q_m^0}{\sum_{m=1}^{M} q_m^0} \right). \tag{3}$$

If the items whose prices are being aggregated are identical—that is, perfectly homogeneous—a unit value index has desirable properties. Balk (2005) identifies it as the target index for homogeneous goods.

Consider the case where the exact same item is sold at different prices during the same period, say lower sales and higher prices in the first week of the month and higher sales and lower prices in the last week of the month. The unit value for the monthly index solves the time aggregation problem and appropriately gives more weight to the lower prices than the higher ones in the aggregate. If the elementary unit value index in equation (3) is used as a price index to deflate a corresponding change in the value, the result is a change in total quantity which is intuitively appropriate, i.e.

$$\frac{\sum_{m=1}^{M} p_m^1 q_m^1}{\sum_{n=1}^{N} p_n^0 q_n^0} \Bigg/ \left[\left(\frac{\sum_{m=1}^{M} p_m^1 q_m^1}{\sum_{m=1}^{M} q_m^1} \right) \Bigg/ \left(\frac{\sum_{m=1}^{M} p_m^0 q_m^0}{\sum_{m=1}^{M} q_m^0} \right) \right] = \frac{\sum_{m=1}^{M} q_m^1}{\sum_{m=1}^{NM} q_m^0} \tag{4}$$

Note that the summation of quantities in the top and bottom of the right-hand-side of equation (2) must be of the exact same type of item for the expression to make sense.

Balk (1998) showed that the unit value index satisfies the conventional index number tests with the exceptions of (i) the *Proportionality Test*: $P(p,\lambda p,q^0,q^t) = \lambda$ for $\lambda > 0$; that is, if all prices are multiplied by the positive number λ, then the new price index is λ. The unit value index only satisfies the proportionality test in the unlikely event that relative quantities do not change; (ii) the *Identity or Constant Prices Test*: $P(p,p,q^0,q^t) = 1$; that is, if the price of every good is identical during the two periods, then the price index should equal unity, no matter what the quantity vectors are. The unit value index only satisfies the identity test if relative quantities, that is the composition of the products compared, do not change; and (iii) *Invariance to Changes in the Units of Measurement (commensurability) Test*: $P(\alpha_1 p_1^0,...,\alpha_n p_n^0;\ \alpha_1 p_1^t,...,\alpha_n p_n^t;\ \alpha_1^{-t} q_1^0,...,\alpha_n^{-t} q_n^0;\ \alpha_1^{-t} q_1^t,...,\alpha_n^{-t} q_n^t) = P(p_1^0,...,p_n^0;\ p_1^t,...,p_n^t;\ q_1^0,...,q_n^0;\ q_1^t,...,q_n^t)$ for all $\alpha_1 > 0, \ldots, \alpha_n > 0$; that is, the price index does not change if the units of measurement for each product are changed. Changes in units *de facto* arise when the quality of items change. However, the commensurability test is satisfied in the homogeneous

component in the right-hand-side expression in equation (1) may have quantities in period b instead of 0. This index is a Lowe index—see ILO *et al.*, 2004 Chapter 15.

case, when items are identical. Moreover, these test were devised for the aggregation of heterogeneous items and are not meaningful for homogeneous items. For example, in the introduction we outlined the case where prices do not change, but a shift in quantities switches the average price to a lower level leading to a fall in the overall price level—there is a meaningful failure of the identity test.

Bradley (2005) takes a cost-of-living index defined in economic theory and compared the bias that results from using unit values as “plug-ins” for prices. He finds that if there is no price dispersion in either the current or reference period compared, the unit value (plug-in) index will not be biased against the theoretical index. This case can be subsumed under that of homogeneous items. If there is price dispersion in the reference (current) period, but not the current period, a unit value “plug-in” would have an upwards (downward) bias, and if there is price dispersion in both periods, there is a guarantee (there is a zero probability that the condition of no bias will hold for any arbitrary data generating process) that there will be a bias in the “plug-in” unit value index, but one cannot sign that bias. Balk (1998) finds the unit value index to be appropriate for a cost of living index only if the underlying preference ordering can be represented by a restrictive simple sum utility function in which the utility from q_1^τ, $q_2^\tau, \ldots, q_m^\tau$ for periods $\tau = 0, t$ is given by the unweighted $\sum_m q_m^\tau$.

C. When to use unit value indexes and when Fisher?

Balk (2005, pages 677-8) recognized the importance of the “homogeneity or heterogeneity” decision with regard to choosing between a unit value or superlative index: “.. the whole problem can be reduced to the rather simple looking operational question: Does it make (economic) sense to add up the quantities of the elements: If the answer to this question is “yes,” then the elementary aggregate is called “homogeneous” and the appropriate, also called target, price index is the unit value index.” If it is difficult to distinguish homogeneous items he advocated using unit value indexes for any subset of items which were homogeneous and a (preferably superlative) price index for the rest. If the splitting into homogeneous and heterogeneous items was not feasible, he advised a price index. Diewert (1995), for CPI compilation, defined a homogeneous item to be a variety *in a specific* outlet and argued that these unit values should be aggregated using a Fisher index. However, he took a pragmatic stance noting that if individual outlet data on transactions were not available or were considered to be too detailed, then unit values for a homogenous commodity over all outlets in a market area might form the lowest level of aggregation., (Diewert, 1995, p.22).[8]

Dálen (2001) and De Haan (2004 and 2007) take a very different view. They argue for quality-adjusted unit value indexes that remove the effect on prices of product heterogeneity. They thus generalize the application of unit value indexes to situations of items for which meaningful quality adjustments can be made, that is, broadly comparable items. We return to their work and this suggestion in Section IV.

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[8] Though work on scanner data finds quite different results when using unit values aggregated over items in a specific outlet (type), than otherwise—see Haan and Opperdoes (1999), Silver and Webb (2002), and Bradley (2005).

The *SNA 2008* holds that if the price dispersion in a period is not due to quality differences—the homogeneous case—a unit value index should be used. Yet it notes an important exception regarding the case of institutionalized price discrimination. If different purchasers of the same good or service, say water or electricity, face different prices and the individual purchasers, say commercial customers and private households, are unable to change from one price to another, then price indexes should be used.[9] The constraint on the availability to the purchaser of different prices must be institutional and not simply an income constraint. Yet even this stance is problematic. The economic theory of producer price index numbers (ILO *et al.*, 2004b, Chapters 17) defines a (fixed input) output price index as the ratio of the two revenues in the periods compared, assuming fixed technologies and inputs. A theoretical cost of living index (COLI) is defined as the ratio of minimum expenditures in the periods compared required to maintain a given level of utility, assuming fixed preferences (ILO *et al.*, 2004a, Chapters 17). From the producer's perspective, a shift in the quantities of identical items sold at differentiated prices effects a change in revenue from fixed inputs[10]—the institutional arrangements matter and indeed were likely devised to enable revenue to be maximized. Unit values should be used. From the purchaser's perspective it make no difference to the ratio of expenditures for a, say, commercial customer if the producer shifts some of its quantities to private households—the institutional arrangements do not matter and unit values should not be used. In other words, from the viewpoints of the purchasers of the above homogeneous product, what counts is his or her (separate from other purchasers) unit value price, not the overall unit value price across all purchasers, which would be the relevant price for the seller.

In choosing between unit value and price indexes it should be borne in mind that the use of unit value indexes for heterogeneous items and price index numbers for homogeneous items are both failings. However, price index number formulas are often used for aggregation across comparable, but not identical, items. Say quantity information is available for comparable items. If quality differences had no effect on price dispersion, then unit value indexes should be used? If the effect of quality differences on price dispersion was small, then there is a case for using unit value indexes, especially if the prices can be adjusted for the quality differences. But what if say only 50 percent of the price dispersion was due to quality differences? There is a sense in which unit value and price indexes can be equally justified. We consider this in more detail in Section IV.

The treatment of quality applies as much to the conditions of sale as to the product. For example, for the aggregation of price changes for the same items across retailers. Similar considerations apply to the measurement of inter-area price indexes, to determine purchasing power parities, producer price indexes, and export and import price indexes. For example, export and import price indexes are calculated by treating exports and imports of the same commodity to/from different countries/customers as different items using conventional price

[9] The empirical evidence is of substantial price discrimination for water—Yoskowitz, D.W. (2002) "Price Dispersion and Price Discrimination: Empirical Evidence from a Spot Market for Water," *Review of Industrial Organization*, 20, 283-289.

[10] We assume the costs of serving the different purchasers are significantly different.

index number formula. Adopting unit value indexes would have major affect on such measures and, in turn, measures of a nation's terms of trade.

There is an exception where unit value indexes are traditionally and wrongly used. There is a literature on bias in import and export price indexes that use unit value indexes from a commodity group defined as a group of items in a classification used by customs documents as proxies for price changes. Such groups can be too widely defined to ensure homogeneity and the findings are that such unit value indexes misrepresent price changes due to compositional changes in quantities and quality mix of what is exported and imported in the category concerned— Angermann (1980), Alterman (1991), Ruffles and Williamson, (1997), and Silver (2007).

Having noted that unit value indexes can give quite different results to price indexes, it is necessary to consider the factors determining such differences.

III. THE DIFFERENCE BETWEEN A UNIT VALUE AND A FISHER INDEX

Párniczky (1974) and Balk (1998) respectively compare unit value indexes to the Paasche and Fisher price indexes. These seminal decompositions, while useful, undertook the decomposition in terms of quantity-weighted covariances. However, quantity weighting implicitly assume homogeneity and, further, the decompositions do not distinguish between levels and changes. Both are issues of concern here. We provide a new decomposition.

We first define Laspeyres and Paasche price indexes, P_L and P_P, and a Laspeyres quantity index, Q_L, respectively as:

$$P_L \equiv \frac{\sum_{m=1}^{M} p_m^t q_m^0}{\sum_{m=1}^{M} p_m^0 q_m^0} = \sum_{m=1}^{M} s_m^0 x_m \,,\; P_P \equiv \frac{\sum_{m=1}^{M} p_m^t q_m^t}{\sum_{m=1}^{M} p_m^0 q_m^t} = \left(\sum_{m=1}^{M} s_m^t x_m^{-1} \right)^{-1} \text{, and } Q_L \equiv \frac{\sum_{m=1}^{M} p_m^0 q_m^t}{\sum_{m=1}^{M} p_m^0 q_m^0} = \sum_{m=1}^{M} s_m^0 y_m \tag{5}$$

where s_m^0 were defined in equation (2) above as period 0 value shares; x_m is the *m*th price relative, and y_m the *m*th quantity relative, defined as: $x_m \equiv \frac{p_m^t}{p_m^0}$; $y_m \equiv \frac{q_m^t}{q_m^0}$, $m = 1,...,\mathrm{M}$.

It follows from equations (1), (3), and (5) that the ratio of a unit value index to a Fisher price index is given by:

$$\frac{P_{UV}}{P_F} = \frac{P_{UV}}{P_L} \times \frac{P_L}{P_F} = \frac{P_{UV}}{P_L} \times \left[\frac{P_L}{P_P} \right]^{\frac{1}{2}} \tag{6}$$

Adopting a Bortkiewicz (1923) decomposition[11] it can be shown that:

$$\frac{P_{UV}}{P_L}=\left[\frac{\sum_{m=1}^{M}p_m^t q_m^t}{\sum_{m=1}^{M}p_m^t q_m^0}\times\frac{\sum_{m=1}^{M}q_m^0}{\sum_{m=1}^{M}q_m^t}\right]=\frac{\sum_{m=1}^{M}s_m^0 x_m y_m}{\sum_{m=1}^{M}s_m^0 x_m}\Bigg/\frac{\sum_{m=1}^{M}q_m^t}{\sum_{m=1}^{M}q_m^0}=\frac{\left(\rho_{x,y}^{s_0}cv^{s_0}(x)cv^{s_0}(y)+1\right)Q_L}{\sum_{m=1}^{M}q_m^t\Big/\sum_{m=1}^{M}q_m^0}\,. \tag{7}$$

where $\rho_{x,y}^{s_0}$ is the s_m^0-weighted correlation coefficient between price and relatives and quantity relatives, x_m and y_m and $cv^{s_0}(x)$ and $cv^{s_0}(y)$ are their s_m^0-weighted respective coefficients of variation, i.e. $\sigma_y/\overline{y}$ and $\sigma_x/\overline{x}$.

Since $$\frac{Q_L}{\sum_{m=1}^{M}q_m^t\Big/\sum_{m=1}^{M}q_m^0}=\frac{\sum_{m=1}^{M}p_m^0 q_m^t/\sum_{m=1}^{M}q_m^t}{\sum_{m=1}^{M}p_m^0 q_m^0/\sum_{m=1}^{M}q_m^0}=\frac{\left(\rho_{p^0,q^t}cv(p^0)cv(q^t)+1\right)}{\left(\rho_{p^0,q^0}cv(p^0)cv(q^0)+1\right)} \tag{8}$$

where ρ_{p^0,q^t}, and ρ_{p^0,q^0} are the correlation coefficients between p_m^0 and q_m^t and between p_m^0 and q_m^0 respectively, it follows from substituting (8) into (7) that:

$$\frac{P_{UV}}{P_L}=\left(\rho_{x,y}^{s_0}cv(x)cv(y)+1\right)\frac{\left(\rho_{p^0,q^t}cv(p^0)cv(q^t)+1\right)}{\left(\rho_{p^0,q^0}cv(p^0)cv(q^0)+1\right)}. \tag{9}$$

$$\frac{P_{UV}}{P_P}=\left[\frac{\sum_{m=1}^{M}p_m^0 q_m^t}{\sum_{m=1}^{M}p_m^0 q_m^0}\times\frac{\sum_{m=1}^{M}q_m^0}{\sum_{m=1}^{M}q_m^t}\right]=\frac{Q_L}{\sum_{m=1}^{M}q_m^t\Big/\sum_{m=1}^{M}q_m^0}=\frac{\left(\rho_{p^0,q^t}cv(p^0)cv(q^t)+1\right)}{\left(\rho_{p^0,q^0}cv(p^0)cv(q^0)+1\right)}\quad\text{from (8)} \tag{10}$$

The substitution effect between Fisher and Laspeyres, and Paasche and Fisher, is:

$$\frac{P_F}{P_L}=\frac{P_P}{P_F}=\left[\frac{P_P}{P_L}\right]^{\frac{1}{2}}=\left[\rho_{x,y}^{s_0}cv^{s_0}(x)cv^{s_0}(y)+1\right]^{\frac{1}{2}}\qquad\text{from (9) and (10).} \tag{11}$$

Substituting (10) and (11) into (6) yields:

$$\frac{P_{UV}}{P_F}=\frac{P_{UV}}{P_L}\times\left[\frac{P_L}{P_P}\right]^{\frac{1}{2}}=\left(\rho_{x,y}^{s_0}cv^{s_0}(x)cv^{s_0}(y)+1\right)\frac{\left(\rho_{p^0,q^t}cv(p^0)cv(q^t)+1\right)}{\left(\rho_{p^0,q^0}cv(p^0)cv(q^0)+1\right)}\times\left[\frac{1}{\left(\rho_{x,y}^{s_0}cv^{s_0}(x)cv^{s_0}(y)+1\right)}\right]^{\frac{1}{2}}$$

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[11] See Bortkiewicz (1923; 374-375) for the first application of this correlation coefficient decomposition technique: we define a correlation coefficient between *u* and *v* as $\rho_{u,v}=\left(\sum uv-m\overline{u}\overline{v}\right)\big/m\sigma_u\sigma_v$. Then $\sum uv/\sum u=\sigma_u\sigma_v\rho_{u,v}/\overline{u}+\overline{v}=\text{cov}(u,v)/\overline{u}+\overline{v}$ and $\sum suv/\sum su$ yield *s*-weighted terms for the decomposition.

$$=\left(\rho_{x,y}^{s_0}cv^{s_0}(x)cv^{s_0}(y)+1\right)^{\frac{1}{2}}\frac{\left(\rho_{p^0,q^t}cv(p^0)cv(q^t)+1\right)}{\left(\rho_{p^0,q^0}cv(p^0)cv(q^0)+1\right)}=\frac{P_F}{P_L}\times\frac{P_{UV}}{P_P}\,. \tag{12}$$

First, the difference between a unit value index and Fisher price index can be seen to depend on the two terms in (12): the first is the substitution bias between Fisher and Laspeyres, as given by (11), $\left(\rho_{x,y}^{s_0}cv^{s_0}(x)cv^{s_0}(y)+1\right)^{\frac{1}{2}}$, also equal to the substitution bias between Paasche and Fisher since $\frac{P_F}{P_L}=\frac{P_P}{P_F}$.[12] The second term has ρ_{p^0,q^t} in the numerator and ρ_{p^0,q^0} in the denominator. Say lower prices in period 0 are associated with higher quantities in period 0, but there is then a shift in period *t* to even higher quantities associated with these lower prices. What might we expect as a realization of this shift in quantities to lower price levels? We would expect $\rho_{p^0,q^0}<\rho_{p^0,q^t}$ and a greater dispersion of quantities in period *t*, $cv(q^0)<cv(q^t)$. This captures the shift in levels and would result in a fall in the unit value index. We noted at the start of this paper the case where prices may remain the same, but quantities shift to products whose absolute levels of prices were lower, leading to a fall in the (average) unit value index for constant prices. In this case, the substitution effect term is unity, but the second term is responsible for the fall in the unit value index.

Second, it is also apparent from (12) that the unit value index will equal the Fisher price index if: all price changes OR quantity changes are equal to each other, OR there is no (weighted) correlation between the base period price and quantity changes; AND all base period prices OR base and current period quantities are equal to each other, OR there is no (unweighted) correlation between the base period prices and base and current period quantities. These are extreme conditions. Having no dispersion in price or quantities or their changes is a negation of the index number problem, and while we do not expect the laws of economics to work perfectly, there is expected to be some relationship between prices and quantities, or their changes.

Third, note that the substitution effect term plays a role for the unit value index compared with a Laspeyres price index, and a countervailing role for a Fisher compared with a Laspeyres price index. The difference between a unit value and Laspeyres price index is comprised of the square of the substitution effect and a change in levels effect.

[12] It follows that if $\rho_{x,y}^{s_0}<0$, Laspeyres>Fisher>Paasche.

Fourth, from (9), (10), and (12) and assuming $\rho_{p^0,q^t} < 0$, $\rho_{p^0,q^0} < 0$ and $\rho^{s_0}_{x,y} < 0$ it can be seen that a unit value index is likely to fall below Laspeyres and Fisher price indexes, but it is difficult to conclude whether it is likely to lie outside the Paasche lower bound.

Fifth, consider the case of product heterogeneity. Assume that the heterogeneity fully accounts for the price dispersion in periods 0 and *t*. Consider further quality-adjustments to the prices of items that for each period adjusts prices to what they would be for a standard reference item. Since product heterogeneity fully accounts for the price dispersion $cv(x)$ and $cv(p^0)$ are zero and the Fisher and unit value indexes will be equal for these heterogeneity-adjusted prices.[13] However, as is apparent from (12), prices becoming less dispersed, as would be the case with heterogeneity-adjusted prices, would have no direct effect on the difference between a Fisher price and a unit value index since the right hand side of (12) has $cv(p^0)$ in both the numerator and denominator. It is only through a possible effect on $cv^{s_0}(x)$ that limited price dispersion may close the gap between the two indexes. It is the increased spread in quantities associated with higher (lower) prices being more strongly associated with lower (higher) quantities in period *t* compared with period), and a substitution effect that makes the difference. If there is a residual dispersion after accounting for price dispersion due to product heterogeneity, there will be a difference between the two formula and neither will be correct.

IV. WHAT TO DO FOR BROADLY COMPARABLE ITEMS

For homogenous items there is no problem: the answer is a unit value index. For heterogeneous items there is no problem: the answer is a superlative index such as a Fisher price index. However, many goods and services are broadly comparable: consumers in most product markets have a selection of differentiated items available to them, even if the differentiation is only due to the services provided by different outlets providing the same item (Hausman and Leibtag, 2008). The index number problem of broadly comparable differentiated goods and services applies to producer, export and import price indexes, as well as consumer ones. Say there are two models, one with a feature worth an additional 5 percent and one without. Assume further that the price of the model with the feature is sold with a 10% premium and prices remain constant. We can expect a shift in quantities to the standard model and an effective drop in the average (unit value) price level. The extent of the

[13] A variant of the law of one price may predict this, but there is much theoretical and empirical work on the failure of the law of one price and the persistence of its failure. The reasons for this include search cost theory—product prices may differ in equilibrium even in markets with symmetric firms selling homogeneous good, if there is a positive, but uncertain, probability that a randomly chosen customer knows only one price, Stigler (1961) and Sorenson (2000);[13] price discrimination, Yoskowitz (2002); random pricing model and price volatility, Varian (1980) and Lach (2002); Signal extraction models, Friedman (1977), Vining and Elwertowsky (1976), and Silver and Ioannidis (2001)); menu cost models, Ball and Mankiw (1994); incomplete pass-through rates of exchange rate fluctuations Feenstra and Kendall (1997) and Engel and Rogers (2001); inventory building models of frequently-purchased-goods Hong *et al.* (2002).

fall would be calculated after stripping out of the prices in both periods the 5 percent difference. Note the answer depends on the price levels, not price changes. Index number theory might argue that the items are heterogeneous, and advise a Fisher answer of no price change. But consumers and producers in the market that are indifferent to the two models, once the 5 percent quality-adjustment to price has been made, experience a fall in average prices that a quality-adjusted unit value index would pick up.

Quality adjustment factors can be applied to prices to render the comparison of prices of differentiated items akin to one of homogeneous items. We make use of (hedonic) quality-adjusted unit value indexes that removes the effects on prices of product heterogeneity, a proposal that goes back to Dálen (2001) and is formalized and empirically examined in De Haan (2004) and reiterated in De Haan (2007).[14] In Section III we provided a decomposition on how the measures differ.

De Haan's (2004) quality-adjusted unit value index solution to the problem of price measurement of broadly comparable items is a most useful and instructive. Since a unit value index is appropriate for homogeneous items, a quality-adjusted unit value index must be appropriate for broadly comparable items. We consider first such a measure.

A hedonic regression (see Triplett, 2004) using data on m = 1,...,M *matched* models for periods $\tau = 0, t$, of the price, p_m^τ , on k = 1,…,K quality characteristics, z_{km}^τ :

$$p_m^\tau = \beta_0^\tau + \sum_{k=1}^{K} \beta_k^\tau z_{km}^\tau + u_m^\tau \tag{13}$$

where u_m^τ are assumed to be normally distributed with mean and variance δ^τ and ξ_τ^2 respectively. The heterogeneity-adjusted prices in each period relative to a reference numeraire item with mean characteristics $\bar{z}_{km}^\tau$ in each period are given by:

$$\hat{p}_m^\tau = p_m^\tau - \sum_{k=1}^{K} \beta_k^\tau \left(z_{km}^\tau - \bar{z}_{km}^\tau\right) \tag{14}$$

Bear in mind the models in each period are matched so that $z_{km}^\tau = z_{km}^0 = z_{km}^t$. Note also that β_k^0 may or may not equal β_k^t and (13) can be estimated on pooled data with a dummy variable for time and with the constraint that $\beta_k^t = \beta_k^0 = \beta_k^\tau$, though it is preferable to estimate (13) separately for each time period without the constraint. The heterogeneity-adjusted unit value index is:

[14] Silver and Heravi (2002) used hedonic regressions to control for heterogeneity in a Dutot index, see also Silver and Heravi (2007).

$$P_U^* = \left(\frac{\sum_{m=1}^{M} \hat{p}_m^t q_m^t}{\sum_{m=1}^{M} q_m^t} \right) \Bigg/ \left(\frac{\sum_{m=1}^{M} \hat{p}_n^0 q_n^0}{\sum_{m=1}^{M} q_n^0} \right). \tag{15}$$

For goods and services with slight product differentiation we would advise a measure based on (15). Of course the quality adjustments need not use hedonic regressions. They may be much simpler due to say the addition of a single feature or option for which cost or market estimates of their value are available. Bear in mind that the items are matched in each period. The above formula has abstracted from the measure between items variation in price *in each period* due to quality differences.

What of items which are comparable, say models of television sets, washing machines, laptop computers, automobiles, whose price dispersion due to product differentiation is significant, as is the price dispersion due to factors that cannot be accounted for by the characteristics of the item? We noted above that there is much empirical evidence that the law of one price does not hold in many markets, reasons for this including price discrimination, menu costs, search costs, signal extraction, inventory holding, and strategic pricing. There is an element in the price measure for which the quality-adjusted unit value reduces the problem to one of homogenous items, but there is also an element for which a Fisher price index is appropriate. We consider a heterogeneity-based expression to capture the price variation in each period due to quality differences, i.e.:

$$\frac{\left[\sum_{k=1}^{K} \beta_k^t \left(z_{km}^{\tau} - \bar{z}_{km}^{\tau} \right) \right]}{\left[\sum_{k=1}^{K} \beta_k^0 \left(z_{km}^{\tau} - \bar{z}_{km}^{\tau} \right) \right]} = \frac{\tilde{p}_m^t}{\tilde{p}_m^0} \tag{16}$$

and the Fisher index[15]:

$$P_F^* \equiv \sqrt{ \frac{\sum_{m=1}^{M} \tilde{p}_m^t q_m^t}{\sum_{m=1}^{M} \tilde{p}_m^0 q_m^t} \times \frac{\sum_{m=1}^{M} \tilde{p}_m^t q_m^0}{\sum_{m=1}^{M} \tilde{p}_m^0 q_m^0} }. \tag{17}$$

[15] In this instance we apply a linear logarithmic hedonic equation (14) to an arithmetic aggregation in (15). Silver and Heravi (2001) and De Haan (2007) have argue for consistency that the same type of aggregator should be applied. Given the advantages of a semi-logarithmic hedonic specification over a linear one (Diewert, 2003), the use of a geometric Törnqvist aggregator as given in (2) might be better matched with a semi-logarithmic specification for (14).

There is a need for a weighted average of (15) and (17), but a problem as to what the weights should be. One approach is to consider what we mean by "comparable."[16] Consider the case for say models of automobiles where a hedonic regression explains just about all of the price variation. The models are very different in this sense, compared to a different data set for automobiles where a hedonic regression explained a very small proportion of the price variation. A Fisher index would be appropriate in the former case and unit value index in the latter. Thus the weights for the heterogeneity-adjusted unit value index (15) might be the ratio of the sum of squared errors from the hedonic regression (SSE) to the total sum of squares (SST) and the weight for (17) the ratio of the (explained) regression sum of squares (SSR) to SST.

A weighted average of the heterogeneity-adjusted unit value index (15) and the heterogeneity-based Fisher index (17), where weights are an average of the proportion of variation explained by the hedonic regressions is given by:

$$P_U^* \bar{w}_U + P_F^*(1-\bar{w}_U) = \frac{\sum_{m=1}^{M} \hat{p}_m^t q_m^t \Big/ \sum_{m=1}^{M} q_m^t}{\sum_{m=1}^{M} \hat{p}_m^0 q_m^0 \Big/ \sum_{m=1}^{M} q_m^0} \times \bar{w}_U + \sqrt{\frac{\sum_{m=1}^{M} \tilde{p}_m^t q_m^t}{\sum_{m=1}^{M} \tilde{p}_m^0 q_m^t} \times \frac{\sum_{m=1}^{M} \tilde{p}_m^t q_m^0}{\sum_{m=1}^{M} \tilde{p}_m^0 q_m^0}} \times (1-\bar{w}_U) \quad (18)$$

where $\bar{w}_U = \frac{SSE}{SST}$ and $(1-\bar{w}_U) = \frac{SSR}{SST} = R^2$. Note that the weights in (18) have a bar over them, they are an arithmetic mean of the weights from period 0 and period t hedonic regressions in equation (13) for $\tau = 0, t$.

An appropriate index should have the property that if all price variation is explained by the hedonic regression, the index is a Fisher index; if none of the price variation is explained by the hedonic regression, the index is a unit value index; as the percentage of price variation explained by the hedonic regression increases, so too will the weight given to the Fisher component. Equation (18) satisfies all these criteria.

The use of such weights is but one proposal. Consider the case of television sets. A hedonic regression could be estimated over all screen sizes with dummies for these screen sizes and variables fir the other quality characteristics. But while the regression would attribute a say 30% premium for a 32 inch screen over a 14 inch one, while controlling for other variables, it is unlikely that consumers would consider the two models as substitutes when an allowance has been made for the screen size, and other variables. Thus the regression should be undertaken for similar goods in the sense that there its some such substitutability.

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[16] Zieschang (1988) in a different context employs a concept of quasi-exchangeability when characteristics completely describe the associated varieties.

It might be argued that substitutability should be the main concept behind the weighting system. However, this is problematic. Substitutability exits for goods and services for which quality-adjustments for unit values are not feasible and the concept of an average price not meaningful, for example, beef and chicken. However, as indicated above, the first step should be to identify a cluster of goods which are comparable and substitutable or exchangeable, for example, television sets of a similar screen size, and then use the hedonic framework.

V. An Empirical Example: Using Scanner Data

The empirical work utilizes monthly scanner data for television sets (TVs) from the bar-code readers of U.K. retail outlets from January 2001 to December 2001. The scanner data were supplemented by data from price collectors from outlets without bar-code readers, though this was negligible. Each observation is a model of a TV in a given month sold in one of four different outlet types: multiple chains, mass merchandisers (department stores), independents and catalogue stores. The analysis demonstrates that differences between unit value and index number formulas can arise even for matched models case, so the complication of price dispersion due to new and old models entering and leaving the market was discounted by employing the restrictive condition of only using models of TVs that were sold in all 12 months of the data. In this sense the data set is illustrative since the prices of new and old models entering and leaving the market, even after adjusting for quality differences, have been shown to differ from those of the matched models (Silver and Heravi, 2005). The data set included series for 448 such models in each month accounting over the period for sales of about 1.4 million TVs worth about £500 million.

The variable set *on each observation* included: **price,** the unit value of a model across all transactions in a month/outlet-type; 38 **brands**—37 dummy variables benchmarked on Sony; the **characteristics** included (i) size of screen—dummy variables for about 19 screen sizes; possession of (ii) Nicam stereo sound; (iii) wide screen; (iv) on-screen text retrieval news and information panels from broadcasting companies, in order of sophistication: teletext, fastext and top fastext—3 dummy variables; (v) 6 types of reception systems—5 dummy variables; (vi) continental monitor style; (vii) Dolby Pro, Dolby SUR/DPL, Dolby Digital sound—3 dummy variables; (viii) Flat & Square, Super-Planar tubes—2 dummy variables; (ix) s-vhs socket; (x) satellite tuner, analogue/digital—2 dummy variables; (xi) digital; (xii) DVD playback or DVD recording—2 dummy variables; (xiii) rear speakers; (xiv) without PC-internet/PC+internet; (xv) real flat tube; (xvi) 100 hertz, doubles refresh rate of picture image; (xvii) vintage and (xviii) DIST—the percentage of outlets in which the model was sold. **Outlet types are** multiple chains, mass merchandisers (department stores), independents and catalogue stores.

The analysis is undertake first for the 94 models with 10-14 inch screen sizes and second for the 354 models with 20-32 inch screen sizes. Figures 1 and 2 provide the respective results for the indices. Unit value indexes can be seen to be more volatile and generally higher than than the price indexes. The differences can be quite substantial: fro example for 14 inch sets the unit value index fell in December by 3.8 percent while the Fisher price index fell by 7.1 percent

Figure 1, Unit value and price indices for 14in TVs

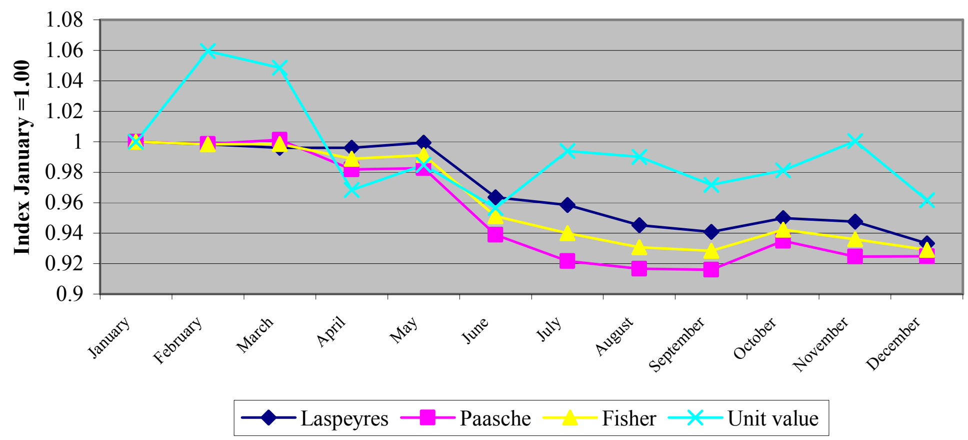
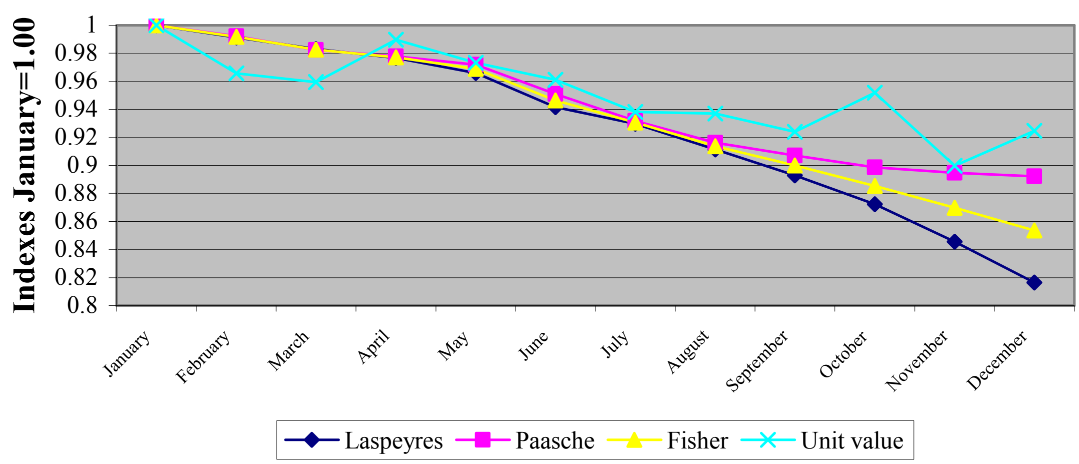


Figure 2, Price and unit value indexes for 20-32 inch TVs



[Being worked on]

VI. CONCLUSIONS

For the aggregation of homogeneous items, the unit value index is the best index and superlative index numbers biased, and for the aggregation of heterogeneous items, superlative index numbers are best index and unit value index numbers biased.

An exception to the use of unit value indexes for homogeneous items is institutionalized price discrimination. If different purchasers of the same good or service, say electricity, face different prices and the individual purchasers, say commercial customers and private households are unable to change from one price to another, then price indexes should be used. Yet for output PPIs and export price indexes from the producer's perspective, unit values should be used, it make a difference to revenues if the producer shifts some of its quantities to higher paying customers.

The factors determining the difference between unit value indices and Laspeyres, Paasche and Fisher price indices were established in Section III. They comprise a substitution bias (for unit value to Laspeyres and a countervailing one from Fisher to Laspeyres) and a levels effect. The conditions for the unit value index to equal the price indexes are established as implausible, though the expected ranking of the indexes is established.

For items that are very similar, a unit value index remains appropriate for it is necessary to capture the effect of a change in price levels, and price indexes do not properly do this. Quality adjustments to the prices to mitigate price dispersion due to the slight product heterogeneity would be appropriate.

The determination of whether or not an item is homogeneous is critical to the choice of index number formula, but in practice many items are broadly comparable, some more than others, and neither a unit value nor a Fisher price index is appropriate. The more similar the items aggregated, the stronger the case for a heterogeneity-adjusted unit value. It follows that an appropriate formula may be based on an average of a heterogeneity-adjusted unit value index and a price index. The weighting ascribed to each should be an indicator of the similarity of the items. A possible indicator explored in this paper is the extent to which the price variation can be explained by price-determining characteristics: the (explained) sum of squares from a hedonic regression. While the discussion has been phrased in terms of hedonic regression analysis the principles apply to simpler quality adjustments.

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