Questioning Some General Wisdom in Axiomatic Index Theory

by

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Content

1. Introduction

- Notation:

  p_i^t = price of item i in period t $\Rightarrow$ price vector $\mathbf{p^t}$

  x_i^t = quantity of item i in period t $\Rightarrow$ quantity vector $\mathbf{x^t}$

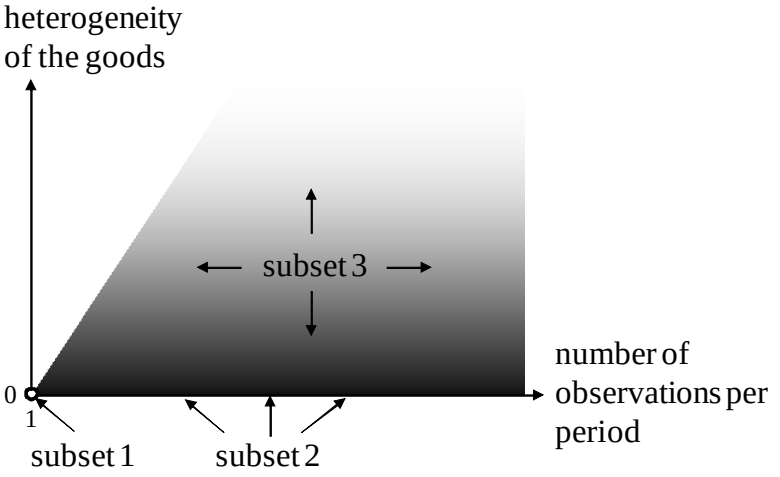
  $t = 0 \leftrightarrow$ base period

  $t = 1 \leftrightarrow$ comparison period

- The level of complexity of a price measurement problem can be viewed as a function of two dimensions:

1. the number of observations per period, and

2. the degree of heterogeneity of the goods considered.


heterogeneity
of the goods
subset 3
0
1
number of
observations per
period
subset 1
subset 2

Subset 1: One Good and Only One Observation

$$P = \frac{p_i^1}{p_i^0} \, .$$

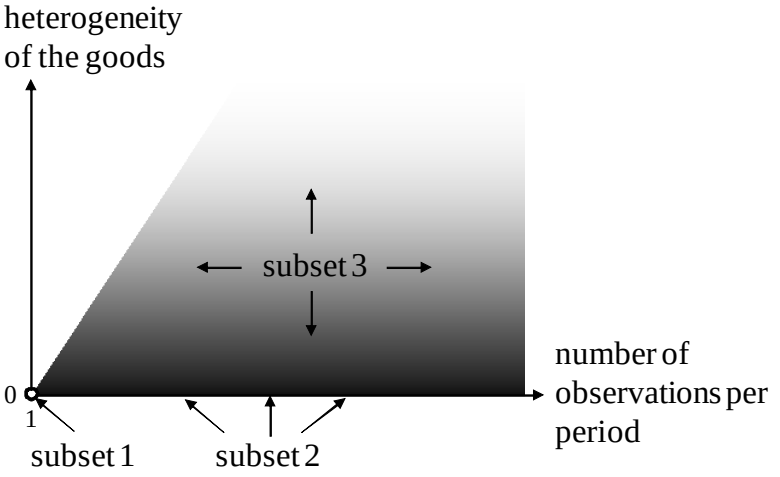
Subset 2: One Good But More Than One Observation

ILO *et al.* (2004, p. 164, par. 9.71) recommends

$$P_{UV} = \frac{\left(\sum p_i^1 x_i^1\right) / \left(\sum x_i^1\right)}{\left(\sum p_i^0 x_i^0\right) / \left(\sum x_i^0\right)} \, , \qquad \text{where } \sum = \sum_{i=1}^{N} \, .$$

Subset 3: Heterogeneous Goods

ILO *et al.* (2004, p. 357, par. 20.18) recommends Fisher, Walsh, or Törnqvist index.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- An Axiom or Test must make sense in the context of all three subsets.
- Example: *Monotonicity test*

Scenario 1:

	base period		comparison period	
	price	quantity	price	quantity
observ. A	8	2	8	3
observ. B	4	2	6	2

Scenario 2:

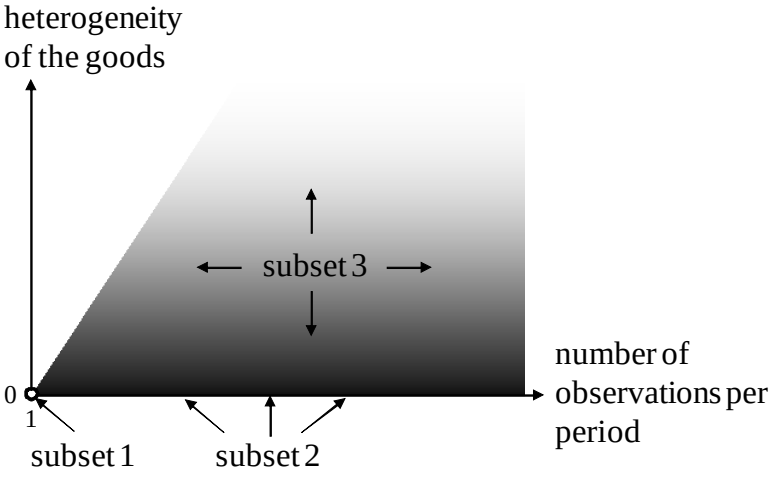
	base period		comparison period	
	price	quantity	price	quantity
observ. A	8	2	10	3
observ. B	4	2	6	2

2. Identity Test

	base period		comparison period	
	price	quantity	price	quantity
observation A	8	57	8	56
observation B	9	50	9	2
observation C	8	43	8	40
observation D	7	50	7	102

- The *identity test* postulates that with constant prices, the quantities should not affect the price index number:

$$P(\mathbf{p^0}, \mathbf{x^0}, \mathbf{p^0}, \mathbf{x^1}) = 1 \ .$$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3. Further Axiomatic Considerations

Proportionality Test:

$$P(\mathbf{p^0}, \mathbf{x^0}, \lambda\mathbf{p^0}, \mathbf{x^1}) = \lambda \ , \quad \text{for all } \lambda > 0 \ .$$

Mean Value Test:

$$\min_i \frac{p_i^1}{p_i^0} \leq P(\mathbf{p^0}, \mathbf{x^0}, \mathbf{p^1}, \mathbf{x^1}) \leq \max_i \frac{p_i^1}{p_i^0} \ .$$

Linear Homogeneity Test:

$$P(\mathbf{p^0}, \mathbf{x^0}, \lambda\mathbf{p^1}, \mathbf{x^1}) = \lambda P(\mathbf{p^0}, \mathbf{x^0}, \mathbf{p^1}, \mathbf{x^1}) = P((1/\lambda)\mathbf{p^0}, \mathbf{x^0}, \mathbf{p^1}, \mathbf{x^1}), \text{ for all } \lambda > 0 \ .$$

4. Concluding Remarks

- Price aggregation problems should be approached as any other problem in economics:

  Start with the simplest case, learn from it,
  and then step by step add complexity.

- This approach reveals that the identity test is flawed.

- In the paper it is also demonstrated that in the context of subset 2 (that is, multiple observations of a single homogeneous good) the overall price change should be computed on the basis of the unit value index.

$$P_{UV} = \frac{\left(\sum p_i^1 x_i^1\right) / \left(\sum x_i^1\right)}{\left(\sum p_i^0 x_i^0\right) / \left(\sum x_i^0\right)}$$