Estimating Production Functions with Heterogeneous Firms

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- e.g., Estimate a production function that explains the input/output behavior of a sample of firms. Use residuals as productivity measures for applied work.
- We present a "new" semiparametric approach to estimation that allows for richer patterns of firm heterogeneity than prevailing approaches of Olley&Pakes (1996) and Levinsohn&Petrin (2002). Focus only on wage and output price heterogeneity today.

The Problem

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- ω_{jt} is endogenous
- Fixed effects impose $\omega_{jt} = \omega_j$
- IV requires good instruments
- The semiparametric approach initiated by OP and pursued further by LP - use a model of firm behavior.

The Scalar Unobservability Assumption

- (ω_{jt}, k_{jt}) is firm j's time t state.
- Firm j makes a time t static input decision $m_{jt} = f_t(\omega_{jt}, k_{jt})$ or dynamic input decision $i_{jt} = g_t(\omega_{jt}, k_{jt})$.
- Take inverse of input demand function

$$\omega_{jt} = \phi_t(k_{jt}, m_{jt})$$

.

- Control for endogeneity nonparametrically

$$y_{jt} = \alpha l_{jt} + \Phi_t(m_{jt}, k_{jt}) + \epsilon_{jt}$$

Evolution of TFP and β

- First order Markov assumption

$$\omega_{jt} = g(\omega_{jt-1}) + \eta_{jt}$$

- Time to build assumption

$$K_{jt} = d(K_{jt-1}, i_{jt-1})$$

- k_{jt} is decided at time $t-1$ and η_{jt} independent of all $t-1$ information implies

$$\eta_{jt} \perp k_{jt}$$

- Use this moment to estimate β

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- Use the first order condition for the firm's static input decision (i.e., labor input decision)
- Use FOC + production function jointly to invert out ω_{jt} and ϵ_{jt} as functions of parameters, i.e., $\omega_{jt}(\alpha, \beta)$ and $\epsilon_{jt}(\alpha, \beta)$

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- Use FOC + production function jointly to invert out ω_{jt} and ϵ_{jt} as functions of parameters, i.e., $\omega_{jt}(\alpha, \beta)$ and $\epsilon_{jt}(\alpha, \beta)$
- Use same moment conditions OP and LP to estimate model

Cobb-Douglas Example

Let the state be $(\omega_{jt}, k_{jt}, P_{jt}, W_{jt})$
Then we have the system

$$\begin{aligned} \ln\left(\frac{P_{jt}Y_{jt}}{W_{jt}L_{jt}}\right) &= -\ln(\alpha) + \varepsilon_{jt} \\ y_{jt} &= \alpha l_{jt} + \beta k_{jt} + \omega_{jt} + \varepsilon_{jt} \end{aligned}$$

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More generally

$$\begin{pmatrix} s_{jt} \\ y_{jt} \end{pmatrix} = \Upsilon(x_{jt}, \omega_{jt}, \varepsilon_{jt})$$

Application to Chilean Data

Plant level Chilean manufacturing panel data from 1979-1996
Same data set used by Levinsohn and Petrin

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Table: Industry 311

Method	Labor	95% CI	Capital	95% CI
OLS	.953	.932,.947	.400	.389,.411
LP	.647	.595,.700	.399	.292,.505
GNR	.414	.402,.425	.362	.274,.391

Robustness Check : The CES

$$Q_{jt} = A_{jt} \left(\alpha L_{jt}^{\rho} + \beta K_{jt}^{\rho} \right)^{\frac{r}{\rho}}$$

For $\alpha + \beta = 1, r > 0, \rho < 1$

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	Estimate	SE
ρ	-.51	.06
α	.14	.06
β	.86	.19
r	.69	.06

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Average Labor Elasticity = .45 (SE=.02)
Average Capital Elasticity = .24 (SE=.02)

The Diewert Production Function

$$Q_{jt} = A_{jt}\left(\alpha L_{jt}^{\rho} + \beta K_{jt}^{\rho} + \gamma L_{jt}^{0.5\rho} K_{jt}^{0.5\rho}\right)^{\frac{r}{\rho}}$$

For $\alpha + \beta + \gamma = 1, r > 0, \rho < 1$

Table: Industry 311

	Estimate	SE
ρ	-1.11	.26
α	.01	.02
β	.85	.12
γ	.14	.10
r	.70	.05

Average Labor Elasticity = .45 (SE=.02)
Average Capital Elasticity = .25 (SE=.01)

Further Applications

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- We can easily allow for more general assumptions on TFP evolution - i.e., higher order Markov assumptions or controlled Markov process assumptions
- Multiple dimensions of unobserved heterogeneity appear prevalent in data