**Changes in the Terms of Trade and Canada's Productivity Performance**

Revised April 14, 2008

W. Erwin Diewert[1]
Discussion Paper 08-05
Department of Economics
University of British Columbia
Vancouver, Canada, V6T 1Z1
Email: diewert@econ.ubc.ca

## Abstract

Using new data from Statistics Canada, the paper shows that the productivity performance of the business sector of the Canadian economy has been reasonably satisfactory over the past 46 years. In particular, traditional gross income Total Factor Productivity (TFP) growth averaged 1.14 percentage points per year over the period 1961-2006 and when a net income framework was used, TFP growth averaged 1.26 percentage points per year. The focus of the study is on the real income generated by the business sector of the Canadian economy. Two concepts of income are used: a gross concept that includes depreciation as a part of income and a more appropriate net concept where depreciation is excluded from income. In both the gross and net income frameworks, the growth of quality adjusted labour input growth was the main driver of growth in real income followed by TFP growth, followed by growth in capital input and then by falling real import prices. However, in recent years, the contribution of falling real import prices turned out to be more than twice as important as capital deepening. The study encountered many data problems which should be addressed in future work on Canadian business sector productivity performance.

**Journal of Economic Literature Classification Numbers**

C43, C67, C82, D24, E22, E43.

**Keywords**

Total factor productivity, real income, terms of trade effects, measurement of capital, measurement of inventory change, user costs, real interest rates.

---

[1] The financial assistance of Industry Canada is gratefully acknowledged. The author also thanks John Baldwin, Serge Coulombe, Wulong Gu, Ulrich Kohli, Danny Leung, Alice Nakamura, Andrew Sharpe and Jianmin Tang for helpful comments. None of the above are responsible for any views expressed in this paper.

## 1. Introduction

Many observers have noted that an improvement in a country's terms of trade has effects that are similar to an improvement in a country's productivity growth. However, it is not straightforward to work out the exact magnitude of each source of gain. Diewert (1983), Diewert and Morrison (1986), Morrison and Diewert (1990) and Kohli (1990) (1991) (2003) (2004a) (2004b) (2006) (2007) developed production theory methodologies which enable one to obtain index number estimates of the contribution of each type of gain. In Appendix 1 below, we adapt this methodology and show how it can be used to measure the determinants of growth in an economy's gross and net real income. In sections 2-4 of the main text, we apply this methodology to the business sector of the Canadian economy over the years 1961-2006.

Appendix 2 below describes how the Canadian business sector data was developed from Statistics Canada sources. Section 2 of the main text aggregates up the data from Appendix 2 and develops conventional measures of Canadian business sector Total Factor Productivity for the years 1961-2006.

However, productivity growth, while perhaps the most important source of growth in living standards, is not the entire story. If a country's export prices increase more rapidly than its import prices, then it is well known that this has an effect that is similar to a productivity improvement.[2] Thus in section 3, we measure the relative contributions of productivity improvements, changes in real export and import prices and growth of labour and capital input to the growth of (gross) real income generated by the business sector in Canada using the methodology explained in sections 1-4 of Appendix 1.

However, this is not the end of the story. GDP is an (imperfect) measure of productive potential, not welfare.[3] For welfare measurement purposes, it is generally conceded that Net Domestic Product (NDP) is a better measure of output, since investment that just meets depreciation means that society is not made any better off from the viewpoint of sustainable final consumption possibilities. Hence, in the second part of the paper, we subtract depreciation off from gross investment and use consumption plus sales to the nonbusiness sector plus *net investment* plus the trade balance as our business sector output concept. Thus depreciation will be treated as an intermediate input in this model of production. Section 5 of Appendix 1 explains this real net product approach and adapts a translog model of production based on the work of Diewert and Morrison (1986) and Kohli (1990) to this new model of market sector real net income generation.[4] This

---

[2] See for example Diewert and Morrison (1986).
[3] For a more extensive discussion of the merits of GDP versus net income, see Diewert (2006a).
[4] For previous implementations of this model of real net income to Japan and Australia, see Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006).

approach is implemented for the Canadian business sector in section 4 of the main text. The main determinants of growth in real net income generated by the business or market sector of the economy are:

- Technical progress or improvements in Total Factor Productivity;
- Growth in domestic output prices or the prices of internationally traded goods and services relative to the price of consumption; and
- Growth in primary inputs.

It turns out that productivity growth becomes a more important factor for explaining real *net* income growth compared to explaining real *gross* income growth. Also the importance of capital deepening is greatly reduced in the net income framework compared to the gross income framework. Somewhat surprisingly, for the years 2000-2006, improvements in the terms of trade made almost the same contribution to real income growth as capital deepening in the gross income framework and in the net income framework, the effects of falling real import prices contributed substantially more to real income growth than capital deepening over the period 2000-2006.

Appendix 3 compares our methodology for determining the effects of improvements in the terms of trade on real income growth with the methodology worked out by Kohli (2006).

Appendix 4 compares our estimates of TFP growth with the business sector Multifactor Productivity Growth estimates recently developed by the Statistics Canada KLEMS program; see Baldwin, Gu and Yan (2007) for a description of the methodology used in the KLEMS program.

Section 5 concludes.[5]

## 2. Output and Input Aggregates and Conventional Productivity Growth for Canada

In Appendix 2, we constructed price and quantity series for 11 net outputs and 8 primary inputs for the business sector of the Canadian economy for the years 1961-2006. The 11 net outputs are:

- Domestic consumption (excluding market residential rents and the services of owner occupied housing);
- Government investment;

[5] The final section of Appendix 1 has some new material on how the real net income model used in the present paper can be extended from a single production sector to the case of many industries. Also the final section of Appendix 4 has some suggestions for improving the measurement of productivity by Statistics Canada but these recommendations may be useful for other official productivity programs. The next revision of the *System of National Accounts 1993* will introduce capital services into the accounts so that price and quantity decompositions of primary inputs will be possible, which will greatly facilitate the measurement of productivity by sector. However, the introduction of capital services into the main production accounts will not be easy and it will be necessary for statistical agencies to do a considerable amount of preparatory work.

- Business sector investment in residential structures;
- Business sector investment in nonresidential structures;
- Business sector investment in machinery and equipment;
- Inventory change;
- Sales of goods and services by the business sector to the nonmarket sector less sales of goods and services from the nonmarket sector to the business sector;
- Exports of goods;
- Exports of services;
- Imports of goods (the quantities are indexed with a minus sign) and
- Imports of services (the quantities are indexed with a minus sign).

The eight primary inputs into the business sector are:

- The labour services of workers with primary or secondary education
- The labour services of workers with some or completed post secondary certificate or diploma;
- The labour services of workers with a university degree or above;[6]
- The stock of machinery and equipment available to the business sector at the start of each year;
- The starting stock of business sector nonresidential structures;
- The stock of nonagricultural, nonresidential land used by the business sector;
- The stock of agricultural land used by the business sector and
- The starting stocks of inventories used by the business sector.

As is explained in Appendix 2, user cost prices for the last five primary inputs were constructed, using balancing or endogenous real rates of return that made the value of net output produced by the business sector equal to the value of primary inputs used by the business sector. All of the price and quantity series for the above 19 outputs produced and inputs used by the Canadian business sector are listed in Appendix 2.

In this section, we will aggregate the above net outputs and primary inputs into D, domestic output, equal to an aggregate of the first seven net outputs listed above; X, exports equal to an aggregate of exports of goods and services; M, imports equal to an aggregate of imports of goods and services; L, labour services equal to an aggregate of the three types of labour and K, capital services equal to an aggregate of the five types of capital services. Once these aggregates have been constructed, we further aggregate the three net outputs, D+X−M, into real gross domestic product Y and aggregate the two inputs, L and K, into domestic input Z and finally construct a conventional measure of productivity Y/Z. The aggregations were performed using chained Törnqvist price indexes.[7] The results are listed in Tables 1 and 2 below.

**Table 1: Prices of Canadian Business Sector Output and Input Aggregates**

[6] These three types of labour input are taken directly from Statistics Canada recent KLEMS program; see Baldwin, Gu and Yan (2007; 26-27) for a description of these data.
[7] More specifically, the chained Divisia option in Shazam was used to do the aggregations.

| Year t | $P_C^t$ | $P_D^t$ | $P_X^t$ | $P_M^t$ | $P_L^t$ | $P_K^t$ | $P_Y^t$ | $P_Z^t$ |
|---|---|---|---|---|---|---|---|---|
| 1961 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 1962 | 1.00538 | 1.00604 | 1.04079 | 1.05429 | 1.03782 | 1.07866 | 1.00119 | 1.05019 |
| 1963 | 1.02055 | 1.02047 | 1.04893 | 1.07529 | 1.06768 | 1.15414 | 1.01219 | 1.09369 |
| 1964 | 1.02437 | 1.03043 | 1.07108 | 1.07801 | 1.11059 | 1.29428 | 1.02740 | 1.16537 |
| 1965 | 1.03690 | 1.05086 | 1.08920 | 1.07432 | 1.18443 | 1.32443 | 1.05415 | 1.22651 |
| 1966 | 1.07553 | 1.08973 | 1.11934 | 1.09489 | 1.25943 | 1.40672 | 1.09618 | 1.30371 |
| 1967 | 1.11050 | 1.12467 | 1.13960 | 1.11152 | 1.33529 | 1.32261 | 1.13258 | 1.33255 |
| 1968 | 1.15168 | 1.16183 | 1.16058 | 1.13325 | 1.41832 | 1.43747 | 1.16968 | 1.42527 |
| 1969 | 1.18980 | 1.20312 | 1.18385 | 1.16514 | 1.52331 | 1.49218 | 1.20821 | 1.51488 |
| 1970 | 1.22208 | 1.24102 | 1.22357 | 1.19496 | 1.61399 | 1.59733 | 1.24955 | 1.61015 |
| 1971 | 1.24828 | 1.28420 | 1.22639 | 1.22459 | 1.72738 | 1.61830 | 1.28315 | 1.69437 |
| 1972 | 1.29847 | 1.34270 | 1.27421 | 1.25190 | 1.86814 | 1.66927 | 1.34913 | 1.80669 |
| 1973 | 1.38744 | 1.44951 | 1.44929 | 1.33495 | 2.04059 | 2.08558 | 1.49015 | 2.05625 |
| 1974 | 1.58382 | 1.66194 | 1.83894 | 1.61653 | 2.35314 | 2.40261 | 1.73605 | 2.37045 |
| 1975 | 1.82198 | 1.88606 | 2.09796 | 1.86125 | 2.70560 | 2.65389 | 1.96247 | 2.69075 |
| 1976 | 1.90726 | 1.99309 | 2.24453 | 1.91066 | 3.10734 | 2.92888 | 2.10552 | 3.05088 |
| 1977 | 2.03175 | 2.11743 | 2.45822 | 2.16399 | 3.39218 | 3.23324 | 2.20694 | 3.34276 |
| 1978 | 2.19264 | 2.28009 | 2.68976 | 2.45197 | 3.53830 | 3.53313 | 2.34394 | 3.54211 |
| 1979 | 2.40645 | 2.50051 | 3.16430 | 2.78074 | 3.78649 | 4.04780 | 2.60589 | 3.88194 |
| 1980 | 2.69497 | 2.78540 | 3.68189 | 3.21157 | 4.12150 | 4.40228 | 2.91090 | 4.22415 |
| 1981 | 2.95335 | 3.07520 | 3.97404 | 3.67151 | 4.59729 | 4.58375 | 3.13761 | 4.59923 |
| 1982 | 3.22860 | 3.33538 | 4.07385 | 3.87032 | 5.02249 | 4.39571 | 3.36479 | 4.80577 |
| 1983 | 3.46323 | 3.53148 | 4.14348 | 3.85771 | 5.22320 | 5.20880 | 3.58558 | 5.23463 |
| 1984 | 3.61506 | 3.66530 | 4.29296 | 4.02889 | 5.48356 | 5.80907 | 3.70980 | 5.62172 |
| 1985 | 3.72257 | 3.77548 | 4.38035 | 4.12592 | 5.75934 | 5.97382 | 3.81494 | 5.85783 |
| 1986 | 3.80422 | 3.86808 | 4.37812 | 4.20095 | 5.90520 | 5.77499 | 3.87756 | 5.87940 |
| 1987 | 3.89726 | 3.97810 | 4.46622 | 4.14146 | 6.11325 | 6.31252 | 4.04688 | 6.20607 |
| 1988 | 4.00205 | 4.08604 | 4.48026 | 4.04492 | 6.51680 | 6.32027 | 4.20445 | 6.47093 |
| 1989 | 4.11690 | 4.20482 | 4.57459 | 4.04389 | 6.79693 | 6.36304 | 4.36602 | 6.66789 |
| 1990 | 4.35206 | 4.36935 | 4.54070 | 4.10175 | 7.05555 | 5.89282 | 4.49949 | 6.66390 |
| 1991 | 4.59099 | 4.51357 | 4.37497 | 4.02502 | 7.34485 | 5.29918 | 4.61930 | 6.62419 |
| 1992 | 4.65258 | 4.56253 | 4.50007 | 4.19541 | 7.48311 | 5.17213 | 4.63861 | 6.66114 |
| 1993 | 4.74252 | 4.64763 | 4.69935 | 4.42139 | 7.46638 | 5.60256 | 4.70077 | 6.81843 |
| 1994 | 4.77089 | 4.72176 | 4.97660 | 4.69349 | 7.42100 | 6.65009 | 4.76752 | 7.19244 |
| 1995 | 4.79147 | 4.75262 | 5.29258 | 4.82937 | 7.54415 | 6.96045 | 4.89248 | 7.38760 |
| 1996 | 4.88952 | 4.82071 | 5.32288 | 4.76732 | 7.63603 | 7.86537 | 5.01269 | 7.78656 |
| 1997 | 4.96547 | 4.86150 | 5.33171 | 4.78760 | 7.85949 | 8.14277 | 5.04667 | 8.03215 |
| 1998 | 5.03224 | 4.91712 | 5.31493 | 4.95489 | 8.08570 | 7.64057 | 4.98891 | 7.98590 |
| 1999 | 5.12045 | 4.98119 | 5.37242 | 4.93315 | 8.30640 | 7.86796 | 5.10149 | 8.21103 |
| 2000 | 5.25425 | 5.09408 | 5.70503 | 5.02898 | 8.72108 | 9.01480 | 5.36189 | 8.90816 |
| 2001 | 5.40970 | 5.21531 | 5.77978 | 5.18877 | 8.96937 | 8.63241 | 5.42583 | 8.91600 |
| 2002 | 5.47743 | 5.29828 | 5.67251 | 5.21950 | 9.09698 | 8.74127 | 5.42233 | 9.03742 |
| 2003 | 5.61543 | 5.37928 | 5.60179 | 4.88445 | 9.26199 | 8.62922 | 5.66586 | 9.09575 |
| 2004 | 5.69263 | 5.46192 | 5.72405 | 4.76048 | 9.48930 | 9.66050 | 5.90257 | 9.63757 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2005 | 5.80796 | 5.59965 | 5.88377 | 4.70288 | 9.87728 | 10.35423 | 6.17961 | 10.14775 |
| 2006 | 5.90800 | 5.72495 | 5.89047 | 4.66574 | 10.26222 | 10.49052 | 6.34295 | 10.43827 |

Note that we have also listed the price of our household consumption aggregate, $P_C{}^t$, in Table 1, which will play a role in subsequent sections. The *productivity level* in year t of the Canadian business sector $T^t$ can be defined as the aggregate year t output, $Q_Y{}^t$ divided by aggregate year t input, $Q_Z{}^t$:[8]

$$(1)\ T^t \equiv Q_Y{}^t/Q_Z{}^t\ ; \qquad t = 1961, ..., 2006.$$

*Productivity growth* for year t, $\tau^t$, is defined as the productivity level in year t divided by the previous year's productivity level:

$$(2)\ \tau^t \equiv T^t/T^{t-1}\ ; \qquad t = 1962, ..., 2006.$$

Table 2 lists the quantities that match up to the prices in Table 1 and it also lists productivity levels and growth rates.

**Table 2: Quantities of Canadian Business Sector Output and Input Aggregates, TFP Levels and TFP Growth Rates**

| Year t | $Q_D{}^t$ | $Q_X{}^t$ | $Q_M{}^t$ | $Q_L{}^t$ | $Q_K{}^t$ | $Q_Y{}^t$ | $Q_Z{}^t$ | $T^t$ | $\tau^t$ |
|---|---|---|---|---|---|---|---|---|---|
| 1961 | 28553 | 7310 | -8180 | 19202 | 8481 | 27683 | 27683 | 1.00000 | ____ |
| 1962 | 30697 | 7639 | -8370 | 20012 | 8566 | 29973 | 28574 | 1.04894 | 1.04894 |
| 1963 | 31767 | 8323 | -8513 | 20542 | 8717 | 31608 | 29253 | 1.08053 | 1.03011 |
| 1964 | 34490 | 9465 | -9602 | 21444 | 8894 | 34384 | 30313 | 1.13429 | 1.04975 |
| 1965 | 37709 | 9890 | -10905 | 22383 | 9191 | 36697 | 31540 | 1.16350 | 1.02575 |
| 1966 | 40538 | 11233 | -12376 | 23547 | 9627 | 39409 | 33135 | 1.18933 | 1.02220 |
| 1967 | 40824 | 12426 | -13055 | 24022 | 10198 | 40230 | 34193 | 1.17656 | 0.98926 |
| 1968 | 42754 | 13925 | -14404 | 24122 | 10642 | 42329 | 34738 | 1.21851 | 1.03566 |
| 1969 | 45856 | 15050 | -16278 | 24696 | 10992 | 44712 | 35660 | 1.25382 | 1.02898 |
| 1970 | 46098 | 16447 | -16005 | 24779 | 11403 | 46583 | 36150 | 1.28858 | 1.02772 |
| 1971 | 48725 | 17212 | -17073 | 25303 | 11782 | 48922 | 37049 | 1.32048 | 1.02475 |
| 1972 | 51835 | 18694 | -19554 | 26041 | 12156 | 51099 | 38158 | 1.33916 | 1.01415 |
| 1973 | 56896 | 20568 | -22397 | 27562 | 12533 | 55285 | 40064 | 1.37989 | 1.03042 |
| 1974 | 61516 | 19655 | -24652 | 28524 | 13073 | 56755 | 41566 | 1.36543 | 0.98952 |
| 1975 | 63743 | 18031 | -23889 | 28504 | 13741 | 57880 | 42214 | 1.37110 | 1.00416 |
| 1976 | 68060 | 19422 | -25376 | 28490 | 14419 | 62103 | 42859 | 1.44899 | 1.05681 |
| 1977 | 71010 | 20646 | -25281 | 28777 | 15090 | 66338 | 43798 | 1.51466 | 1.04532 |
| 1978 | 72172 | 22694 | -26058 | 29989 | 15736 | 68990 | 45653 | 1.51117 | 0.99770 |
| 1979 | 75729 | 23523 | -27910 | 31726 | 16318 | 71447 | 47961 | 1.48968 | 0.98578 |
| 1980 | 75289 | 23748 | -27013 | 32803 | 17082 | 72278 | 49807 | 1.45115 | 0.97414 |
| 1981 | 78730 | 24170 | -27702 | 33691 | 17795 | 75362 | 51412 | 1.46584 | 1.01012 |
| 1982 | 70788 | 23789 | -23204 | 32044 | 18716 | 72281 | 50608 | 1.42825 | 0.97436 |
| 1983 | 74223 | 25195 | -25549 | 32267 | 19086 | 74730 | 51188 | 1.45991 | 1.02217 |
| 1984 | 78814 | 29833 | -29968 | 33481 | 19386 | 79846 | 52690 | 1.51537 | 1.03799 |

[8] This is also known as Multifactor Productivity or Total Factor Productivity.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1985 | 83764 | 31267 | -32486 | 34856 | 19824 | 83663 | 54486 | 1.53550 | 1.01328 |
| 1986 | 86839 | 32607 | -34820 | 36368 | 20368 | 85719 | 56533 | 1.51626 | 0.98747 |
| 1987 | 92299 | 33566 | -36671 | 38149 | 20912 | 90247 | 58848 | 1.53354 | 1.01140 |
| 1988 | 98137 | 36570 | -41604 | 39886 | 21617 | 94316 | 61281 | 1.53907 | 1.00360 |
| 1989 | 102477 | 36930 | -44024 | 40932 | 22568 | 96612 | 63260 | 1.52722 | 0.99231 |
| 1990 | 99919 | 38654 | -44876 | 40977 | 23574 | 95128 | 64231 | 1.48103 | 0.96976 |
| 1991 | 95966 | 39352 | -45982 | 39695 | 24283 | 90973 | 63439 | 1.43403 | 0.96826 |
| 1992 | 95238 | 42175 | -48154 | 39296 | 24793 | 91038 | 63396 | 1.43602 | 1.00139 |
| 1993 | 96615 | 46743 | -51718 | 40147 | 25038 | 93608 | 64535 | 1.45049 | 1.01008 |
| 1994 | 101167 | 52672 | -55933 | 41702 | 25236 | 100113 | 66360 | 1.50864 | 1.04009 |
| 1995 | 102221 | 57152 | -59169 | 42913 | 25690 | 102719 | 68026 | 1.50999 | 1.00090 |
| 1996 | 107694 | 60353 | -62199 | 44189 | 26249 | 108503 | 69850 | 1.55337 | 1.02873 |
| 1997 | 117236 | 65384 | -71100 | 45611 | 26977 | 114561 | 71979 | 1.59157 | 1.02459 |
| 1998 | 119529 | 71347 | -74731 | 47066 | 28283 | 119598 | 74714 | 1.60073 | 1.00575 |
| 1999 | 122519 | 78969 | -80562 | 48740 | 29521 | 124890 | 77594 | 1.60953 | 1.00550 |
| 2000 | 130510 | 86010 | -87116 | 50524 | 30704 | 133798 | 80534 | 1.66138 | 1.03221 |
| 2001 | 130835 | 83474 | -82635 | 51230 | 32034 | 135654 | 82552 | 1.64325 | 0.98908 |
| 2002 | 136452 | 84475 | -84039 | 52278 | 32939 | 140807 | 84482 | 1.66670 | 1.01427 |
| 2003 | 139200 | 82558 | -87489 | 53274 | 33666 | 138361 | 86187 | 1.60536 | 0.96320 |
| 2004 | 148475 | 86537 | -94812 | 55151 | 34326 | 144844 | 88710 | 1.63278 | 1.01708 |
| 2005 | 156678 | 88443 | -101927 | 55936 | 35336 | 148613 | 90500 | 1.64213 | 1.00573 |
| 2006 | 164655 | 89077 | -107002 | 56885 | 36637 | 152627 | 92746 | 1.64565 | 1.00214 |

The average rate of TFP growth over the 45 years 1962-2006 is 1.14% per year,[9] which is much higher than the 0.5 to 0.7% per year range that Diewert and Lawrence (2000) found over the period 1962-1996. The present 1.14% average rate of TFP growth can also be compared with Statistics Canada's recent KLEMS program average Multifactor Productivity Growth over the same years of 0.43% per year,[10] which is a rather substantial difference! In Appendix 4 below, we attempt to determine why our results are so different from the official Statistics Canada results.[11]

Over the golden years 1962-1973, TFP growth averaged 2.73% per year; over the dismal years, 1974-1991, TFP growth averaged only 0.25% per year but over the remainder of

[9] This rate of TFP growth is reasonably close to the average rate of productivity growth for Australia obtained by Diewert and Lawrence (2006) using a similar methodology and over a similar period. The Diewert and Lawrence market sector average rate of TFP growth for Australia over the period 1961-2004 was 1.49% per year. However, there is an upward bias in the Diewert and Lawrence results due to the fact that they essentially used hours worked as their measure of labour input instead of a quality adjusted measure of labour input for Australia (which was not available).

[10] See CANSIM II series V41712881, Canada, Multifactor Productivity, Business Sector, table 3830021, Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub-Sectors. Comparing levels of TFP with the starting level being 1 in 1961, our TFP ended up at 1.65 in 2006 whereas KLEMS Multifactor Productivity ended up at 1.20 in 2006. This is a very substantial difference.

[11] Our measures of business sector output and capital input were different from the KLEMS measures because we excluded rental housing from our measure of value added and we excluded the land and residential structures inputs from our measure of capital services, whereas the KLEMS measures included rental housing in their output and capital input measures. Our measures of labour input were identical and it turned out that the average rate of growth of our business sector value added measure was very close to the corresponding KLEMS average growth rate. The capital services growth rates differed substantially.

the nineties, 1992-1999, TFP growth nicely recovered to average a very respectable 1.46% per year. During the naughts, 2000-2006, TFP growth again fell off to average only 0.34% per year. There were two recent poor productivity growth years, 2001 and 2003, where drops of 1.09% and 3.68% occurred and if these years are excluded, the average TFP growth rate during the remaining years of the naughts is 1.43% per year. Hopefully the Canadian productivity recovery since the recession of 1991 is not a statistical mirage and it will continue into the future.

However, productivity growth is not necessarily the entire story behind the growth in living standards: if the price of Canadian exports increases more rapidly than the price of Canadian imports, then the real income generated by the business sector should increase. This terms of trade effect is not taken into account in the above productivity computations. Thus in the following section, we implement the translog real income methodology explained in sections 1- 4 of Appendix 1 below and this approach will enable us to assess the contribution to Canadian living standards of improvements in Canada's terms of trade.

## 3. Explaining Real Income Growth Generated by the Canadian Business Sector: the Gross Output Approach

The basic methodology used in this section can easily be explained in nontechnical terms. The business sector faces (exogenous) domestic and international prices for the net outputs it produces: domestic outputs, exports and (minus) imports. The business sector also utilizes inputs of labour and capital in order to produce its outputs. The value of outputs produced by the business sector less the value of imports used (value added) must eventually flow back to the labour and capital primary inputs that were used to produce value added. This is the (gross) income generated by the business sector. We divide this gross income in year t by the price of consumption in year t, $P_C^t$, in order to turn this nominal income into *real income* $\rho^t$; this real income is the number of consumption bundles that *could* be purchased by the owners of the labour and capital inputs that were used in year t by the Canadian business sector. We also divide each of the prices $P_D^t$, $P_X^t$, $P_M^t$, $P_L^t$ and $P_K^t$ by the price of consumption, $P_C^t$, in order to form the corresponding real output and input prices facing the Canadian business sector in each year. Our measure of the (gross) real income generated by the business sector $\rho^t$ and the corresponding real output and input prices are listed in Table 3.

**Table 3: Gross Real Income Generated by the Canadian Business Sector and Real Output and Input Prices**

| Year t | $\rho^t$ | $P_D^t/P_C^t$ | $P_X^t/P_C^t$ | $P_M^t/P_C^t$ | $P_L^t/P_C^t$ | $P_K^t/P_C^t$ |
|---|---|---|---|---|---|---|
| 1961 | 27683 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 1962 | 29848 | 1.00065 | 1.03522 | 1.04865 | 1.03226 | 1.07288 |
| 1963 | 31349 | 0.99992 | 1.02781 | 1.05363 | 1.04618 | 1.13090 |
| 1964 | 34486 | 1.00592 | 1.04560 | 1.05236 | 1.08417 | 1.26349 |
| 1965 | 37307 | 1.01347 | 1.05044 | 1.03608 | 1.14228 | 1.27730 |
| 1966 | 40165 | 1.01320 | 1.04074 | 1.01800 | 1.17099 | 1.30793 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1967 | 41030 | 1.01276 | 1.02620 | 1.00092 | 1.20243 | 1.19101 |
| 1968 | 42990 | 1.00882 | 1.00773 | 0.98400 | 1.23152 | 1.24815 |
| 1969 | 45404 | 1.01119 | 0.99500 | 0.97927 | 1.28031 | 1.25414 |
| 1970 | 47630 | 1.01550 | 1.00122 | 0.97781 | 1.32069 | 1.30705 |
| 1971 | 50289 | 1.02878 | 0.98247 | 0.98102 | 1.38380 | 1.29643 |
| 1972 | 53093 | 1.03407 | 0.98132 | 0.96413 | 1.43872 | 1.28557 |
| 1973 | 59377 | 1.04473 | 1.04458 | 0.96217 | 1.47076 | 1.50319 |
| 1974 | 62210 | 1.04932 | 1.16108 | 1.02065 | 1.48574 | 1.51697 |
| 1975 | 62343 | 1.03517 | 1.15147 | 1.02155 | 1.48498 | 1.45660 |
| 1976 | 68558 | 1.04500 | 1.17684 | 1.00178 | 1.62921 | 1.53565 |
| 1977 | 72059 | 1.04217 | 1.20990 | 1.06509 | 1.66959 | 1.59135 |
| 1978 | 73750 | 1.03988 | 1.22672 | 1.11827 | 1.61371 | 1.61136 |
| 1979 | 77368 | 1.03908 | 1.31492 | 1.15554 | 1.57347 | 1.68206 |
| 1980 | 78069 | 1.03355 | 1.36621 | 1.19169 | 1.52933 | 1.63352 |
| 1981 | 80063 | 1.04126 | 1.34561 | 1.24317 | 1.55664 | 1.55205 |
| 1982 | 75330 | 1.03307 | 1.26180 | 1.19876 | 1.55562 | 1.36149 |
| 1983 | 77370 | 1.01971 | 1.19642 | 1.11391 | 1.50819 | 1.50403 |
| 1984 | 81938 | 1.01390 | 1.18752 | 1.11447 | 1.51687 | 1.60691 |
| 1985 | 85739 | 1.01421 | 1.17670 | 1.10835 | 1.54714 | 1.60476 |
| 1986 | 87372 | 1.01679 | 1.15086 | 1.10429 | 1.55228 | 1.51805 |
| 1987 | 93711 | 1.02074 | 1.14599 | 1.06266 | 1.56860 | 1.61973 |
| 1988 | 99086 | 1.02099 | 1.11949 | 1.01071 | 1.62836 | 1.57926 |
| 1989 | 102458 | 1.02136 | 1.11117 | 0.98227 | 1.65098 | 1.54559 |
| 1990 | 98351 | 1.00397 | 1.04334 | 0.94248 | 1.62120 | 1.35403 |
| 1991 | 91534 | 0.98314 | 0.95295 | 0.87672 | 1.59984 | 1.15426 |
| 1992 | 90764 | 0.98064 | 0.96722 | 0.90174 | 1.60838 | 1.11167 |
| 1993 | 92784 | 0.97999 | 0.99090 | 0.93229 | 1.57435 | 1.18135 |
| 1994 | 100042 | 0.98970 | 1.04312 | 0.98378 | 1.55547 | 1.39389 |
| 1995 | 104884 | 0.99189 | 1.10458 | 1.00791 | 1.57450 | 1.45267 |
| 1996 | 111236 | 0.98593 | 1.08863 | 0.97501 | 1.56171 | 1.60862 |
| 1997 | 116434 | 0.97906 | 1.07376 | 0.96418 | 1.58283 | 1.63988 |
| 1998 | 118568 | 0.97712 | 1.05617 | 0.98463 | 1.60678 | 1.51832 |
| 1999 | 124427 | 0.97280 | 1.04921 | 0.96342 | 1.62220 | 1.53658 |
| 2000 | 136539 | 0.96952 | 1.08579 | 0.95713 | 1.65982 | 1.71572 |
| 2001 | 136059 | 0.96407 | 1.06841 | 0.95916 | 1.65802 | 1.59573 |
| 2002 | 139391 | 0.96729 | 1.03562 | 0.95291 | 1.66081 | 1.59587 |
| 2003 | 139604 | 0.95795 | 0.99757 | 0.86983 | 1.64938 | 1.53670 |
| 2004 | 150185 | 0.95947 | 1.00552 | 0.83625 | 1.66695 | 1.69702 |
| 2005 | 158122 | 0.96413 | 1.01305 | 0.80973 | 1.70065 | 1.78277 |
| 2006 | 163864 | 0.96902 | 0.99703 | 0.78973 | 1.73700 | 1.77565 |

Thus the gross real income generated by the Canadian business sector has grown from \$27,683 million dollars worth of 1961 consumption bundles in 1961 to \$163,864 million in 2006, a 5.92 fold increase. Looking at the change in real input and output prices, the real price of domestic output has fallen to .969 times the starting level (due to the fact that machinery and equipment prices have risen less rapidly than the price of consumption) and the real price of exports has fallen slightly to .997 times the starting level. However, the real price of imports has fallen substantially to .790 times the starting level. This probably reflects the "China factor"; i.e., the growth of lower priced imports from Asia in recent years. The quality adjusted real wages of business sector workers have risen to 1.74 times their initial 1961 levels. The real price of capital services has risen 1.78 fold, reflecting rapidly rising prices of agricultural land and nonagricultural business land as well as upward trends in machinery and equipment depreciation rates and in real rates of return; see Appendix 2 for details.[12]

There are six quantitative factors that can be used to explain the real income $\rho^t$ generated by the business sector in year t:

- The price of domestic production (an aggregate of C+I+G) relative to the price of consumption in year t, $P_D^t/P_C^t$;
- The price of exports relative to the price of consumption in year t, $P_X^t/P_C^t$;
- The price of imports relative to the price of consumption in year t, $P_M^t/P_C^t$;
- The quantity of labour used by the business sector in year t, $Q_L^t$;
- The quantity of capital used by the business sector in year t, $Q_K^t$ and
- The level of technology of the business sector in year t.

The formal model outlined in Appendix 1, based on the work of Diewert and Morrison (1986) and Kohli (1990), allows us to decompose the growth of real income from year t−1 to t, $\rho^t/\rho^{t-1}$, into multiplicative year to year contribution factors $\alpha_D^t$, $\alpha_X^t$, $\alpha_M^t$, $\beta_L^t$, $\beta_K^t$ and $\tau^t$ that describe the effects of changes in the six explanatory variables listed above going from year t−1 to t. The model outlined in Appendix 1 leads to the following equation which decomposes the year to year growth in real income generated by the business sector, $\rho^t/\rho^{t-1}$, into a product of six year to year explanatory contribution factors:[13]

$$(3)\ \rho^t/\rho^{t-1} = \tau^t\,\alpha_D^t\,\alpha_X^t\,\alpha_M^t\,\beta_L^t\,\beta_K^t\,; \qquad t = 1962, 1963,...,2006.$$

Thus if $\alpha_D^t$ is greater than one, this means that the domestic price of output grew faster than the price of consumption going from year t−1 to t and $\alpha_D^t$ measures the contribution of rising real domestic output prices to the growth in real income. Similarly, if $\alpha_X^t$ is greater than one, this means that Canadian export prices grew faster than the price of consumption going from year t−1 to t and $\alpha_X^t$ measures the contribution of rising real export prices to the growth in real income generated by the Canadian business sector. However, if if $\alpha_M^t$ is greater than one, this means that Canadian import prices did not

[12] The volatility of the real price of capital services reflects the fact that we have used balancing real rates of return in our user costs and these real rates are subject to a considerable amount of measurement error. One would expect the aggregate real price of capital services to decline, reflecting the decline in the real price of machinery and equipment, but this decline is offset by a large increase in the real price of land services.

[13] See equations (42), (51) and (56) in Appendix 1 in order to derive this equation. All of the variables in equation (3) can be identified using the data in Appendix 2.

increase as quickly as the price of consumption going from year t−1 to t and $\alpha_M^t$ measures the contribution of falling real import prices to the growth in real income generated by the Canadian business sector. If $\beta_L^t$ is greater than one, then business sector labour input increased going from year t−1 to t and $\beta_L^t$ measures the contribution of the increase in labour input to the growth in real income generated by the Canadian business sector. Similarly, if $\beta_K^t$ is greater than one, then business sector capital services input increased going from year t−1 to t and $\beta_K^t$ measures the contribution of the increase in capital input to the growth in real income generated by the Canadian business sector. Finally, if $\tau^t$ is greater than one, then the efficiency of the Canadian business sector increased from year t−1 to t and $\tau^t$ measures the contribution of the efficiency increase to the growth in real income generated by the Canadian business sector. These year to year contribution factors are listed in Table 4 along with the averages of these contribution factors in the last two rows of the Table.[14]

**Table 4: Business Sector Year to Year Growth in Real Income and Year to Year Contribution Factors**

| Year t | $\rho^t/\rho^{t-1}$ | $\tau^t$ | $\alpha_D^t$ | $\alpha_X^t$ | $\alpha_M^t$ | $\beta_L^t$ | $\beta_K^t$ | $\alpha_{XM}^t$ |
|---|---|---|---|---|---|---|---|---|
| 1962 | 1.07821 | 1.04894 | 1.00067 | 1.00920 | 0.98609 | 1.02905 | 1.00306 | 0.99516 |
| 1963 | 1.05029 | 1.03011 | 0.99925 | 0.99807 | 0.99863 | 1.01817 | 1.00547 | 0.99670 |
| 1964 | 1.10006 | 1.04975 | 1.00605 | 1.00481 | 1.00035 | 1.02964 | 1.00643 | 1.00517 |
| 1965 | 1.08182 | 1.02575 | 1.00762 | 1.00131 | 1.00466 | 1.02955 | 1.01060 | 1.00597 |
| 1966 | 1.07661 | 1.02220 | 0.99973 | 0.99736 | 1.00544 | 1.03541 | 1.01465 | 1.00279 |
| 1967 | 1.02153 | 0.98927 | 0.99956 | 0.99578 | 1.00536 | 1.01396 | 1.01772 | 1.00112 |
| 1968 | 1.04777 | 1.03566 | 0.99609 | 0.99423 | 1.00554 | 1.00293 | 1.01297 | 0.99974 |
| 1969 | 1.05614 | 1.02898 | 1.00239 | 0.99584 | 1.00164 | 1.01642 | 1.00996 | 0.99747 |
| 1970 | 1.04903 | 1.02772 | 1.00427 | 1.00211 | 1.00051 | 1.00235 | 1.01137 | 1.00262 |
| 1971 | 1.05583 | 1.02475 | 1.01294 | 0.99357 | 0.99891 | 1.01456 | 1.01014 | 0.99249 |
| 1972 | 1.05576 | 1.01415 | 1.00516 | 0.99960 | 1.00599 | 1.02036 | 1.00938 | 1.00559 |
| 1973 | 1.11836 | 1.03041 | 1.01037 | 1.02234 | 1.00073 | 1.04019 | 1.00940 | 1.02309 |
| 1974 | 1.04771 | 0.98952 | 1.00448 | 1.03927 | 0.97762 | 1.02367 | 1.01349 | 1.01601 |
| 1975 | 1.00214 | 1.00416 | 0.98587 | 0.99710 | 0.99965 | 0.99952 | 1.01609 | 0.99675 |
| 1976 | 1.09970 | 1.05681 | 1.00996 | 1.00729 | 1.00748 | 0.99967 | 1.01562 | 1.01482 |
| 1977 | 1.05105 | 1.04532 | 0.99720 | 1.00947 | 0.97745 | 1.00676 | 1.01503 | 0.98670 |
| 1978 | 1.02348 | 0.99770 | 0.99775 | 1.00501 | 0.98144 | 1.02767 | 1.01430 | 0.98636 |
| 1979 | 1.04906 | 0.98577 | 0.99922 | 1.02735 | 0.98678 | 1.03732 | 1.01276 | 1.01377 |
| 1980 | 1.00906 | 0.97414 | 0.99464 | 1.01572 | 0.98731 | 1.02171 | 1.01642 | 1.00283 |
| 1981 | 1.02555 | 1.01012 | 1.00753 | 0.99378 | 0.98234 | 1.01749 | 1.01447 | 0.97623 |
| 1982 | 0.94088 | 0.97436 | 0.99216 | 0.97446 | 1.01464 | 0.96754 | 1.01739 | 0.98873 |
| 1983 | 1.02708 | 1.02215 | 0.98739 | 0.97925 | 1.02743 | 1.00449 | 1.00696 | 1.00611 |

[14] The fifth row from the bottom gives the average over the years 1962-2006 and the remaining rows give the averages over the years 1962-1973, 1974-1991, 1992-1999 and 2000-2006. The careful reader will notice that the productivity growth rates $\tau^t$ listed in Table 4 do not quite agree with those listed in Table 2. The reason for these small differences is that when calculating $\tau^t$ in Table 4, the input aggregate is a direct Törnqvist quantity index whereas in Table 2, the input aggregate was an implicit quantity index; i.e., the value of inputs was deflated by the Törnqvist input price index.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1984 | 1.05903 | 1.03799 | 0.99443 | 0.99694 | 0.99980 | 1.02334 | 1.00587 | 0.99674 |
| 1985 | 1.04639 | 1.01328 | 1.00031 | 0.99607 | 1.00228 | 1.02544 | 1.00843 | 0.99834 |
| 1986 | 1.01904 | 0.98748 | 1.00254 | 0.99051 | 1.00158 | 1.02744 | 1.00986 | 0.99208 |
| 1987 | 1.07256 | 1.01140 | 1.00392 | 0.99822 | 1.01658 | 1.03119 | 1.00947 | 1.01477 |
| 1988 | 1.05736 | 1.00360 | 1.00024 | 0.99041 | 1.02128 | 1.02923 | 1.01177 | 1.01149 |
| 1989 | 1.03403 | 0.99231 | 1.00037 | 0.99697 | 1.01215 | 1.01717 | 1.01486 | 1.00909 |
| 1990 | 0.95991 | 0.96976 | 0.98263 | 0.97480 | 1.01777 | 1.00073 | 1.01460 | 0.99212 |
| 1991 | 0.93069 | 0.96826 | 0.97873 | 0.96354 | 1.03198 | 0.97848 | 1.00939 | 0.99435 |
| 1992 | 0.99159 | 1.00139 | 0.99739 | 1.00641 | 0.98716 | 0.99300 | 1.00636 | 0.99348 |
| 1993 | 1.02225 | 1.01008 | 0.99932 | 1.01154 | 0.98351 | 1.01486 | 1.00307 | 0.99486 |
| 1994 | 1.07823 | 1.04006 | 1.01001 | 1.02729 | 0.97166 | 1.02559 | 1.00264 | 0.99817 |
| 1995 | 1.04840 | 1.00090 | 1.00217 | 1.03350 | 0.98654 | 1.01867 | 1.00632 | 1.01959 |
| 1996 | 1.06056 | 1.02872 | 0.99422 | 0.99136 | 1.01865 | 1.01871 | 1.00795 | 1.00986 |
| 1997 | 1.04673 | 1.02459 | 0.99324 | 0.99182 | 1.00635 | 1.01984 | 1.01045 | 0.99812 |
| 1998 | 1.01833 | 1.00576 | 0.99805 | 0.98983 | 0.98739 | 1.01994 | 1.01770 | 0.97734 |
| 1999 | 1.04942 | 1.00550 | 0.99570 | 0.99570 | 1.01364 | 1.02250 | 1.01568 | 1.00929 |
| 2000 | 1.09734 | 1.03221 | 0.99682 | 1.02340 | 1.00405 | 1.02271 | 1.01485 | 1.02755 |
| 2001 | 0.99648 | 0.98908 | 0.99479 | 0.98925 | 0.99873 | 1.00864 | 1.01628 | 0.98800 |
| 2002 | 1.02449 | 1.01427 | 1.00314 | 0.98020 | 1.00379 | 1.01271 | 1.01054 | 0.98391 |
| 2003 | 1.00153 | 0.96320 | 0.99081 | 0.97747 | 1.05240 | 1.01188 | 1.00820 | 1.02869 |
| 2004 | 1.07580 | 1.01708 | 1.00152 | 1.00465 | 1.02134 | 1.02172 | 1.00739 | 1.02609 |
| 2005 | 1.05285 | 1.00573 | 1.00462 | 1.00429 | 1.01706 | 1.00862 | 1.01146 | 1.02142 |
| 2006 | 1.03631 | 1.00214 | 1.00489 | 0.99120 | 1.01306 | 1.01018 | 1.01449 | 1.00415 |
| A62-06 | 1.0410 | 1.0114 | 0.99934 | 0.99974 | 1.0028 | 1.0160 | 1.0111 | 1.0023 |
| A62-73 | 1.0660 | 1.0273 | 1.0037 | 1.0012 | 1.0012 | 1.0210 | 1.0101 | 1.0023 |
| A74-91 | 1.0253 | 1.0025 | 0.99663 | 0.99756 | 1.0025 | 1.0133 | 1.0126 | 0.99985 |
| A92-99 | 1.0394 | 1.0146 | 0.99876 | 1.0059 | 0.99436 | 1.0166 | 1.0088 | 1.0001 |
| A00-06 | 1.0407 | 1.0034 | 0.99951 | 0.99578 | 1.0158 | 1.0138 | 1.0119 | 1.0114 |

Looking at the sample averages listed in the fifth last row of Table 4, it can be seen that the (gross) real income generated by the Canadian business sector over the entire sample period grew at 4.10 percent per year over the 46 years 1961-2005. The biggest contributor to this growth was the growth of quality adjusted labour input at 1.6 percentage points per year. Next was Total Factor Productivity growth, $\tau^t$, which contributed on average 1.14 percentage points per year, followed by capital services growth (1.11 percentage points per year) and declines in real import prices (0.28 percentage points per year). Declines in real domestic output prices and real export prices gave rise to negative average contribution factors, – 0.07 and –0.03 percentage points per year respectively. The last column in Table 4 gives the product of the real export and real import price contribution factors, $\alpha_{XM}{}^t$, defined as:

(4) $\alpha_{XM}{}^t \equiv \alpha_X{}^t \alpha_M{}^t$.

Roughly speaking, $\alpha_{XM}{}^t$ is a *terms of trade contribution factor*; it gives the contribution to real income growth of the combined effects of real changes in the international prices facing the Canadian business sector.[15] It can be seen that the effects of changing real international prices are not negligible for Canada: on average, changing real export and import prices contributed 0.23 percentage points per year to real income growth over the entire sample period.[16] However, for shorter periods, the effects of changing real international prices can be far more important in explaining changes in the real income generated by the market sector of an economy. Thus if we restrict our attention to the period 2000-2006, it can be seen by looking at the last row of Table 4 that the effects of improvements in Canada's terms of trade become almost as important as the effects of capital deepening; i.e., during this period, the average annual growth in the real income generated by the Canadian business sector was 4.07 percent per year and the following factors explained this growth rate: decreases in the real price of imports (1.58), increases in quality adjusted labour input (1.38), increases in capital services input, 1.19) and improvements in TFP (0.34). There were small negative contributors to market sector real income growth during the naughts: decreases in the real price of domestically produced goods and services (−0.05) and decreases in the real price of exports (−0.42). Thus decreases in the real price of imports proved to be the most important factor in explaining the growth in real income generated by the market sector during this period. Overall, the joint effects of changes in real export and import prices contributed about 1.14 percentage points per year on average to the growth of market sector real income during the naughts, which was very close to the contribution of capital deepening over this period (which was 1.19 percentage points per year on average).[17]

The last four rows of Table 4 present the various growth factors for 4 subperiods:

- The 12 golden years for the Canadian economy, 1962-1973, when the real income generated by the business sector grew by 6.60% per year and TFP growth was a stellar 2.73% per year;
- The 18 dismal years for the Canadian economy, 1974-1991, characterized by stagflation, oil shocks and rapidly increasing tax rates when the real income generated by the business sector grew by 2.53% per year and TFP growth was a disappointing 0.25% per year;
- The 8 years in the nineties after the recession of 1991, 1992-1999, when real income growth recovered to 3.94% per year and TFP growth also recovered to 1.46% per year and
- The 7 years in the present century, 2000-2006, when TFP growth dropped off to 0.34% per year but real income growth was still strong at 4.07% per year due to the very strong contribution made by falling real import prices during this period, which contributed on average 1.58% per year to real income growth.

[15] Ulrich Kohli has pointed out that this is a slight abuse of terminology. Strictly speaking, the terms of trade is the price of exports over the price of imports and hence involves only two prices. Our definition of $\alpha_{XM}{}^t$ involves three prices: the price of exports, the price of imports and the price of domestic consumption. Our terms of trade contribution factor is the rate of change counterpart to Kohli's (2006; 50) *trading gains factor*. See Appendix 3 for a discussion of the differences between our approach and that of Kohli.

[16] Thus the contribution of falling real import prices outweighs the effects of falling real export prices.

[17] These results are very similar to the results obtained for Australia using a similar framework by Diewert and Lawrence (2006); i.e., both Australia and Canada have had very favourable changes in their terms of trade in recent years which contributed greatly to real income growth during the naughts.

The annual change information in Table 4 can be converted into levels using equations (46) in Appendix 1 (with obvious extensions to multiple inputs and outputs). Thus let $T^t$, $A_D^t$, $A_X^t$, $A_M^t$, $B_L^t$, $B_K^t$ and $A_{XM}^t$ be the cumulated products of the annual link factors $\tau^t$, $\alpha_D^t$, $\alpha_X^t$, $\alpha_M^t$, $\beta_L^t$, $\beta_K^t$ and $\alpha_{XM}^t$ respectively. Using these definitions and cumulating equations (3) leads to the following equation, which explains the cumulative growth in real gross income generated by the Canadian business sector relative to the base year 1961:

$$(5)\ \rho^t/\rho^{1961} = T^t A_D^t A_X^t A_M^t B_L^t B_K^t\ ; \qquad t = 1962, 1963, \ldots , 2006.$$

The cumulated variables that appear in (5) above are reported in Table 5 below along with the cumulated terms of trade contribution factor, $A_{XM}^t$ defined to be the product of the two cumulated international price factors, $A_X^t$ and $A_M^t$.

**Table 5: Business Sector Cumulated Growth in Real Income and Cumulated Contribution Factors**

| Year t | $\rho^t/\rho^{1961}$ | $T^t$ | $A_D^t$ | $A_X^t$ | $A_M^t$ | $B_L^t$ | $B_K^t$ | $A_{XM}^t$ |
|---|---|---|---|---|---|---|---|---|
| 1961 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 1962 | 1.07821 | 1.04894 | 1.00067 | 1.00920 | 0.98609 | 1.02905 | 1.00306 | 0.99516 |
| 1963 | 1.13243 | 1.08053 | 0.99992 | 1.00725 | 0.98474 | 1.04774 | 1.00855 | 0.99188 |
| 1964 | 1.24574 | 1.13429 | 1.00598 | 1.01210 | 0.98508 | 1.07880 | 1.01503 | 0.99700 |
| 1965 | 1.34766 | 1.16350 | 1.01365 | 1.01342 | 0.98967 | 1.11068 | 1.02579 | 1.00295 |
| 1966 | 1.45090 | 1.18933 | 1.01337 | 1.01075 | 0.99506 | 1.15001 | 1.04083 | 1.00575 |
| 1967 | 1.48214 | 1.17656 | 1.01292 | 1.00648 | 1.00039 | 1.16605 | 1.05927 | 1.00687 |
| 1968 | 1.55294 | 1.21852 | 1.00896 | 1.00067 | 1.00594 | 1.16947 | 1.07301 | 1.00661 |
| 1969 | 1.64012 | 1.25382 | 1.01136 | 0.99651 | 1.00758 | 1.18867 | 1.08370 | 1.00406 |
| 1970 | 1.72054 | 1.28858 | 1.01568 | 0.99861 | 1.00810 | 1.19146 | 1.09602 | 1.00669 |
| 1971 | 1.81659 | 1.32048 | 1.02883 | 0.99219 | 1.00700 | 1.20882 | 1.10713 | 0.99914 |
| 1972 | 1.91789 | 1.33916 | 1.03413 | 0.99179 | 1.01304 | 1.23343 | 1.11752 | 1.00472 |
| 1973 | 2.14490 | 1.37988 | 1.04486 | 1.01395 | 1.01378 | 1.28301 | 1.12802 | 1.02792 |
| 1974 | 2.24723 | 1.36542 | 1.04954 | 1.05377 | 0.99109 | 1.31337 | 1.14324 | 1.04438 |
| 1975 | 2.25203 | 1.37110 | 1.03470 | 1.05071 | 0.99074 | 1.31274 | 1.16163 | 1.04098 |
| 1976 | 2.47656 | 1.44898 | 1.04501 | 1.05837 | 0.99815 | 1.31230 | 1.17978 | 1.05641 |
| 1977 | 2.60299 | 1.51465 | 1.04208 | 1.06839 | 0.97564 | 1.32117 | 1.19751 | 1.04236 |
| 1978 | 2.66411 | 1.51116 | 1.03974 | 1.07374 | 0.95753 | 1.35773 | 1.21464 | 1.02814 |
| 1979 | 2.79480 | 1.48967 | 1.03893 | 1.10311 | 0.94487 | 1.40840 | 1.23014 | 1.04230 |
| 1980 | 2.82011 | 1.45114 | 1.03336 | 1.12045 | 0.93288 | 1.43898 | 1.25034 | 1.04525 |
| 1981 | 2.89215 | 1.46583 | 1.04115 | 1.11348 | 0.91641 | 1.46416 | 1.26843 | 1.02040 |
| 1982 | 2.72118 | 1.42824 | 1.03299 | 1.08504 | 0.92983 | 1.41662 | 1.29050 | 1.00890 |
| 1983 | 2.79487 | 1.45988 | 1.01996 | 1.06253 | 0.95533 | 1.42298 | 1.29948 | 1.01507 |
| 1984 | 2.95987 | 1.51533 | 1.01428 | 1.05927 | 0.95514 | 1.45619 | 1.30711 | 1.01176 |
| 1985 | 3.09719 | 1.53546 | 1.01459 | 1.05510 | 0.95732 | 1.49323 | 1.31813 | 1.01008 |
| 1986 | 3.15615 | 1.51623 | 1.01717 | 1.04509 | 0.95884 | 1.53420 | 1.33112 | 1.00207 |
| 1987 | 3.38516 | 1.53351 | 1.02116 | 1.04324 | 0.97473 | 1.58205 | 1.34373 | 1.01688 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1988 | 3.57932 | 1.53904 | 1.02141 | 1.03323 | 0.99548 | 1.62829 | 1.35954 | 1.02856 |
| 1989 | 3.70111 | 1.52720 | 1.02178 | 1.03010 | 1.00758 | 1.65624 | 1.37974 | 1.03791 |
| 1990 | 3.55274 | 1.48101 | 1.00403 | 1.00414 | 1.02548 | 1.65746 | 1.39989 | 1.02973 |
| 1991 | 3.30651 | 1.43401 | 0.98267 | 0.96752 | 1.05828 | 1.62179 | 1.41304 | 1.02391 |
| 1992 | 3.27871 | 1.43600 | 0.98011 | 0.97372 | 1.04469 | 1.61044 | 1.42203 | 1.01724 |
| 1993 | 3.35165 | 1.45047 | 0.97944 | 0.98496 | 1.02746 | 1.63437 | 1.42639 | 1.01200 |
| 1994 | 3.61385 | 1.50858 | 0.98925 | 1.01183 | 0.99834 | 1.67620 | 1.43015 | 1.01016 |
| 1995 | 3.78877 | 1.50994 | 0.99140 | 1.04573 | 0.98490 | 1.70749 | 1.43920 | 1.02994 |
| 1996 | 4.01820 | 1.55331 | 0.98567 | 1.03670 | 1.00327 | 1.73944 | 1.45064 | 1.04009 |
| 1997 | 4.20598 | 1.59151 | 0.97901 | 1.02822 | 1.00964 | 1.77394 | 1.46579 | 1.03814 |
| 1998 | 4.28306 | 1.60067 | 0.97710 | 1.01777 | 0.99691 | 1.80932 | 1.49173 | 1.01462 |
| 1999 | 4.49471 | 1.60947 | 0.97291 | 1.01339 | 1.01051 | 1.85003 | 1.51513 | 1.02404 |
| 2000 | 4.93225 | 1.66132 | 0.96981 | 1.03711 | 1.01461 | 1.89205 | 1.53764 | 1.05226 |
| 2001 | 4.91489 | 1.64318 | 0.96475 | 1.02596 | 1.01332 | 1.90839 | 1.56267 | 1.03963 |
| 2002 | 5.03524 | 1.66664 | 0.96778 | 1.00564 | 1.01716 | 1.93264 | 1.57914 | 1.02290 |
| 2003 | 5.04294 | 1.60530 | 0.95888 | 0.98299 | 1.07046 | 1.95560 | 1.59208 | 1.05225 |
| 2004 | 5.42519 | 1.63271 | 0.96034 | 0.98756 | 1.09331 | 1.99809 | 1.60385 | 1.07970 |
| 2005 | 5.71189 | 1.64206 | 0.96478 | 0.99179 | 1.11196 | 2.01530 | 1.62223 | 1.10283 |
| 2006 | 5.91929 | 1.64558 | 0.96949 | 0.98307 | 1.12648 | 2.03582 | 1.64574 | 1.10741 |

Looking at the last row of Table 5, it can be seen that the gross real income generated by the business sector grew 5.92 fold over the years 1961-2006. The main factors explaining this growth are growth of quality adjusted labour input (cumulative growth factor 2.04), productivity increases (cumulative growth factor 1.65), growth of capital services (cumulative growth factor 1.65) and lower real import prices (cumulative growth factor 1.13). There were negative contributions from declining real domestic output prices (cumulative growth factor 0.97) and declining real export prices (cumulative growth factor .98). In recent years, the real prices of Canada's raw materials exports have increased dramatically. However, these increases do not show up in the $A_X^t$ column of Table 5; i.e., the overall real price of Canadian exports has remained relatively constant in recent years. This apparent contradiction can be explained by falling real prices for Canadian exports of manufactured goods. As already noted above, the effects of falling real import prices in recent years have been substantial.

As is noted in section 5 of Appendix 1, the income concept used in this section is biased upwards. The problem is that depreciation payments are part of the user cost of capital for each asset but depreciation does not provide households with any sustainable purchasing power. Hence the measure of real income $\rho^t$ that is used in this section is overstated. In the following section, we implement the *net real income model* that is described in more detail in section 5 of Appendix 1.

## 4. Explaining Real Income Growth Generated by the Canadian Business Sector: the Net Output Approach

The overstatement of income problem that is implicit in the approach used in the previous section can readily be remedied: all we need to do is to take the user cost formula for an asset that has investment price $P_I^t$ in year t and decompose it into two parts:

- One part that represents depreciation and foreseen obsolescence, $\delta P_I^t K^t$, and
- The remaining part that is the reward for postponing consumption, $r^t P_I^t K^t$.

The depreciation part $\delta P_I^t K^t$, will be removed from the user cost and treated as an intermediate input as an offset to gross investment. We now explain this rather simple idea in more detail below.

In our empirical work thus far (described in detail in Appendix 2), our user costs took the following form:

$$(6)\ U^t = (r^t + \delta^t + \tau_B^t)P_I^t$$

where $r^t$ is the balancing period t real rate of interest, $\delta^t$ is a geometric depreciation rate for period t, $\tau_B^t$ is an appropriate business taxation rate on the asset (including property taxes if applicable) and $P_I^t$ is the period t investment price for the asset. However, in the net product approach to the measurement of income,[18] we split up each (gross product) user cost times the beginning of the period stock $K^t$ into the depreciation component $\delta^t P_I^t K^t$ and the remaining term $(r^t+\tau_B^t)P_I^t K^t$ and we regard the second term as a genuine income component but we treat the first term as an intermediate input cost for the business sector and treat it as an offset to gross investment made by the business sector during the year under consideration. Thus in the present section, our new aggregate for domestic output will aggregate the same C+I+G components as before, but now we add the depreciation series for business structures and for machinery and equipment as negative outputs of the business sector. As noted above, the machinery and equipment and nonresidential structures user costs are also changed since the depreciation terms are now omitted. Thus the new investment aggregate I is a *net investment aggregate* (gross investment components were indexed with a positive sign in the aggregate and depreciation components were indexed with a negative sign in the aggregate) and the new capital services aggregate is now a "reward for waiting" capital services aggregate.[19]

The above changes mean that our aggregate data series have changed somewhat. The new net product counterparts to the old gross product Tables 1-3 are presented below as Tables 6-8.[20]

---

[18] See Diewert (2006a) for a more detailed discussion of the net income approach to income measurement.

[19] This approach seems to be broadly consistent with an approach advocated by Rymes (1968) (1983), who stressed the role of waiting services: "Second, one can consider the 'waiting' or 'abstinence' associated with the net returns to capital as the nonlabour primary input." T.K. Rymes (1968; 362). Denison (1974) also advocated a net product approach to productivity measurement.

[20] The TFP growth rates $\tau^t$ in Tables 7 and 9 differ slightly because when calculating $\tau^t$ in Table 9, the input aggregate is a direct Törnqvist quantity index whereas in Table 7, the input aggregate was an implicit quantity index; i.e., the value of inputs was deflated by the Törnqvist input price index. Both the direct and implicit Törnqvist indexes are superlative and hence will generally approximate each other very closely; see Diewert (1978).

**Table 6: Prices of Canadian Business Sector (Net) Output and Input Aggregates**

| Year t | $P_C^t$ | $P_D^t$ | $P_X^t$ | $P_M^t$ | $P_L^t$ | $P_K^t$ | $P_Y^t$ | $P_Z^t$ |
|---|---|---|---|---|---|---|---|---|
| 1961 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 1962 | 1.00538 | 1.00481 | 1.04079 | 1.05429 | 1.03782 | 1.13234 | 0.99914 | 1.05550 |
| 1963 | 1.02055 | 1.01850 | 1.04893 | 1.07529 | 1.06768 | 1.25342 | 1.00886 | 1.10220 |
| 1964 | 1.02437 | 1.02524 | 1.07108 | 1.07801 | 1.11059 | 1.48424 | 1.02160 | 1.17944 |
| 1965 | 1.03690 | 1.04293 | 1.08920 | 1.07432 | 1.18443 | 1.50052 | 1.04642 | 1.24303 |
| 1966 | 1.07553 | 1.08091 | 1.11934 | 1.09489 | 1.25943 | 1.61052 | 1.08795 | 1.32441 |
| 1967 | 1.11050 | 1.11672 | 1.13960 | 1.11152 | 1.33529 | 1.42008 | 1.12545 | 1.35241 |
| 1968 | 1.15168 | 1.15707 | 1.16058 | 1.13325 | 1.41832 | 1.62202 | 1.16575 | 1.45745 |
| 1969 | 1.18980 | 1.19616 | 1.18385 | 1.16514 | 1.52331 | 1.66773 | 1.20166 | 1.55145 |
| 1970 | 1.22208 | 1.23158 | 1.22357 | 1.19496 | 1.61399 | 1.81176 | 1.24096 | 1.65228 |
| 1971 | 1.24828 | 1.27322 | 1.22639 | 1.22459 | 1.72738 | 1.79381 | 1.27167 | 1.74041 |
| 1972 | 1.29847 | 1.33168 | 1.27421 | 1.25190 | 1.86814 | 1.82877 | 1.33864 | 1.86078 |
| 1973 | 1.38744 | 1.43978 | 1.44929 | 1.33495 | 2.04059 | 2.54131 | 1.48567 | 2.13608 |
| 1974 | 1.58382 | 1.65156 | 1.83894 | 1.61653 | 2.35314 | 2.93564 | 1.73550 | 2.46420 |
| 1975 | 1.82198 | 1.87786 | 2.09796 | 1.86125 | 2.70560 | 3.20817 | 1.96471 | 2.80141 |
| 1976 | 1.90726 | 1.98271 | 2.24453 | 1.91066 | 3.10734 | 3.61741 | 2.11062 | 3.20415 |
| 1977 | 2.03175 | 2.10583 | 2.45822 | 2.16399 | 3.39218 | 4.07520 | 2.20736 | 3.52340 |
| 1978 | 2.19264 | 2.26575 | 2.68976 | 2.45197 | 3.53830 | 4.46713 | 2.33767 | 3.71940 |
| 1979 | 2.40645 | 2.48747 | 3.16403 | 2.78074 | 3.78649 | 5.26303 | 2.60711 | 4.07796 |
| 1980 | 2.69497 | 2.77726 | 3.68189 | 3.21157 | 4.12150 | 5.68230 | 2.92025 | 4.42943 |
| 1981 | 2.95335 | 3.06138 | 3.97404 | 3.67151 | 4.59729 | 5.64832 | 3.13072 | 4.80147 |
| 1982 | 3.22860 | 3.32497 | 4.07385 | 3.87032 | 5.02249 | 4.93900 | 3.35533 | 4.98799 |
| 1983 | 3.46323 | 3.55107 | 4.14348 | 3.85771 | 5.22320 | 6.59291 | 3.60985 | 5.50398 |
| 1984 | 3.61506 | 3.69913 | 4.29296 | 4.02889 | 5.48356 | 7.77771 | 3.74623 | 5.96273 |
| 1985 | 3.72257 | 3.81598 | 4.38035 | 4.12592 | 5.75934 | 8.01538 | 3.85697 | 6.23004 |
| 1986 | 3.80422 | 3.91688 | 4.37812 | 4.20095 | 5.90520 | 7.51533 | 3.92264 | 6.24048 |
| 1987 | 3.89726 | 4.03952 | 4.46622 | 4.14146 | 6.11325 | 8.59723 | 4.11486 | 6.62880 |
| 1988 | 4.00205 | 4.15535 | 4.48026 | 4.04492 | 6.51680 | 8.51247 | 4.28886 | 6.93400 |
| 1989 | 4.11690 | 4.28175 | 4.57459 | 4.04389 | 6.79693 | 8.48331 | 4.46568 | 7.15222 |
| 1990 | 4.35206 | 4.46407 | 4.54070 | 4.10175 | 7.05555 | 7.34498 | 4.61185 | 7.12429 |
| 1991 | 4.59099 | 4.66280 | 4.37497 | 4.02502 | 7.34485 | 6.21899 | 4.78406 | 7.11331 |
| 1992 | 4.65258 | 4.72122 | 4.50007 | 4.19541 | 7.48311 | 5.91272 | 4.80690 | 7.15453 |
| 1993 | 4.74252 | 4.81034 | 4.69935 | 4.42139 | 7.46638 | 6.75739 | 4.86804 | 7.32683 |
| 1994 | 4.77089 | 4.87349 | 4.97660 | 4.69349 | 7.42100 | 8.89929 | 4.92183 | 7.75232 |
| 1995 | 4.79147 | 4.90428 | 5.29258 | 4.82937 | 7.54415 | 9.52932 | 5.06589 | 7.98286 |
| 1996 | 4.88952 | 4.98029 | 5.32288 | 4.76732 | 7.63603 | 11.50727 | 5.20438 | 8.47016 |
| 1997 | 4.96547 | 5.02092 | 5.33171 | 4.78760 | 7.85949 | 12.04730 | 5.23680 | 8.76036 |
| 1998 | 5.03224 | 5.07276 | 5.31493 | 4.95489 | 8.08570 | 10.76390 | 5.15280 | 8.67227 |
| 1999 | 5.12045 | 5.15890 | 5.37242 | 4.93315 | 8.30640 | 11.33176 | 5.29682 | 8.96636 |
| 2000 | 5.25425 | 5.28840 | 5.70503 | 5.02898 | 8.72108 | 13.90167 | 5.60301 | 9.83266 |
| 2001 | 5.40970 | 5.42560 | 5.77978 | 5.18877 | 8.96937 | 12.90648 | 5.67061 | 9.82070 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2002 | 5.47743 | 5.51553 | 5.67251 | 5.21950 | 9.09698 | 13.04603 | 5.65565 | 9.95111 |
| 2003 | 5.61543 | 5.64652 | 5.60179 | 4.88445 | 9.26199 | 13.01170 | 5.98321 | 10.07433 |
| 2004 | 5.69263 | 5.75059 | 5.72405 | 4.76048 | 9.48930 | 15.46161 | 6.27518 | 10.76826 |
| 2005 | 5.80796 | 5.91458 | 5.88377 | 4.70288 | 9.87728 | 17.06379 | 6.60960 | 11.41094 |
| 2006 | 5.90800 | 6.06936 | 5.89047 | 4.66574 | 10.26222 | 17.38831 | 6.81227 | 11.78403 |

Comparing Table 6 with Table 1, we see that the 2006 price of domestic absorption, $P_D$, has increased to 6.07 from its earlier 2006 gross approach level of 5.72. This is due to the fact that net investment is considerably smaller than gross investment and so the relatively low inflation price of machinery and equipment gets a much smaller weight in net domestic absorption compared to its weight in gross domestic absorption. The other striking difference between Tables 1 and 6 is that the price of waiting services, $P_K^t$, in Table 6 grew 17.4 fold over the sample period whereas the price of traditional capital services, $P_K^t$, in Table 1, grew only 10.5 fold. This difference in growth rates is explained by the fact that the price of machinery and equipment services gets a much lower weight in the Table 6 capital services aggregate compared to its weight in the Table 1 capital services aggregate because the corresponding user cost for the net concept of capital services now excludes the very large depreciation term in the net user cost. Thus the price of agricultural land and business nonagricultural land gets a much higher weight in the net user cost compared to the gross concept user cost.[21] However, even though the land components now get a much higher weight in weighting services compared to machinery and equipment, the overall price increase in input prices has only increased to 11.8 fold (compared to the gross output model 10.4 fold increase in input prices) over the sample period due to the fact that the importance of capital services dramatically shrinks relative to labour services in the net output framework.

**Table 7: Quantities of Canadian Business Sector Net Output and Input Aggregates, TFP Levels and TFP Growth Rates**

| Year t | $Q_D^t$ | $Q_X^t$ | $Q_M^t$ | $Q_L^t$ | $Q_K^t$ | $Q_Y^t$ | $Q_Z^t$ | $T^t$ | $\tau^t$ |
|---|---|---|---|---|---|---|---|---|---|
| 1961 | 24559 | 7310 | -8180 | 19202 | 4487 | 23689 | 23689 | 1.00000 | _____ |
| 1962 | 26652 | 7639 | -8370 | 20012 | 4537 | 25929 | 24544 | 1.05641 | 1.05641 |
| 1963 | 27654 | 8323 | -8513 | 20542 | 4635 | 27498 | 25169 | 1.09252 | 1.03419 |
| 1964 | 30284 | 9465 | -9602 | 21444 | 4730 | 30183 | 26144 | 1.15451 | 1.05674 |
| 1965 | 33336 | 9890 | -10905 | 22383 | 4874 | 32324 | 27211 | 1.18789 | 1.02891 |
| 1966 | 35916 | 11233 | -12376 | 23547 | 5085 | 34787 | 28576 | 1.21735 | 1.02480 |
| 1967 | 35830 | 12426 | -13055 | 24022 | 5342 | 35241 | 29327 | 1.20166 | 0.98712 |
| 1968 | 37450 | 13925 | -14404 | 24122 | 5522 | 37032 | 29620 | 1.25023 | 1.04041 |
| 1969 | 40333 | 15050 | -16278 | 24696 | 5683 | 39193 | 30357 | 1.29109 | 1.03268 |
| 1970 | 40319 | 16447 | -16005 | 24779 | 5885 | 40819 | 30658 | 1.33146 | 1.03127 |
| 1971 | 42695 | 17212 | -17073 | 25303 | 6051 | 42905 | 31350 | 1.36860 | 1.02789 |
| 1972 | 45559 | 18694 | -19554 | 26041 | 6213 | 44829 | 32250 | 1.39005 | 1.01568 |
| 1973 | 50360 | 20568 | -22397 | 27562 | 6365 | 48744 | 33903 | 1.43778 | 1.03434 |
| 1974 | 54582 | 19655 | -24652 | 28524 | 6581 | 49807 | 35078 | 1.41988 | 0.98755 |

[21] From Table 10 in Appendix 2, we estimate that the price of agricultural land increased 18.6 fold and the price of business nonagricultural land increased 48.5 fold over the period 1961-2006. For comparison purposes, the price of residential land increased 78.1 fold over this period.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1975 | 56328 | 18031 | -23889 | 28504 | 6864 | 50461 | 35390 | 1.42587 | 1.00421 |
| 1976 | 60134 | 19422 | -25376 | 28490 | 7135 | 54172 | 35684 | 1.51811 | 1.06469 |
| 1977 | 62599 | 20646 | -25281 | 28777 | 7424 | 57928 | 36291 | 1.5962 | 1.05144 |
| 1978 | 63312 | 22694 | -26058 | 29989 | 7720 | 60145 | 37801 | 1.59107 | 0.99678 |
| 1979 | 66440 | 23523 | -27910 | 31726 | 7973 | 62173 | 39748 | 1.56417 | 0.98309 |
| 1980 | 65412 | 23748 | -27013 | 32803 | 8298 | 62443 | 41167 | 1.51680 | 0.96972 |
| 1981 | 68222 | 24170 | -27702 | 33691 | 8553 | 64904 | 42320 | 1.53366 | 1.01112 |
| 1982 | 59483 | 23789 | -23204 | 32044 | 8898 | 61063 | 41076 | 1.48659 | 0.96931 |
| 1983 | 62511 | 25195 | -25549 | 32267 | 8991 | 63109 | 41391 | 1.52471 | 1.02564 |
| 1984 | 66809 | 29833 | -29968 | 33481 | 9112 | 67926 | 42676 | 1.59166 | 1.04391 |
| 1985 | 71421 | 31267 | -32486 | 34856 | 9322 | 71420 | 44215 | 1.61527 | 1.01483 |
| 1986 | 74066 | 32607 | -34820 | 36368 | 9558 | 73060 | 45924 | 1.59089 | 0.98491 |
| 1987 | 79016 | 33566 | -36671 | 38149 | 9772 | 77093 | 47856 | 1.61094 | 1.01261 |
| 1988 | 84187 | 36570 | -41604 | 39886 | 10039 | 80531 | 49810 | 1.61675 | 1.00360 |
| 1989 | 87664 | 36930 | -44024 | 40932 | 10380 | 82017 | 51210 | 1.60160 | 0.99063 |
| 1990 | 84354 | 38654 | -44876 | 40977 | 10741 | 79795 | 51655 | 1.54478 | 0.96452 |
| 1991 | 79923 | 39352 | -45982 | 39695 | 10966 | 75197 | 50574 | 1.48688 | 0.96252 |
| 1992 | 78795 | 42175 | -48154 | 39296 | 11115 | 74845 | 50286 | 1.48839 | 1.00102 |
| 1993 | 79844 | 46743 | -51718 | 40147 | 11147 | 77048 | 51192 | 1.50509 | 1.01122 |
| 1994 | 84040 | 52672 | -55933 | 41702 | 11204 | 83135 | 52781 | 1.57509 | 1.04651 |
| 1995 | 84708 | 57152 | -59169 | 42913 | 11378 | 85308 | 54136 | 1.57580 | 1.00045 |
| 1996 | 89578 | 60353 | -62199 | 44189 | 11595 | 90473 | 55590 | 1.62750 | 1.03281 |
| 1997 | 98340 | 65384 | -71100 | 45611 | 11910 | 95853 | 57300 | 1.67284 | 1.02786 |
| 1998 | 99610 | 71347 | -74731 | 47066 | 12417 | 99795 | 59295 | 1.68302 | 1.00608 |
| 1999 | 101508 | 78969 | -80562 | 48740 | 12853 | 103931 | 61396 | 1.69278 | 1.00580 |
| 2000 | 108143 | 86010 | -87116 | 50524 | 13226 | 111456 | 63512 | 1.75489 | 1.03669 |
| 2001 | 107390 | 83474 | -82635 | 51230 | 13702 | 112218 | 64796 | 1.73186 | 0.98688 |
| 2002 | 111934 | 84475 | -84039 | 52278 | 13977 | 116330 | 66115 | 1.75950 | 1.01596 |
| 2003 | 113963 | 82558 | -87489 | 53274 | 14234 | 113423 | 67362 | 1.68377 | 0.95696 |
| 2004 | 122094 | 86537 | -94812 | 55151 | 14407 | 118897 | 69287 | 1.71601 | 1.01915 |
| 2005 | 129046 | 88443 | -101927 | 55936 | 14755 | 121683 | 70483 | 1.72642 | 1.00607 |
| 2006 | 135575 | 89077 | -107002 | 56885 | 15215 | 124528 | 71989 | 1.72982 | 1.00197 |

Thus the level of business sector Total Factor Productivity using the net approach increased 1.73 fold over the period 1961-2006 and the average rate of net TFP growth was 1.26 percent per year. Recall that using the gross approach, the level of business sector Total Factor Productivity increased 1.65 fold over the period 1961-2006 and the average rate of gross product TFP growth was 1.14 percent per year. Thus switching to the more appropriate net approach increases Canadian business sector TFP growth by about 0.12 percentage points per year, which is not large but it does represent a 10% increase in the average rate of TFP growth. For a more detailed breakdown of net TFP growth, see the last rows of Table 9 below, which is the net product counterpart to Table 4 above.

The net counterpart to Table 3 above is Table 8 below; $\rho^t$ now represents the *net real income* generated by the Canadian business sector in year t.

**Table 8: Net Real Income Generated by the Canadian Business Sector and Real Output and Input Prices**

| Year t | $\rho^t$ | $P_D^t/P_C^t$ | $P_X^t/P_C^t$ | $P_M^t/P_C^t$ | $P_L^t/P_C^t$ | $P_K^t/P_C^t$ |
|---|---|---|---|---|---|---|
| 1961 | 23689 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 1962 | 25768 | 0.99944 | 1.03522 | 1.04865 | 1.03226 | 1.12628 |
| 1963 | 27183 | 0.99800 | 1.02781 | 1.05363 | 1.04618 | 1.22818 |
| 1964 | 30102 | 1.00085 | 1.04560 | 1.05236 | 1.08417 | 1.44893 |
| 1965 | 32621 | 1.00582 | 1.05044 | 1.03608 | 1.14228 | 1.44712 |
| 1966 | 35188 | 1.00500 | 1.04074 | 1.01800 | 1.17099 | 1.49742 |
| 1967 | 35716 | 1.00560 | 1.02620 | 1.00092 | 1.20243 | 1.27878 |
| 1968 | 37484 | 1.00468 | 1.00773 | 0.98400 | 1.23152 | 1.40839 |
| 1969 | 39584 | 1.00535 | 0.99500 | 0.97927 | 1.28031 | 1.40169 |
| 1970 | 41450 | 1.00777 | 1.00122 | 0.97781 | 1.32069 | 1.48252 |
| 1971 | 43709 | 1.01998 | 0.98247 | 0.98102 | 1.38380 | 1.43702 |
| 1972 | 46216 | 1.02558 | 0.98132 | 0.96413 | 1.43872 | 1.40840 |
| 1973 | 52196 | 1.03772 | 1.04458 | 0.96217 | 1.47076 | 1.83165 |
| 1974 | 54577 | 1.04277 | 1.16108 | 1.02065 | 1.48574 | 1.85352 |
| 1975 | 54414 | 1.03067 | 1.15147 | 1.02155 | 1.48498 | 1.76081 |
| 1976 | 59948 | 1.03956 | 1.17684 | 1.00178 | 1.62921 | 1.89666 |
| 1977 | 62935 | 1.03646 | 1.20990 | 1.06509 | 1.66959 | 2.00576 |
| 1978 | 64123 | 1.03334 | 1.22672 | 1.11827 | 1.61371 | 2.03733 |
| 1979 | 67357 | 1.03367 | 1.31492 | 1.15554 | 1.57347 | 2.18705 |
| 1980 | 67663 | 1.03053 | 1.36621 | 1.19169 | 1.52933 | 2.10848 |
| 1981 | 68802 | 1.03658 | 1.34561 | 1.24317 | 1.55664 | 1.91251 |
| 1982 | 63460 | 1.02985 | 1.26180 | 1.19876 | 1.55562 | 1.52976 |
| 1983 | 65781 | 1.02536 | 1.19642 | 1.11391 | 1.50819 | 1.90369 |
| 1984 | 70391 | 1.02325 | 1.18752 | 1.11447 | 1.51687 | 2.15148 |
| 1985 | 73998 | 1.02509 | 1.17670 | 1.10835 | 1.54714 | 2.15318 |
| 1986 | 75334 | 1.02961 | 1.15086 | 1.10429 | 1.55228 | 1.97553 |
| 1987 | 81397 | 1.03650 | 1.14599 | 1.06266 | 1.56860 | 2.20597 |
| 1988 | 86302 | 1.03830 | 1.11949 | 1.01071 | 1.62836 | 2.12703 |
| 1989 | 88966 | 1.04004 | 1.11117 | 0.98227 | 1.65098 | 2.06061 |
| 1990 | 84559 | 1.02574 | 1.04334 | 0.94248 | 1.62120 | 1.68770 |
| 1991 | 78360 | 1.01564 | 0.95295 | 0.87672 | 1.59984 | 1.35461 |
| 1992 | 77328 | 1.01475 | 0.96722 | 0.90174 | 1.60838 | 1.27085 |
| 1993 | 79087 | 1.01430 | 0.99090 | 0.93229 | 1.57435 | 1.42485 |
| 1994 | 85765 | 1.02151 | 1.04312 | 0.98378 | 1.55547 | 1.86533 |
| 1995 | 90194 | 1.02354 | 1.10458 | 1.00791 | 1.57450 | 1.98881 |
| 1996 | 96299 | 1.01856 | 1.08863 | 0.97501 | 1.56171 | 2.35346 |
| 1997 | 101091 | 1.01117 | 1.07376 | 0.96418 | 1.58283 | 2.42622 |
| 1998 | 102186 | 1.00805 | 1.05617 | 0.98463 | 1.60678 | 2.13899 |
| 1999 | 107510 | 1.00751 | 1.04921 | 0.96342 | 1.62220 | 2.21304 |
| 2000 | 118854 | 1.00650 | 1.08579 | 0.95713 | 1.65982 | 2.64579 |
| 2001 | 117631 | 1.00294 | 1.06841 | 0.95916 | 1.65802 | 2.38580 |
| 2002 | 120115 | 1.00696 | 1.03562 | 0.95291 | 1.66081 | 2.38178 |
| 2003 | 120851 | 1.00554 | 0.99757 | 0.86983 | 1.64938 | 2.31713 |
| 2004 | 131065 | 1.01018 | 1.00552 | 0.83625 | 1.66695 | 2.71608 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 2005 | 138478 | 1.01836 | 1.01305 | 0.80973 | 1.70065 | 2.93800 |
| 2006 | 143589 | 1.02731 | 0.99703 | 0.78973 | 1.73700 | 2.94318 |

Note that from Table 8, the starting level of net real income in 1961, $23,689 million, is less than the corresponding starting level of gross real income in 1961 from Table 3, which was $27,683 million. This makes sense since we now subtract depreciation from the previous estimates of gross income. Net real income generated by the Canadian business sector grew 6.06 fold over the period 1961-2006, which is 2.4 percent greater than the 5.92 fold growth of gross real income. The real price of waiting capital services grew 2.94 fold, which is more rapid than the previous 1.78 fold increase in the real price of gross capital services from Table 3. This difference is due to the fact that depreciation gave the price of machinery and equipment (which decreases in real terms) a larger role in the price of gross capital services but when depreciation is regarded as an intermediate input, the price of land (which increases in real terms) gets a much bigger weight in the price of waiting capital services.

The same translog contributions methodology explained in Appendix 1 can be applied to the net output model used in the present section. Thus equation (3) in the previous section is applicable to our new measure of real income generated by the Canadian business sector and Table 9 below is the net income counterpart to Table 4 in the previous section.

**Table 9: Business Sector Year to Year Growth in Net Real Income and Net Year to Year Contribution Factors**

| Year t | $\rho^t/\rho^{t-1}$ | $\tau^t$ | $\alpha_D{}^t$ | $\alpha_X{}^t$ | $\alpha_M{}^t$ | $\beta_L{}^t$ | $\beta_K{}^t$ | $\alpha_{XM}{}^t$ |
|---|---|---|---|---|---|---|---|---|
| 1962 | 1.08774 | 1.05641 | 0.99942 | 1.01071 | 0.98384 | 1.03388 | 1.00213 | 0.99438 |
| 1963 | 1.05493 | 1.03418 | 0.99852 | 0.99777 | 0.99841 | 1.02103 | 1.00437 | 0.99619 |
| 1964 | 1.10737 | 1.05672 | 1.00290 | 1.00554 | 1.00040 | 1.03414 | 1.00443 | 1.00594 |
| 1965 | 1.08368 | 1.02891 | 1.00505 | 1.00150 | 1.00533 | 1.03390 | 1.00670 | 1.00684 |
| 1966 | 1.07871 | 1.02480 | 0.99916 | 0.99698 | 1.00622 | 1.04056 | 1.00921 | 1.00319 |
| 1967 | 1.01498 | 0.98714 | 1.00061 | 0.99517 | 1.00614 | 1.01600 | 1.01010 | 1.00128 |
| 1968 | 1.04952 | 1.04041 | 0.99907 | 0.99338 | 1.00636 | 1.00336 | 1.00662 | 0.99970 |
| 1969 | 1.05601 | 1.03268 | 1.00068 | 0.99523 | 1.00188 | 1.01886 | 1.00590 | 0.99710 |
| 1970 | 1.04714 | 1.03127 | 1.00241 | 1.00242 | 1.00058 | 1.00270 | 1.00720 | 1.00300 |
| 1971 | 1.05451 | 1.02789 | 1.01197 | 0.99261 | 0.99875 | 1.01676 | 1.00571 | 0.99137 |
| 1972 | 1.05736 | 1.01568 | 1.00551 | 0.99954 | 1.00689 | 1.02345 | 1.00515 | 1.00643 |
| 1973 | 1.12938 | 1.03426 | 1.01192 | 1.02558 | 1.00084 | 1.04608 | 1.00500 | 1.02644 |
| 1974 | 1.04562 | 0.98755 | 1.00497 | 1.04484 | 0.97455 | 1.02699 | 1.00748 | 1.01825 |
| 1975 | 0.99702 | 1.00421 | 0.98776 | 0.99668 | 0.99960 | 0.99945 | 1.00944 | 0.99628 |
| 1976 | 1.10171 | 1.06469 | 1.00910 | 1.00834 | 1.00856 | 0.99962 | 1.00870 | 1.01698 |
| 1977 | 1.04982 | 1.05144 | 0.99691 | 1.01084 | 0.97424 | 1.00774 | 1.00921 | 0.98480 |
| 1978 | 1.01887 | 0.99678 | 0.99691 | 1.00575 | 0.97873 | 1.03182 | 1.00949 | 0.98436 |
| 1979 | 1.05043 | 0.98309 | 1.00032 | 1.03150 | 0.98482 | 1.04301 | 1.00814 | 1.01584 |
| 1980 | 1.00454 | 0.96972 | 0.99694 | 1.01812 | 0.98540 | 1.02504 | 1.01041 | 1.00326 |
| 1981 | 1.01684 | 1.01113 | 1.00594 | 0.99279 | 0.97957 | 1.02030 | 1.00753 | 0.97251 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1982 | 0.92235 | 0.96932 | 0.99353 | 0.97005 | 1.01722 | 0.96195 | 1.00898 | 0.98676 |
| 1983 | 1.03658 | 1.02551 | 0.99578 | 0.97553 | 1.03249 | 1.00531 | 1.00248 | 1.00722 |
| 1984 | 1.07008 | 1.04390 | 0.99800 | 0.99642 | 0.99977 | 1.02736 | 1.00360 | 0.99619 |
| 1985 | 1.05125 | 1.01483 | 1.00176 | 0.99543 | 1.00265 | 1.02960 | 1.00628 | 0.99807 |
| 1986 | 1.01805 | 0.98492 | 1.00441 | 0.98901 | 1.00183 | 1.03188 | 1.00655 | 0.99082 |
| 1987 | 1.08048 | 1.01260 | 1.00675 | 0.99794 | 1.01919 | 1.03613 | 1.00573 | 1.01709 |
| 1988 | 1.06026 | 1.00361 | 1.00175 | 0.98898 | 1.02450 | 1.03367 | 1.00693 | 1.01322 |
| 1989 | 1.03087 | 0.99063 | 1.00171 | 0.99652 | 1.01399 | 1.01977 | 1.00817 | 1.01046 |
| 1990 | 0.95046 | 0.96455 | 0.98592 | 0.97089 | 1.02060 | 1.00085 | 1.00781 | 0.99088 |
| 1991 | 0.92669 | 0.96255 | 0.98987 | 0.95763 | 1.03738 | 0.97496 | 1.00419 | 0.99342 |
| 1992 | 0.98683 | 1.00102 | 0.99909 | 1.00751 | 0.98498 | 0.99181 | 1.00252 | 0.99237 |
| 1993 | 1.02275 | 1.01121 | 0.99954 | 1.01355 | 0.98068 | 1.01746 | 1.00055 | 0.99397 |
| 1994 | 1.08443 | 1.04638 | 1.00719 | 1.03199 | 0.96693 | 1.03000 | 1.00113 | 0.99786 |
| 1995 | 1.05165 | 1.00045 | 1.00196 | 1.03912 | 0.98434 | 1.02178 | 1.00382 | 1.02285 |
| 1996 | 1.06768 | 1.03277 | 0.99536 | 0.99000 | 1.02165 | 1.02172 | 1.00505 | 1.01143 |
| 1997 | 1.04976 | 1.02786 | 0.99299 | 0.99058 | 1.00733 | 1.02291 | 1.00767 | 0.99784 |
| 1998 | 1.01083 | 1.00610 | 0.99697 | 0.98825 | 0.98543 | 1.02309 | 1.01145 | 0.97385 |
| 1999 | 1.05211 | 1.00580 | 0.99948 | 0.99502 | 1.01582 | 1.02612 | 1.00908 | 1.01077 |
| 2000 | 1.10551 | 1.03667 | 0.99906 | 1.02703 | 1.00468 | 1.02623 | 1.00803 | 1.03183 |
| 2001 | 0.98971 | 0.98688 | 0.99676 | 0.98762 | 0.99854 | 1.00996 | 1.01016 | 0.98618 |
| 2002 | 1.02112 | 1.01596 | 1.00371 | 0.97710 | 1.00439 | 1.01474 | 1.00553 | 0.98139 |
| 2003 | 1.00613 | 0.95696 | 0.99867 | 0.97396 | 1.06092 | 1.01377 | 1.00503 | 1.03329 |
| 2004 | 1.08451 | 1.01913 | 1.00436 | 1.00535 | 1.02460 | 1.02504 | 1.00346 | 1.03008 |
| 2005 | 1.05656 | 1.00607 | 1.00765 | 1.00491 | 1.01954 | 1.00986 | 1.00733 | 1.02454 |
| 2006 | 1.03690 | 1.00197 | 1.00844 | 0.98996 | 1.01492 | 1.01163 | 1.00963 | 1.00473 |
| A62-06 | 1.0418 | 1.0126 | 1.0006 | 0.99968 | 1.0032 | 1.0185 | 1.0065 | 1.0027 |
| A62-73 | 1.0684 | 1.0309 | 1.0031 | 1.0014 | 1.0013 | 1.0242 | 1.0060 | 1.0027 |
| A74-91 | 1.0240 | 1.0023 | 0.99880 | 0.99707 | 1.0031 | 1.0153 | 1.0073 | 0.99980 |
| A92-99 | 1.0408 | 1.0164 | 0.99907 | 1.0070 | 0.99340 | 1.0194 | 1.0052 | 1.0001 |
| A00-06 | 1.0429 | 1.0034 | 0.0027 | 0.99513 | 1.0182 | 1.0159 | 1.0070 | 1.0131 |

The net real income generated by the Canadian business sector grew at an annual rate of 4.18 percent on average over the 46 year period 1961-2006, which is a bit more than the gross real income growth rate of 4.10 percent. Real domestic output prices averaged a tiny positive contribution to the growth in real net income of 0.06 per year and falling real export prices made a tiny negative contribution of −0.03 per year. Positive average contributions to the growth of real net income were due to productivity improvements (1.26 per year compared to 1.14 in the gross income framework), growth of labour input (1.85 per year compared to the previous gross income 1.60), growth of capital input (0.65 per year compared to the previous 1.11) and falls in real import prices (0.32 per year compared to the previous 0.28). Comparing these average contribution growth rates in the gross and net real income frameworks leads to the following important observations:

- The role of productivity improvements is *magnified* in the net income framework compared to the gross income framework;[22]
- The role of increases in labour input is also *magnified* in the net income framework;
- The role of increases in capital input (capital deepening) is greatly *diminished* in the net income framework and
- The role of falling real import prices is also *magnified* in the net income framework.

During the naughts, the average contribution factor for changes in real export and import prices together was 1.14 % per year in the gross framework and 1.31 % per year in the net framework. The corresponding contribution factor for capital growth during the naughts was 1.19% in the gross framework and 0.70 % in the net framework. Looking at the contribution of falling import prices alone in the net income framework, during the entire sample period, falling import prices contributed about 0.32 percentage points per year to real income growth whereas the effects of net capital accumulation contributed about 0.65 percentage points per year. During the years of the present decade, falling import prices contributed a very large 1.82 percentage points per year to real income growth whereas the effects of net capital accumulation contributed only 0.70 percentage points per year and TFP improvements contributed only 0.34 percentage points per year. Thus for short periods, changes in the real export or import prices that a country faces can have substantially larger effects on living standards than the effects of (net) capital accumulation or improvements in TFP.[23]

The average annual rate of TFP growth in the net income framework was a satisfactory 1.26% per year. As usual, there are considerable variations in this average rate over different periods. During the golden years, 1962-1973, TFP growth averaged a spectacular 3.09% per year. During the dismal years 1974-1991, TFP growth averaged only 0.23% per year but over the period, 1992-1999, TFP growth recovered to average a respectable 1.64% per year. However, during the years 2000-2006, net TFP growth fell to 0.34% per year, but all of this decline is explained by two poor performance years (2001 and 2003).

As in the previous section, the year over year growth factors listed in Table 9 above can be cumulated and the decomposition given by equation (5) in the previous section will apply to our new net data. The cumulated variables that appear in the new version of equation (5) are reported below in Table 10.

---

[22] This phenomenon is reasonably well known and is explained in Schreyer (2001): as the input denominator in a total factor productivity measure shrinks (by treating inputs as negative outputs and placing them in the net output numerator), the resulting measure of TFP will increase. This magnification effect shows up most clearly during periods of large productivity growth; i.e., during the period 1962-1973, the average net TFP growth rate was 3.09% per year compared to the average gross rate of 2.73% and during the period 1992-1999, the average net TFP growth rate was 1.64% per year compared to the average gross rate of 1.46 %.

[23] However, improvements in a country's terms of trade are unlikely to be sustainable over longer periods. In the long run, improvements in TFP are likely to be the most important factor explaining rising living standards.

**Table 10: Business Sector Cumulated Growth in Net Real Income and Cumulated Contribution Factors**

| Year t | $\rho^t/\rho^{1961}$ | $T^t$ | $A_D{}^t$ | $A_X{}^t$ | $A_M{}^t$ | $B_L{}^t$ | $B_K{}^t$ | $A_{XM}{}^t$ |
|---|---|---|---|---|---|---|---|---|
| 1961 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 1962 | 1.08774 | 1.05641 | 0.99942 | 1.01071 | 0.98384 | 1.03388 | 1.00213 | 0.99438 |
| 1963 | 1.14749 | 1.09252 | 0.99794 | 1.00846 | 0.98228 | 1.05562 | 1.00651 | 0.99058 |
| 1964 | 1.27069 | 1.15449 | 1.00083 | 1.01404 | 0.98267 | 1.09166 | 1.01097 | 0.99647 |
| 1965 | 1.37703 | 1.18787 | 1.00588 | 1.01556 | 0.98791 | 1.12866 | 1.01775 | 1.00328 |
| 1966 | 1.48542 | 1.21733 | 1.00504 | 1.01249 | 0.99406 | 1.17444 | 1.02712 | 1.00647 |
| 1967 | 1.50767 | 1.20168 | 1.00566 | 1.0076 | 1.00016 | 1.19323 | 1.03749 | 1.00776 |
| 1968 | 1.58233 | 1.25024 | 1.00472 | 1.00093 | 1.00653 | 1.19724 | 1.04436 | 1.00746 |
| 1969 | 1.67096 | 1.29110 | 1.00540 | 0.99615 | 1.00842 | 1.21982 | 1.05052 | 1.00454 |
| 1970 | 1.74973 | 1.33147 | 1.00783 | 0.99856 | 1.00901 | 1.22310 | 1.05808 | 1.00756 |
| 1971 | 1.84511 | 1.36861 | 1.01990 | 0.99118 | 1.00775 | 1.24361 | 1.06413 | 0.99886 |
| 1972 | 1.95093 | 1.39007 | 1.02551 | 0.99073 | 1.01469 | 1.27277 | 1.06961 | 1.00529 |
| 1973 | 2.20335 | 1.43770 | 1.03773 | 1.01607 | 1.01555 | 1.33142 | 1.07496 | 1.03187 |
| 1974 | 2.30386 | 1.41980 | 1.04289 | 1.06164 | 0.98970 | 1.36736 | 1.08300 | 1.05070 |
| 1975 | 2.29699 | 1.42578 | 1.03013 | 1.05812 | 0.98930 | 1.36661 | 1.09322 | 1.04679 |
| 1976 | 2.53062 | 1.51802 | 1.03951 | 1.06695 | 0.99777 | 1.36609 | 1.10273 | 1.06457 |
| 1977 | 2.65670 | 1.59611 | 1.03629 | 1.07851 | 0.97206 | 1.37666 | 1.11289 | 1.04838 |
| 1978 | 2.70683 | 1.59097 | 1.03310 | 1.08472 | 0.95139 | 1.42046 | 1.12345 | 1.03199 |
| 1979 | 2.84335 | 1.56407 | 1.03343 | 1.11888 | 0.93695 | 1.48156 | 1.13259 | 1.04834 |
| 1980 | 2.85625 | 1.51670 | 1.03027 | 1.13916 | 0.92328 | 1.51866 | 1.14438 | 1.05176 |
| 1981 | 2.90436 | 1.53358 | 1.03638 | 1.13095 | 0.90441 | 1.54948 | 1.15299 | 1.02284 |
| 1982 | 2.67883 | 1.48654 | 1.02968 | 1.09708 | 0.91999 | 1.49053 | 1.16335 | 1.00929 |
| 1983 | 2.77682 | 1.52446 | 1.02533 | 1.07023 | 0.94987 | 1.49843 | 1.16623 | 1.01659 |
| 1984 | 2.97142 | 1.59139 | 1.02328 | 1.06640 | 0.94965 | 1.53943 | 1.17043 | 1.01271 |
| 1985 | 3.12371 | 1.61499 | 1.02508 | 1.06152 | 0.95217 | 1.58501 | 1.17778 | 1.01075 |
| 1986 | 3.18010 | 1.59064 | 1.02961 | 1.04986 | 0.95392 | 1.63553 | 1.18550 | 1.00148 |
| 1987 | 3.43604 | 1.61068 | 1.03656 | 1.04770 | 0.97222 | 1.69461 | 1.19229 | 1.01859 |
| 1988 | 3.64308 | 1.61649 | 1.03838 | 1.03616 | 0.99604 | 1.75168 | 1.20055 | 1.03206 |
| 1989 | 3.75554 | 1.60135 | 1.04015 | 1.03255 | 1.00998 | 1.78630 | 1.21036 | 1.04285 |
| 1990 | 3.56950 | 1.54458 | 1.02550 | 1.00249 | 1.03078 | 1.78782 | 1.21981 | 1.03334 |
| 1991 | 3.30782 | 1.48673 | 1.01511 | 0.96001 | 1.06930 | 1.74305 | 1.22493 | 1.02654 |
| 1992 | 3.26426 | 1.48824 | 1.01419 | 0.96722 | 1.05324 | 1.72877 | 1.22801 | 1.01871 |
| 1993 | 3.33854 | 1.50492 | 1.01373 | 0.98032 | 1.03289 | 1.75897 | 1.22869 | 1.01257 |
| 1994 | 3.62041 | 1.57472 | 1.02102 | 1.01169 | 0.99873 | 1.81174 | 1.23008 | 1.01040 |
| 1995 | 3.80740 | 1.57543 | 1.02302 | 1.05127 | 0.98309 | 1.85119 | 1.23478 | 1.03349 |
| 1996 | 4.06510 | 1.62706 | 1.01827 | 1.04075 | 1.00438 | 1.89140 | 1.24102 | 1.04530 |
| 1997 | 4.26739 | 1.67239 | 1.01112 | 1.03094 | 1.01174 | 1.93474 | 1.25053 | 1.04304 |
| 1998 | 4.31359 | 1.68259 | 1.00806 | 1.01883 | 0.99700 | 1.97942 | 1.26485 | 1.01577 |
| 1999 | 4.53836 | 1.69235 | 1.00754 | 1.01375 | 1.01278 | 2.03112 | 1.27634 | 1.02671 |
| 2000 | 5.01721 | 1.75440 | 1.00659 | 1.04116 | 1.01751 | 2.08441 | 1.28659 | 1.05939 |
| 2001 | 4.96557 | 1.73138 | 1.00333 | 1.02827 | 1.01603 | 2.10517 | 1.29967 | 1.04475 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2002 | 5.07042 | 1.75902 | 1.00706 | 1.00471 | 1.02049 | 2.13620 | 1.30686 | 1.02530 |
| 2003 | 5.10153 | 1.68331 | 1.00572 | 0.97855 | 1.08266 | 2.16562 | 1.31342 | 1.05944 |
| 2004 | 5.53267 | 1.71551 | 1.01011 | 0.98379 | 1.10929 | 2.21984 | 1.31797 | 1.09131 |
| 2005 | 5.84562 | 1.72592 | 1.01783 | 0.98862 | 1.13096 | 2.24173 | 1.32763 | 1.11809 |
| 2006 | 6.06134 | 1.72932 | 1.02642 | 0.97869 | 1.14784 | 2.26780 | 1.34041 | 1.12338 |

The net real income generated by the business sector grew 6.06 fold over the years 1961-2006. The main factors explaining this growth are growth of labour input (cumulative growth factor 2.27), productivity increases (cumulative growth factor 1.73), growth of waiting capital services (cumulative growth factor 1.34), lower real import prices (cumulative growth factor 1.15)[24] and higher real domestic output prices (cumulative growth factor 1.03). There was a small negative contribution from declining real export prices (cumulative growth factor .98).

**5. Conclusion**

There are four major conclusions that we can draw from the above results.

First, using new data from Statistics Canada, we have shown that the productivity performance of the business sector of the Canadian economy has been reasonably satisfactory over the past 46 years. In particular, traditional gross income Total Factor Productivity growth averaged 1.14 percent per year over the period 1962-2006[25] and when the net income framework was used, TFP growth averaged 1.26 percent per year. However, there was a long period (1974-1991) where the productivity performance of the Canadian business sector was decidedly unsatisfactory.

Second, we have shown that the role of explanatory factors for growth in the real income generated by the business sector of the Canadian economy changes substantially when we shift from the standard gross product growth accounting model to a theoretically more appropriate net product growth accounting framework. In general, the main positive drivers of real income growth (growth in labour input, TFP growth and declining real import prices) are *magnified* but the effects of capital services input growth are greatly *diminished* when we switch to the net framework as compared to the gross product framework.[26] An important implication of this result is that improvements in TFP probably become the most important factor for explaining improvements in per capita living standards in the long run and the favourable effects of capital deepening are not as big as they appear to be in the traditional gross income growth accounting methodology.

Third, the results presented here show that over short periods of time, changes in the external price environment facing an economy can have substantial effects on living

---

[24] Note that most of this growth took place over the years 2001-2006.
[25] The corresponding Statistics Canada average Multifactor Productivity growth rate over this period was only 0.43 percent per year. We attempt to explain this puzzling discrepancy in Appendix 4 below but without complete success.
[26] Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006) found similar results for Japan and Australia using a similar net income framework.

standards. Thus during the years of the present decade, the real (net) income generated by the Canadian business sector grew at an average rate of 4.29 percent per year and declines in real import prices (the China effect) contributed 1.82 percentage points to this increase, which was greater than the effects of quality adjusted labour input growth (1.59 percentage points per year), increases in waiting services (0.70 percentage points per year).[27]

Finally, the study uncovered many data problems which should be addressed in future work on Canadian productivity performance. A discussion of these data problems can be found in section 6 of Appendix 2 below. More generally, it is evident that statistical agencies are able to provide reasonably accurate data on the prices and quantities of the outputs produced and intermediate inputs used by the various industries in the economy. This is in large part due to the fact that the *System of National Accounts 1993* used by most statistical agencies has developed an adequate methodology for the treatment of gross outputs and intermediate inputs. However, the corresponding methodology for the treatment of primary inputs was not well developed.[28] In particular, the treatment of capital services was absent the *System of National Accounts 1993* and will only be introduced in the next international version of the System of National Accounts. This absence of a standard methodology for the treatment of capital services means that national statistical agencies have not been able to deliver a generally accepted treatment of capital services in their productivity accounts. Thus detailed data on capital stocks and flows by industry is either not available from national statistical agencies or is not provided due to the lack of information on capital inputs. Given the importance of accurate information on productivity growth, it is important that international agencies provide guidance on acceptable methods for measuring primary input prices and volumes and that national statistical agencies provide more details on how they construct their estimates of primary inputs in their productivity accounts. National departments that have an interest in better productivity measurement (e.g., central banks, departments of finance and industry departments) should support initiatives that will improve the measurement of primary input growth.

## Appendix 1: Explaining Real Income Growth: The Translog Approach

### 1. The Production Theory Framework

In this section, we present the production theory framework which will be used in the main text of the paper.[29] The main reference is Diewert and Morrison (1986)[30] but we

[27] The Canadian experience with improvements in the terms of trade during the past decade is similar to the Australian experience; see Diewert and Lawrence (2006).

[28] The *System of National Accounts 1993* has a good chapter on wage indexes but does not provide a standard methodology for the treatment of self employment labour input. The recent preliminary Manual on the measurement of capital by Schreyer (2007) fills in an important methodological gap in the existing SNA.

[29] With the exception of the last section of this Appendix, this material is drawn from Diewert (2005b), Diewert, Mizobuchi and Nomura (2005) and Diewert and Lawrence (2006).

also draw on the theory of the output price index, which was developed by Fisher and Shell (1972) and Archibald (1977). This theory is the producer theory counterpart to the theory of the cost of living index for a single consumer (or household) that was first developed by the Russian economist, A. A. Konüs (1924). These economic approaches to price indexes rely on the assumption of (competitive) *optimizing behavior* on the part of economic agents (consumers or producers). Thus we consider only the market sector of the economy in what follows; i.e., that part of the economy that is motivated by profit maximizing behavior. In our empirical work, we define the market sector to be the Canadian business sector of the economy less the rental and owner occupied housing sectors.[31]

Initially, we assume that the market sector of the economy produces quantities of M (net)[32] outputs, $y \equiv [y_1,...,y_M]$, which are sold at the positive producer prices $P \equiv [P_1,...,P_M]$. We further assume that the market sector of the economy uses positive quantities of N primary inputs, $x \equiv [x_1,...,x_N]$ which are purchased at the positive primary input prices $W \equiv [W_1,...,W_N]$. In period t, we assume that there is a feasible set of output vectors y that can be produced by the market sector if the vector of primary inputs x is utilized by the market sector of the economy; denote this period t production possibilities set by $S^t$. We assume that $S^t$ is a closed convex cone that exhibits a free disposal property.[33]

[30] The theory also draws on Samuelson (1953), Diewert (1974; 133-141) (1980) (1983; 1077-1100), Fox and Kohli (1998), Kohli (1978) (1990) (1991) (2003) (2004a) (2004b) (2006) (2007), Morrison and Diewert (1990), Samuelson (1953) and Sato (1976).

[31] The Canadian business sector excludes all of the general government sectors such as schools, hospitals, universities, defence and public administration where no independent measures of output can be obtained. For owner occupied housing, output is equal to input and hence no productivity improvements can be generated by this sector according to SNA conventions. Due to the difficulties involved in splitting up the residential housing stock into the rental and owner occupied portions, we omit the entire residential housing stock and the consumption of residential housing services in our data. However, we do include investment in residential housing, since that investment is part of the output of the market production sector.

[32] If the mth commodity is an import (or other produced input) into the market sector of the economy, then the corresponding quantity $y_m$ is indexed with a negative sign. We will follow Kohli (1978) (1991) and Woodland (1982) in assuming that imports flow through the domestic production sector and are "transformed" (perhaps only by adding transportation, wholesaling and retailing margins) by the domestic production sector. The recent textbook by Feenstra (2004; 76) also uses this approach.

[33] For a more explanation for the meaning of these properties, see Diewert (1973) (1974; 134) or Woodland (1982) or Kohli (1978) (1991). The assumption that $S^t$ is a cone means that the technology is subject to constant returns to scale. This is an important assumption since it implies that the value of outputs should equal the value of inputs in equilibrium. In our empirical work, we use an ex post rate of return in our user costs of capital, which forces the value of inputs to equal the value of outputs for each period. The function $g^t$ is known as the *GDP function* or the *national product function* in the international trade literature (see Kohli (1978)(1991), Woodland (1982) and Feenstra (2004; 76). It was introduced into the economics literature by Samuelson (1953). Alternative terms for this function include: (i) the *gross profit function*; see Gorman (1968); (ii) the *restricted profit function*; see Lau (1976) and McFadden (1978); and (iii) the *variable profit function*; see Diewert (1973) (1974).

Given a vector of output prices P and a vector of available primary inputs x, we define *the period t market sector GDP function*, $g^t(P,x)$, as follows:[34]

$$(1)\ g^t(P,x) \equiv \max_y \{P\cdot y : (y,x) \text{ belongs to } S^t\}\ ; \qquad t = 0,1,2, \ldots .$$

Thus market sector GDP depends on t (which represents the period t technology set $S^t$), on the vector of output prices P that the market sector faces and on x, the vector of primary inputs that is available to the market sector.

If $P^t$ is the period t output price vector and $x^t$ is the vector of inputs used by the market sector during period t and if the GDP function is differentiable with respect to the components of P at the point $P^t,x^t$, then the period t vector of market sector outputs $y^t$ will be equal to the vector of first order partial derivatives of $g^t(P^t,x^t)$ with respect to the components of P; i.e., we will have the following equations for each period t:[35]

$$(2)\ y^t = \nabla_P\, g^t(P^t,x^t)\ ; \qquad t = 0,1,2, \ldots .$$

Thus the period t market sector supply vector $y^t$ can be obtained by differentiating the period t market sector GDP function with respect to the components of the period t output price vector $P^t$.

If the GDP function is differentiable with respect to the components of x at the point $P^t,x^t$, then the period t vector of input prices $W^t$ will be equal to the vector of first order partial derivatives of $g^t(P^t,x^t)$ with respect to the components of x; i.e., we will have the following equations for each period t:[36]

$$(3)\ W^t = \nabla_x\, g^t(P^t,x^t)\ ; \qquad t = 0,1,2, \ldots .$$

Thus the period t market sector input prices $W^t$ paid to primary inputs can be obtained by differentiating the period t market sector GDP function with respect to the components of the period t input quantity vector $x^t$.

The constant returns to scale assumption on the technology sets $S^t$ implies that the value of outputs will equal the value of inputs in period t; i.e., we have the following relationships:

$$(4)\ g^t(P^t,x^t) = P^t\cdot y^t = W^t\cdot x^t\ ; \qquad t = 0,1,2, \ldots .$$

[34] The function $g^t(P,x)$ will be linearly homogeneous and convex in the components of P and linearly homogeneous and concave in the components of x; see Diewert (1973) (1974; 136). Notation: $P\cdot y \equiv \sum_{m=1}^M P_m y_m$.

[35] These relationships are due to Hotelling (1932; 594). Note that $\nabla_P\, g^t(P^t,x^t) \equiv [\partial g^t(P^t,x^t)/\partial P_1, \ldots, \partial g^t(P^t,x^t)/\partial P_M]$.

[36] These relationships are due to Samuelson (1953) and Diewert (1974; 140). Note that $\nabla x\, g^t(P^t,x^t) \equiv [\partial g^t(P^t,x^t)/\partial x_1, \ldots, \partial g^t(P^t,x^t)/\partial x_N]$.

The above material will be useful in what follows but of course, our focus is not on GDP; instead our focus is on the income generated by the market sector or more precisely, on *the real income generated by the market sector*. However, since market sector GDP (the value of market sector production) is distributed to the factors of production used by the market sector, nominal market sector GDP will be equal to nominal market sector income; i.e., from (4), we have $g^t(P^t,x^t) = P^t\cdot y^t = W^t\cdot x^t$. As an approximate welfare measure that can be associated with market sector production,[37] we will choose to measure the *real income generated by the market sector in period t*, $r^t$, in terms of the number of consumption bundles that the nominal income could purchase in period t; i.e., define $\rho^t$ as follows:

$$(5)\ \rho^t \equiv W^t\cdot x^t/P_C^{\,t}\,; \qquad t = 0,1,2, ...$$
$$= w^t\cdot x^t$$
$$= p^t\cdot y^t$$
$$= g^t(p^t,x^t)$$

where $P_C^{\,t} > 0$ is the *period t consumption expenditures deflator* and the market sector period t *real output price* $p^t$ and *real input price* $w^t$ vectors are defined as the corresponding nominal price vectors deflated by the consumption expenditures price index; i.e., we have the following definitions:[38]

$$(6)\ p^t \equiv P^t/P_C^{\,t}\,;\ w^t \equiv W^t/P_C^{\,t}\,; \qquad t = 0,1,2, ...\ .$$

The first and last equality in (5) imply that period t real income, $\rho^t$, is equal to the period t GDP function, evaluated at the period t real output price vector $p^t$ and the period t input vector $x^t$, $g^t(p^t,x^t)$. Thus *the growth in real income over time can be explained by three main factors: t (Technical Progress or Total Factor Productivity growth), growth in real output prices and the growth of primary inputs*. We will shortly give formal definitions for these three growth factors.

Using the linear homogeneity properties of the GDP functions $g^t(P,x)$ in P and x separately, we can show that the following counterparts to the relations (2) and (3) hold using the deflated prices p and w:[39]

---

[37] Since some of the primary inputs used by the market sector can be owned by foreigners, our measure of *domestic* welfare generated by the market production sector is only an approximate one. Moreover, our suggested welfare measure is not sensitive to the distribution of the income that is generated by the market sector.

[38] Our approach is similar to the approach advocated by Kohli (2004b; 92), except he essentially deflates nominal GDP by the domestic expenditures deflator rather than just the domestic (household) expenditures deflator; i.e., he deflates by the deflator for C+G+I, whereas we suggest deflating by the deflator for C. Another difference in his approach compared to the present approach is that we restrict our analysis to the market sector GDP, whereas Kohli deflates all of GDP (probably due to data limitations). Our treatment of the balance of trade surplus or deficit is also different.

[39] If producers in the market sector of the economy are solving the profit maximization problem that is associated with $g^t(P,x)$, which uses the original output prices P, then they will also solve the profit maximization problem that uses the normalized output prices $p \equiv P/P_C$; i.e., they will also solve the problem defined by $g^t(p,x)$.

(7) $y^t = \nabla_p g^t(p^t,x^t)$ ; $t = 0,1,2, \ldots$

(8) $w^t = \nabla_x g^t(p^t,x^t)$ ; $t = 0,1,2, \ldots$ .

Now we are ready to define a family of *period t productivity growth factors or technical progress shift factors* $\tau(p,x,t)$:[40]

(9) $\tau(p,x,t) \equiv g^t(p,x)/g^{t-1}(p,x)$ ; $t = 1,2, \ldots$ .

Thus $\tau(p,x,t)$ measures the proportional change in the real income produced by the market sector at the reference real output prices p and reference input quantities used by the market sector x where the numerator in (9) uses the period t technology and the denominator in (9) uses the period t−1 technology. Thus each choice of reference vectors p and x will generate a possibly different measure of the shift in technology going from period t−1 to period t. Note that we are using the chain system to measure the shift in technology.

It is natural to choose special reference vectors for the measure of technical progress defined by (9): a *Laspeyres type measure* $\tau_L^t$ that chooses the period t−1 reference vectors $p^{t-1}$ and $x^{t-1}$ and a *Paasche type measure* $\tau_P^t$ that chooses the period t reference vectors $p^t$ and $x^t$:

(10) $\tau_L^t \equiv \tau(p^{t-1},x^{t-1},t) = g^t(p^{t-1},x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})$ ; $t = 1,2, \ldots$ ;

(11) $\tau_P^t \equiv \tau(p^t,x^t,t) \quad = g^t(p^t,x^t)/g^{t-1}(p^t,x^t)$ ; $t = 1,2, \ldots$ .

Since both measures of technical progress are equally valid, it is natural to average them to obtain an overall measure of technical change. If we want to treat the two measures in a symmetric manner and we want the measure to satisfy the time reversal property from index number theory[41] (so that the estimate going backwards is equal to the reciprocal of the estimate going forwards), then the geometric mean will be the best simple average to take.[42] Thus we define the geometric mean of (10) and (11) as follows:[43]

(12) $\tau^t \equiv [\tau_L^t \, \tau_P^t]^{1/2}$ ; $t = 1,2, \ldots$ .

At this point, it is not clear how we will obtain empirical estimates for the theoretical productivity growth indexes defined by (10)-(12). One obvious way would be to assume a functional form for the GDP function $g^t(p,x)$, collect data on output and input prices and quantities for the market sector for a number of years (and for the consumption

[40] This measure of technical progress is due to Diewert and Morrison (1986; 662). A special case of it was defined earlier by Diewert (1983; 1063).

[41] See Fisher (1922; 64).

[42] See the discussion in Diewert (1997) on choosing the "best" symmetric average of Laspeyres and Paasche indexes that will lead to the satisfaction of the time reversal test by the resulting average index.

[43] The theoretical productivity change indexes defined by (10)-(12) were first defined by Diewert and Morrison (1986; 662-663) in the nominal GDP context. See Diewert (1993) for properties of symmetric means.

expenditures deflator), add error terms to equations (7) and (8) and use econometric techniques to estimate the unknown parameters in the assumed functional form. However, econometric techniques are generally not completely straightforward: different econometricians will make different stochastic specifications and will choose different functional forms.[44] Moreover, as the number of outputs and inputs grows, it will be impossible to estimate a flexible functional form. Thus we will suggest methods for implementing measures like (12) in this Appendix that are based on exact index number techniques.

We turn now to the problem of defining theoretical indexes for the effects on real income due to changes in real output prices. Define a family of *period t real output price growth factors* $\alpha(p^{t-1},p^t,x,s)$:[45]

(13) $\alpha(p^{t-1},p^t,x,s) \equiv g^s(p^t,x)/g^s(p^{t-1},x)$ ; $s = 1,2, ...$ .

Thus $\alpha(p^{t-1},p^t,x,s)$ measures the proportional change in the real income produced by the market sector that is induced by the change in real output prices going from period t−1 to t, using the technology that is available during period s and using the reference input quantities x. Thus each choice of the reference technology s and the reference input vector x will generate a possibly different measure of the effect on real income of a change in real output prices going from period t−1 to period t.

Again, it is natural to choose special reference vectors for the measures defined by (13): a *Laspeyres type measure* $\alpha_L^t$ that chooses the period t−1 reference technology and reference input vector $x^{t-1}$ and a *Paasche type measure* $\alpha_P^t$ that chooses the period t reference technology and reference input vector $x^t$:

(14) $\alpha_L^t \equiv \alpha(p^{t-1},p^t,x^{t-1},t-1) = g^{t-1}(p^t,x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})$ ; $t = 1,2, ...$ ;
(15) $\alpha_P^t \equiv \alpha(p^{t-1},p^t,x^t,t) = g^t(p^t,x^t)/g^t(p^{t-1},x^t)$ ; $t = 1,2, ...$ .

Since both measures of real output price change are equally valid, it is natural to average them to obtain an overall measure of the effects on real income of the change in real output prices:[46]

(16) $\alpha^t \equiv [\alpha_L^t \alpha_P^t]^{1/2}$ ; $t = 1,2, ...$ .

[44] "The estimation of GDP functions such as (19) can be controversial, however, since it raises issues such as estimation technique and stochastic specification. ... We therefore prefer to opt for a more straightforward index number approach." Ulrich Kohli (2004a; 344).
[45] This measure of real output price change was essentially defined by Fisher and Shell (1972; 56-58), Samuelson and Swamy (1974; 588-592), Archibald (1977; 60-61), Diewert (1980; 460-461) (1983; 1055) and Balk (1998; 83-89). Readers who are familiar with the theory of the true cost of living index will note that the real output price index defined by (13) is analogous to the Konüs (1924) *true cost of living index* which is a ratio of cost functions, say $C(u,p^t)/C(u,p^{t-1})$ where u is a reference utility level: $g^s$ replaces C and the reference utility level u is replaced by the vector of reference variables x.
[46] The indexes defined by (13)-(16) were defined by Diewert and Morrison (1986; 664) in the nominal GDP function context.

Finally, we look at the problem of defining theoretical indexes for the effects on real income due to changes in input quantities. Define a family of *period t real input quantity growth factors* $\beta(x^{t-1},x^t,p,s)$:[47]

(17) $\beta(x^{t-1},x^t,p,s) \equiv g^s(p,x^t)/g^s(p,x^{t-1})$ ; $s = 1,2, \ldots$ .

Thus $\beta(x^{t-1},x^t,p,s)$ measures the proportional change in the real income produced by the market sector that is induced by the change in input quantities used by the market sector going from period t–1 to t, using the technology that is available during period s and using the reference real output prices p. Thus each choice of the reference technology s and the reference real output price vector p will generate a possibly different measure of the effect on real income of a change in input quantities going from period t−1 to period t.

Again, it is natural to choose special reference vectors for the measures defined by (17): a *Laspeyres type measure* $\beta_L{}^t$ that chooses the period t−1 reference technology and reference real output price vector $p^{t-1}$ and a *Paasche type measure* $\beta_P{}^t$ that chooses the period t reference technology and reference real output price vector $p^t$:

(18) $\beta_L{}^t \equiv \beta(x^{t-1},x^t,p^{t-1},t-1) = g^{t-1}(p^{t-1},x^t)/g^{t-1}(p^{t-1},x^{t-1})$ ; $t = 1,2, \ldots$ ;
(19) $\beta_P{}^t \equiv \beta(x^{t-1},x^t,p^t,t) = g^t(p^t,x^t)/g^t(p^t,x^{t-1})$ ; $t = 1,2, \ldots$ .

Since both measures of real input growth are equally valid, it is natural to average them to obtain an overall measure of the effects of input growth on real income:[48]

(20) $\beta^t \equiv [\beta_L{}^t \beta_P{}^t]^{1/2}$ ; $t = 1,2, \ldots$ .

Recall that market sector real income for period t was defined by (5) as $\rho^t$ equal to nominal period t factor payments $W^t \cdot x^t$ deflated by the household consumption price deflator $P_C{}^t$. It is convenient to define $\gamma^t$ as the *period t chain rate of growth factor for real income*:

(21) $\gamma^t \equiv \rho^t/\rho^{t-1}$ ; $t = 1,2, \ldots$ .

It turns out that the definitions for $\gamma^t$ and the technology, output price and input quantity growth factors $\tau(p,x,t)$, $\alpha(p^{t-1},p^t,x,s)$, $\beta(x^{t-1},x^t,p,s)$ defined by (9), (13) and (17) respectively satisfy some interesting identities, which we will now develop. We have:

(22) $\gamma^t \equiv \rho^t/\rho^{t-1}$ ; $t = 1,2, \ldots$ .
$= g^t(p^t,x^t)/g^{t-1}(p^{t-1},x^{t-1})$ using definitions (4) and (5)
$= [g^t(p^t,x^t)/g^{t-1}(p^t,x^t)][g^{t-1}(p^t,x^t)/g^{t-1}(p^{t-1},x^t)][g^{t-1}(p^{t-1},x^t)/g^{t-1}(p^{t-1},x^{t-1})]$
$= \tau_P{}^t\, \alpha(p^{t-1},p^t,x^t,t-1)\, \beta_L{}^t$ using definitions (11), (13) and (18).

[47] This type of index was defined as a true index of value added by Sato (1976; 438) and as a real input index by Diewert (1980; 456).
[48] The theoretical indexes defined by (17)-(20) were defined in Diewert and Morrison (1986; 665) in the nominal GDP context.

In a similar fashion, we can establish the following companion identity:

(23) $\gamma^t \equiv \tau_L{}^t\, \alpha(p^{t-1},p^t,x^{t-1},t)\, \beta_P{}^t$ using definitions (10), (13) and (19).

Thus multiplying (22) and (23) together and taking positive square roots of both sides of the resulting identity and using definitions (12) and (20), we obtain the following identity:

(24) $\gamma^t \equiv \tau^t\, [\alpha(p^{t-1},p^t,x^t,t-1)\alpha(p^{t-1},p^t,x^{t-1},t)]^{1/2}\, \beta^t$ ; $t = 1,2, \ldots$ .

In a similar fashion, we can derive the following alternative decomposition for $\gamma^t$ into growth factors:

(25) $\gamma^t \equiv \tau^t\, \alpha^t\, [\beta(x^{t-1},x^t,p^t,t-1)\beta(x^{t-1},x^t,p^{t-1},t)]^{1/2}$ ; $t = 1,2, \ldots$ .

It is quite likely that the real output price growth factor $[\alpha(p^{t-1},p^t,x^t,t-1)\alpha(p^{t-1},p^t,x^{t-1},t)]^{1/2}$ is fairly close to $\alpha^t$ defined by (16) and it is quite likely that the input growth factor $[\beta(x^{t-1},x^t,p^t,t-1)\beta(x^{t-1},x^t,p^{t-1},t)]^{1/2}$ is quite close to $\beta^t$ defined by (20); i.e., we have the following approximate equalities:

(26) $[\alpha(p^{t-1},p^t,x^t,t-1)\alpha(p^{t-1},p^t,x^{t-1},t)]^{1/2} \approx \alpha^t$ ; $t = 1,2, \ldots$ ;
(27) $[\beta(x^{t-1},x^t,p^t,t-1)\beta(x^{t-1},x^t,p^{t-1},t)]^{1/2} \approx \beta^t$ ; $t = 1,2, \ldots$ .

Substituting (26) and (27) into (24) and (25) respectively leads to the following approximate decompositions for the growth of real income into explanatory factors:

(28) $\gamma^t \approx \tau^t\, \alpha^t\, \beta^t$ ; $t = 1,2, \ldots$

where $\tau^t$ is a technology growth factor, $\alpha^t$ is a growth in real output prices factor and $\beta^t$ is a growth in primary inputs factor.

Rather than look at explanatory factors for the growth in real market sector income, it is sometimes convenient to express the level of real income in period t in terms of an *index of the technology level* or of Total Factor Productivity in period t, $T^t$, of the *level of real output prices* in period t, $A^t$, and of the *level of primary input quantities* in period t, $B^t$.[49] Thus we use the growth factors $\tau^t$, $\alpha^t$ and $\beta^t$ as follows to define the levels $T^t$, $A^t$ and $B^t$:

(29) $T^0 \equiv 1$ ; $T^t \equiv T^{t-1}\, \tau^t$ ; $t = 1,2, \ldots$ ;
(30) $A^0 \equiv 1$ ; $A^t \equiv A^{t-1}\alpha^t$ ; $t = 1,2, \ldots$ ;
(31) $B^0 \equiv 1$ ; $B^t \equiv B^{t-1}\beta^t$ ; $t = 1,2, \ldots$ .

---

[49] This type of levels presentation of the data is quite instructive when presented in graphical form. It was suggested by Kohli (1990) and used extensively by him; see Kohli (1991), (2003) (2004a) (2004b) and Fox and Kohli (1998).

Using the approximate equalities (28) for the chain links that appear in (29)-(31), we can establish the following approximate relationship for the level of real income in period t, $\rho^t$, and the period t levels for technology, real output prices and input quantities:

(32) $\rho^t/\rho^0 \approx T^t A^t B^t$ ; $t = 0,1,2, \ldots$ .

In the following section, we note a set of assumptions on the technology sets that will ensure that the approximate real income growth decompositions (28) and (32) hold as exact equalities.

**3. The Translog GDP Function Approach**

We now follow the example of Diewert and Morrison (1986; 663) and assume that the log of the period t (deflated) GDP function, $g^t(p,x)$, has the following translog functional form:[50]

(33) $\ln g^t(p,x) \equiv a_0^t + \sum_{m=1}^M a_m^t \ln p_m^t + (1/2) \sum_{m=1}^M \sum_{k=1}^M a_{mk} \ln p_m^t \ln p_k^t$
$+ \sum_{n=1}^N b_n^t \ln x_n^t + (1/2)\sum_{n=1}^N \sum_{j=1}^N b_{nj} \ln x_n^t \ln x_j^t + \sum_{m=1}^M \sum_{n=1}^M c_{mn} \ln p_m^t \ln x_n^t$ ;
$t = 0,1,2, \ldots$ .

Note that the coefficients for the quadratic terms are assumed to be constant over time. The coefficients must satisfy the following restrictions in order for $g^t$ to satisfy the linear homogeneity properties that we have assumed in section 2 above:[51]

(34) $\sum_{m=1}^M a_m^t = 1$ for $t = 0,1,2, \ldots$;
(35) $\sum_{n=1}^N b_n^t = 1$ for $t = 0,1,2, \ldots$;
(36) $a_{mk} = a_{km}$ for all k,m ;
(37) $b_{nj} = b_{jn}$ for all n,j ;
(38) $\sum_{k=1}^M a_{mk} = 0$ for $m = 1,\ldots,M$ ;
(39) $\sum_{j=1}^N b_{nj} = 0$ for $n = 1,\ldots,N$ ;
(40) $\sum_{n=1}^N c_{mn} = 0$ for $m = 1,\ldots,M$ ;
(41) $\sum_{m=1}^M c_{mn} = 0$ for $n = 1,\ldots,N$ .

Recall the approximate decomposition of real income growth going from period t−1 to t given by (28) above, $\gamma^t \approx \tau^t \alpha^t \beta^t$. Diewert and Morrison (1986; 663) showed that[52] if $g^{t-1}$ and $g^t$ are defined by (33)-(41) above and there is competitive profit maximizing behavior

[50] This functional form was first suggested by Diewert (1974; 139) as a generalization of the translog functional form introduced by Christensen, Jorgenson and Lau (1971). Diewert (1974; 139) indicated that this functional form was flexible.
[51] There are additional restrictions on the parameters which are necessary to ensure that $g^t(p,x)$ is convex in p and concave in x. Note that when we divide the original prices by one of the prices, then one of the scaled prices will be identically equal to one and hence its logarithm will be identically equal to zero.
[52] Diewert and Morrison established their proof using the nominal GDP function $g^t(P,x)$. However, it is easy to rework their proof using the deflated GDP function $g^t(p,x)$ using the fact that $g^t(p,x) = g^t(P/P_C,x) = g^t(P,x)/P_C$ using the linear homogeneity property of $g^t(P,x)$ in P.

on the part of all market sector producers for all periods t, then (28) holds as an exact equality; i.e., we have

(42) $\gamma^t = \tau^t \alpha^t \beta^t$ ; $t = 1,2, \ldots$ .

In addition, Diewert and Morrison (1986; 663-665) showed that $\tau^t$, $\alpha^t$ and $\beta^t$ could be calculated using empirically observable price and quantity data for periods t−1 and t as follows:

(43) $\ln\alpha^t = \sum_{m=1}^{M} (1/2)[(p_m^{t-1}y_m^{t-1}/p^{t-1}\cdot y^{t-1}) + (p_m^t y_m^t/p^t\cdot y^t)] \ln(p_m^t/p_m^{t-1})$
$= \ln P_T(p^{t-1},p^t,y^{t-1},y^t)$;
(44) $\ln\beta^t = \sum_{n=1}^{N} (1/2)[(w_n^{t-1}x_n^{t-1}/w^{t-1}\cdot x^{t-1}) + (w_n^t x_n^t/w^t\cdot x^t)] \ln(x_n^t/x_n^{t-1})$
$= \ln Q_T(w^{t-1},w^t,x^{t-1},x^t)$;
(45) $\tau^t = \gamma^t/\alpha^t \beta^t$

where $P_T(p^{t-1},p^t,y^{t-1},y^t)$ is the Törnqvist (1936) and Törnqvist and Törnqvist (1937) output price index and $Q_T(w^{t-1},w^t,x^{t-1},x^t)$ is the Törnqvist input quantity index.

Since equations (42) now hold as exact identities under our present assumptions, equations (32), the cumulated counterparts to equations (28), will also hold as exact decompositions; i.e., under our present assumptions, we have

(46) $\rho^t/\rho^0 = T^t A^t B^t$ ; $t = 1,2, \ldots$ .

We will implement the real income decompositions (42) and (46) in the main text.

## 4. The Translog GDP Function Approach and Changes in the Terms of Trade

For some purposes, it is convenient to decompose the aggregate period t contribution factor due to changes in all deflated output prices $\alpha^t$ into separate effects for each change in each output price. Similarly, it can sometimes be useful to decompose the aggregate period t contribution factor due to changes in all market sector primary input quantities $\beta^t$ into separate effects for each change in each input quantity. In this section, we indicate how this can be done, making the same assumptions on the technology that were made in the previous section.

We first model the effects of a change in a single (deflated) output price, say $p_m$, going from period t−1 to t. Counterparts to the theoretical Laspeyres and Paasche type price indexes defined by (14) and (15) above for changes in all (deflated) output prices are the following *Laspeyres type measure* $\alpha_{Lm}^t$ that chooses the period t−1 reference technology and holds constant other output prices at their period t−1 levels and holds inputs constant at their period t−1 levels $x^{t-1}$ and a *Paasche type measure* $\alpha_{Pm}^t$ that chooses the period t reference technology and reference input vector $x^t$ and holds constant other output prices at their period t levels:

$$(47)\ \alpha_{Lm}^{t} \equiv g^{t-1}(p_1^{t-1},\ldots,p_{m-1}^{t-1},p_m^{t},p_{m+1}^{t-1},\ldots,p_M^{t-1},x^{t-1})/g^{t-1}(p^{t-1},x^{t-1})\ ; \qquad m = 1,\ldots,M;\ t = 1,2,\ldots ;$$

$$(48)\ \alpha_{Pm}^{t} \equiv g^{t}(p^{t},x^{t})/g^{t}(p_1^{t},\ldots,p_{m-1}^{t},p_m^{t-1},p_{m+1}^{t},\ldots,p_M^{t},x^{t})\ ; \qquad m = 1,\ldots,M;\ t = 1,2,\ldots .$$

Since both measures of real output price change are equally valid, it is natural to average them to obtain an *overall measure of the effects on real income of the change in the real price of output m*:[53]

$$(49)\ \alpha_m^{t} \equiv [\alpha_{Lm}^{t}\,\alpha_{Pm}^{t}]^{1/2}\ ; \qquad m = 1,\ldots,M\ ;\ t = 1,2,\ldots .$$

Under the assumption that the deflated GDP functions $g^t(p,x)$ have the translog functional forms as defined by (33)-(41) in the previous section, the arguments of Diewert and Morrison (1986; 666) can be adapted to give us the following result:

$$(50)\ \ln\alpha_m^{t} = (1/2)[(p_m^{t-1}y_m^{t-1}/p^{t-1}\cdot y^{t-1}) + (p_m^{t}y_m^{t}/p^{t}\cdot y^{t})]\ \ln(p_m^{t}/p_m^{t-1})\ ; \qquad m = 1,\ldots,M\ ;\ t = 1,2,\ldots .$$

Note that $\ln\alpha_m^t$ is equal to the mth term in the summation of the terms on the right hand side of (43). This observation means that we have the following exact decomposition of the period t aggregate real output price contribution factor $\alpha^t$ into a product of separate price contribution factors; i.e., we have under present assumptions:

$$(51)\ \alpha^{t} = \alpha_1^{t}\alpha_2^{t}\ldots\alpha_M^{t}\ ; \qquad t = 1,2,\ldots .$$

The above decomposition is useful for analyzing how real changes in the price of exports (i.e., a change in the price of exports relative to the price of domestic consumption) and in the price of imports impact on the real income generated by the market sector. In the empirical illustration which follows later, we let M equal three. The three net outputs are:

- Domestic sales (C+I+G);
- Exports (X) and
- Imports (M).

Since commodities 1 and 2 are outputs, $y_1$ and $y_2$ will be positive but since commodity 3 is an input into the market sector, $y_3$ will be negative. Hence an increase in the real price of exports will *increase* real income but an increase in the real price of imports will *decrease* the real income generated by the market sector, as is evident by looking at the contribution terms defined by (50) for m = 2 (where $y_m^t > 0$) and for m = 3 (where $y_m^t < 0$).

[53] The indexes defined by (47)-(49) were defined by Diewert and Morrison (1986; 666) in the nominal GDP function context.

As mentioned above, it is also useful to have a decomposition of the aggregate contribution of input growth to the growth of real income into separate contributions for each important class of primary input that is used by the market sector. We now model the effects of a change in a single input quantity, say $x_n$, going from period t−1 to t. Counterparts to the theoretical Laspeyres and Paasche type quantity indexes defined by (18) and (19) above for changes in input n are the following *Laspeyres type measure* $\beta_{Ln}^t$ that chooses the period t−1 reference technology and holds constant other input quantities at their period t−1 levels and holds real output prices at their period t−1 levels $p^{t-1}$ and a *Paasche type measure* $\beta_{Pn}^t$ that chooses the period t reference technology and reference real output price vector $p^t$ and holds constant other input quantities at their period t levels:

(52) $\beta_{Ln}^t \equiv g^{t-1}(p^{t-1},x_1^{t-1},...,x_{n-1}^{t-1},x_n^t,x_{n+1}^{t-1},..., x_N^{t-1})/g^{t-1}(p^{t-1},x^{t-1})$ ; n = 1,...,N; t = 1,2, ... ;

(53) $\beta_{Pn}^t \equiv g^t(p^t,x^t)/g^t(p^t,x_1^t,...,x_{n-1}^t,x_n^{t-1},x_{n+1}^t,..., p_N^t)$ ; n = 1,...,N; t = 1,2, ... .

Since both measures of input change are equally valid, as usual, we average them to obtain *an overall measure of the effects on real income of the change in the quantity of input n*:[54]

(54) $\beta_n^t \equiv [\beta_{Pn}^t\,\beta_{Pn}^t]^{1/2}$ ; n = 1,...,N ; t = 1,2, ... .

Under the assumption that the deflated GDP functions $g^t(p,x)$ have the translog functional forms as defined by (33)-(41) in the previous section, the arguments of Diewert and Morrison (1986; 667) can be adapted to give us the following result:

(55) $\ln\beta_n^t = (1/2)[(w_n^{t-1}x_n^{t-1}/w^{t-1}\cdot x^{t-1}) + (w_n^t x_n^t/w^t\cdot x^t)] \ln(x_n^t/x_n^{t-1})$ ; n = 1,...,N ; t = 1,2, ... .

Note that $\ln\beta_n^t$ is equal to the nth term in the summation of the terms on the right hand side of (44). This observation means that we have the following exact decomposition of the period t aggregate input growth contribution factor $\beta^t$ into a product of separate input quantity contribution factors; i.e., we have under present assumptions:

(56) $\beta^t = \beta_1^t\beta_2^t... \beta_N^t$ ; t = 1,2, ... .

## 5. The Deflated NDP Translog Approach

There is a severe flaw with all of the analysis presented in the previous sections of this Appendix.. The problem is that depreciation payments are part of the user cost of capital for each asset but depreciation does not provide households with any sustainable purchasing power. Hence our real income measure defined by (5) above is overstated.

---

[54] The indexes defined by (52)-(54) were defined by Diewert and Morrison (1986; 667) in the nominal GDP function context.

To see why Gross Domestic Product overstates income, consider the model of production that is described by the following quotations:

"We must look at the production process during a period of time, with a beginning and an end. It starts, at the commencement of the Period, with an Initial Capital Stock; to this there is applied a Flow Input of labour, and from it there emerges a Flow Output called Consumption; then there is a Closing Stock of Capital left over at the end. If Inputs are the things that are put in, the Outputs are the things that are got out, and the production of the Period is considered in isolation, then the Initial Capital Stock is an Input. A Stock Input to the Flow Input of labour; and further (what is less well recognized in the tradition, but is equally clear when we are strict with translation), the Closing Capital Stock is an Output, a Stock Output to match the Flow Output of Consumption Goods. Both input and output have stock and flow components; capital appears both as input and as output" John R. Hicks (1961; 23).

"The business firm can be viewed as a receptacle into which factors of production, or inputs, flow and out of which outputs flow...The total of the inputs with which the firm can work within the time period specified includes those inherited from the previous period and those acquired during the current period. The total of the outputs of the business firm in the same period includes the amounts of outputs currently sold and the amounts of inputs which are bequeathed to the firm in its succeeding period of activity." Edgar O. Edwards and Philip W. Bell (1961; 71-72).

Hicks and Edwards and Bell obviously had the same model of production in mind: in each accounting period, the business unit combines the capital stocks and goods in process that it has inherited from the previous period with "flow" inputs purchased in the current period (such as labour, materials, services and additional durable inputs) to produce current period "flow" outputs as well as end of the period depreciated capital stock components which are regarded as outputs from the perspective of the current period (but will be regarded as inputs from the perspective of the next period).[55]

All of the "flow" inputs that are purchased during the period and all of the "flow" outputs that are sold during the period are the inputs and outputs that appear in the usual definition of cash flow. These are the flow inputs and outputs that are very familiar to national income accountants. But this is not the end of the story: the firm inherits an endowment of assets at the beginning of the production period and at the end of the period, the firm will have the net profit or loss that has occurred due to its sales of outputs and its purchases of inputs during the period. As well, *it will have a stock of assets that it can use when it starts production in the following period*. Just focusing on the flow transactions that occur within the production period will not give a complete picture of the firm's productive activities. Hence, to get a complete picture of the firm's production activities over the course of a period, it is necessary to add the value of the closing stock of assets less the beginning of the period stock of assets to the cash flow that accrued to the firm from its sales and purchases of market goods and services during the accounting period.

We illustrate the above theory by considering a very simple two output, two input model of the market sector. One of the outputs is output in period t, $Y^t$ and the other output is

[55] For more on this model of production and additional references to the literature, see the Appendices in Diewert (1977) (1980). The usual user cost of capital can be derived from this framework if depreciation is independent of use.

an investment good, $I^t$. One of the inputs is the flow of noncapital primary input $X^t$ and the other input is $K^t$, capital services. Suppose that the average prices during period t of a unit of $Y^t$, $X^t$ and $I^t$ are $P_Y^t$, $P_X^t$ and $P_I^t$ respectively. Suppose further that the interest rate prevailing at the beginning of period t is $r^t$. The value of the beginning of period t capital stock is assumed to be $P_I^t$, the investment price for period t. In order to induce households to let the business sector use the initial stock of capital, firms have to pay households interest equal to $r^t P_I^t K^t$. Then neglecting balance sheet items, the market sector's period t *cash flow* is:[56]

(57) $CF^t \equiv P_Y^t Y^t + P_I^t I^t - P_X^t X^t - r^t P_I^t K^t$.

$K^t$ is interpreted as the firm's beginning of period t stock of capital it has at its disposal and its end of period stock of capital is defined to be $K^{t+1}$. These capital stocks are valued at the balance sheet prices prevailing at the beginning and end of period t, $P_I^t$ and $P_I^{t+1}$ respectively.

The market sector period t *pure profit* is defined as its cash flow plus the value of its end of period t capital stock less the value of its beginning of period t capital stock:

(58) $\prod^t \equiv CF^t + P_I^{t+1} K^{t+1} - P_I^t K^t$.

Now the end of period depreciated stock of capital is related to the beginning of the period stock by the following equation:

(59) $K^{t+1} = (1 - \delta)K^t$

where $0 < \delta < 1$ denotes the depreciation rate.

Now substitute (57) and (59) into the definition of pure profits (58) and we obtain the following expression:

$$
\begin{aligned}
(60)\ \prod^t &\equiv P_Y^t Y^t + P_I^t I^t - P_X^t X^t - r^t P_I^t K^t + P_I^{t+1}(1-\delta)K^t - P_I^t K^t \\
&= P_Y^t Y^t + P_I^t I^t - P_X^t X^t - \{r^t P_I^t + \delta P_I^{t+1} - (P_I^{t+1} - P_I^t)\}K^t.
\end{aligned}
$$

The expression that precedes the capital stock $K^t$, $\{r^t P_I^t + \delta P_I^{t+1} - (P_I^{t+1} - P_I^t)\}$, can be recognized as the *user cost of capital*;[57] it is the gross rental price that must be paid to a capitalist in order to induce him or her to loan the services of a unit of the capital stock to the production sector.

Some simplifications for (60) occur if we make two additional assumptions:

---

[56] For equity financed firms, we need to include an imputed return for equity capital.

[57] See Christensen and Jorgenson (1969) for a derivation in continuous time and Diewert (1980; 471) for a derivation in discrete time.

- Assume that producers and households expect price level stability so that the end of the period price for a new unit of capital $P_I^{t+1}$ is expected to be equal to the beginning of the period price for a new unit of capital $P_I^t$; in this case, we can interpret $r^t$ as the period t real interest rate;[58]
- Assume that pure profits are zero so that $\Pi^t$ equals zero.

Substituting these two assumptions into equation (60) leads to the following expression:

(61) $\Pi^t = P_Y^t Y^t + P_I^t I^t - P_X^t X^t - \{r^t P_I^t + \delta P_I^t\}K^t = 0.$

Equation (61) can be rearranged to yield the following value of output equals value of input equation:

(62) $P_Y^t Y^t + P_I^t I^t = P_X^t X^t + \{r^t P_I^t + \delta P_I^t\}K^t.$

Equation (62) is essentially the closed economy counterpart to the (gross) value of outputs equals (gross) value of primary inputs equation (4), $P^t \cdot y^t = W^t \cdot x^t$, that we have been using thus far in this Appendix. We now come to the point of this rather long digression: *the (gross) payments to primary inputs that is defined by the right hand side of (62) is not income*, in the sense of Hicks.[59] The owner of a unit of capital cannot spend the entire period t gross rental income $\{r^t P_I^t + \delta P_I^t\}$ on consumption during period t because the depreciation portion of the rental, $\delta P_I^t$, is required in order to keep his or her capital intact. Thus the owner of a new unit of capital at the beginning of period t loans the unit to the market sector and gets the gross return $\{r^t P_I^t + \delta P_I^t\}$ at the end of the period plus the depreciated unit of the initial capital stock, which is worth only $(1 - \delta)P_I^t$. Thus $\delta P_I^t$ of this gross return must be set aside in order to restore the lender of the capital services to his or her original wealth position at the beginning of period t. This means that *period t Hicksian market sector income* is not the value of payments to primary inputs, $P_X^t X^t + \{r^t P_I^t + \delta P_I^t\}K^t$; instead it is the value of payments to labour $P_X^t X^t$ plus the reward for waiting, $r^t P_I^t K^t$. Using this definition of market sector (net) Hicksian income, we can rearrange equation (62) as follows:

(63) Hicksian market sector income $\equiv P_X^t X^t + r^t P_I^t K^t$
$= P_Y^t Y^t + P_I^t I^t - \delta P_I^t K^t$
= Value of consumption + value of gross investment − value of depreciation.

Thus in this Hicksian net income framework, our new output concept is equal to our old output concept less the value of depreciation. We take the price of depreciation to be the

[58] This assumption can be relaxed somewhat and we can still end up with much the same model; see Diewert (2006a).
[59] We will use Hicks' third concept of income here: "Income No. 3 must be defined as the maximum amount of money which the individual can spend this week, and still be able to expect to spend this week, and still be able to expect to spend the same amount *in real terms* in each ensuing week." J.R. Hicks (1946; 174).

corresponding investment price $P_I^t$ and the quantity of depreciation is taken to be the depreciation rate times the beginning of the period stock, $\delta K^t$.

Hence the overstatement of income problem that is implicit in the approaches used in previous sections can readily be remedied: all we need to do is to take the user cost formula for an asset and decompose it into two parts:

- One part that represents depreciation and foreseen obsolescence, $\delta P_I^t K^t$, and
- The remaining part that is the reward for postponing consumption, $r^t P_I^t K^t$.

In our empirical work, our user costs in the gross product approach took the following form:

(64) $u^t = (r^t + \delta^t + \tau^t)P_I^t$

where $r^t$ is the balancing period t real rate of interest, $\delta^t$ is a geometric depreciation rate for period t, $\tau^t$ is an average capital taxation rate on the asset and $P_I^t$ is the period t investment price for the asset. However, when we used the net product approach, we split up each (gross product) user cost times the beginning of the period stock $K^t$ into the depreciation component $\delta^t P_I^t K^t$ and the remaining term $(r^t+\tau^t)P_I^t K^t$ and we regarded the second term as a genuine income component but the first term was treated as an intermediate input cost for the market sector and was an offset to gross investment made by the market sector during the period under consideration. In the main text, when the net approach was used, the investment aggregate I was a *net investment aggregate* (gross investment components were indexed with a positive sign in the aggregate and depreciation components were indexed with a negative sign in the aggregate). The capital services aggregate in the net approach was a "reward for waiting" capital services aggregate rather than the gross return aggregate that was used in the gross product approach.[60]

## 6. Sectoral Contributions to Real Income Growth

The above theory applied to the market sector as a whole. However, it is of considerable interest to determine which separate industries contributed the most to the overall growth of real income generated by the market sector of the economy. Hence, in this section, we outline how this can be done if industry data on outputs, inputs and the corresponding prices are available.[61] However, at the outset, it should be noted that in general, we will

[60] This approach seems to be broadly consistent with an approach advocated by Rymes (1968) (1983), who stressed the role of waiting services: "Second, one can consider the 'waiting' or 'abstinence' associated with the net returns to capital as the nonlabour primary input." T.K. Rymes (1968; 362). Denison (1974) also advocated a net product approach to productivity measurement.

[61] In Canada, such data are available from the Input-Output and Productivity Divisions of Statistics Canada. However, these data for the past 5 years are not available at present.

not be able to single out the effects of changes in real international prices as we were able to do when the entire business sector is treated as a single industry.[62]

We assume that there are I industries in the market sector of the economy. As in section 2, we assume that there is a common list of M (net) outputs which each industry produces or uses as intermediate inputs. The net output vector for industry i in period t is $y^{it} \equiv [y_1^{it},...,y_M^{it}]$, which are sold at the positive producer prices for industry i in period t, $P^{it} \equiv [P_1^{it},...,P_M^{it}]$, for i = 1,...,I. There is also a common list of N primary inputs used by each industry. In period t, we assume that industry i uses nonnegative quantities of N primary inputs, $x^{it} \equiv [x_1^{it},...,x_N^{it}]$ which are purchased at the positive primary input prices $W^{it} \equiv [W_1^{it},...,W_N^{it}]$ for i = 1,...,I. In each period t, we assume that there is a feasible set of net output vectors $y^i$ that can be produced by industry i if the vector of primary inputs $x^i$ is utilised by that industry; denote this period t production possibilities set by $S^{it}$. We assume that $S^{it}$ is a closed convex cone that exhibits a free disposal property. We shall take the net product point of view developed in the previous section for each industry in what follows.

Given a vector of industry i net output prices $P^{it}$ and a vector of available primary inputs $x^{it}$ for that industry, we define *the industry i period t net product function*, $g^{it}(P^{it},x^{it})$, as follows:

$$(65)\quad g^{it}(P^{it},x^{it}) \equiv \max_y \{P^{it}\cdot y : (y,x^{it}) \text{ belongs to } S^{it}\} = P^{it}\cdot y^{it} ; \qquad i = 1,...,I ; t = 0,1,2, ... .$$

Since we have assumed constant returns to scale for each industry, it is natural to assume that the income generated by industry i in period t, $W^{it}\cdot x^{it}$, is equal to the corresponding value of net product, $P^{it}\cdot y^{it}$; i.e., we have:

$$(66)\quad P^{it}\cdot y^{it} = W^{it}\cdot x^{it} ; \qquad i = 1,...,I ; t = 0,1,2, ... .$$

Define the *period t, industry i real input and output price vectors*, $w^{it}$ and $p^{it}$ respectively, as follows:

$$(67)\quad w^{it} \equiv W^{it}/P_C^{t} ; \; p^{it} \equiv P^{it}/P_C^{t} ; \qquad i = 1,...,I ; t = 0,1,2, ... .$$

[62] The problem is not methodological; it is a data problem. In order to determine the effects of changing real import and export prices on the real income generated by an industry, we require information on the value and price of exports produced by the industry and on the value and price of imports used by the industry. However, the *System of National Accounts 1993* does not set up the production accounts so that the exports produced and imports used by an industry are recorded in the recommended system of production accounts. In theory, this problem can be remedied simply by distinguishing industry outputs as being either exported or delivered to domestic users and by distinguishing industry inputs as being either imports or supplied by a domestic producer; see Diewert (2007b) (2007c) for the details of the resulting modified industry accounts. In practice, it will be extremely difficult to collect the required information. For further discussion of these issues, see section 1 of Appendix 2 below.

As in section 2 of this Appendix, we can define the *real income generated by industry i in period t*, $\rho^{it}$, as the nominal income generated by industry i in period t, $W^{it}\cdot x^{it}$, divided by the consumption price deflator for period t, $P_C^{\,t}$. Using (65)-(67), we have:

$$(68)\ \rho^{it} \equiv W^{it}\cdot x^{it}/P_C^{\,t} \qquad i = 1,...,I\ ;\ t = 0,1,2, ...\ .$$
$$= w^{it}\cdot x^{it}$$
$$= P^{it}\cdot y^{it}/P_C^{\,t}$$
$$= p^{it}\cdot y^{it}$$
$$= g^{it}(p^{it},x^{it})$$

where the last equality follows using (65)-(67) and the linear homogeneity of $g^{it}(P^{it},x^{it})$ in $P^{it}$.

We now rework the theoretical analysis presented in sections 2-4 above, except we apply it at the industry level instead of the economy wide market sector level. Thus define $\gamma^{it}$ as the *period t chain link rate of growth factor for the real income generated by industry i*:

$$(69)\ \gamma^{it} \equiv \rho^{it}/\rho^{it-1}\ ; \qquad i = 1,...,I\ ;\ t = 1,2,...\ .$$

Now assume that the industry i, period t (deflated) GDP function, $g^{it}(p,x)$, has a translog functional form analogous to that defined above by (33)-(41). Repeat the analysis at the national level that led up to equation (42), except now apply it at the industry level. We can derive the following industry counterparts to the national equation (42):

$$(70)\ p^{it}\cdot y^{it}/p^{it-1}\cdot y^{it-1} = \rho^{it}/\rho^{it-1} = \gamma^{it} = \tau^{it}\ \alpha^{it}\ \beta^{it}\ ; \qquad i = 1,...,I\ ;\ t = 0,1,2, ...$$

where the *period t, industry i chain link technical progress growth rate* $\tau^{it}$, *output price growth rate* $\alpha^{it}$ and *input quantity growth rate* $\beta^{it}$ can be calculated using the period t and t−1 price and quantity data for industry i as follows, for i = 1,...,I ; t = 0,1,2, ... :

$$(71)\ \ln\alpha^{it} \equiv \sum_{m=1}^{M} (1/2)[(p_m^{\,it-1}y_m^{\,it-1}/p^{it-1}\cdot y^{it-1}) + (p_m^{\,it}y_m^{\,it}/p^{it}\cdot y^{it})]\ \ln(p_m^{\,it}/p_m^{\,it-1})$$
$$= \ln P_T(p^{it-1},p^{it},y^{it-1},y^{it});$$
$$(72)\ \ln\beta^{it} \equiv \sum_{n=1}^{N} (1/2)[(w_n^{\,it-1}x_n^{\,it-1}/w^{it-1}\cdot x^{it-1}) + (w_n^{\,it}x_n^{\,it}/w^{it}\cdot x^{it})]\ \ln(x_n^{\,it}/x_n^{\,it-1})$$
$$= \ln Q_T(w^{it-1},w^{it},x^{it-1},x^{it});$$
$$(73)\quad \tau^{it} \equiv \gamma^{it}/\alpha^{it}\ \beta^{it}$$

where $P_T(p^{it-1},p^{it},y^{it-1},y^{it})$ is the period t, industry i Törnqvist output price index and $Q_T(w^{it-1},w^{it},x^{it-1},x^{it})$ is the period t, industry i Törnqvist input quantity index.

Recall that in section 2, we defined cumulated counterparts to the chain link equations (42). We can do the same type of operation for the industry data. Thus define the industry i *level of total factor productivity* in period t relative to period 0 as $T^{it}$, the industry i *level of real output prices* in period t relative to period 0 as $A^{it}$ and the industry i level of *primary input* in period t relative to period 0 as $B^{it}$. These industry levels can be defined in terms of the corresponding industry chain link factors, $\tau^{it}$,$\alpha^{it}$ and $\beta^{it}$ as follows:

(74) $T^{i0} \equiv 1$ ; $T^{it} \equiv T^{it-1} \tau^{it}$ ; $t = 1,2, \ldots$ ;
(75) $A^{i0} \equiv 1$ ; $A^{it} \equiv A^{it-1}\alpha^{it}$ ; $t = 1,2, \ldots$ ;
(76) $B^{i0} \equiv 1$ ; $B^{it} \equiv B^{it-1}\beta^{it}$ ; $t = 1,2, \ldots$ .

Since equations (70) hold as exact identities under our present assumptions, the following cumulated counterparts to these equations will also hold as exact decompositions:

(77) $p^{it}\cdot y^{it}/p^{i0}\cdot y^{i0} = \rho^{it}/\rho^{i0} = T^{it} A^{it} B^{it}$ ; $i = 1,\ldots,I$ ; $t = 1,2, \ldots$ .

Thus three factors contribute to the period t level of real income generated by industry i relative to the period 0 level: the level of period t total factor productivity of industry i in period t (relative to period 0), $T^{it}$, the growth in real output prices for industry i going from period 0 to t, $A^{it}$, and the growth in primary inputs utilized by industry i going from period 0 to t, $B^{it}$.

The nominal value of market sector output in period t is the corresponding sum of industry nominal values, $\sum_{i=1}^{I} P^{it}\cdot y^{it}$, which can be converted into the *period t real income generated by the market sector*, $\rho^t$, by dividing this sum by the period t consumption price deflator, $P_C^t$:

(78) $\rho^t \equiv \sum_{i=1}^{I} P^{it}\cdot y^{it}/P_C^t = \sum_{i=1}^{I} p^{it}\cdot y^{it} = \sum_{i=1}^{I} \rho^{it}$ ; $t = 0,1,\ldots$

where the last equality follows using (68). Define industry i's share of market sector nominal (or real) net output in period 0 as

(79) $s_i^0 \equiv \rho^{i0}/\rho^0$ ; $i = 1,\ldots,I$.

Using the above definitions, we can decompose the growth in market sector real income, going from period 0 to t, as follows:

$$
\begin{aligned}
(80)\ \rho^t/\rho^0 &= [\textstyle\sum_{i=1}^{I} \rho^{it}]/\rho^0 && \text{using (78)} \\
&= \textstyle\sum_{i=1}^{I} [\rho^{it}/\rho^{i0}][\rho^{i0}/\rho^0] && \\
&= \textstyle\sum_{i=1}^{I} s_i^0 [\rho^{it}/\rho^{i0}] && \text{using (79)} \\
&= \textstyle\sum_{i=1}^{I} s_i^0 T^{it} A^{it} B^{it} && \text{using (77).}
\end{aligned}
$$

Equation (80) shows the factors that determine the evolution of market sector real income growth over time. There are four sets of factors at work:

- The industrial structure of net product in the base period; i.e., the base period industry shares of market sector net output, $s_i^0$;
- The total factor productivity performance of industry i cumulated from the base period to the current period; i.e., the industry productivity factors, $T^{it}$;

- The growth in industry output prices (deflated by the price of the consumption aggregate) going from period 0 to t; i.e., the industry real output price factors, $A^{it}$ and
- The growth in primary inputs utilized by industry i going from period 0 to t; i.e., the industry primary input growth factors, $B^{it}$.

Note that if an industry i experiences growth in its (net) output prices relative to the price of consumption, then the corresponding real output price factor $A^{it}$ will be greater than one and this effect will contribute to overall real income growth. It is this type of factor that is missing in traditional Total Factor Productivity decompositions; i.e., the traditional analysis ignores favourable (or unfavourable) output price effects.[63]

## Appendix 2: Business Sector Data on Outputs and Inputs for Canada 1961-2006

### 1. Introduction

The basic approach to measuring productivity growth in this paper is to use recently released information on business sector outputs and inputs from Statistics Canada's KLEMS data base along with information on aggregate final demand expenditures in order to construct "top down" measures of the productivity performance of the Canadian business sector. We also make extensive use of Statistics Canada's National Balance Sheet estimates for information on various capital inputs used by the business sector. Thus the present approach to productivity measurement is an aggregate "top down" approach as opposed to the usual industry "bottom up" approach which makes use of detailed data on inputs used and outputs produced by industrial sectors and aggregates up sectoral productivity growth rates in order to obtain national business sector estimates.[64] With reliable data, the two approaches should give very similar answers.[65] Unfortunately, data on industry inputs and outputs are not likely to be as reliable as the corresponding national data for a variety of reasons[66] so it is useful to provide a check on the industry approach to productivity measurement by using the national aggregate approach.

There is another reason for undertaking a productivity study using final demand data and this reason is that the effects of changes in a country's terms of trade can be measured in this framework whereas these effects cannot be measured in the industry accounts framework using the existing System of National Accounts 1993 (SNA 1993); see Chapter 15 in Eurostat, IMF, OECD, UN and World Bank (1993). In particular, the Input

---

[63] Improvements in the country's terms of trade are also ignored by the traditional methodology. This does not mean that the traditional emphasis on pure efficiency improvements is "wrong"; it just does not answer the question that we are focusing on, which is: what is the rate of growth in consumption equivalents that the market sector of the economy is generating?

[64] The bottom up approach is used by the Statistics Canada KLEMS program; see Baldwin, Wu and Yan (2007) for an overview and Baldwin and Yu (2007) for additional information on the construction of the Statistics Canada KLEMS capital services aggregates.

[65] In fact, if indirect tax effects could be ignored and if nominal and real input output tables were perfectly consistent, the two approaches should give exactly the same answer; see Diewert (2006b) (2007c).

[66] For a detailed discussion of these reasons, see Diewert (2001).

Output accounts as outlined in Table 15.1 in the SNA 1993 do not show the role of international trade in goods and services by industry. Exports and imports enter the main supply and use tables (Table 15.1) as additions (or subtractions) to total net supply or to total domestic final demand in the familiar C+I+G+X−M setup. This means that Table 15.1 in the main production accounts of *SNA 1993* does not elaborate on which industries are actually using the imports or on which industries are actually doing the exporting by commodity.[67] Thus at present, data difficulties prevent us from looking at the effects of changes in the terms of trade using the bottom up industry aggregation approach.

Diewert and Lawrence (2000) undertook a study of Canada's business sector productivity using the national approach for the years 1962-1996 and Diewert (2002) extended their data to cover the years 1962-1998. The present study is an extension of these previous studies but there are some differences:

- Statistics Canada has provided new data on national expenditure aggregates back to 1961 using annual chained index numbers and so it is no longer necessary to work with the old fixed base data on the most disaggregated level possible and then use chain indexes to aggregate up these data.
- Statistics Canada has also provided new data on the outputs produced and inputs used by the Canadian business sector back to 1961 using chained Fisher indexes as part of their KLEMS productivity measurement program. In particular, we will use the KLEMS estimates of labour input, which are a big improvement over the estimates of labour input used by Diewert and Lawrence.
- Diewert and Lawrence (2000) worked with a rather narrow definition of the government sector; their definition included only the public administration industry. In this study, we adopt the Statistics Canada definition of the nonbusiness sector (except that we add to it the residential rental housing industry) and include the general government sector and the publicly funded defence, hospital and education sectors in the nonbusiness sector.[68] Since output in the nonbusiness sector is measured by input, the use of the broader definition of the government sector should lead to higher estimates of productivity growth in the business sector compared to the estimates tabled in Diewert and Lawrence (2000) and Diewert (2002).
- Statistics Canada has reorganized its information on indirect taxes (less subsidies) into two categories: taxes that fall primarily on outputs and taxes

[67] It should be noted that *SNA 1993* does have a recommended optional Table 15.5 which is exactly suited to our present needs; i.e., this table provides the detail for imports by commodity and by industry. However, *SNA 1993* does not provide a recommendation for a corresponding commodity by industry table for exports.

[68] The nonbusiness sector consists of the following industries: (1) Government funding of hospitals; (2) Government funding of residential care; (3) Government funding of universities; (4) Government funding of other education; (5) Defence services; (6) Other municipal government services; (7) Other provincial government services and (8) Other federal government services.

that fall primarily on inputs. This new information is very useful in making adjustments to output prices for indirect tax effects.[69]

In section 2 of this Appendix, we will list the basic final demand expenditure series that were used in this study. Section 3 simply lists the three published business sector measures of quality adjusted labour input for the Canadian business sector that are available on CANSIM as part of the Statistics Canada KLEMS program. Section 4 studies the problems associated with forming estimates for capital inputs. Section 5 concludes by forming estimates of tax rates on primary inputs. This information is used to calculate estimates of balancing after tax real rates of return. Then this information is used along with the information developed in previous sections in order to calculate user costs for five classes of capital input: machinery and equipment, nonresidential structures, agricultural land, nonagricultural and nonresidential business land and inventories. Section 6 concludes with some observations on the weak points in the data and recommendations for further work on developing a set of productivity accounts for Canada.

## 2. Estimates of Canadian Final Demand Expenditures

Much of the information tabled in this section is updated information that can be found in the *Canadian Economic Observer*, Statistics Canada (2007), Table 1: Gross Domestic Product (GDP) by Income and Expenditure (millions of dollars and in chained 2002 dollars). The series in this Table are annual series and run from 1961 to 2006. The most recent version of these data were used using the Statistics Canada online data service CANSIM II, which were frequently listed as quarterly data. If the quarterly data were seasonally adjusted, then the data for a year were summed and divided by 4 in order to obtain annual data. If the quarterly data were not seasonally adjusted, then they were simply summed in order to obtain annual data. In what follows, we will use the CANSIM individual series label to identify the exact series used.

The first two series are Personal Expenditures on Goods and Services in current and constant chained 2002 dollars, CANSIM II series V498087 and V1992044 respectively. Dividing the current dollar series $V_{CT}$ by the constant dollar series $Q_{CT}$ gives us an implicit price series $P_{CT}$ for personal consumption.

We would like to exclude the imputed expenditures on Owner Occupied Housing (OOH) from the above series since there is no possibility of productivity gains occurring in this sector. However, if we exclude imputed rent from the business sector output series, we also need to exclude the services of the owner occupied housing capital stock as an input into the business sector. Unfortunately, we are not able to construct a reliable measure of

[69] In early studies of the Total Factor Productivity of an economy like those done by Solow (1957) and Jorgenson and Griliches (1967), outputs were priced at final demand prices, which include indirect taxes. However, Jorgenson and Griliches (1972; 85) noted that this treatment was not consistent with competitive price taking behavior on the part of producers, since producers do not derive any benefit from indirect taxes that fall on their outputs and thus these taxes should be removed.

the Owner Occupied Housing capital stock from available data; we can only construct a more reliable residential housing capital stock which includes the housing capital stock that is rented. We also were not able to split residential land input into reliable owner occupied and rental components.[70] Hence we excluded both imputed and paid rents from our list of business sector outputs and we excluded the entire residential housing stock and the associated land as an inputs into the business sector.[71] Information on current dollar expenditures on imputed rents and paid rents (this is the series $V_{PR}$ in Table 1 below) for the years 1961-2006 is available from CANSIM II series V498532 and V498533 respectively. The corresponding information on chained 1997 constant dollar expenditures on imputed rents and paid rents ($Q_{PR}$) is available from CANSIM II series V1992078 and V1992079 only for the years 1981-2006. We divide $V_{PR}$ by $Q_{PR}$ in order to form a price index for paid rents, $P_{PR}$. We could follow the same strategy to form a price index for imputed rents for the years 1981-2006.[72] However, an alternative series on the imputed value of OOH services for the years 1961-2004 is available from the industry accounts. This series is CANSIM II series V3859926, Business Sector: Owner Occupied Dwellings, from Table 370023: Gross Domestic Product (GDP) at Basic Prices in Current Dollars, System of National Accounts, Benchmark Values, by North American Industry Classification System (NAICS). This series is listed as $V_{IMR}$ in Table 1 below.[73] The final demand value series for imputed rents (not listed) is about 13% higher than its industry counterpart, $V_{IMR}$. We use the industry series for imputed rents rather than the final demand series because we want our business sector value added to closely approximate the Statistics Canada KLEMS program business sector value added, except that our aggregate will not include paid residential rents.[74]

We now describe how we estimated a price index for the paid rents series for the years 1961-1981 and how we formed a price index for the industry value added series for the imputed rents for OOH for the years 1961-2006. An old series for the industry value added generated by OOH, CANSIM II series V334072, Canada: Current Prices; Business

---

[70] The determination of the structures and land inputs into the production of rented residential housing is a difficult task since the investment data on residential housing is not decomposed into owned and rented investments. This lack of information was also a problem for the Statistics Canada KLEMS program: "Data on investment in rental residential buildings are not available. For the annual MFP programs, we divide the total investment in residential building into rental building and owner-occupied dwelling using paid rents for rental buildings and imputed rents for owner occupied dwelling as the split ratios. The investment in residential buildings and paid and imputed rents are available from the Income and Expenditure Accounts. On average, we find that about 30% of total rents are paid rents and the remaining 70% are imputed rents." Baldwin, Wu and Yan (2007; 43).

[71] This means our productivity estimates will be biased downward slightly since the inputs that are used in the rental housing market are included in our estimates but the corresponding outputs are not included.

[72] We did construct the corresponding expenditure based price series for imputed rents for the period 1981-2006 and compared this price index with the corresponding industry based price index for imputed rents described below for the years 1981-2004 and found that the movements were similar. We used the expenditure based price index for the years 2004-2006 to extend the industry based price index from 2004 to 2006.

[73] We explain below how this industry based value series for imputed rents was extended from 2004 to 2006.

[74] The KLEMS business sector value added aggregate excludes imputed rents whereas our business sector value added aggregate will exclude both imputed and paid rents. Our treatment of inventory change is also different.

Sector; Owner Occupied Dwellings, from Table 3790001, Gross Domestic Product (GDP) at Factor Cost, System of National Accounts Benchmark Values, by Industry, is available for the years 1961-1997. The corresponding series in constant 1992 dollars is available for the years 1961-2000 as CANSIM II Series V328857 in Table 3790004. We use these two series to form a price index for imputed rent for the years 1961-1997, $P_{IMR}^t$ in Table 1 below. A constant dollar industry series for the services of OOH for the years 1997-2006 can be obtained from CANSIM II Series V14183160, Canada; Seasonally Adjusted at Annual Rates; Chained 1997 Dollars; Owner Occupied Dwellings in Table 3790018, Gross Domestic Product (GDP) at Basic Prices by NAICS.[75] Dividing $V_{IMR}^t$ by this constant dollar series will give us a price index for imputed rents running from 1997 to 2006 and we link this series to the earlier $P_{IMR}^t$ series that ran from 1961 to 1997. We then normalized the price series to equal 1 in 1961 and formed the quantity series $Q_{IMR}^t$ as $V_{IMR}^t$ divided by $P_{IMR}^t$. $V_{IMR}^t$, $Q_{IMR}^t$ and $P_{IMR}^t$ are listed in Table 1 below. Recall that we have a value series for paid rents, $V_{PR}^t$, that covers the years 1961-2006 but the corresponding price index series, $P_{PR}^t$, covers only the years 1981-2006. We extend $P_{PR}^t$ back to 1961 using the movements in $P_{IMR}^t$. The resulting price series is normalized to equal 1 in 1961 and a quantity series for paid rents, $Q_{PR}^t$, is obtained by dividing $V_{PR}^t$ by $P_{PR}^t$. These three series are also listed in Table 1 below.

**Table 1: Housing Value, Quantity and Price Series for Imputed and Paid Rents[76]**

| Year t | $V_{IMR}^t$ | $Q_{IMR}^t$ | $V_{PR}^t$ | $Q_{PR}^t$ | $P_{IMR}^t$ | $P_{PR}^t$ |
|---|---|---|---|---|---|---|
| 1961 | 2292 | 2292 | 1107 | 1107 | 1.00000 | 1.00000 |
| 1962 | 2436 | 2380 | 1176 | 1149 | 1.02350 | 1.02350 |
| 1963 | 2660 | 2412 | 1290 | 1170 | 1.10275 | 1.10275 |
| 1964 | 2832 | 2477 | 1396 | 1221 | 1.14316 | 1.14316 |
| 1965 | 2976 | 2531 | 1503 | 1278 | 1.17565 | 1.17565 |
| 1966 | 3249 | 2620 | 1658 | 1337 | 1.23992 | 1.23992 |
| 1967 | 3585 | 2678 | 1860 | 1390 | 1.33856 | 1.33856 |
| 1968 | 3985 | 2707 | 2091 | 1420 | 1.47212 | 1.47212 |
| 1969 | 4416 | 2784 | 2342 | 1476 | 1.58633 | 1.58633 |
| 1970 | 4897 | 2833 | 2645 | 1530 | 1.72855 | 1.72855 |
| 1971 | 5388 | 2864 | 2918 | 1551 | 1.88118 | 1.88118 |
| 1972 | 5757 | 2866 | 3183 | 1584 | 2.00889 | 2.00889 |
| 1973 | 6307 | 2862 | 3451 | 1566 | 2.20366 | 2.20366 |
| 1974 | 7107 | 2923 | 3787 | 1558 | 2.43126 | 2.43126 |
| 1975 | 8313 | 2992 | 4290 | 1544 | 2.77854 | 2.77854 |
| 1976 | 10038 | 3072 | 4842 | 1482 | 3.26746 | 3.26746 |
| 1977 | 12126 | 3084 | 5443 | 1384 | 3.93199 | 3.93199 |

[75] Somewhat mysteriously, this constant dollar series extends all the way to 2006 whereas the corresponding current dollar series ends at 2004. As noted above, we extended the industry price index for imputed rents from 2004 to 2006 using the movements in the corresponding expenditure based price index for imputed rents over the years 2004-2006. Given this extended price index plus the industry based constant dollar series for imputed rents, the industry based value series for imputed rents was extended to 2006.

[76] The units for all value and quantity series in this Appendix are millions of current dollars for the V series and millions of 1961 dollars for the Q series.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1978 | 14090 | 3051 | 6106 | 1322 | 4.61807 | 4.61807 |
| 1979 | 15797 | 2996 | 6829 | 1295 | 5.27283 | 5.27283 |
| 1980 | 17869 | 3053 | 7686 | 1313 | 5.85278 | 5.85278 |
| 1981 | 20512 | 3159 | 8822 | 1359 | 6.49322 | 6.49322 |
| 1982 | 23489 | 3213 | 10082 | 1410 | 7.31046 | 7.15154 |
| 1983 | 26285 | 3256 | 11295 | 1444 | 8.07270 | 7.82159 |
| 1984 | 28446 | 3294 | 12181 | 1471 | 8.63567 | 8.28079 |
| 1985 | 30694 | 3360 | 12967 | 1500 | 9.13517 | 8.64482 |
| 1986 | 33386 | 3463 | 13955 | 1539 | 9.64089 | 9.06928 |
| 1987 | 36117 | 3573 | 15090 | 1599 | 10.10837 | 9.43653 |
| 1988 | 39587 | 3801 | 16419 | 1662 | 10.41493 | 9.87670 |
| 1989 | 44078 | 4011 | 18201 | 1726 | 10.98935 | 10.54481 |
| 1990 | 48016 | 4221 | 19786 | 1798 | 11.37552 | 11.00446 |
| 1991 | 51779 | 4469 | 21133 | 1853 | 11.58636 | 11.40566 |
| 1992 | 54872 | 4627 | 22269 | 1899 | 11.8590 | 11.72872 |
| 1993 | 57263 | 4770 | 23108 | 1943 | 12.00486 | 11.89235 |
| 1994 | 60557 | 4887 | 24056 | 1982 | 12.39142 | 12.13504 |
| 1995 | 63613 | 5001 | 24869 | 2016 | 12.72013 | 12.33820 |
| 1996 | 65418 | 5116 | 25632 | 2049 | 12.78691 | 12.51068 |
| 1997 | 67405 | 5245 | 26425 | 2097 | 12.85127 | 12.59838 |
| 1998 | 69835 | 5389 | 27223 | 2139 | 12.95872 | 12.72809 |
| 1999 | 72144 | 5557 | 28173 | 2187 | 12.98263 | 12.87911 |
| 2000 | 74582 | 5704 | 29059 | 2231 | 13.07545 | 13.02515 |
| 2001 | 77093 | 5843 | 30092 | 2279 | 13.19410 | 13.20509 |
| 2002 | 80895 | 6074 | 31491 | 2341 | 13.31831 | 13.44940 |
| 2003 | 83916 | 6250 | 32829 | 2413 | 13.42651 | 13.60407 |
| 2004 | 87614 | 6482 | 34133 | 2487 | 13.51648 | 13.72248 |
| 2005 | 91502 | 6730 | 35435 | 2561 | 13.59609 | 13.83837 |
| 2006 | 96665 | 6985 | 37137 | 2638 | 13.83893 | 14.07831 |

Recall the price and quantity series for a consumption aggregate (which includes all rents, paid and imputed), $P_{CT}$ and $Q_{CT}$, along with the two price and quantity series for imputed and paid rents in Table 1 above. We changed the sign of the rent quantity series from plus to minus and then calculated a chained Fisher net consumption aggregate by aggregating all consumption (plus sign on the quantities) and rents (negative sign on the quantities). The resulting price and quantity series should closely approximate the price and quantity of consumption excluding housing services. However, the price series includes indirect taxes (less subsidies) on outputs but for productivity measurement purposes, as mentioned earlier, these tax wedges should be excluded. Statistics Canada has a series for indirect taxes less subsidies on products $V_{IT}{}^{t}$, CANSIM II series V1997473, for the years 1961-2006. We subtracted two other tax series from this indirect tax series because these other tax series will be taken into account separately in the price of exports of goods (this is the Oil Export Tax series, CANSIM series V499746) and in the price of imports of goods (this is the Customs Import Duties series, CANSIM series V499741). The resulting indirect taxes less subsidies on products (less trade taxes) series was used to remove the tax wedges on the price of consumption series. The resulting price and quantity of consumption series, $P_C^t$ and $Q_C^t$, are listed in Tables 2 and 3 below.[77]

We turn our attention to the investment components of final demand. Current dollar government gross fixed capital formation is available as CANSIM II series V498093 for the years 1961-2006. The corresponding chained 2002 dollar series is CANSIM II series V1992050 and we use these two series to form price and quantity series for general government sector investment, $P_{IG}^t$ and $Q_{IG}^t$, which are listed in Tables 1 and 2 below.[78]

The current and constant chained dollar series for the years 1961-2006 for residential structures investment can be obtained as CANSIM II series V498096 and V1992053 respectively, the current and constant chained dollar series for nonresidential structures investment can be obtained as CANSIM II series V498098 and V1992053 respectively and the current and constant chained dollar series for machinery and equipment investment can be obtained as CANSIM II series V498099 and V1992056 respectively. The resulting price and quantity series are listed as $P_{IR}^t$ , $P_{INR}^t$, $P_{IME}^t$ and $Q_{IR}^t$, $Q_{INR}^t$, $Q_{IME}^t$ in Tables 2 and 3 below. The price and quantity series for inventory change are $P_{II}^t$ and $Q_{II}^t$ respectively but the description of how they were constructed is deferred until we discuss how we formed estimates of the beginning of the year stocks of inventories.

**Table 2: Price Indexes for Business Sector Outputs: Consumption and Investment**

| Year t | $P_C^t$ | $P_{IG}^t$ | $P_{IR}^t$ | $P_{IME}^t$ | $P_{INR}^t$ | $P_{II}^t$ |
|---|---|---|---|---|---|---|
| 1961 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 1962 | 1.00538 | 1.00855 | 1.00504 | 1.01973 | 1.00592 | 1.01546 |
| 1963 | 1.02055 | 1.03939 | 1.02769 | 1.03352 | 1.03251 | 1.00186 |
| 1964 | 1.02437 | 1.06231 | 1.07312 | 1.06837 | 1.06158 | 1.04471 |
| 1965 | 1.03690 | 1.13926 | 1.13368 | 1.09293 | 1.12281 | 1.06126 |
| 1966 | 1.07553 | 1.21007 | 1.20765 | 1.11917 | 1.19323 | 1.08837 |
| 1967 | 1.11050 | 1.23160 | 1.28518 | 1.13235 | 1.24188 | 1.11396 |
| 1968 | 1.15168 | 1.23844 | 1.31431 | 1.14980 | 1.25227 | 1.13529 |
| 1969 | 1.18980 | 1.28464 | 1.38118 | 1.19542 | 1.32495 | 1.15519 |
| 1970 | 1.22208 | 1.33877 | 1.42615 | 1.24395 | 1.39058 | 1.17977 |
| 1971 | 1.24828 | 1.40873 | 1.53179 | 1.28144 | 1.46812 | 1.18180 |
| 1972 | 1.29847 | 1.48528 | 1.67349 | 1.32335 | 1.55098 | 1.20129 |
| 1973 | 1.38744 | 1.64838 | 1.97123 | 1.37155 | 1.71873 | 1.31746 |
| 1974 | 1.58382 | 2.01078 | 2.36134 | 1.52083 | 2.03419 | 1.56585 |
| 1975 | 1.82198 | 2.23421 | 2.56072 | 1.70347 | 2.27337 | 1.91110 |
| 1976 | 1.90726 | 2.34499 | 2.76853 | 1.82172 | 2.40093 | 1.95883 |
| 1977 | 2.03175 | 2.48990 | 2.87768 | 1.95440 | 2.52980 | 1.98462 |
| 1978 | 2.19264 | 2.66206 | 3.04069 | 2.13274 | 2.71145 | 2.08729 |

[77] We renormalize all price and quantity series so that the normalized price is 1 in 1961. The units for quantity and value series in this Appendix are in millions of current and 1961 dollars respectively.
[78] The price series for investment should be adjusted for indirect taxes that fall on investment outputs. Since these taxes are relatively small and it is difficult to collect consistent information on these taxes over our sample period, we neglect these indirect tax wedges on investment components of final expenditure.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1979 | 2.40645 | 2.88522 | 3.28046 | 2.31511 | 2.96312 | 2.31871 |
| 1980 | 2.69497 | 3.19858 | 3.55455 | 2.49752 | 3.32520 | 2.69159 |
| 1981 | 2.95335 | 3.68313 | 3.99273 | 2.79726 | 3.68676 | 3.05380 |
| 1982 | 3.22860 | 3.92658 | 4.08226 | 3.01411 | 3.96113 | 3.34545 |
| 1983 | 3.46323 | 4.01498 | 4.25350 | 3.08751 | 3.93090 | 3.39571 |
| 1984 | 3.61506 | 4.17063 | 4.41785 | 3.09060 | 4.08142 | 3.49878 |
| 1985 | 3.72257 | 4.20827 | 4.55564 | 3.12859 | 4.21351 | 3.58540 |
| 1986 | 3.80422 | 4.20267 | 4.90827 | 3.16319 | 4.27520 | 3.61840 |
| 1987 | 3.89726 | 4.22375 | 5.40819 | 3.11015 | 4.47320 | 3.66603 |
| 1988 | 4.00205 | 4.33769 | 5.78293 | 3.06780 | 4.72840 | 3.77235 |
| 1989 | 4.11690 | 4.43728 | 6.13195 | 3.07778 | 4.92520 | 3.93005 |
| 1990 | 4.35206 | 4.53066 | 6.11231 | 3.09245 | 5.08853 | 3.95564 |
| 1991 | 4.59099 | 4.31837 | 6.32257 | 2.93829 | 5.00311 | 3.94467 |
| 1992 | 4.65258 | 4.31896 | 6.39710 | 2.95425 | 4.97541 | 3.81266 |
| 1993 | 4.74252 | 4.34342 | 6.58445 | 3.01578 | 5.03758 | 3.96305 |
| 1994 | 4.77089 | 4.42033 | 6.76485 | 3.11452 | 5.20497 | 4.09862 |
| 1995 | 4.79147 | 4.51572 | 6.76717 | 3.12739 | 5.27332 | 4.17589 |
| 1996 | 4.88952 | 4.53812 | 6.75581 | 3.10234 | 5.43035 | 4.41960 |
| 1997 | 4.96547 | 4.57906 | 6.87512 | 3.10301 | 5.56694 | 4.04774 |
| 1998 | 5.03224 | 4.59706 | 6.95993 | 3.14384 | 5.71450 | 3.74178 |
| 1999 | 5.12045 | 4.57201 | 7.13210 | 3.04811 | 5.82995 | 3.74178 |
| 2000 | 5.25425 | 4.68967 | 7.29782 | 3.02212 | 6.02775 | 3.89921 |
| 2001 | 5.40970 | 4.68012 | 7.48766 | 3.06002 | 6.07934 | 3.93949 |
| 2002 | 5.47743 | 4.72977 | 7.81242 | 3.08815 | 6.18175 | 3.93949 |
| 2003 | 5.61543 | 4.72659 | 8.21290 | 2.88779 | 6.30506 | 3.88823 |
| 2004 | 5.69263 | 4.79584 | 8.71303 | 2.76004 | 6.69854 | 4.00905 |
| 2005 | 5.80796 | 4.89284 | 9.11300 | 2.67406 | 7.07710 | 4.08228 |
| 2006 | 5.90800 | 5.02988 | 9.77854 | 2.57738 | 7.39198 | 4.16648 |

**Table 3: Quantity Indexes for Business Sector Outputs: Consumption and Investment**

| Year t | $Q_C^t$ | $Q_{IG}^t$ | $Q_{IR}^t$ | $Q_{IME}^t$ | $Q_{INR}^t$ | $Q_{II}^t$ |
|---|---|---|---|---|---|---|
| 1961 | 20265 | 1887 | 2211 | 2144 | 2618 | -105 |
| 1962 | 21331 | 2094 | 2271 | 2322 | 2545 | 776 |
| 1963 | 22290 | 2101 | 2354 | 2556 | 2637 | 474 |
| 1964 | 23529 | 2141 | 2715 | 3027 | 3050 | 548 |
| 1965 | 24974 | 2426 | 2825 | 3611 | 3320 | 1144 |
| 1966 | 26240 | 2668 | 2699 | 4337 | 3802 | 1152 |
| 1967 | 27228 | 2718 | 2754 | 4350 | 3613 | 300 |
| 1968 | 28525 | 2758 | 3132 | 3984 | 3593 | 695 |
| 1969 | 29923 | 2700 | 3551 | 4307 | 3592 | 1546 |
| 1970 | 30450 | 2645 | 3254 | 4419 | 3946 | 416 |
| 1971 | 32321 | 2985 | 3728 | 4537 | 4089 | 351 |
| 1972 | 34891 | 2938 | 4066 | 4940 | 4074 | 103 |
| 1973 | 37676 | 2781 | 4371 | 5993 | 4396 | 472 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1974 | 39789 | 2845 | 4464 | 6737 | 4675 | 1519 |
| 1975 | 41468 | 2962 | 4386 | 7121 | 5286 | 669 |
| 1976 | 43911 | 2855 | 5172 | 7363 | 5168 | 1598 |
| 1977 | 45480 | 2916 | 5242 | 7260 | 5479 | 2099 |
| 1978 | 47255 | 2875 | 5291 | 7492 | 5626 | 1138 |
| 1979 | 48694 | 2803 | 5251 | 8526 | 6337 | 1627 |
| 1980 | 49521 | 2869 | 4977 | 9054 | 7055 | -965 |
| 1981 | 49914 | 2967 | 5279 | 10142 | 7620 | 270 |
| 1982 | 48210 | 3095 | 4340 | 8597 | 6929 | -2785 |
| 1983 | 49567 | 2991 | 5079 | 8207 | 6361 | -509 |
| 1984 | 51955 | 3124 | 5131 | 8696 | 6288 | 1572 |
| 1985 | 54842 | 3497 | 5578 | 9652 | 6590 | 1313 |
| 1986 | 56973 | 3485 | 6267 | 10605 | 6210 | 961 |
| 1987 | 59433 | 3621 | 7190 | 12171 | 6454 | 1135 |
| 1988 | 61835 | 3789 | 7340 | 14394 | 7110 | 985 |
| 1989 | 63760 | 4184 | 7640 | 15424 | 7345 | 1446 |
| 1990 | 64001 | 4463 | 6835 | 14706 | 7346 | -387 |
| 1991 | 62103 | 4692 | 5824 | 14271 | 7075 | -873 |
| 1992 | 62821 | 4621 | 6238 | 14120 | 5960 | -1660 |
| 1993 | 63783 | 4560 | 6024 | 13731 | 5993 | -338 |
| 1994 | 65822 | 4894 | 6271 | 15058 | 6533 | 1135 |
| 1995 | 67161 | 4740 | 5340 | 16239 | 6574 | 1756 |
| 1996 | 68967 | 4536 | 5852 | 17230 | 6696 | 4044 |
| 1997 | 72495 | 4390 | 6330 | 21703 | 7881 | 5117 |
| 1998 | 74536 | 4361 | 6106 | 23575 | 7906 | 3 071 |
| 1999 | 77521 | 5039 | 6324 | 25951 | 8101 | 447 |
| 2000 | 80901 | 5229 | 6656 | 27580 | 8266 | 3484 |
| 2001 | 82720 | 5830 | 7363 | 26758 | 8712 | -1102 |
| 2002 | 85721 | 6044 | 8403 | 25995 | 8195 | 1165 |
| 2003 | 88311 | 6370 | 8854 | 27991 | 8651 | -2683 |
| 2004 | 91247 | 6690 | 9517 | 30592 | 9268 | 944 |
| 2005 | 94798 | 7418 | 9853 | 33884 | 10269 | 1782 |
| 2006 | 98870 | 8019 | 10061 | 36394 | 11593 | 1495 |

All of the outputs described above can be regarded as outputs produced by the business sector and sold to final demanders. However, the business sector also sells goods and services to the nonbusiness sector and it also purchases smaller amounts of goods and services from the nonbusiness sector. We now describe how we formed price and quantity estimates for the net sales of the business sector to the nonbusiness sector.

For the years 1961-2006 from the National Income and Expenditure Accounts, CANSIM II series V498092; Government Current Expenditure on Goods and Services, Table 3800002, we have estimates of total government gross current expenditure on goods and services (less sales of goods and services to the business sector) in current dollars. From the same Table and for the same years, CANSIM II series V1992049; Government Current Expenditure on Goods and Services, Table 3800002, we have estimates of total government gross current expenditure on goods and services (less sales of goods and services to the business sector) in chained 2002 dollars. We use these two series to form price and quantity series for final demand government sector expenditures, $P_G^t$ and $Q_G^t$, which are listed in Table 4.

Recall that the Statistics Canada KLEMS productivity program business sector value added aggregate *includes* rental residential housing but *excludes* the services of owned residential housing (whereas our business sector value added aggregate *excludes* all forms of residential rents). The Industry Division of Statistics Canada produces yet another business sector estimate of nominal and real value added (at factor cost) which *includes* all residential rents, both imputed and paid. We will denote this value added aggregate by $V_B^t$ in year t. Statistics Canada also produces a companion nonbusiness sector value added aggregate (at factor cost) which we will denote by $V_N^t$ in year t. If the value of indirect taxes less subsidies on products for year t, $V_{IT}^t$, is added to the sum of these two industry value added aggregates, we get an estimate of the value of GDP at final demand prices in year t; i.e., we have the following identity:

(1) $V_B^t + V_N^t + V_{IT}^t = V_{GDP}^t$ .

We will now describe how we formed estimates for $V_B^t$ and $V_N^t$ along with the corresponding price and quantity decompositions. From Table 3790024, Gross Domestic Product (GDP) at Basic Prices in Current Dollars, SNA, Benchmark Values, Special Industry Aggregations Based on the North American Industry Classification System (NAICS), we can obtain the $V_B^t$ series (title is Canada: Business Sector Industries) for the years 1961-2004 from CANSIM II Series V3860037. From the same Table 3790024, we can obtain the $V_N^t$ series (title is Canada: Non-Business Sector Industries) for the years 1961-2004 from CANSIM II Series V3860040. We can obtain price indexes $P_B^t$, $P_N^t$ and quantity indexes $Q_B^t$,$Q_N^t$ for $V_B^t$ and $V_N^t$ for the years 1961-1997 by using the series V334562, V335071, V334565 and V335074 from CANSIM Table 3790002, Gross Domestic Product (GDP) at Factor Cost, System of National Accounts Benchmark Values by Industry (Special Aggregations). These series give business and nonbusiness sector value added at basic prices in current dollars and in constant 1992 dollars. Using CANSIM Table 3790020, we can find estimates for $Q_B^t$ (Series V14182646) and for $Q_N^t$ (Series V14182651) in chained 1997 dollars for the years 1997-2006. Hence using these series in conjunction with our earlier value series $V_B^t$ and $V_N^t$ which run from 1961 to 2004, we can obtain price series for business and nonbusiness sector value added at basic prices for the years 1997-2004. These price series can be linked to our earlier price series $P_B^t$ and $P_N^t$ which extended to 1997 so that the resulting price series will run from 1961 to 2004. However, we still do not have price or value series for the B and N sectors for 2004-2006, although we do have quantity series for these years. We extend the price series $P_N^t$ from 2004 using the movements in the overall Producer Price Index over the years 2004-2006; see CANSIM II Series V1574377, Industrial Product Price Index; Canada; Total; All Commodities in Table 3290039: Industry Price Indexes, by Major Commodity Aggregation and Stage of Processing. It turns out that the total of $V_B^t$ and $V_N^t$ is available in another CANSIM II V3860274. Canada, Gross Domestic Product (GDP) at Basic Prices in Table 3800030: GDP and GNP at Market Prices and Net

National Income at Basic Prices. Thus we have enough information to deduce the price $P_B^t$ and the value of business sector output $V_B^t$ for the years 2004-2006. The business and nonbusiness sector price and quantity series, $P_B^t$, $P_N^t$ and $Q_B^t$, $Q_N^t$ for real value added at basic prices are listed in Table 4 below.

**Table 4: Business Sector, Nonbusiness Sector, Government Final Demand and KLEMS Business Sector Price and Quantity Aggregates**

| Year t | $Q_B^t$ | $Q_N^t$ | $Q_G^t$ | $Q_{BKLEMS}^t$ | $P_B^t$ | $P_N^t$ | $P_G^t$ | $P_{BKLEMS}^t$ |
|---|---|---|---|---|---|---|---|---|
| 1961 | 33097 | 5204 | 6624 | 30805 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 1962 | 35338 | 5480 | 6928 | 33059 | 1.00919 | 1.03863 | 1.02916 | 1.00509 |
| 1963 | 37217 | 5713 | 7164 | 35013 | 1.02992 | 1.08205 | 1.05990 | 1.01881 |
| 1964 | 39810 | 5952 | 7542 | 37567 | 1.04877 | 1.14227 | 1.09761 | 1.03600 |
| 1965 | 42658 | 6120 | 7883 | 40122 | 1.07554 | 1.21527 | 1.15160 | 1.06938 |
| 1966 | 45529 | 6409 | 8581 | 42827 | 1.12248 | 1.34490 | 1.24333 | 1.11745 |
| 1967 | 46616 | 6870 | 9334 | 43728 | 1.16053 | 1.45671 | 1.32836 | 1.15516 |
| 1968 | 49335 | 7263 | 9944 | 46133 | 1.19304 | 1.54780 | 1.41575 | 1.18948 |
| 1969 | 51965 | 7585 | 10376 | 48537 | 1.23452 | 1.71317 | 1.53846 | 1.23074 |
| 1970 | 52968 | 7962 | 11287 | 49889 | 1.29437 | 1.84146 | 1.64279 | 1.27609 |
| 1971 | 55844 | 8255 | 11631 | 51843 | 1.33615 | 1.95964 | 1.75921 | 1.33535 |
| 1972 | 59086 | 8549 | 11995 | 54998 | 1.40300 | 2.12810 | 1.89268 | 1.40260 |
| 1973 | 63467 | 8887 | 12559 | 59206 | 1.54872 | 2.31234 | 2.04912 | 1.55366 |
| 1974 | 65346 | 9295 | 13357 | 61310 | 1.79635 | 2.65779 | 2.33927 | 1.79872 |
| 1975 | 65545 | 9790 | 14251 | 62061 | 2.03754 | 3.05325 | 2.65962 | 2.01795 |
| 1976 | 70082 | 10097 | 14525 | 66118 | 2.17572 | 3.45337 | 2.99471 | 2.15434 |
| 1977 | 72425 | 10348 | 15205 | 68823 | 2.32657 | 3.73913 | 3.24567 | 2.27212 |
| 1978 | 74875 | 10644 | 15473 | 71979 | 2.51505 | 3.96850 | 3.45708 | 2.42047 |
| 1979 | 77878 | 10805 | 15635 | 75134 | 2.80145 | 4.29236 | 3.77635 | 2.69350 |
| 1980 | 79169 | 11138 | 16169 | 76938 | 3.12982 | 4.66836 | 4.14895 | 2.98837 |
| 1981 | 81847 | 11496 | 16441 | 80244 | 3.40152 | 5.22919 | 4.64970 | 3.21385 |
| 1982 | 78970 | 11693 | 16767 | 77088 | 3.65996 | 5.83630 | 5.18504 | 3.44460 |
| 1983 | 81077 | 11952 | 17045 | 79192 | 3.89493 | 6.12278 | 5.48059 | 3.65571 |
| 1984 | 86041 | 12198 | 17222 | 84752 | 4.03944 | 6.37296 | 5.69544 | 3.76522 |
| 1985 | 90944 | 12471 | 17959 | 89260 | 4.13895 | 6.57967 | 5.90593 | 3.87314 |
| 1986 | 93580 | 12708 | 18283 | 91514 | 4.19906 | 6.81314 | 6.09362 | 3.92907 |
| 1987 | 97824 | 12840 | 18525 | 96022 | 4.38354 | 7.17180 | 6.36273 | 4.08969 |
| 1988 | 102723 | 13057 | 19370 | 100981 | 4.58086 | 7.53056 | 6.60377 | 4.26785 |
| 1989 | 105427 | 13224 | 19903 | 103685 | 4.75619 | 8.03351 | 6.95695 | 4.41096 |
| 1990 | 106128 | 13541 | 20605 | 103235 | 4.85194 | 8.60166 | 7.34874 | 4.52286 |
| 1991 | 104194 | 13849 | 21208 | 99027 | 4.90597 | 9.01919 | 7.64969 | 4.63908 |
| 1992 | 105171 | 14045 | 21414 | 99628 | 4.92162 | 9.36188 | 7.88200 | 4.64471 |
| 1993 | 108151 | 14150 | 21422 | 102483 | 4.97722 | 9.50865 | 7.98997 | 4.69377 |
| 1994 | 113766 | 14218 | 21156 | 108795 | 5.08269 | 9.55910 | 8.11082 | 4.75837 |
| 1995 | 117124 | 14279 | 21034 | 112401 | 5.23638 | 9.61980 | 8.19895 | 4.89045 |
| 1996 | 119744 | 14025 | 20786 | 114956 | 5.33957 | 9.72826 | 8.23447 | 4.99301 |
| 1997 | 125797 | 13787 | 20579 | 121417 | 5.40168 | 9.95401 | 8.34626 | 5.04144 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1998 | 131475 | 13890 | 21240 | 127127 | 5.37429 | 10.07510 | 8.44225 | 5.00882 |
| 1999 | 139515 | 14320 | 21687 | 135542 | 5.47529 | 10.18237 | 8.57899 | 5.10349 |
| 2000 | 147808 | 14614 | 22356 | 144108 | 5.71191 | 10.65177 | 8.94989 | 5.34163 |
| 2001 | 149733 | 14926 | 23229 | 146512 | 5.80825 | 10.88586 | 9.11375 | 5.41622 |
| 2002 | 153895 | 15241 | 23802 | 150269 | 5.82601 | 11.29659 | 9.42889 | 5.42824 |
| 2003 | 156933 | 15608 | 24551 | 153124 | 6.02555 | 11.73665 | 9.71085 | 5.62807 |
| 2004 | 162130 | 15921 | 25174 | 158233 | 6.23215 | 11.96951 | 9.88601 | 5.83194 |
| 2005 | 167081 | 16154 | 25725 | 162891 | 6.49092 | 12.14802 | 10.20999 | 6.09614 |
| 2006 | 171718 | 16519 | 26578 | 167249 | 6.66061 | 12.43401 | 10.52761 | 6.26057 |

It is also of some interest to compare the price and quantity of the above Industry Division business sector prices and quantities $P_B^t$ and $Q_B^t$ with the corresponding business sector prices and quantities $P_{BKLEMS}^t$ and $Q_{BKLEMS}^t$ that originate with the Statistics Canada productivity program.[79] These series are also listed in Table 4. The source for $Q_{BKLEMS}^t$ for the years 1961-2006 is CANSIM II series V41712932: Canada, Real Gross Domestic Product (GDP), Business Sector from Table 3830021: Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Subsectors by the North American Industry Classification (NAICS). The corresponding nominal value added series $V_{BKLEMS}^t$ is available in the same table for the years 1961-2003 as CANSIM II series V41713153: Canada: Gross Domestic Product (GDP), Business Sector. The values $V_{BKLEMS}^t$ for the missing years 2004-2006 can be obtained by adding the value of imputed rents, $V_{IMR}^t$, to the Industry Division value added for the Business Sector, $V_B^t$. Finally, $P_{BKLEMS}^t$ can be obtained by dividing $V_{BKLEMS}^t$ by $Q_{BKLEMS}^t$. As usual, we normalized the resulting price and quantity series so that $P_{BKLEMS}^t$ equals 1 when t equals 1961. The resulting $P_{BKLEMS}^t$ and $Q_{BKLEMS}^t$ are listed in Table 4.

Recall the GDP identity defined by (1) above, which expressed the nominal value of GPD, $V_{GDP}^t$, at final demand prices as being equal to the value added of the Industry Division business sector value added at basic prices, $V_B^t$, plus nonbusiness sector value added, $V_N^t$, plus the value of indirect taxes less subsidies on products, $V_{IT}^t$. We can also express the value of GDP at final demand prices as the familiar sum of final demand values; i.e., as the following sum of final demand expenditures on consumption plus investment plus government expenditures on goods and services plus exports less imports:

(2) $V_{GDP}^t = V_{CT}^t + V_I^t + V_G^t + V_X^t - V_M^t$.

Define a new consumption aggregate at basic prices $V_{CN}^t$ as the value of consumption at final demand prices, $V_{CT}^t$, less indirect taxes less subsidies on products, $V_{IT}^t$:

(3) $V_{CN}^t \equiv V_{CT}^t - V_{IT}^t$ .

[79] Recall that the Productivity Program business sector value added aggregate $V_{KLEMS}^t$ should be equal to the Industry Division value added aggregate $V_B^t$ less the value of imputed rents from the Industry Division, $V_{IMR}^t$.

Now equate the two expressions for the value of GDP given by (1) and (2) and use the resulting equation to express business sector value added $V_B{}^t$ in terms of final demand components and the value of nonbusiness sector value added $V_N{}^t$. Making use of (3), the resulting equation is the following one:[80]

(4) $V_B{}^t = V_{CN}{}^t + V_I{}^t + V_X{}^t - V_M{}^t + (V_G{}^t - V_N{}^t)$.

Conceptually, the aggregate $V_G{}^t - V_N{}^t$ should be equal to the sales of the business sector of goods and services to the nonbusiness sector less the purchases of intermediate inputs of the business sector from the nonbusiness sector. Put another way, the business sector's net sales of goods and services should equal its net deliveries to final demand sectors ($V_C{}^t + V_I{}^t + V_X{}^t - V_M{}^t$) plus its net deliveries to the nonbusiness sector ($V_G{}^t - V_N{}^t$).

Recall that we did not use the Industry Division's concept of Business Sector value added; we subtracted the value of imputed and paid residential rent from our business sector aggregate. Let $V_R{}^t$ be equal to the sum of imputed residential rent $V_{IMR}{}^t$ and paid residential rent $V_{IMR}{}^t$ (see Table 1 for these series). Conceptually, if we subtract rents $V_R{}^t$ from $V_{CN}{}^t$, we should get $V_C{}^t$, the consumption aggregate whose price and quantity is listed in Tables 2 and 3 above. Thus subtracting $V_R{}^t$ from both sides of (4) leads to the following identity:

(5) $V_B{}^t - V_R{}^t = V_C{}^t + V_I{}^t + V_X{}^t - V_M{}^t + (V_G{}^t - V_N{}^t)$.

Thus our business sector value added aggregate can be formed using either the left or right hand sides of the identity (5). We will use the right hand side of (5) to form our value measure of business sector net output since we want to focus on the effects of changing international prices on the performance of the business sector.

How should the corresponding real quantities that correspond to the value aggregates on either side of (5) be calculated? Obviously, each cell in the supply and use tables that correspond to the value aggregate on the left hand side of (5) could be aggregated up using a chained superlative index number formula provided that an appropriate price deflator were available for each cell.[81] On the other hand, the value cells that are components on the right hand side of (5) that correspond to final demand components (at basic prices) could be aggregated up using a chained superlative index number formula. We can then ask: under what conditions would the corresponding quantity aggregates be equal? This question is addressed by Moyer, Reinsdorf and Yuskavage (2006) and in more detail by Diewert (2006b) (2007b) (2007c). The answer to this question is that if the detailed data are constructed in an appropriate manner and the Fisher formula is used, then the direct industry aggregation and the aggregation of final demand component

[80] The identity (4) is not quite consistent with our treatment of indirect taxes less subsidies since we also made some indirect tax adjustments to the prices of exports and imports as explained above; i.e., since we used a slight modification of (3) to adjust final demand consumption prices for indirect tax wedges, we used a corresponding slight modification of the identity (4).

[81] Quantities in the Make matrix would have a positive sign while quantities in the Use matrix would have a negative sign.

approaches are perfectly consistent.[82] In addition, if two stage aggregation procedures are used and a superlative index number formula is used at each stage of aggregation, then the theoretical and empirical results in Diewert (1978) show that the commonly used single stage superlative indexes will approximate their two or more stage counterparts to a high degree of approximation if the chain principle is used.[83]

Using the above results, we will construct our measure of business sector real value added by aggregating up the value components on the right hand side of (5). Rather than work with both $V_G^t$ and $V_N^t$ as final demand components, we will aggregate over these two components to form the value aggregate $V_{GN}^t$ equal to $(V_G^t - V_N^t)$, and conceptually, this value aggregate should be equal to the net deliveries of goods and services of our business sector to the nonbusiness sector less the purchases of intermediate inputs by our business sector from the nonbusiness sector. The year t price and quantity aggregates, $P_{GN}^t$ and $Q_{GN}^t$, that correspond to these value aggregates $V_{GN}^t$ are calculated using chained Fisher indexes with $Q_N^t$ getting a negative weight in the index number formula. $P_{GN}^t$ and $Q_{GN}^t$ are listed in Tables 5 and 6 below.

We now turn our attention to the export and import components of final demand. Current dollar exports of goods are available as CANSIM II series V498104 for the years 1961-2006. The corresponding chained 2002 dollar series is CANSIM II series V1992061 and we use these two series to form price and quantity series for the exports of goods. However, during the years 1974-1985, Canada imposed an export tax on its energy exports, which is included in the price of exports. However, producers do not receive this export tax revenue and so it must be subtracted from the export price. This adjustment of the export price index for exports of goods can be accomplished using the Oil Export Tax series, CANSIM series V499746 from the National Income and Expenditure Accounts. After making this adjustment, the resulting price and quantity series are $P_{XG}^t$ and $Q_{XG}^t$, which are listed in Tables 5 and 6 below. Current dollar exports of services are available as CANSIM II series V498105 for the years 1961-2006. The corresponding chained 2002 dollar series is CANSIM II series V1992062 and we use these two series to form price and quantity series for the exports of services, $P_{XS}^t$ and $Q_{XS}^t$, which are listed in Tables 5 and 6 below.

A series on the current dollar imports of goods is available as CANSIM II series V498107 for the years 1961-2006. The corresponding chained 2002 dollar series is CANSIM II series V1992064 and we use these two series to form price and quantity series for the imports of goods. However, the price of imports does not include import duties that are added to the international cost of these imported goods. Hence we must add these import duties to the price of imports. The series on customs import duties is CANSIM II series V499741 and after adjusting the price of imports using this series, the resulting price and quantity series for the imports of goods are $P_{MG}^t$ and $Q_{MG}^t$, which are listed in Tables 5 and 6 below.

[82] See Diewert (2006b) (2007b) and the numerical examples in Diewert (2007c) in particular.

[83] The results of Hill (2006) show that these approximation results will not necessarily hold for mean of order r superlative indexes if r is large in magnitude.

Current dollar information on imports of services can be found as CANSIM II series V498108 for the years 1961-2006 and the corresponding constant 2002 chained dollar series is CANSIM II series V1992065. We use these two series to form price and quantity series for the imports of services, $P_{MS}^t$ and $Q_{MS}^t$, which are listed in Tables 5 and 6. Note that since imported goods and services are inputs into the business sector, when we form a value added aggregate, we need to append a minus sign to the quantity series $Q_{MG}^t$ and $Q_{MS}^t$.

**Table 5: Price Indexes for Business Sector Outputs: Net Sales to the Nonbusiness Sector, Exports and Imports**

| Year t | $P_{GN}^t$ | $P_{XG}^t$ | $P_{XS}^t$ | $P_{MG}^t$ | $P_{MS}^t$ |
|---|---|---|---|---|---|
| 1961 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 1962 | 0.99388 | 1.04455 | 1.01858 | 1.05474 | 1.05228 |
| 1963 | 0.97566 | 1.04973 | 1.04402 | 1.07448 | 1.07921 |
| 1964 | 0.92683 | 1.07005 | 1.07736 | 1.07395 | 1.09777 |
| 1965 | 0.91239 | 1.08404 | 1.12159 | 1.06213 | 1.13619 |
| 1966 | 0.88328 | 1.10919 | 1.18364 | 1.08026 | 1.17024 |
| 1967 | 0.89087 | 1.11893 | 1.26762 | 1.09002 | 1.22610 |
| 1968 | 0.96139 | 1.13150 | 1.34554 | 1.10503 | 1.28761 |
| 1969 | 0.96769 | 1.14924 | 1.41327 | 1.12868 | 1.36902 |
| 1970 | 1.00582 | 1.18288 | 1.50120 | 1.15320 | 1.42900 |
| 1971 | 1.10291 | 1.17700 | 1.57903 | 1.17572 | 1.49900 |
| 1972 | 1.14412 | 1.2203 | 1.66505 | 1.20080 | 1.53952 |
| 1973 | 1.22092 | 1.39836 | 1.79018 | 1.28178 | 1.63282 |
| 1974 | 1.35752 | 1.80131 | 2.02338 | 1.58601 | 1.75102 |
| 1975 | 1.48446 | 2.05662 | 2.29526 | 1.83178 | 1.98130 |
| 1976 | 1.64265 | 2.19269 | 2.51896 | 1.87866 | 2.04366 |
| 1977 | 1.78697 | 2.40482 | 2.72893 | 2.12290 | 2.34119 |
| 1978 | 1.92861 | 2.64048 | 2.90258 | 2.39739 | 2.69834 |
| 1979 | 2.18540 | 3.13733 | 3.14587 | 2.72170 | 3.04229 |
| 1980 | 2.48896 | 3.67058 | 3.49696 | 3.16197 | 3.39335 |
| 1981 | 2.79431 | 3.94012 | 3.94719 | 3.62497 | 3.81611 |
| 1982 | 3.10614 | 4.00326 | 4.34103 | 3.80741 | 4.10352 |
| 1983 | 3.37412 | 4.04346 | 4.66353 | 3.75688 | 4.30819 |
| 1984 | 3.48567 | 4.18504 | 4.87299 | 3.90211 | 4.63802 |
| 1985 | 3.67204 | 4.25442 | 5.13320 | 3.96293 | 4.98855 |
| 1986 | 3.74218 | 4.22373 | 5.40392 | 4.00581 | 5.29276 |
| 1987 | 3.79747 | 4.29990 | 5.59379 | 3.93924 | 5.28864 |
| 1988 | 3.78607 | 4.30102 | 5.72724 | 3.85763 | 5.09254 |
| 1989 | 3.82606 | 4.38240 | 5.93246 | 3.85625 | 5.09401 |
| 1990 | 3.86395 | 4.32868 | 6.08151 | 3.89841 | 5.24914 |
| 1991 | 3.92576 | 4.12347 | 6.29162 | 3.79761 | 5.31731 |
| 1992 | 3.94383 | 4.25649 | 6.33223 | 3.94324 | 5.63381 |
| 1993 | 3.96988 | 4.45524 | 6.51684 | 4.11472 | 6.19826 |
| 1994 | 4.17941 | 4.74286 | 6.66734 | 4.35323 | 6.68231 |

| | | | | | |
|---|---|---|---|---|---|
| 1995 | 4.29853 | 5.06556 | 6.88429 | 4.47580 | 6.90224 |
| 1996 | 4.20205 | 5.08273 | 7.03680 | 4.39309 | 7.01253 |
| 1997 | 4.10280 | 5.07304 | 7.22032 | 4.38567 | 7.26074 |
| 1998 | 4.13974 | 5.04123 | 7.34563 | 4.50744 | 7.80221 |
| 1999 | 4.29033 | 5.09188 | 7.46192 | 4.46610 | 7.97505 |
| 2000 | 4.43225 | 5.42723 | 7.72519 | 4.54728 | 8.18543 |
| 2001 | 4.45850 | 5.50634 | 7.74816 | 4.67149 | 8.64576 |
| 2002 | 4.56662 | 5.37580 | 7.87210 | 4.68373 | 8.84642 |
| 2003 | 4.56905 | 5.29147 | 7.93621 | 4.37054 | 8.39644 |
| 2004 | 4.62443 | 5.40859 | 8.09379 | 4.25298 | 8.24472 |
| 2005 | 5.04191 | 5.56207 | 8.29473 | 4.20188 | 8.14153 |
| 2006 | 5.30928 | 5.55557 | 8.43143 | 4.15888 | 8.17293 |

**Table 6: Quantity Indexes for Business Sector Outputs: Net Sales to the Nonbusiness Sector, Exports and Imports**

| Year t | $Q_{GN}{}^t$ | $Q_{XG}{}^t$ | $Q_{XS}{}^t$ | $Q_{MG}{}^t$ | $Q_{MS}{}^t$ |
|---|---|---|---|---|---|
| 1961 | 1420 | 6274 | 1036 | 6645 | 1535 |
| 1962 | 1447 | 6508 | 1132 | 6890 | 1480 |
| 1963 | 1446 | 7119 | 1204 | 7046 | 1467 |
| 1964 | 1596 | 8179 | 1286 | 7983 | 1619 |
| 1965 | 1798 | 8530 | 1360 | 9220 | 1692 |
| 1966 | 2320 | 9744 | 1492 | 10539 | 1850 |
| 1967 | 2684 | 10543 | 1865 | 11136 | 1935 |
| 1968 | 2950 | 12500 | 1499 | 12425 | 2014 |
| 1969 | 3068 | 13466 | 1657 | 13969 | 2337 |
| 1970 | 3858 | 14770 | 1767 | 13521 | 2472 |
| 1971 | 3885 | 15591 | 1747 | 14626 | 2476 |
| 1972 | 3942 | 17172 | 1721 | 17071 | 2586 |
| 1973 | 4247 | 18894 | 1893 | 19647 | 2888 |
| 1974 | 4819 | 17712 | 2095 | 21507 | 3278 |
| 1975 | 5397 | 16164 | 1998 | 20434 | 3550 |
| 1976 | 5254 | 17528 | 2048 | 21488 | 3971 |
| 1977 | 5963 | 18776 | 2052 | 21488 | 3883 |
| 1978 | 5833 | 20619 | 2273 | 22347 | 3824 |
| 1979 | 5796 | 21162 | 2556 | 24450 | 3637 |
| 1980 | 6062 | 21289 | 2658 | 23402 | 3760 |
| 1981 | 5845 | 21635 | 2738 | 23994 | 3860 |
| 1982 | 6019 | 21489 | 2508 | 19714 | 3594 |
| 1983 | 5998 | 22894 | 2535 | 22003 | 3690 |
| 1984 | 5838 | 27476 | 2685 | 26455 | 3775 |
| 1985 | 6539 | 28767 | 2839 | 28907 | 3905 |
| 1986 | 6635 | 29636 | 3254 | 30877 | 4268 |
| 1987 | 6789 | 30578 | 3295 | 32464 | 4536 |
| 1988 | 7814 | 33372 | 3546 | 36779 | 5185 |
| 1989 | 8423 | 33535 | 3704 | 38515 | 5792 |

| | | | | | |
|---|---|---|---|---|---|
| 1990 | 9043 | 35128 | 3857 | 38591 | 6406 |
| 1991 | 9508 | 35812 | 3893 | 39405 | 6664 |
| 1992 | 9458 | 38404 | 4157 | 41605 | 6739 |
| 1993 | 9223 | 42694 | 4519 | 45231 | 6865 |
| 1994 | 8537 | 48108 | 5093 | 49936 | 6755 |
| 1995 | 8165 | 52380 | 5396 | 53414 | 6763 |
| 1996 | 8263 | 55104 | 5851 | 56146 | 7111 |
| 1997 | 8414 | 59802 | 6264 | 65408 | 7374 |
| 1998 | 9511 | 64897 | 7085 | 69397 | 7367 |
| 1999 | 9380 | 72476 | 7400 | 75266 | 7684 |
| 2000 | 10022 | 79115 | 7937 | 81739 | 8114 |
| 2001 | 11040 | 76409 | 7967 | 77064 | 7954 |
| 2002 | 11443 | 77018 | 8276 | 78370 | 8091 |
| 2003 | 12088 | 75427 | 7983 | 80811 | 8831 |
| 2004 | 12607 | 79330 | 8189 | 87921 | 9391 |
| 2005 | 13172 | 81225 | 8270 | 95061 | 9816 |
| 2006 | 14015 | 82025 | 8185 | 100021 | 10188 |

We turn our attention to forming estimates of business sector labour input.

## 3. Business Sector Labour Input Estimates

Quality adjusted measures of the quantity of three types of labour for the years 1961-2006 are available from the Statistics Canada KLEMS productivity program; see CANSIM Table 3830021 which has the title: Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub-Sectors, by the North American Industry Classification System (NAICS). The three series are V41713000 (the title is Canada: Labour Input of Workers with Primary or Secondary Education; Business Sector), V41713017 (Labour Input of workers with Some or Completed Post-Secondary Certificate or Diploma; Business Sector) and V41713034 (Labour Input of Workers with University Degree or Above, Business Sector). The corresponding value of labour input or labour compensation series are found in the same Table and their CANSIM series numbers are V41713187, V41713204 and V41713221. These value series only cover the years 1961-2003.[84] These KLEMS labour series allowed us to construct the three business sector labour input series $Q_{L1}^t$, $Q_{L2}^t$ and $Q_{L3}^t$ for the years 1961-2006 (see Table 7 below for a listing of these data) and the corresponding wage index series $P_{L1}^t$, $P_{L2}^t$ and $P_{L3}^t$ for the years 1961-2003 (see Table 7).

The Statistics Canada productivity program aggregate labour input measure is described as follows:

"The labour input is an aggregate of the hours worked of all persons classified by their education, work experience and class of employment (paid versus self-employed workers). This aggregate labour input measure is constructed by aggregating hours at work data for each of 56 types of workers classified by their

[84] This is very puzzling: the quantity series run from 1961 to 2006; why stop the corresponding value series at 2003?

educational attainment (4), work experience (7) and class of workers (2) using an annual chained-Fisher index. The effect of Fisher aggregation is to produce a measure of labour input that reflects both changes in total hours of work and changes in the composition of workers." John R. Baldwin, Wulong Gu and Beiling Yan (2007; 37).

Baldwin, Gu and Yan (2007; 26) describe their more disaggregated measures of labour input as follows:

"Labour input for MFP measures reflects the compositional shifts of workers by education, experience and class of workers (paid versus self-employed). The growth of labour input (labour services) is an aggregate of the growth of hours worked by different classes of workers, weighted by the hourly wages of each class."

Thus each of the three types of labour classified by educational attainment $Q_{L1}{}^{t}$, $Q_{L2}{}^{t}$ and $Q_{L3}{}^{t}$ is a Fisher quantity aggregate over the other characteristics, holding constant the relevant educational levels. Baldwin, Gu and Yan (2007; 26) also comment on the difficulties associated with breaking up the net operating surplus generated by the self employed into labour and capital compensation components:

"We have modified the assumptions about the share of labour going to the self-employed workers to reflect changes that occurred during the 1990s. In the past, it had been assumed that the self employed essentially earned incomes similar to the employed. The Census of Population up to 1990 showed that this was a reasonable assumption; however, during the 1990s, self-employed income fell behind that of production workers. The new measure of self-employed for calculating labour input assumes that the hourly earning of self-employed workers is proportional to that of paid workers with the same level of education and experience. The proportional or scaling factor for each level of education and experience is based on the relative hourly earnings of paid versus self-employed workers derived from the Census of Population."

Overall, we believe that Statistics Canada has done an excellent job in constructing their new measures of labour input and we will use these measures in the present study.[85] The effect of using the Statistics Canada measures of quality adjusted labour input is to increase the growth of labour input by about 37% over the sample period compared to using hours worked as the measure of labour input.[86] Basically, there was a big shift in labour inputs from less skilled and less educated workers to more educated workers over this period which served to greatly increase quality adjusted labour input compared to unweighted hours worked by all types of labour.

As noted above, the KLEMS estimates of real labour input for the three types of labour run from 1961-2006 but the corresponding value series stop at 2003. Hence we need to

[85] The labour input that is used in the residential rental of housing industry should be deducted from our measure of labour input (since we exclude all residential housing outputs from our definition of the business sector while the KLEMS program business sector excludes only the services of Owner Occupied Housing). However, the KLEMS data base that is available in CANSIM does not include information on the three types of labour input that is used in the residential housing rental industry so we were not able to deduct these labour inputs from total business sector labour input. Thus our productivity estimates will have a tiny downward bias due to this factor.

[86] Estimates of total hours worked in the KLEMS business sector for the years 1961-2006 are available from CANSIM II series V41712966, (Canada, Hours Worked, Business Sector) in Table 3830021 (Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub Sectors, by North American Industry Classification System (NAICS)).

estimate either wages or values for the three types of labour for the years 2003-2006. In order to accomplish this task, we formed our own estimates of the total value of labour input over the years 1961-2006. Estimates of wages, salaries and supplementary labour income for the business sector are available from CANSIM II series V498167 for the years 1961-2006. However, this measure of business sector payments for labour services neglects the labour input of the self employed (and unpaid family workers); i.e., it includes only the gross wages of employees. The value of the labour services rendered by the self employed are part of the gross operating surplus of the household sector, which includes also the returns to the capital and land used by the self employed. An upper bound to the value of self employed labour services is the sum of unincorporated business net income which is available for 1961-2006 as CANSIM II series V498170. We assume that two thirds of unincorporated net income is a return to labour and one third is the return to capital. We added this imputed labour income of the self employed to the labour income of employees in the business sector and compared this measure of total business sector labour compensation to the corresponding total labour compensation from the KLEMS data base[87] and found that these two series were very close until about 1995 and then they gradually diverged to end up about 4% apart in 2003. We used the rates of growth of our imperfect measure of business sector labour income growth to extend the official KLEMS business sector labour compensation series from 2003 to 2006. We then divided this extended measure of total labour compensation by the KLEMS business sector measure of aggregate labour input[88] in order to obtain an implicit wage rate for aggregate business sector labour for the years 2003-2006. We used the movements in this implicit wage rate to extend the KLEMS wage indexes $P_{L1}^t$, $P_{L2}^t$ and $P_{L3}^t$ from 2003 to 2006; see Table 7 below for the results of these manipulations.

**Table 7: Price and Quantity Indexes for Three Types of Business Sector Labour**

| Year t | $P_{L1}^t$ | $P_{L2}^t$ | $P_{L3}^t$ | $Q_{L1}^t$ | $Q_{L2}^t$ | $Q_{L3}^t$ |
|---|---|---|---|---|---|---|
| 1961 | 1.00000 | 1.00000 | 1.00000 | 17122 | 710 | 1370 |
| 1962 | 1.03079 | 1.18632 | 1.02090 | 17328 | 1216 | 1435 |
| 1963 | 1.06063 | 1.21353 | 1.05505 | 17242 | 1723 | 1473 |
| 1964 | 1.10549 | 1.24560 | 1.09363 | 17483 | 2230 | 1564 |
| 1965 | 1.18372 | 1.29313 | 1.16917 | 17758 | 2762 | 1641 |
| 1966 | 1.26529 | 1.33357 | 1.24754 | 18154 | 3345 | 1784 |
| 1967 | 1.35102 | 1.36529 | 1.32399 | 18051 | 3852 | 1835 |
| 1968 | 1.44391 | 1.40535 | 1.41896 | 17724 | 4282 | 1835 |
| 1969 | 1.56080 | 1.46440 | 1.53809 | 17724 | 4789 | 1913 |
| 1970 | 1.66473 | 1.50843 | 1.64090 | 17380 | 5195 | 1965 |
| 1971 | 1.77951 | 1.68033 | 1.59906 | 17191 | 5752 | 2158 |
| 1972 | 1.93628 | 1.82564 | 1.61617 | 17173 | 6386 | 2313 |
| 1973 | 2.13893 | 1.97704 | 1.64029 | 17707 | 7222 | 2494 |
| 1974 | 2.49742 | 2.23896 | 1.79972 | 17810 | 7931 | 2714 |

[87] See the CANSIM II series V41713170, Canada, Labour Compensation, Business Sector, in Table 3830021, Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub-Sectors, by NAICS.
[88] See the CANSIM II series V41712949 with the title Canada, Labour Input, Business Sector.

| | | | | | | |
|---|---|---|---|---|---|---|
| 1975 | 2.90667 | 2.52086 | 2.00380 | 17311 | 8362 | 2856 |
| 1976 | 3.39634 | 2.82284 | 2.17107 | 16915 | 8793 | 2882 |
| 1977 | 3.75767 | 3.03202 | 2.23829 | 16657 | 9300 | 3050 |
| 1978 | 3.90890 | 3.15548 | 2.41283 | 16950 | 10110 | 3296 |
| 1979 | 4.17362 | 3.37618 | 2.63121 | 17483 | 11149 | 3619 |
| 1980 | 4.51981 | 3.71500 | 2.84849 | 17638 | 11960 | 3877 |
| 1981 | 5.01370 | 4.09367 | 3.45273 | 17827 | 12492 | 4162 |
| 1982 | 5.50913 | 4.47193 | 3.64072 | 16760 | 11985 | 4123 |
| 1983 | 5.58078 | 4.79839 | 3.95408 | 16726 | 12188 | 4226 |
| 1984 | 5.95403 | 4.90205 | 4.17036 | 17053 | 12771 | 4666 |
| 1985 | 6.20419 | 5.19867 | 4.42028 | 17500 | 13455 | 5028 |
| 1986 | 6.33684 | 5.30441 | 4.68367 | 17948 | 14241 | 5441 |
| 1987 | 6.64403 | 5.43184 | 4.73239 | 18533 | 15153 | 5868 |
| 1988 | 7.12576 | 5.79003 | 4.91345 | 19032 | 16040 | 6398 |
| 1989 | 7.27373 | 5.97576 | 5.74126 | 19256 | 16623 | 6760 |
| 1990 | 7.26520 | 6.50634 | 6.01208 | 18894 | 16901 | 6966 |
| 1991 | 7.42295 | 6.73848 | 6.70637 | 17845 | 16547 | 7096 |
| 1992 | 7.57456 | 6.90166 | 6.72111 | 17242 | 16471 | 7406 |
| 1993 | 7.70013 | 6.83616 | 6.50220 | 16846 | 17079 | 8194 |
| 1994 | 7.69494 | 6.84991 | 6.26389 | 16795 | 18346 | 8711 |
| 1995 | 7.81858 | 7.02559 | 6.24296 | 16692 | 19511 | 8995 |
| 1996 | 7.93673 | 6.96835 | 6.58531 | 16812 | 20322 | 9474 |
| 1997 | 8.05121 | 7.13006 | 7.07001 | 16330 | 21918 | 10004 |
| 1998 | 8.26570 | 7.31536 | 7.34334 | 16382 | 22679 | 10792 |
| 1999 | 8.45786 | 7.48968 | 7.64746 | 16967 | 23414 | 11244 |
| 2000 | 8.86036 | 7.87341 | 8.03823 | 17311 | 24224 | 12007 |
| 2001 | 9.04675 | 8.09776 | 8.35847 | 17001 | 24731 | 12614 |
| 2002 | 9.12863 | 8.18971 | 8.58573 | 17208 | 25339 | 12925 |
| 2003 | 9.33401 | 8.37686 | 8.61459 | 16829 | 26302 | 13467 |
| 2004 | 9.56309 | 8.58245 | 8.82602 | 17191 | 27189 | 14230 |
| 2005 | 9.95409 | 8.93335 | 9.18688 | 17156 | 27113 | 15186 |
| 2006 | 10.34202 | 9.28150 | 9.54491 | 17397 | 27189 | 15871 |

We now turn our attention to the problems associated with the estimation of beginning of the year capital stocks for the business sector.

## 4. Business Sector Capital Stock Estimates

Our general strategy in this section will be to use estimates from the National Balance Sheets to obtain estimates of inventory and land stocks used by the business sector; see Statistics Canada (1997). This balance sheet information is also used to calibrate estimates of depreciation for reproducible capital stocks used by the business sector.

For the years 1962-2007, beginning of the year estimates of various national wealth components can be obtained from the CANSIM II data base. National totals for the value of various assets can be obtained from CANSIM table 3780004 (National Balance Sheet

Accounts, by Sectors) for residential structures (see series V34675), nonresidential structures (V34676), machinery and equipment (V34677), inventories (V34679) and land (V34680). The same table has the corresponding asset values for the persons and unincorporated business sector; for residential structures (see series V33464), nonresidential structures (V33465), machinery and equipment (V33466), inventories (V33468) and land (V33469). Table 3780004 also has the corresponding asset values for corporations and government business enterprises; for residential structures (see series V31693), nonresidential structures (V31694), machinery and equipment (V31695), inventories (V31696) and land (V31697). Finally, table 3780004 has the corresponding asset values for the government sector; for residential structures (see series V32575), nonresidential structures (V32576), machinery and equipment (V32577), inventories (V32578) and land (V32579). We subtract the government sector value of nonresidential structures, machinery and equipment and inventories from the corresponding total economy asset values in order to obtain business sector estimates of the value of beginning of the year t business sector nonresidential structure stocks $VK_{NR}^t$, business machinery and equipment stocks $VK_{ME}^t$, and business inventory stocks $VK_{BI}^t$; see Table 8 below for a listing of these business sector stock values. Although residential structures are not part of our domain of definition for business sector output, it will prove useful to have some information on the value of residential structures and residential land for comparison purposes. Thus the total value of residential structures from the national balance sheets for Canada, $VK_{RS}^t$, is also listed in Table 8.

Determining the value of business sector land is difficult. The problem is that the household sector owns a considerable amount of land that is used for business purposes; i.e., unincorporated persons own farm land and rental business properties and the land used in these enterprises should appear as inputs into the business sector. The corporate business sector also owns some land associated with residential rental properties and we are trying to exclude these inputs from our measure of business sector input. We will make some rough approximations in an attempt to solve these difficulties.

We first find estimates for the price and quantity of agricultural land, $P_{AL}^t$ and $QK_{AL}^t$. Estimates of the area of agricultural land are available for the Census years 1981,1986, 1991, 1996, 2001 and 2006 from CANSIM II series V32166910 and we interpolated the quantity of land in use in agriculture between these years using constant rates of growth (geometric interpolation). From Leacy (1983), series M-23, Area of Land in Farm Holdings, Census Data in thousands of acres, we can obtain estimates of the area of farm land for 1961 and 1971. After converting from acres to hectares, these data can be appended to the previous data and again geometric interpolation between the various census years can be used to complete our estimates for $QK_{AL}^t$; see Table ? for a listing.[89] CANSIM table 20020 (Balance Sheet of the Agricultural Sector at December 31) has asset value data for the end of the year for 1981-2006, which is beginning of the year values for the years 1982-2007. The two series that are of interest to us from this table are V157698 (the value of farm real estate) and V157699 (the value of farm land), which we denote by $VK_{AL}^t$ for year t. Thus for the years 1982-2007, the price of agricultural land, the price series $P_{AL}^t$ can be obtained by dividing $VK_{AL}^t$ by $QK_{AL}^t$. For the years

[89] As usual, the listed data are normalized so that the corresponding price is unity in 1961.

1961-1980, we link $P_{AL}^t$ to CANSIM series V381831 (the title is Canada, Value per Acre) in Table 20003, Value per Acre of Farm Land and Buildings. This last series runs from 1961 to 2006 and we found that it was quite close to $P_{AL}^t$ for the overlap years 1981-2006. With estimates for the price and quantity of agricultural land for the years 1961-1980, we can form estimates for the corresponding values, $VK_{AL}^t$; see Table 8 below. The price and quantity of agricultural land, $P_{AL}^t$ and $QK_{AL}^t$, are listed in Tables 10 and 11 below.

We assume that agricultural land is an input into our business sector. We also assume that the value of residential land, $VK_{RL}^t$, is equal to the total value of household and unincorporated business land less the value of agricultural land. Finally, we assume that the value of nonagricultural business land is equal to equal to the value of corporate enterprise land, $VK_{BL}^t$; see Table 8 below.

**Table 8: Beginning of Year Asset Values for Residential Structures and Land and Five Business Sector Capital Stocks**

| Year t | $VK_{RS}^t$ | $VK_{RL}^t$ | $VK_{ME}^t$ | $VK_{NR}^t$ | $VK_{BI}^t$ | $VK_{BL}^t$ | $VK_{AL}^t$ |
|---|---|---|---|---|---|---|---|
| 1962 | 29923 | 11426 | 17855 | 29388 | 13698 | 6820 | 6200 |
| 1963 | 31587 | 12086 | 18372 | 30885 | 14292 | 7281 | 6570 |
| 1964 | 33987 | 12551 | 18955 | 32950 | 15398 | 7840 | 7300 |
| 1965 | 37359 | 13678 | 20280 | 35287 | 16224 | 8537 | 8160 |
| 1966 | 41490 | 14976 | 22072 | 39040 | 17884 | 9621 | 9130 |
| 1967 | 45923 | 16366 | 24533 | 44066 | 19588 | 10971 | 10280 |
| 1968 | 49751 | 17717 | 26756 | 48425 | 20303 | 12138 | 11300 |
| 1969 | 54004 | 19826 | 28452 | 51605 | 21462 | 13161 | 11480 |
| 1970 | 59460 | 22134 | 31047 | 57053 | 23742 | 14717 | 11530 |
| 1971 | 65375 | 24497 | 33523 | 62717 | 24275 | 16421 | 11710 |
| 1972 | 74240 | 28058 | 35940 | 70605 | 25097 | 18614 | 12540 |
| 1973 | 88826 | 32835 | 38860 | 77459 | 27660 | 21533 | 15090 |
| 1974 | 110379 | 40033 | 43373 | 88640 | 33614 | 26117 | 19530 |
| 1975 | 130374 | 47562 | 52661 | 109758 | 43928 | 32759 | 24580 |
| 1981 | 147006 | 53407 | 63294 | 129682 | 46336 | 38954 | 28880 |
| 1977 | 164762 | 59191 | 72012 | 143549 | 50117 | 43763 | 33350 |
| 1978 | 181443 | 66438 | 80735 | 158378 | 57091 | 49318 | 39670 |
| 1979 | 202530 | 73635 | 91348 | 176465 | 66060 | 55774 | 49190 |
| 1980 | 227329 | 83604 | 104812 | 202525 | 81062 | 64557 | 62180 |
| 1981 | 258231 | 101063 | 114828 | 235694 | 89024 | 75224 | 70110 |
| 1982 | 288954 | 118822 | 125468 | 276642 | 98428 | 86636 | 70200 |
| 1983 | 308722 | 117237 | 142809 | 308729 | 90451 | 93532 | 68840 |
| 1984 | 330534 | 127041 | 152652 | 313983 | 91417 | 96021 | 65970 |
| 1985 | 351148 | 139328 | 162140 | 329229 | 99318 | 101438 | 62110 |
| 1986 | 378416 | 142791 | 173948 | 346345 | 104983 | 105796 | 57730 |
| 1987 | 423013 | 165711 | 183517 | 363395 | 109889 | 113337 | 54270 |
| 1988 | 477398 | 195769 | 191914 | 382426 | 117358 | 122547 | 53320 |
| 1989 | 527764 | 223324 | 202595 | 411344 | 126135 | 134213 | 56790 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1990 | 570809 | 262596 | 219586 | 440852 | 132675 | 145366 | 61390 |
| 1991 | 600762 | 258675 | 231684 | 466856 | 130781 | 155133 | 63030 |
| 1992 | 635502 | 287090 | 238486 | 467116 | 123077 | 159092 | 61850 |
| 1993 | 667367 | 307043 | 236266 | 464897 | 121352 | 162090 | 62230 |
| 1994 | 707914 | 330457 | 245686 | 473082 | 124117 | 169691 | 64710 |
| 1995 | 739526 | 352715 | 259323 | 491487 | 131198 | 179044 | 69750 |
| 1996 | 749702 | 343615 | 270003 | 503709 | 146615 | 185095 | 75540 |
| 1997 | 770434 | 352942 | 273986 | 521819 | 150648 | 193888 | 81550 |
| 1998 | 798876 | 374632 | 294979 | 544709 | 158409 | 202313 | 86380 |
| 1999 | 829677 | 394368 | 320778 | 567081 | 169901 | 211188 | 89500 |
| 2000 | 871382 | 425256 | 338450 | 590748 | 178794 | 221506 | 92140 |
| 2001 | 906034 | 452800 | 362419 | 621801 | 194366 | 234213 | 95020 |
| 2002 | 958361 | 502346 | 380507 | 642754 | 190023 | 243534 | 97610 |
| 2003 | 1031276 | 573836 | 390917 | 663346 | 192080 | 254448 | 99910 |
| 2004 | 1122515 | 633359 | 371844 | 688967 | 187291 | 270107 | 102200 |
| 2005 | 1215119 | 732815 | 372654 | 736403 | 194566 | 292041 | 104630 |
| 2006 | 1314745 | 834365 | 382309 | 767916 | 206003 | 309350 | 107320 |
| 2007 | 1465798 | 947275 | 398386 | 789765 | 215587 | 328583 | 110680 |

We assume that the quantity of residential land $QK_{RL}^t$ and the quantity of business nonagricultural land $QK_{BL}^t$ are constant over the sample period and hence the corresponding price series $P_{RL}^t$ and $P_{BL}^t$ are proportional to the corresponding value series $VK_{RL}^t$ and $VK_{BL}^t$ for the years 1962-2007. We extend these two price series back to 1961 using the movement from 1961 to 1962 in another land price series; namely series S319 in Leacy (1983): Average Land Cost per Dwelling Unit, NHA, Single Detached. These land price series are listed in Table 10 below and the corresponding quantity series are listed in Table 11. The price series $P_{RL}^t$ and $P_{BL}^t$ are listed in Table 10 below and the corresponding quantity of land series $QK_{RL}^t$ and $QK_{BL}^t$ are listed in Table 11.

From Table 2 above, we have price deflators for machinery and equipment investment and for nonresidential structures for year t, $P_{IME}^t$ and $P_{INR}^t$ respectively, and we use these deflators to divide $VK_{ME}^t$ and $VK_{NR}^t$ by $P_{IME}^t$ and $P_{INR}^t$ respectively in order to obtain preliminary beginning of the year capital stock quantity series $QK_{ME}^t$ and $QK_{NR}^t$;[90] see columns 2 and 3 in Table 9 below for a listing of these data.

Using the geometric or declining balance depreciation model of depreciation, the starting capital stock of a generic asset in period t+1, $QK^{t+1}$, is equal to one minus the depreciation rate in period t, $\delta^t$, times the previous period's starting stock, $QK^t$, plus the new investment in the previous period, $Q_I^t$; i.e., we have:

$$(6)\ QK^{t+1} = (1-\delta^t)QK^t + Q_I^t .$$

[90] The use of these prices (which are average prices over the year) for stock deflation purposes is not quite appropriate because conceptually, we should be using the prices that prevail for these stock components at the beginning of the year rather than the average prices in the year which follows. However, for our purposes, the errors made here will not be material.

Given information on beginning of the year capital stocks and investment during each year, the above equation can be solved for a balancing depreciation rate, $\delta^t$, that reconciles the investment information with the balance sheet information:

(7) $\delta^t = [QK^t - QK^{t+1} + Q_I^t]/QK^t$ .

We used the above equation (7) for the years t = 1962-2003 for machinery and equipment and for nonresidential structures using the investment data $Q_{IME}^t$ and $Q_{INR}^t$ listed in Table 3 above and the deflated balance sheet beginning of the year preliminary capital stocks $QK_{ME}^t$ and $QK_{NR}^t$ just described above. The resulting balancing depreciation rates $\delta_{ME}^t$ and $\delta_{NR}^t$ are listed in columns 3 and 4 of Table 9 below.

**Table 9: Actual and Smoothed Business Sector Depreciation Rates for Machinery and Equipment and Nonresidential Structures Implied by the Balance Sheets and Investment Flow Data**

| | Original Data | | | | Smoothed Data | | | |
|---|---|---|---|---|---|---|---|---|
| Year t | $QK_{NR}^t$ | $QK_{ME}^t$ | $\delta_{NR}^t$ | $\delta_{ME}^t$ | $QK_{NR}^t$ | $QK_{ME}^t$ | $\delta_{NR}^t$ | $\delta_{ME}^t$ |
| 1961 | | | | | 28295 | 17662 | 0.06 | 0.130 |
| 1962 | 29215 | 17510 | 0.06324 | 0.11740 | 29215 | 17510 | 0.06 | 0.131 |
| 1963 | 29912 | 17776 | 0.05052 | 0.14572 | 30007 | 17538 | 0.06 | 0.132 |
| 1964 | 31039 | 17742 | 0.08574 | 0.12476 | 30844 | 17779 | 0.06 | 0.133 |
| 1965 | 31427 | 18556 | 0.06458 | 0.13178 | 32043 | 18442 | 0.06 | 0.134 |
| 1966 | 32718 | 19722 | 0.03170 | 0.12135 | 33441 | 19582 | 0.06 | 0.135 |
| 1967 | 35483 | 21666 | 0.01202 | 0.12673 | 35237 | 21275 | 0.06 | 0.136 |
| 1968 | 38670 | 23270 | 0.08570 | 0.14841 | 36736 | 22732 | 0.06 | 0.137 |
| 1969 | 38949 | 23801 | 0.03883 | 0.13234 | 38124 | 23602 | 0.06 | 0.138 |
| 1970 | 41028 | 24958 | 0.05495 | 0.12889 | 39429 | 24652 | 0.06 | 0.139 |
| 1971 | 42719 | 26160 | 0.03009 | 0.13529 | 41009 | 25645 | 0.06 | 0.140 |
| 1972 | 45523 | 27158 | 0.09948 | 0.13866 | 42637 | 26591 | 0.06 | 0.141 |
| 1973 | 45068 | 28333 | 0.13066 | 0.20495 | 44152 | 27783 | 0.06 | 0.142 |
| 1974 | 43575 | 28519 | -0.00069 | 0.15226 | 45900 | 29831 | 0.06 | 0.143 |
| 1975 | 48280 | 30914 | -0.00927 | 0.10645 | 47820 | 32302 | 0.06 | 0.144 |
| 1976 | 54013 | 34744 | 0.04513 | 0.15142 | 50236 | 34771 | 0.06 | 0.145 |
| 1977 | 56743 | 36846 | 0.06716 | 0.16964 | 52390 | 37092 | 0.06 | 0.146 |
| 1978 | 58411 | 37855 | 0.07675 | 0.15558 | 54725 | 38936 | 0.06 | 0.147 |
| 1979 | 59554 | 39457 | 0.08370 | 0.15248 | 57067 | 40704 | 0.06 | 0.148 |
| 1980 | 60906 | 41966 | 0.06619 | 0.23757 | 59981 | 43206 | 0.06 | 0.149 |
| 1981 | 63930 | 41050 | 0.02676 | 0.23302 | 63437 | 45822 | 0.06 | 0.150 |
| 1982 | 69839 | 41627 | -0.02536 | 0.09537 | 67251 | 49091 | 0.06 | 0.151 |
| 1983 | 78539 | 46254 | 0.10148 | 0.10957 | 70144 | 50275 | 0.06 | 0.152 |
| 1984 | 76930 | 49392 | 0.06605 | 0.12680 | 72296 | 50840 | 0.06 | 0.153 |
| 1985 | 78137 | 51825 | 0.04754 | 0.12514 | 74247 | 51757 | 0.06 | 0.154 |
| 1986 | 81013 | 54991 | 0.07387 | 0.11986 | 76382 | 53438 | 0.06 | 0.155 |
| 1987 | 81238 | 59006 | 0.08387 | 0.14608 | 78009 | 55761 | 0.06 | 0.156 |
| 1988 | 80878 | 62558 | 0.05527 | 0.17786 | 79782 | 59233 | 0.06 | 0.157 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1989 | 83518 | 65825 | 0.05061 | 0.15560 | 82105 | 64328 | 0.06 | 0.158 |
| 1990 | 86636 | 71007 | 0.00772 | 0.09665 | 84523 | 69588 | 0.06 | 0.159 |
| 1991 | 93313 | 78850 | 0.06969 | 0.15719 | 86798 | 73230 | 0.06 | 0.160 |
| 1992 | 93885 | 80726 | 0.08052 | 0.20444 | 88665 | 75784 | 0.06 | 0.161 |
| 1993 | 92286 | 78343 | 0.08006 | 0.16837 | 89305 | 77703 | 0.06 | 0.162 |
| 1994 | 90890 | 78884 | 0.04643 | 0.13972 | 89940 | 78846 | 0.06 | 0.163 |
| 1995 | 93203 | 82920 | 0.07531 | 0.14625 | 91076 | 81052 | 0.06 | 0.164 |
| 1996 | 92758 | 87032 | 0.06165 | 0.18344 | 92186 | 83999 | 0.06 | 0.165 |
| 1997 | 93735 | 88297 | 0.06716 | 0.18316 | 93350 | 87369 | 0.06 | 0.166 |
| 1998 | 95321 | 93828 | 0.06248 | 0.12964 | 95630 | 94569 | 0.06 | 0.167 |
| 1999 | 97270 | 105238 | 0.07573 | 0.18243 | 97798 | 102351 | 0.06 | 0.168 |
| 2000 | 98005 | 111991 | 0.04071 | 0.18871 | 100031 | 111107 | 0.06 | 0.169 |
| 2001 | 102281 | 118437 | 0.06861 | 0.18558 | 102296 | 119910 | 0.06 | 0.170 |
| 2002 | 103976 | 123215 | 0.06696 | 0.11233 | 104870 | 126283 | 0.06 | 0.171 |
| 2003 | 105208 | 135369 | 0.10461 | 0.21153 | 106773 | 130683 | 0.06 | 0.172 |
| 2004 | 102853 | 134724 | 0.07843 | 0.19267 | 109018 | 136196 | 0.06 | 0.173 |
| 2005 | 104054 | 139359 | 0.10032 | 0.17875 | 111744 | 143226 | 0.06 | 0.174 |
| 2006 | 103885 | 148333 | | | 115309 | 152189 | 0.06 | 0.175 |

It is evident that the information that is contained in the national balance sheets and in the System of National Accounts investment data is not completely consistent; the implied balancing depreciation rates are far too variable, especially the nonresidential structures rate $\delta_{NR}^{t}$, which are negative for the years 1974, 1975 and 1982. However, it is likely that the general trends in each data source are reasonably accurate. Thus for nonresidential structures, we take the sample average balancing depreciation rate (which was .060)[91] along with the starting stock for the asset in 1962 as the "truth" and calculate constant dollar declining balance business sector nonresidential capital stocks using equation (6) and we also denote the resulting estimates by $QK_{NR}^{t}$; see column 6 of Table 9 above for a listing. The corresponding price series is $P_{INR}^{t}$ listed in Table 2 above.

It is also evident that the balancing depreciation rates for machinery and equipment $\delta_{ME}^{t}$ listed in column 5 of Table 9 have an upward trend and this trend should be taken into account. Thus we regressed these balancing depreciation rates $\delta_{ME}^{t}$ on a constant and a time trend and found that the least squares estimates for the constant and the trend were 0.13047 (standard error 0.01004) and 0.0010014 (standard error 0.000389) respectively.[92] We rounded these estimates to 0.13 and 0.001 and used these estimates to

[91] This is a rather high depreciation rate for structures by international standards. We also applied the same methodology to residential construction and found that the average balancing depreciation rate was $\delta_{RS}$ = 0.040, which is again somewhat high by international standards.
[92] The sample average depreciation rate for machinery and equipment that we obtained was 0.153 which again is high by international standards. However, the depreciation rates used in the recent KLEMS data are even higher; the arithmetic average of the depreciation rates for the 15 equipment asset classes considered by Baldwin, Gu and Yan (2007; 42) was 0.233, which does not seem to be consistent with the rates implied by the balance sheet data. The arithmetic average of the 13 structures depreciation rates used by Baldwin, Gu and Yan (2007; 42) was 0.0785, with the lowest rate being 0.06. Again, this rather high average rate for structures does not seem to be consistent with the balance sheet data. These observations

generate a new trending series for the depreciation rate for business sector machinery and equipment, which is reported as the last column $\delta_{ME}{}^t$ in Table 9. We use these new depreciation rate estimates in equation (6) along with the starting stock for the machinery and equipment capital stock from the balance sheets in 1962 as the "truth" and calculate constant dollar declining balance business sector machinery and equipment capital stocks, which we denote by $QK_{ME}{}^t$; see column 7 of Table 9 above for a listing of this series. The corresponding price series is $P_{IME}{}^t$ listed in Table 2 above.

Although residential structures and land are excluded from our measure of business sector input, it is useful for comparison purposes to have estimates of the prices and quantities of these stocks. For residential structures, we followed exactly the same strategy as was used above for nonresidential structures. We found that the average balancing depreciation rate was 0.04. The resulting smoothed beginning of the year stocks of residential structures is the series $QK_{RS}{}^t$ which is listed in Table 11 below. The corresponding year t price $P_{RS}{}^t$ is listed in Table 10 below; it is equal to the corresponding year t investment price for residential structures, $P_{INR}{}^t$, which was listed in Table 2 above.

End of the year current market value starting stocks of inventories for the entire economy and for the government sector are available from the National Balance Sheet Accounts; see CANSIM series V34679 and V32578 (Table 3780004) for the years 1961-2006. Subtracting the government inventory stocks from the total inventory stocks will give us estimates for the value of the business sector beginning of the year inventory stocks for the years 1962-2007, $VK_{BI}{}^t$. We can subtract the value of inventory change for 1961 (see CANSIM II series V498100; table 3800002; Canada, Current Prices, Business Investment in Inventories) from the starting stock of inventories in 1962 in order to extend the value of inventory stock series back to 1961. Diewert (2002), drawing on Diewert and Lawrence (2000), used older national balance sheet information to construct current and constant dollar estimates of beginning of the year stocks of inventories for the years 1962-1999. These series may be used to construct a price of inventory series $P_{BI}{}^t$ for the years 1962-1999. We extend this price series to the years 1961 and 2000-2005 by using the Industrial Product Price Index for Canada and for All Commodities, CANSIM II series V1574377, table 3290039. The inventory value series $VK_{BI}{}^t$ can be divided by the inventory stock price series $P_{BI}{}^t$, in order to obtain a real beginning of the year business sector stock of inventories, $QK_{BI}{}^t$. The resulting price and quantity series (after normalization so that the price is unity in 1961) are listed in Table 10 for $P_{BI}{}^t$ and Table 11 for $QK_{BI}{}^t$.

**Table 10: Residential Structures and Land Price Indexes and Business Sector Inventory, Nonagricultural Land and Agricultural Land Price Indexes**

| Year t | $P_{RS}{}^t$ | $P_{RL}{}^t$ | $P_{BI}{}^t$ | $P_{BL}{}^t$ | $P_{AL}{}^t$ |
|---|---|---|---|---|---|
| 1961 | 1.00000 | 1.00000 | 1.00000 | 1.00000 | 1.00000 |
| 1962 | 1.00504 | 1.06956 | 1.01546 | 1.06956 | 1.04000 |

do not make the KLEMS depreciation rates incorrect since they are based on reliable survey data on discards. However, as we shall see later, the high KLEMS depreciation rates appear to lead to some anomalies.

| | | | | | |
|---|---|---|---|---|---|
| 1963 | 1.02769 | 1.13134 | 1.00186 | 1.14186 | 1.10000 |
| 1964 | 1.07312 | 1.17487 | 1.04471 | 1.22953 | 1.22000 |
| 1965 | 1.13368 | 1.28037 | 1.06126 | 1.33883 | 1.36000 |
| 1966 | 1.20765 | 1.40187 | 1.08837 | 1.50884 | 1.52000 |
| 1967 | 1.28518 | 1.53198 | 1.11396 | 1.72055 | 1.72000 |
| 1968 | 1.31431 | 1.65845 | 1.13529 | 1.90357 | 1.90000 |
| 1969 | 1.38118 | 1.85587 | 1.15519 | 2.06400 | 1.94000 |
| 1970 | 1.42615 | 2.07191 | 1.17977 | 2.30803 | 1.96000 |
| 1971 | 1.53179 | 2.29311 | 1.18180 | 2.57526 | 2.00000 |
| 1972 | 1.67349 | 2.62645 | 1.20129 | 2.91918 | 2.16000 |
| 1973 | 1.97123 | 3.07361 | 1.31746 | 3.37696 | 2.62000 |
| 1974 | 2.36134 | 3.74740 | 1.56585 | 4.09586 | 3.42000 |
| 1975 | 2.56072 | 4.45217 | 1.91110 | 5.13750 | 4.34000 |
| 1976 | 2.76853 | 4.99931 | 1.95883 | 6.10905 | 5.14000 |
| 1977 | 2.87768 | 5.54073 | 1.98462 | 6.86323 | 5.92000 |
| 1978 | 3.04069 | 6.21911 | 2.08729 | 7.73441 | 7.02000 |
| 1979 | 3.28046 | 6.89280 | 2.31871 | 8.74688 | 8.68000 |
| 1980 | 3.55455 | 7.82598 | 2.69159 | 10.12430 | 10.94000 |
| 1981 | 3.99273 | 9.46028 | 3.05380 | 11.79717 | 12.30000 |
| 1982 | 4.08226 | 11.12266 | 3.34545 | 13.58689 | 12.28000 |
| 1983 | 4.2535 | 10.97429 | 3.39571 | 14.66837 | 12.00746 |
| 1984 | 4.41785 | 11.89202 | 3.49878 | 15.05871 | 11.47452 |
| 1985 | 4.55564 | 13.04218 | 3.58540 | 15.90824 | 10.77083 |
| 1986 | 4.90827 | 13.36634 | 3.61840 | 16.59170 | 9.98330 |
| 1987 | 5.40819 | 15.51183 | 3.66603 | 17.77433 | 9.38642 |
| 1988 | 5.78293 | 18.32549 | 3.77235 | 19.21871 | 9.22342 |
| 1989 | 6.13195 | 20.90485 | 3.93005 | 21.04826 | 9.82631 |
| 1990 | 6.11231 | 24.58101 | 3.95564 | 22.79735 | 10.62426 |
| 1991 | 6.32257 | 24.21398 | 3.94467 | 24.32908 | 10.91042 |
| 1992 | 6.39710 | 26.87384 | 3.81266 | 24.94996 | 10.69755 |
| 1993 | 6.58445 | 28.74160 | 3.96305 | 25.42013 | 10.75269 |
| 1994 | 6.76485 | 30.93333 | 4.09862 | 26.61217 | 11.17139 |
| 1995 | 6.76717 | 33.01685 | 4.17589 | 28.07898 | 12.03048 |
| 1996 | 6.75581 | 32.16502 | 4.41960 | 29.02794 | 13.01832 |
| 1997 | 6.87512 | 33.03810 | 4.04774 | 30.40692 | 14.07653 |
| 1998 | 6.95993 | 35.06845 | 3.74178 | 31.72819 | 14.93456 |
| 1999 | 7.13210 | 36.91589 | 3.74178 | 33.12003 | 15.49944 |
| 2000 | 7.29782 | 39.80725 | 3.89921 | 34.73818 | 15.98327 |
| 2001 | 7.48766 | 42.38558 | 3.93949 | 36.73098 | 16.50975 |
| 2002 | 7.81242 | 47.02347 | 3.93949 | 38.19277 | 16.95600 |
| 2003 | 8.21290 | 53.71548 | 3.88823 | 39.90438 | 17.34990 |
| 2004 | 8.71303 | 59.28730 | 4.00905 | 42.36014 | 17.74345 |
| 2005 | 9.11300 | 68.59715 | 4.08228 | 45.79999 | 18.16070 |
| 2006 | 9.77854 | 78.10301 | 4.16648 | 48.51451 | 18.62402 |

**Table 11: Residential Structures and Land Price Indexes and Business Sector Inventory, Nonagricultural Land and Agricultural Land Quantity Indexes**

| Year t | $QK_{RS}{}^{t}$ | $QK_{RL}{}^{t}$ | $QK_{BI}{}^{t}$ | $QK_{BL}{}^{t}$ | $QK_{AL}{}^{t}$ |
|---|---|---|---|---|---|
| 1961 | 28710 | 10683 | 13594 | 6376 | 5950 |
| 1962 | 29773 | 10683 | 13489 | 6376 | 5960 |
| 1963 | 30853 | 10683 | 14266 | 6376 | 5980 |
| 1964 | 31972 | 10683 | 14739 | 6376 | 5990 |
| 1965 | 33409 | 10683 | 15287 | 6376 | 6000 |
| 1966 | 34898 | 10683 | 16432 | 6376 | 6010 |
| 1967 | 36201 | 10683 | 17584 | 6376 | 5980 |
| 1968 | 37507 | 10683 | 17884 | 6376 | 5950 |
| 1969 | 39139 | 10683 | 18579 | 6376 | 5920 |
| 1970 | 41124 | 10683 | 20124 | 6376 | 5880 |
| 1971 | 42733 | 10683 | 20541 | 6376 | 5850 |
| 1972 | 44752 | 10683 | 20892 | 6376 | 5810 |
| 1973 | 47028 | 10683 | 20995 | 6376 | 5760 |
| 1974 | 49518 | 10683 | 21467 | 6376 | 5710 |
| 1975 | 52001 | 10683 | 22986 | 6376 | 5660 |
| 1976 | 54307 | 10683 | 23655 | 6376 | 5620 |
| 1977 | 57307 | 10683 | 25253 | 6376 | 5630 |
| 1978 | 60257 | 10683 | 27352 | 6376 | 5650 |
| 1979 | 63137 | 10683 | 28490 | 6376 | 5670 |
| 1980 | 65863 | 10683 | 30117 | 6376 | 5680 |
| 1981 | 68205 | 10683 | 29152 | 6376 | 5700 |
| 1982 | 70756 | 10683 | 29421 | 6376 | 5720 |
| 1983 | 72266 | 10683 | 26637 | 6376 | 5730 |
| 1984 | 74455 | 10683 | 26128 | 6376 | 5750 |
| 1985 | 76607 | 10683 | 27701 | 6376 | 5770 |
| 1986 | 79121 | 10683 | 29014 | 6376 | 5780 |
| 1987 | 82223 | 10683 | 29975 | 6376 | 5780 |
| 1988 | 86124 | 10683 | 31110 | 6376 | 5780 |
| 1989 | 90019 | 10683 | 32095 | 6376 | 5780 |
| 1990 | 94058 | 10683 | 33541 | 6376 | 5780 |
| 1991 | 97130 | 10683 | 33154 | 6376 | 5780 |
| 1992 | 99069 | 10683 | 32281 | 6376 | 5780 |
| 1993 | 101344 | 10683 | 30621 | 6376 | 5790 |
| 1994 | 103314 | 10683 | 30283 | 6376 | 5790 |
| 1995 | 105453 | 10683 | 31418 | 6376 | 5800 |
| 1996 | 106574 | 10683 | 33174 | 6376 | 5800 |
| 1997 | 108164 | 10683 | 37218 | 6376 | 5790 |
| 1998 | 110167 | 10683 | 42335 | 6376 | 5780 |
| 1999 | 111867 | 10683 | 45406 | 6376 | 5770 |
| 2000 | 113715 | 10683 | 45854 | 6376 | 5760 |
| 2001 | 115823 | 10683 | 49338 | 6376 | 5760 |
| 2002 | 118553 | 10683 | 48235 | 6376 | 5760 |

| | | | | | |
|---|---|---|---|---|---|
| 2003 | 122214 | 10683 | 49400 | 6376 | 5760 |
| 2004 | 126179 | 10683 | 46717 | 6376 | 5760 |
| 2005 | 130649 | 10683 | 47661 | 6376 | 5760 |
| 2006 | 135276 | 10683 | 49443 | 6376 | 5760 |

It is possible to generate an alternative value of inventory stock series by cumulating information on the value of inventory change from the System of National Accounts. Thus the CANSIM II series V498100 estimates the current value of business investment in inventories, which conceptually, should equal the value of inventory change over the year. Using the balance sheet estimates of the starting stock of inventories for 1962 (which was $13,698 million) and the above series, we can cumulate inventory changes and obtain an alternative SNA based estimated value of inventory change, which ended up at $91,709 million at the start of 2007. However, using the balance sheet estimates for the beginning of 2007 value of business inventories, we obtain the estimate $215,587 million, which is 2.35 times as big as the implied SNA estimate. Thus the SNA based estimates basically say that the business sector *real* stock of inventories increased by only 64% over a 45 year period, whereas real output grew 5.5 fold over this period. This does not seem plausible. It is true that inventory to output ratios have been falling due to just in time delivery and other inventory management techniques but the number of goods that are being produced has also been growing, which implies an increasing need for inventories. In any case, we will take the balance sheet estimates of inventory stocks as the "truth".[93]

Recalling Table 2 in this Appendix, the price series for inventory change $P_{II}^t$ in year t is set equal to $P_{BI}^t$ listed in Table 10. The quantity of inventory change in year t listed in Table 3 above, $Q_{II}^t$, is set equal to the stock at the beginning of year t+1, $QK_{BI}^{t+1}$, less the stock at the beginning of year t, $QK_{BI}^t$.

## 5. Primary Input Tax Rates, Balancing Real Rates of Return and User Costs

Nonresidential structures (office buildings, factories, etc.) and business land have to pay property taxes on these inputs whereas machinery and equipment and inventory stocks are generally exempt from paying these taxes. Thus it is necessary to take into account property taxes when constructing user costs of capital for business nonresidential structures and business land. Information on property taxes for the years 1961-2006 is available from Statistics Canada; see CANSIM II series V499942, table 3800035 (Real Property Taxes of Local Governments) and CANSIM II series V499841, table 3800033 (Real Property Taxes of Provincial Governments). We approximate the asset base on which these taxes fall as the total beginning of the year national value of land, residential structures and nonresidential structures. Data on these values are available for the years 1962-2007 from the National Balance Sheets and these data are described at the beginning of section 4. These series were summed and the sum was used as the tax base

[93] This choice will lead to an increase in measured Total Factor Productivity compared to estimates that rely on the SNA estimates of inventory change. See Diewert and Smith (1994) for a detailed accounting framework for inventories that is consistent with the Hicks (1961) and Edwards and Bell (1961) model of production and Diewert (2005b) for a critical review of SNA conventions for measuring inventory change.

for the sum of the two property tax series, V499942 plus V499841. The resulting property tax rates are reported as the series $\tau_P^t$ in Table 12 below[94] and it will be used in the construction of the user costs of business sector land and nonresidential structures.[95]

It is of some interest to calculate the average business tax rate for taxes that apply to the use of financial capital in the business sector so we provide estimates for this tax rate by year. These business taxes that fall on the return to capital are defined to be the sum of the following taxes:

- Taxes less subsidies on factors of production (CANSIM II series V1992216, table 3800001) less local government and Provincial government property taxes;
- Total government taxes on income from corporations and government business enterprises (CANSIM II series V499131, table 3800007 ) and
- Total government taxes on income from nonresidents (CANSIM II series V499132, table 3800007).

The sum of the above three sources of general business taxes that fall on capital stock components was divided by the corresponding sum of the beginning of the year value of assets for our four types of business sector asset; i.e., the above sum of taxes for year t was divided by $P_{IM}^t \times QK_{ME}^t$ (year t starting value of machinery and equipment) plus $P_{IN}^t \times QK_{NR}^t$ (year t starting value of nonresidential structures) plus $P_{BL}^t \times K_{BL}^t$ (year t starting value of business sector land) plus $P_{AL}^t \times K_{AL}^t$ (year t starting value of agricultural land) plus $P_{BI}^t \times QK_{BI}^t$ (year t value of starting stocks of inventories) and the resulting year t general business tax rate is denoted as $\tau_B^t$, which is listed in Table 12 below.

Using the property tax rates $\tau_P^t$, the general business tax rates $\tau_B^t$, the machinery and equipment depreciation rates $\delta_{ME}^t$ and the nonresidential structures depreciation rates $\delta_{NR}^t$, the *user costs* of machinery and equipment, nonresidential structures, business land , agricultural land and inventories, $U_{ME}^t$, $U_{NR}^t$, $U_{BL}^t$, $U_{AL}^t$ and $U_{BI}^t$ respectively, can be defined as follows:[96]

(8) $U_{ME}^t \equiv [r^t + \tau_B^t + \delta_{ME}^t]\, P_{IME}^t$ ;
(9) $U_{NR}^t \equiv [r^t + \tau_B^t + \tau_P^t + \delta_{NR}^t]\, P_{INR}^t$ ;
(10) $U_{BL}^t \equiv [r^t + \tau_B^t + \tau_P^t]\, P_{BL}^t$ ;
(11) $U_{AL}^t \equiv [r^t + \tau_B^t + \tau_P^t]\, P_{AL}^t$ ;
(12) $U_{BI}^t \equiv [r^t + \tau_B^t]\, P_{BI}^t$

[94] The tax rate for 1961 was set equal to the corresponding rate for 1962.
[95] This is a very rough approximation to the actual property tax rates on business sector land and nonresidential structures since actual property tax rates are different across different sectors and assets. For example, business sector property assets are generally taxed more heavily than household property assets.
[96] For additional material on user costs and many historical references, see Jorgenson (1989) (1996a) (1996b) and Diewert (2005a) and (2006a).

where $r^t$ is suitable real rate of return that applies to the business sector in year t. In the present study, we will follow national income accounting conventions and will take $r^t$ to be the *balancing real rate of return*;[97] i.e., it is the rate of return that is consistent with the year t value of business sector net output being equal to the value of primary inputs used by the business sector in year t, where the user costs (8)-(12) are used as prices for the beginning of the year capital inputs. Thus $r^t$ can be determined as the solution to the following linear in $r^t$ equation:

$$\begin{aligned}(13)\ & P_C{}^tQ_C{}^t + P_{IG}{}^tQ_{IG}{}^t + P_{IR}{}^tQ_{IR}{}^t + P_{INR}{}^tQ_{INR}{}^t + P_{IME}{}^tQ_{IME}{}^t + P_{II}{}^tQ_{II}{}^t + P_{GN}{}^tQ_{GN}{}^t \\ & + P_{XG}{}^tQ_{XG}{}^t + P_{XS}{}^tQ_{XS}{}^t + P_{MG}{}^tQ_{MG}{}^t + P_{MS}{}^tQ_{MS}{}^t \\ = {} & P_{L1}{}^tQ_{L1}{}^t + P_{L2}{}^tQ_{L2}{}^t + P_{L3}{}^tQ_{L3}{}^t + [r^t + \tau_B{}^t + \delta_{ME}{}^t]\, P_{IME}{}^tQK_{ME}{}^t \\ & + [r^t + \tau_B{}^t + \tau_P{}^t + \delta_{NR}{}^t]\, P_{INR}{}^tQK_{NR}{}^t + [r^t + \tau_B{}^t + \tau_P{}^t]\, P_{BL}{}^tQK_{BL}{}^t \\ & + [r^t + \tau_B{}^t + \tau_P{}^t]\, P_{AL}{}^tQK_{AL}{}^t + [r^t + \tau_B{}^t]\, P_{BI}{}^tQK_{BI}{}^t\end{aligned}$$

where the various price and quantity series are defined in the above Appendix 2 tables. The resulting series of balancing real rates of return is listed in Table 12 below. Once $r^t$ has been determined, then the four series of user costs defined by (8)-(12) can also be calculated; these series are also listed in Table 12. Note that $r^t$ is a *real after tax rate of return* because we do not include a capital gains term in our user costs and all user costs are evaluated at the average prices for the corresponding investment good for year t.

**Table 12: Business Sector Tax Rates, Balancing Real Rates of Return and User Costs**

| Year t | $\tau_P{}^t$ | $\tau_B{}^t$ | $r^t$ | $U_{ME}{}^t$ | $U_{NR}{}^t$ | $U_{BL}{}^t$ | $U_{AL}{}^t$ | $U_{BI}{}^t$ |
|---|---|---|---|---|---|---|---|---|
| 1961 | 0.01528 | 0.03007 | 0.02373 | 0.18379 | 0.12907 | 0.06907 | 0.06907 | 0.05379 |
| 1962 | 0.01528 | 0.03092 | 0.02978 | 0.19548 | 0.13678 | 0.08126 | 0.07902 | 0.06164 |
| 1963 | 0.01534 | 0.03183 | 0.03446 | 0.20494 | 0.14623 | 0.09321 | 0.08979 | 0.06641 |
| 1964 | 0.01546 | 0.03337 | 0.04294 | 0.22363 | 0.16112 | 0.11284 | 0.11197 | 0.07973 |
| 1965 | 0.01566 | 0.03264 | 0.04022 | 0.22609 | 0.16676 | 0.11852 | 0.12039 | 0.07733 |
| 1966 | 0.01568 | 0.03187 | 0.04197 | 0.23373 | 0.17841 | 0.13507 | 0.13607 | 0.08036 |
| 1967 | 0.01547 | 0.03016 | 0.03038 | 0.22255 | 0.16891 | 0.13078 | 0.13074 | 0.06744 |

[97] For most purposes, it is probably preferable to use an exogenous real rate of return in the user costs (3)-(6) since the resulting prices will probably approximate market rental prices better. For discussion of this topic, see Diewert (2006a). However, in the present study, there was little difference in the empirical results if the sample average real rate of return (4.95 %) was used in place of the balancing real rate; i.e., in the gross income model, average TFP growth changed from 1.14 % to 1.13 % per year and in the net income model, average TFP growth changed from 1.26 % to 1.25 % per year This is similar to results obtained by Diewert and Lawrence (2005) (2006) for Australia. Their first study used the sample average balancing real rate for Australia whereas their second study used the year by year balancing real rates of return. However, Baldwin and Gu (2007; 27) found substantial differences for the Canadian business sector in their TFP growth rates for the period 1961-1981 where their estimated average TFP growth rates increased from the 0.90 to 1.01 % per year range using balancing or endogenous interest rates to the 1.18 to 1.26 % range using an exogenous interest rate. The differences that Baldwin and Gu (2007; 28) found for the 1981-2001 period were not nearly as large: an increase from the 0.30-0.38 % range to the 0.32-0.43 % range. Baldwin and Gu (2007; 18) mention that they used a constant real rate of interest equal to 5.1 % in their exogenous interest rate models, which is very close to the 4.95 % real rate that we used in our exogenous real rate computations.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1968 | 0.01588 | 0.03270 | 0.03508 | 0.23545 | 0.17989 | 0.15924 | 0.15894 | 0.07694 |
| 1969 | 0.01642 | 0.03404 | 0.03200 | 0.24391 | 0.18875 | 0.17019 | 0.15997 | 0.07629 |
| 1970 | 0.01612 | 0.03146 | 0.03759 | 0.25880 | 0.20187 | 0.19657 | 0.16693 | 0.08146 |
| 1971 | 0.01565 | 0.03209 | 0.03302 | 0.26284 | 0.20665 | 0.20798 | 0.16152 | 0.07695 |
| 1972 | 0.01511 | 0.03387 | 0.02890 | 0.26966 | 0.21385 | 0.22736 | 0.16823 | 0.07541 |
| 1973 | 0.01416 | 0.03771 | 0.04409 | 0.30695 | 0.26805 | 0.32404 | 0.25141 | 0.10776 |
| 1974 | 0.01318 | 0.04125 | 0.03896 | 0.33947 | 0.31204 | 0.38254 | 0.31942 | 0.12561 |
| 1975 | 0.01244 | 0.03677 | 0.03854 | 0.37359 | 0.33589 | 0.45081 | 0.38083 | 0.14392 |
| 1976 | 0.01290 | 0.03334 | 0.04508 | 0.40701 | 0.36331 | 0.55787 | 0.46938 | 0.15361 |
| 1977 | 0.01328 | 0.03138 | 0.05130 | 0.44693 | 0.39454 | 0.65859 | 0.56808 | 0.16409 |
| 1978 | 0.01319 | 0.03117 | 0.05189 | 0.49067 | 0.42368 | 0.74449 | 0.67572 | 0.17338 |
| 1979 | 0.01218 | 0.03235 | 0.05657 | 0.54848 | 0.47734 | 0.88424 | 0.87748 | 0.20616 |
| 1980 | 0.01221 | 0.03265 | 0.05116 | 0.58144 | 0.51879 | 0.97212 | 1.05045 | 0.22558 |
| 1981 | 0.01239 | 0.03138 | 0.04175 | 0.62413 | 0.53647 | 1.00880 | 1.05180 | 0.22330 |
| 1982 | 0.01221 | 0.02764 | 0.03027 | 0.62967 | 0.51541 | 0.95267 | 0.86104 | 0.19372 |
| 1983 | 0.01246 | 0.02857 | 0.05015 | 0.71233 | 0.59425 | 1.33736 | 1.09476 | 0.26729 |
| 1984 | 0.01244 | 0.03193 | 0.06045 | 0.75837 | 0.67270 | 1.57845 | 1.20275 | 0.32322 |
| 1985 | 0.01251 | 0.03178 | 0.06143 | 0.77342 | 0.69826 | 1.68181 | 1.13869 | 0.33419 |
| 1986 | 0.01276 | 0.03076 | 0.05507 | 0.76178 | 0.67799 | 1.63574 | 0.98423 | 0.31056 |
| 1987 | 0.01277 | 0.03307 | 0.06372 | 0.78622 | 0.75848 | 1.94737 | 1.02839 | 0.35484 |
| 1988 | 0.01272 | 0.03315 | 0.05888 | 0.76397 | 0.77900 | 2.01313 | 0.96614 | 0.34716 |
| 1989 | 0.01285 | 0.03315 | 0.05429 | 0.75542 | 0.78947 | 2.11098 | 0.98550 | 0.34365 |
| 1990 | 0.01301 | 0.03233 | 0.03937 | 0.71343 | 0.73637 | 1.93120 | 0.90000 | 0.28363 |
| 1991 | 0.01347 | 0.03043 | 0.02908 | 0.64497 | 0.66528 | 1.77538 | 0.79617 | 0.23472 |
| 1992 | 0.01396 | 0.03029 | 0.02569 | 0.64099 | 0.64647 | 1.74482 | 0.74811 | 0.21341 |
| 1993 | 0.01404 | 0.03235 | 0.03160 | 0.68140 | 0.69511 | 1.98240 | 0.83855 | 0.25342 |
| 1994 | 0.01377 | 0.03471 | 0.04926 | 0.76919 | 0.82103 | 2.60109 | 1.09190 | 0.34416 |
| 1995 | 0.01338 | 0.03668 | 0.05191 | 0.78995 | 0.85412 | 2.86324 | 1.22676 | 0.36995 |
| 1996 | 0.01341 | 0.04010 | 0.06536 | 0.83905 | 0.97131 | 3.45048 | 1.54745 | 0.46608 |
| 1997 | 0.01336 | 0.04342 | 0.06577 | 0.85391 | 1.01624 | 3.72634 | 1.72507 | 0.44197 |
| 1998 | 0.01354 | 0.03921 | 0.05557 | 0.82302 | 0.96190 | 3.43701 | 1.61781 | 0.35467 |
| 1999 | 0.01362 | 0.04447 | 0.05444 | 0.81356 | 1.00583 | 3.72691 | 1.74411 | 0.37009 |
| 2000 | 0.01288 | 0.04934 | 0.07089 | 0.87409 | 1.16402 | 4.62402 | 2.12754 | 0.46880 |
| 2001 | 0.01258 | 0.03947 | 0.06954 | 0.85378 | 1.10395 | 4.46616 | 2.00744 | 0.42945 |
| 2002 | 0.01223 | 0.03787 | 0.07060 | 0.86305 | 1.11705 | 4.60993 | 2.04662 | 0.42732 |
| 2003 | 0.01211 | 0.03980 | 0.06838 | 0.80910 | 1.13673 | 4.80003 | 2.08699 | 0.42062 |
| 2004 | 0.01191 | 0.04270 | 0.08315 | 0.82483 | 1.32468 | 5.83541 | 2.44429 | 0.50453 |
| 2005 | 0.01152 | 0.04316 | 0.09174 | 0.82602 | 1.46087 | 6.70611 | 2.65912 | 0.55071 |
| 2006 | 0.01125 | 0.04291 | 0.09113 | 0.79652 | 1.51751 | 7.04871 | 2.70590 | 0.55848 |
| Average | 0.01357 | 0.03483 | 0.04950 | 0.57038 | 0.61723 | 1.8755 | 0.95158 | 0.25139 |

Note that the sample average of the balancing after tax real rates of return $r^t$ was a rather large 4.950% per year.[98] The average property tax rate $\tau_P{}^t$ was 1.357% while the

[98] The corresponding balancing real rate of return for Australia averaged around 3 percent; see Diewert and Lawrence (2006). Normally, after tax real rates of return are in the 1 to 3 percent rate whereas our average

average business tax rate on assets was 3.483%. Thus the before business tax real rate of return averaged 8.433%. Thus it appears that governments are taking about 41% of the before tax return to capital assets on average.[99] However, it must be kept in mind that these balancing rates of return may not be very reliable; they contain the net effect of all the measurement errors that were made in constructing this data set. The volatility in the above real rates of return is a source of concern since it is likely that a considerable proportion of the volatility is caused by various measurement errors. The volatility in the real rates of return also causes volatility in the user costs and possible volatility in productivity growth rates. However, we repeated our productivity calculations using a constant after tax real rate of return (equal to the sample average real rate of 4.95 %) and found no material difference in our productivity growth rates; see Tables 13 and 14 below which are counterparts to Tables 4 and 9, except that the constant after tax real rate of 4.95 % was used in place of the endogenous or balancing real rates used in Tables 4 and 9. Hence the volatility in the productivity growth rates is mainly due to volatility in our output measures.

Table 13 below is the constant real interest rate counterpart to Table 4 in the main text.

**Table 13: Business Sector Year to Year Growth in Real Gross Income and Year to Year Contribution Factors Using a Constant Real Interest Rate**

| Year t | $\rho^t/\rho^{t-1}$ | $\tau^t$ | $\alpha_D{}^t$ | $\alpha_X{}^t$ | $\alpha_M{}^t$ | $\beta_L{}^t$ | $\beta_K{}^t$ | $\alpha_{XM}{}^t$ |
|---|---|---|---|---|---|---|---|---|
| 1962 | 1.07821 | 1.04838 | 1.00067 | 1.0092 | 0.98609 | 1.02905 | 1.00360 | 0.99516 |
| 1963 | 1.05029 | 1.02916 | 0.99925 | 0.99807 | 0.99863 | 1.01817 | 1.00640 | 0.99670 |
| 1964 | 1.10006 | 1.04920 | 1.00605 | 1.00481 | 1.00035 | 1.02964 | 1.00696 | 1.00517 |
| 1965 | 1.08182 | 1.02518 | 1.00762 | 1.00131 | 1.00466 | 1.02955 | 1.01117 | 1.00597 |
| 1966 | 1.07661 | 1.02133 | 0.99973 | 0.99736 | 1.00544 | 1.03541 | 1.01551 | 1.00279 |
| 1967 | 1.02153 | 0.98768 | 0.99956 | 0.99578 | 1.00536 | 1.01396 | 1.01935 | 1.00112 |
| 1968 | 1.04777 | 1.03426 | 0.99609 | 0.99423 | 1.00554 | 1.00293 | 1.01434 | 0.99974 |
| 1969 | 1.05614 | 1.02785 | 1.00239 | 0.99584 | 1.00164 | 1.01642 | 1.01107 | 0.99747 |
| 1970 | 1.04903 | 1.02644 | 1.00427 | 1.00211 | 1.00051 | 1.00235 | 1.01264 | 1.00262 |
| 1971 | 1.05583 | 1.02381 | 1.01294 | 0.99357 | 0.99891 | 1.01456 | 1.01108 | 0.99249 |
| 1972 | 1.05576 | 1.01300 | 1.00516 | 0.99960 | 1.00599 | 1.02036 | 1.01053 | 1.00559 |
| 1973 | 1.11836 | 1.02967 | 1.01037 | 1.02234 | 1.00073 | 1.04019 | 1.01012 | 1.02309 |

rate is close to 5 percent. This suggests that our estimates of the value of output are too high or that the value of labour input are too low or that our estimated asset values for business sector capital inputs are too small. We think that the last possibility is the most probable one. Using the data tabled in this appendix, we calculated a business sector nominal and real value of business sector output and we also calculated the corresponding business sector nominal and real capital stock inputs where the real measures were calculated using chained Fisher indexes. We found that the nominal business sector capital output ratio fell from 2.60 in 1961 to 1.93 in 2006 while the real capital output ratio fell from 2.60 in 1961 to 1.64 in 2006. These falls in the capital output ratio seem unlikely. See Diewert and Fox (2001) for a discussion of output mismeasurement problems.

[99] This relatively high rate of business taxation has two negative effects: (i) it raises the user cost of capital and hence lessens the beneficial effects of capital deepening and (ii) the high rates lead to a relatively large loss of productive efficiency; i.e., the deadweight losses of such large tax rates are likely to be large. See Diewert and Lawrence (2002) for a methodology for estimating the deadweight losses due to capital taxation.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1974 | 1.04771 | 0.98892 | 1.00448 | 1.03927 | 0.97762 | 1.02367 | 1.01410 | 1.01601 |
| 1975 | 1.00214 | 1.00308 | 0.98587 | 0.99710 | 0.99965 | 0.99952 | 1.01718 | 0.99675 |
| 1976 | 1.09970 | 1.05606 | 1.00996 | 1.00729 | 1.00748 | 0.99967 | 1.01634 | 1.01482 |
| 1977 | 1.05105 | 1.04519 | 0.99720 | 1.00947 | 0.97745 | 1.00676 | 1.01516 | 0.98670 |
| 1978 | 1.02348 | 0.99789 | 0.99775 | 1.00501 | 0.98144 | 1.02767 | 1.01410 | 0.98636 |
| 1979 | 1.04906 | 0.98613 | 0.99922 | 1.02735 | 0.98678 | 1.03732 | 1.01240 | 1.01377 |
| 1980 | 1.00906 | 0.97455 | 0.99464 | 1.01572 | 0.98731 | 1.02171 | 1.01599 | 1.00283 |
| 1981 | 1.02555 | 1.00990 | 1.00753 | 0.99378 | 0.98234 | 1.01749 | 1.01470 | 0.97623 |
| 1982 | 0.94088 | 0.97297 | 0.99216 | 0.97446 | 1.01464 | 0.96754 | 1.01885 | 0.98873 |
| 1983 | 1.02708 | 1.02194 | 0.98739 | 0.97925 | 1.02743 | 1.00449 | 1.00716 | 1.00611 |
| 1984 | 1.05903 | 1.03817 | 0.99443 | 0.99694 | 0.99980 | 1.02334 | 1.00569 | 0.99674 |
| 1985 | 1.04639 | 1.01391 | 1.00031 | 0.99607 | 1.00228 | 1.02544 | 1.00780 | 0.99834 |
| 1986 | 1.01904 | 0.98799 | 1.00254 | 0.99051 | 1.00158 | 1.02744 | 1.00934 | 0.99208 |
| 1987 | 1.07256 | 1.01191 | 1.00392 | 0.99822 | 1.01658 | 1.03119 | 1.00896 | 1.01477 |
| 1988 | 1.05736 | 1.00433 | 1.00024 | 0.99041 | 1.02128 | 1.02923 | 1.01104 | 1.01149 |
| 1989 | 1.03403 | 0.99283 | 1.00037 | 0.99697 | 1.01215 | 1.01717 | 1.01432 | 1.00909 |
| 1990 | 0.95991 | 0.96954 | 0.98263 | 0.97480 | 1.01777 | 1.00073 | 1.01483 | 0.99212 |
| 1991 | 0.93069 | 0.96754 | 0.97873 | 0.96354 | 1.03198 | 0.97848 | 1.01015 | 0.99435 |
| 1992 | 0.99159 | 1.00068 | 0.99739 | 1.00641 | 0.98716 | 0.99300 | 1.00707 | 0.99348 |
| 1993 | 1.02225 | 1.00995 | 0.99932 | 1.01154 | 0.98351 | 1.01486 | 1.00319 | 0.99486 |
| 1994 | 1.07823 | 1.03995 | 1.01001 | 1.02729 | 0.97166 | 1.02559 | 1.00275 | 0.99817 |
| 1995 | 1.04840 | 1.00094 | 1.00217 | 1.03350 | 0.98654 | 1.01867 | 1.00628 | 1.01959 |
| 1996 | 1.06056 | 1.02912 | 0.99422 | 0.99136 | 1.01865 | 1.01871 | 1.00756 | 1.00986 |
| 1997 | 1.04673 | 1.02559 | 0.99324 | 0.99182 | 1.00635 | 1.01984 | 1.00947 | 0.99812 |
| 1998 | 1.01833 | 1.00681 | 0.99805 | 0.98983 | 0.98739 | 1.01994 | 1.01664 | 0.97734 |
| 1999 | 1.04942 | 1.00593 | 0.99570 | 0.99570 | 1.01364 | 1.02250 | 1.01525 | 1.00929 |
| 2000 | 1.09734 | 1.03303 | 0.99682 | 1.02340 | 1.00405 | 1.02271 | 1.01405 | 1.02755 |
| 2001 | 0.99648 | 0.99061 | 0.99479 | 0.98925 | 0.99873 | 1.00864 | 1.01472 | 0.98800 |
| 2002 | 1.02449 | 1.01514 | 1.00314 | 0.98020 | 1.00379 | 1.01271 | 1.00968 | 0.98391 |
| 2003 | 1.00153 | 0.96393 | 0.99081 | 0.97747 | 1.05240 | 1.01188 | 1.00743 | 1.02869 |
| 2004 | 1.07580 | 1.01772 | 1.00152 | 1.00465 | 1.02134 | 1.02172 | 1.00676 | 1.02609 |
| 2005 | 1.05285 | 1.00753 | 1.00462 | 1.00429 | 1.01706 | 1.00862 | 1.00965 | 1.02142 |
| 2006 | 1.03631 | 1.00467 | 1.00489 | 0.99120 | 1.01306 | 1.01018 | 1.01194 | 1.00415 |
| A62-06 | 1.0410 | 1.0113 | 0.99934 | 0.99974 | 1.0028 | 1.0160 | 1.0112 | 1.0023 |
| A62-73 | 1.0660 | 1.0263 | 1.0037 | 1.0012 | 1.0012 | 1.0210 | 1.0111 | 1.0023 |
| A74-91 | 1.0253 | 1.0024 | 0.99663 | 0.99756 | 1.0025 | 1.0133 | 1.0127 | 0.99985 |
| A92-99 | 1.0394 | 1.0149 | 0.99876 | 1.0059 | 0.99436 | 1.0166 | 1.0085 | 1.0001 |
| A00-06 | 1.0407 | 1.0047 | 0.99951 | 0.99578 | 1.0158 | 1.0138 | 1.0106 | 1.0114 |

Comparing the sample average growth rates in the above Table (see the row labeled A62-06) and comparing these rates with the corresponding rates listed in Table 4 of the main text shows that the overall sample average rates differ by at most 0.0001 percentage points. For the subperiods, some of the differences are larger; for example, during the period 2000-2006, the average rate of TFP growth using constant real rates in the user costs was 0.47 % per year whereas using endogenous real rates, the average TFP growth rate was only 0.34 % per year.

Table 13 below is the constant after tax real interest rate counterpart to Table 9 in the main text.

**Table 14: Business Sector Year to Year Growth in Net Real Income and Year to Year Contribution Factors Using a Constant Real Interest Rate**

| Year t | $\rho^t/\rho^{t-1}$ | $\tau^t$ | $\alpha_D{}^t$ | $\alpha_X{}^t$ | $\alpha_M{}^t$ | $\beta_L{}^t$ | $\beta_K{}^t$ | $\alpha_{XM}{}^t$ |
|---|---|---|---|---|---|---|---|---|
| 1962 | 1.08774 | 1.05575 | 0.99942 | 1.01071 | 0.98384 | 1.03388 | 1.00275 | 0.99438 |
| 1963 | 1.05493 | 1.03308 | 0.99852 | 0.99777 | 0.99841 | 1.02103 | 1.00544 | 0.99619 |
| 1964 | 1.10737 | 1.05609 | 1.00290 | 1.00554 | 1.00040 | 1.03414 | 1.00504 | 1.00594 |
| 1965 | 1.08368 | 1.02825 | 1.00505 | 1.00150 | 1.00533 | 1.03390 | 1.00735 | 1.00684 |
| 1966 | 1.07871 | 1.02381 | 0.99916 | 0.99698 | 1.00622 | 1.04056 | 1.01019 | 1.00319 |
| 1967 | 1.01498 | 0.98532 | 1.00061 | 0.99517 | 1.00614 | 1.01600 | 1.01197 | 1.00128 |
| 1968 | 1.04952 | 1.03880 | 0.99907 | 0.99338 | 1.00636 | 1.00336 | 1.00818 | 0.99970 |
| 1969 | 1.05601 | 1.03139 | 1.00068 | 0.99523 | 1.00188 | 1.01886 | 1.00716 | 0.99710 |
| 1970 | 1.04714 | 1.02979 | 1.00241 | 1.00242 | 1.00058 | 1.00270 | 1.00865 | 1.00300 |
| 1971 | 1.05451 | 1.02680 | 1.01197 | 0.99261 | 0.99875 | 1.01676 | 1.00679 | 0.99137 |
| 1972 | 1.05736 | 1.01436 | 1.00551 | 0.99954 | 1.00689 | 1.02345 | 1.00646 | 1.00643 |
| 1973 | 1.12938 | 1.03341 | 1.01192 | 1.02558 | 1.00084 | 1.04608 | 1.00583 | 1.02644 |
| 1974 | 1.04562 | 0.98688 | 1.00497 | 1.04484 | 0.97455 | 1.02699 | 1.00817 | 1.01825 |
| 1975 | 0.99702 | 1.00298 | 0.98776 | 0.99668 | 0.99960 | 0.99945 | 1.01068 | 0.99628 |
| 1976 | 1.10171 | 1.06384 | 1.00910 | 1.00834 | 1.00856 | 0.99962 | 1.00951 | 1.01698 |
| 1977 | 1.04982 | 1.05129 | 0.99691 | 1.01084 | 0.97424 | 1.00774 | 1.00935 | 0.98480 |
| 1978 | 1.01887 | 0.99701 | 0.99691 | 1.00575 | 0.97873 | 1.03182 | 1.00926 | 0.98436 |
| 1979 | 1.05043 | 0.98350 | 1.00032 | 1.03150 | 0.98482 | 1.04301 | 1.00772 | 1.01584 |
| 1980 | 1.00454 | 0.97018 | 0.99694 | 1.01812 | 0.98540 | 1.02504 | 1.00992 | 1.00326 |
| 1981 | 1.01684 | 1.01086 | 1.00594 | 0.99279 | 0.97957 | 1.02030 | 1.00780 | 0.97251 |
| 1982 | 0.92235 | 0.96767 | 0.99353 | 0.97005 | 1.01722 | 0.96195 | 1.01070 | 0.98676 |
| 1983 | 1.03658 | 1.02525 | 0.99578 | 0.97553 | 1.03249 | 1.00531 | 1.00273 | 1.00722 |
| 1984 | 1.07008 | 1.04412 | 0.99800 | 0.99642 | 0.99977 | 1.02736 | 1.00339 | 0.99619 |
| 1985 | 1.05125 | 1.01556 | 1.00176 | 0.99543 | 1.00265 | 1.02960 | 1.00555 | 0.99807 |
| 1986 | 1.01805 | 0.98551 | 1.00441 | 0.98901 | 1.00183 | 1.03188 | 1.00595 | 0.99082 |
| 1987 | 1.08048 | 1.01319 | 1.00675 | 0.99794 | 1.01919 | 1.03613 | 1.00515 | 1.01709 |
| 1988 | 1.06026 | 1.00444 | 1.00175 | 0.98898 | 1.02450 | 1.03367 | 1.00610 | 1.01322 |
| 1989 | 1.03087 | 0.99123 | 1.00171 | 0.99652 | 1.01399 | 1.01977 | 1.00756 | 1.01046 |
| 1990 | 0.95046 | 0.96429 | 0.98592 | 0.97089 | 1.02060 | 1.00085 | 1.00808 | 0.99088 |
| 1991 | 0.92669 | 0.96170 | 0.98987 | 0.95763 | 1.03738 | 0.97496 | 1.00508 | 0.99342 |
| 1992 | 0.98683 | 1.00019 | 0.99909 | 1.00751 | 0.98498 | 0.99181 | 1.00335 | 0.99237 |
| 1993 | 1.02275 | 1.01106 | 0.99954 | 1.01355 | 0.98068 | 1.01746 | 1.00070 | 0.99397 |
| 1994 | 1.08443 | 1.04625 | 1.00719 | 1.03199 | 0.96693 | 1.03000 | 1.00126 | 0.99786 |
| 1995 | 1.05165 | 1.00050 | 1.00196 | 1.03912 | 0.98434 | 1.02178 | 1.00378 | 1.02285 |
| 1996 | 1.06768 | 1.03324 | 0.99536 | 0.99000 | 1.02165 | 1.02172 | 1.00460 | 1.01143 |
| 1997 | 1.04976 | 1.02901 | 0.99299 | 0.99058 | 1.00733 | 1.02291 | 1.00654 | 0.99784 |
| 1998 | 1.01083 | 1.00731 | 0.99697 | 0.98825 | 0.98543 | 1.02309 | 1.01024 | 0.97385 |
| 1999 | 1.05211 | 1.00630 | 0.99948 | 0.99502 | 1.01582 | 1.02612 | 1.00858 | 1.01077 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2000 | 1.10551 | 1.03761 | 0.99906 | 1.02703 | 1.00468 | 1.02623 | 1.00712 | 1.03183 |
| 2001 | 0.98971 | 0.98864 | 0.99676 | 0.98762 | 0.99854 | 1.00996 | 1.00837 | 0.98618 |
| 2002 | 1.02112 | 1.01696 | 1.00371 | 0.9771 | 1.00439 | 1.01474 | 1.00454 | 0.98139 |
| 2003 | 1.00613 | 0.95781 | 0.99867 | 0.97396 | 1.06092 | 1.01377 | 1.00414 | 1.03329 |
| 2004 | 1.08451 | 1.01986 | 1.00436 | 1.00535 | 1.0246 | 1.02504 | 1.00275 | 1.03008 |
| 2005 | 1.05656 | 1.00812 | 1.00765 | 1.00491 | 1.01954 | 1.00986 | 1.00527 | 1.02454 |
| 2006 | 1.0369 | 1.00486 | 1.00844 | 0.98996 | 1.01492 | 1.01163 | 1.00673 | 1.00473 |
| A62-06 | 1.0418 | 1.0125 | 1.0006 | 0.99968 | 1.0032 | 1.0185 | 1.0066 | 1.0027 |
| A62-73 | 1.0684 | 1.0297 | 1.0031 | 1.0014 | 1.0013 | 1.0242 | 1.0072 | 1.0027 |
| A74-91 | 1.0240 | 1.0022 | 0.99880 | 0.99707 | 1.0031 | 1.0153 | 1.0074 | 0.99980 |
| A92-99 | 1.0408 | 1.0167 | 0.99907 | 1.0070 | 0.99340 | 1.0194 | 1.0049 | 1.0001 |
| A00-06 | 1.0429 | 1.0048 | 0.0027 | 0.99513 | 1.0182 | 1.0159 | 1.0056 | 1.0131 |

Comparing the sample average growth rates in the above Table (see the row labeled A62-06) and comparing these rates with the corresponding rates listed in Table 9 of the main text shows that the overall sample average rates differ by at most 0.0001 percentage points. For the subperiods, a few of the differences are larger; for example, during the period 2000-2006, the average rate of TFP growth using constant real rates in the user costs and the net income model was 0.48 % per year whereas using endogenous real rates, the average TFP growth rate using the net income model was only 0.34 % per year.

## 6. Sources of Error

There are many problems with the data constructed in this Appendix. Some of the more important possible sources of error are listed as follows:

- Our adjustments for converting final demand prices (those facing the final demanders of the goods and services produced by the business sector) into basic prices (prices facing the producers of the goods and services) were rather crude and some aggregation error will be associated with our procedures. In particular, only crude adjustments for the effects of indirect taxes on the components of consumption were made. Also our method for estimating the net supplies of the business sector to the nonbusiness sector are rather indirect and subject to some error.[100]
- Our tax adjustments for the price of imports and exports were also not completely satisfactory due to various aggregation errors; i.e., we were not able to assign taxes accurately to the various components of imports and exports.
- Our measure of labour input relies on the Statistics Canada KLEMS program estimates for quality adjusted labour and their may be some amount of error in these estimates. In particular, it is very difficult to account for the hours of work and labour compensation for the self employed.
- It proved to be difficult to reconcile balance sheet information with investment information.[101] Our treatment of investment and capital services was highly

[100] In particular, we did not have access to *chained* price indexes for the nonbusiness sector for the years prior to 1997 and this will lead to some aggregation errors.
[101] Recall the volatility in the balancing depreciation rates listed in columns 3 and 4 of Table 9.

aggregated and hence contains some aggregation errors. We also relied heavily on the Statistics Canada Balance Sheet estimates and these estimates are highly aggregated; in particular, there is not enough detail on the allocation of land. Moreover, the Balance Sheet stocks appear to give asset values that are too small.[102]

- Our treatment of property taxes is very approximate.
- Our user costs of capital were constructed using a particular set of assumptions (no capital gains and endogenous real rates of return) and these assumptions are not universally accepted.
- The roles of infrastructure capital and R&D investments were not taken into account.
- The role of resource depletion was also not taken into account.

The next international version of the System of National Accounts will recognize capital services in the production accounts. This will be a big step forward since it will allow inputs in the SNA production accounts to be decomposed into price and quantity components and hence the revised SNA will facilitate the development of productivity accounts for each country that implements the revised SNA. However, just introducing capital services into the SNA will not be sufficient in order to develop accurate sectoral productivity accounts. The revised SNA also needs to consider the following problems:

- More attention needs to be given to the development of basic prices by industry and by commodity; i.e., we need accurate information on the exact location of indirect taxes (and commodity subsidies) by commodity and industry on both outputs and intermediate inputs.
- In order to deal adequately with the complications introduced by international trade, the existing Input Output production accounts need to be reworked so that the role of traded goods and services can be tracked by industry.
- The treatment of inventory change in the present SNA seems inadequate for the needs of productivity accounts. Inventory change should be integrated with the balance sheet accounts and the user cost accounts.
- The investment accounts need to be integrated with the corresponding balance sheet accounts, both in nominal and real terms.
- The treatment of land in the balance sheets requires additional work; i.e., there are problems in obtaining information on the quantity of land used by each industry and sector and valuing the land appropriately.[103]
- Difficult decisions must be made on the exact form of the user cost formula to be used when measuring capital services; i.e., the revised SNA should make specific

[102] Evidence of this possible undercounting of asset values in the Balance Sheet accounts are the declining capital output ratios that are implied by our data. Moreover, the assessed value of real property (land and structures) in British Columbia for 2007 was just over one trillion dollars. If we add up the value of land and structures in the National Balance Sheets for the beginning of 2007, we get a value of about 4 trillion dollars. If we multiply the British Columbia value by a factor of 8, it seems that the national value of real property should be equal to about 8 trillion instead of the 4 trillion in the accounts.

[103] There are some difficult conceptual and practical problems involved in separating structure value from land value; see Diewert (2007) for a discussion of some of these problems.

recommendations on how user costs should be constructed so that some measure of international comparability can be achieved in the accounts.
- The problems involved in making imputations for the labour input of the self employed (and unpaid family workers) should also be addressed.

The introduction of capital services into the SNA will provide challenges for statistical agencies. However, as national statistical agencies make productivity accounts a part of their regular production of the national accounts, there will be benefits to the statistical system as a whole since a natural output of the new system of accounts will be balancing real rates of return by sector or industry. These balancing real rates of return will provide a check on the accuracy of the sectoral data: if the rates are erratic or very large or very small, this can indicate measurement error in the sectoral data and hence will give the statistical agency an early indication of problems with the data.

Statistics Canada already has an extensive productivity program. It is to be hoped that as the program evolves in the future, the data will be presented to the public in some detail and hopefully, at some level of aggregation, revised series will be made available back to 1961.[104]

## Appendix 3: Kohli's Treatment of the Gains from Trade

Ulrich Kohli, the chief economist for the Swiss National Bank, has long had an interest in adjusting income measures for changes in a country's terms of trade using production theory; see Kohli (1990) (2003) (2004a) (2004b) (2008) and Fox and Kohli (1998). His latest methodology is conveniently laid out in Kohli (2006) and this paper also has an application to Canada so it should be possible to compare his empirical results with the results presented here.

For our purposes, there are four main differences in Kohli's methodology for determining the welfare effects of changes in a country's terms of trade:

- Kohli's production sector is the entire economy whereas our production sector is just the business sector;
- Kohli uses final demand prices to value outputs whereas we use the prices that producers face; i.e., our methodology follows Jorgenson and Griliches (1972; 85) and adjusts prices for indirect tax wedges;
- Kohli divides the nominal income produced by his production sector by the price of domestic absorption (the price of C + G + I) in order to obtain his real income concept whereas we divide the nominal income produced by the market sector of the economy by the price of consumption (the price of C) in order to obtain our real income concept and finally
- Kohli's methodology requires information only on the prices and volumes (quantities) of the components of final demand whereas our methodology

[104] It is important to have data back to the early 1960's since the 1950's and 1960's were decades of very high productivity growth. Hence if we want to explain the productivity slowdown that took place in the 1970's, it is important to have comparable data for the 1960's.

seemingly requires information on the prices and volumes of primary inputs as well, which is a strike against the use of our methodology, since information on the prices and quantities of primary inputs used by the economy is much harder to obtain than the comparable information on outputs produced by the economy.

The first methodological difference will only be important empirically if the nonbusiness sector grows faster or slower than the business sector and the second difference will only be important if the ratio of indirect taxes on products to GDP is changing. The second factor is not likely to be important in the case of Canada but the first factor is important, particularly since our definition of the business sector totally excludes residential housing from outputs and inputs.

The effects of the third factor can readily be determined. Our basic methodology explained in Appendix 1 is not affected by the choice of deflator; i.e., instead of dividing by the price of consumption $P_C$, we can just as easily divide by our domestic price $P_D$ (or any other price that we think is relevant for welfare evaluation purposes) and our basic production theory methodology is not affected. Below, we will divide by $P_D$ instead of $P_C$ and we will find that using our definition of the business sector, it does not make a lot of difference whether we divide by the price of domestic consumption or by the price of domestic absorption. However, the question raised by Kohli's methodology is: which deflator is the "right" one? We would argue that the price of consumption has a closer connection with welfare than the price of absorption and is easier to understand but we concede that "reasonable" economists might well opt for Kohli's alternative.[105]

The fourth factor is the most interesting one from a methodological point of view. At first glance, it would appear that Kohli's methodology for determining the effects of changes in the terms of trade on real income growth has a clear advantage over our methodology, since our methodology evidently requires price and quantity information on primary inputs, whereas his methodology requires information only on the prices and quantities of final demand components. However, this apparent methodological difference is illusory; we will show below that our methodology is actually equivalent to that of Kohli, except that he divides his nominal income by $P_D$ (the price of C + G + I0 whereas in section 2 of the main text, we divided our nominal income by $P_C$ (the price of C).

We now rework our (gross) real income methodology, explained in sections 2 and 3 of the main text, but in this Appendix, we will substitute the price of business sector domestic value added $P_D^t$ for the price of consumption produced by the business sector $P_C^t$ as our deflator for the nominal income generated by the Canadian business sector. Referring back to section 2 in the main text, recall that the year t price of consumption was $P_C^t$, the price of domestic sales was $P_D^t$, the price of exports was $P_X^t$, the price of imports was $P_M^t$, the price of labour services was $P_L^t$, the price of capital services was $P_K^t$, the price of business sector value added was $P_Y^t$ and the price of business sector primary input was $P_Z^t$. The corresponding quantity aggregates were defined as $Q_C^t$, $Q_D^t$, $Q_X^t$, $Q_M^t$,

[105] Kohli (2006; 49) presents some additional arguments justifying his preference for the price of absorption as a deflator over other alternatives.

$Q_L^t$, $Q_K^t$, $Q_Y^t$ and $Q_Z^t$ respectively. We use the same definitions here but in order to apply the translog methodology developed in Appendix 1, we need to define $P_Y^t$ as the Törnqvist price index for the components of business sector value added (D, X and −M) and $Q_Y^t$ as the corresponding implicit output quantity index. Using these definitions and the material in Appendix 1, the year t TFP growth for the business sector can be defined as (one plus) output growth divided by (one plus) input growth as follows:[106]

(1) $\tau^t \equiv [Q_Y^t/Q_Y^{t-1}]/[Q_Z^t/Q_Z^{t-1}]$ ; t = 1962, 1963,..., 2006.

Year t nominal income generated by the business sector can still be defined either as $P_Y^tQ_Y^t$ (output side definition) or $P_Z^tQ_Z^t$ (input side definition). Recall that in section 3 of the main text, we divided year t nominal income by $P_C^t$, the year t price of consumption. Now we will follow Kohli's example and divide nominal income by the domestic price index $P_D^t$. Thus define the year t *Kohli type real income*, $\rho^t$, as follows:

(2) $\rho^t \equiv P_Y^tQ_Y^t/P_D^t$ ; t = 1961, ..., 2006.

The formal model outlined in Appendix 1, based on the work of Diewert and Morrison (1986) and Kohli (1990), again allows us to decompose the growth of Kohli type real income from year t−1 to t, $\rho^t/\rho^{t-1}$, into multiplicative year to year contribution factors $\alpha_D^t$, $\alpha_X^t$, $\alpha_M^t$, $\beta_L^t$, $\beta_K^t$ and $\tau^t$ that describe the effects of changes in these six explanatory variables going from year t−1 to t. Thus the model outlined in Appendix 1 leads to the following equation which decomposes the year to year growth in Kohli type real income generated by the business sector, $\rho^t/\rho^{t-1}$, into a product of six year to year explanatory contribution factors:[107]

(3) $\rho^t/\rho^{t-1} = \tau^t\, \alpha_D^t\, \alpha_X^t\, \alpha_M^t\, \beta_L^t\, \beta_K^t$ ; t = 1962, 1963,...,2006.

It should be noted that the TFP growth factor $\tau^t$ which appears in (3) is equal to the same $\tau^t$ which appears in equation (1) above and hence can be estimated empirically if data on output and input prices and quantities are available. We have all of the necessary data for the Canadian business sector so we can calculate all of the terms in equation (3) above for Canada for the years 1962-2006. The results in Table 1 below are Kohli type counterparts to our Table 4 in section 3 of the main text.

**Table 1: Business Sector Year to Year Growth in Kohli Type Real Income and Year to Year Contribution Factors**

| Year t | $\rho^t/\rho^{t-1}$ | $Q_Y^t/Q_Y^{t-1}$ | $\tau^t$ | $\alpha_X^t$ | $\alpha_M^t$ | $\beta_L^t$ | $\beta_K^t$ | $\alpha_{XM}^t$ |
|---|---|---|---|---|---|---|---|---|
| 1962 | 1.07751 | 1.08272 | 1.04894 | 1.00902 | 0.98628 | 1.02905 | 1.00306 | 0.99518 |
| 1963 | 1.05106 | 1.05456 | 1.03011 | 0.99827 | 0.99841 | 1.01817 | 1.00547 | 0.99668 |

[106] It is traditional to use X rather than Z to denote an input aggregate but unfortunately, we have already used X to denote aggregate exports.
[107] See equations (42), (51) and (56) in Appendix 1 in order to derive this equation.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1964 | 1.09350 | 1.08782 | 1.04975 | 1.00313 | 1.00208 | 1.02964 | 1.00643 | 1.00522 |
| 1965 | 1.07376 | 1.06726 | 1.02575 | 0.99919 | 1.00690 | 1.02955 | 1.01060 | 1.00608 |
| 1966 | 1.07689 | 1.07390 | 1.02220 | 0.99744 | 1.00536 | 1.03541 | 1.01465 | 1.00278 |
| 1967 | 1.02198 | 1.02084 | 0.98927 | 0.99591 | 1.00522 | 1.01396 | 1.01772 | 1.00111 |
| 1968 | 1.05186 | 1.05216 | 1.03566 | 0.99546 | 1.00427 | 1.00293 | 1.01297 | 0.99972 |
| 1969 | 1.05366 | 1.05630 | 1.02898 | 0.99507 | 1.00244 | 1.01642 | 1.00996 | 0.99750 |
| 1970 | 1.04458 | 1.04184 | 1.02772 | 1.00067 | 1.00196 | 1.00235 | 1.01137 | 1.00263 |
| 1971 | 1.04220 | 1.05022 | 1.02475 | 0.98918 | 1.00321 | 1.01456 | 1.01014 | 0.99236 |
| 1972 | 1.05037 | 1.04451 | 1.01415 | 0.99786 | 1.00777 | 1.02036 | 1.00938 | 1.00561 |
| 1973 | 1.10694 | 1.08190 | 1.03041 | 1.01864 | 1.00443 | 1.04019 | 1.00940 | 1.02315 |
| 1974 | 1.04312 | 1.02660 | 0.98952 | 1.03762 | 0.97926 | 1.02367 | 1.01349 | 1.01610 |
| 1975 | 1.01584 | 1.01982 | 1.00416 | 1.00185 | 0.99426 | 0.99952 | 1.01609 | 0.99610 |
| 1976 | 1.08935 | 1.07296 | 1.05681 | 1.00412 | 1.01111 | 0.99967 | 1.01562 | 1.01528 |
| 1977 | 1.05391 | 1.06820 | 1.04532 | 1.01040 | 0.97646 | 1.00676 | 1.01503 | 0.98662 |
| 1978 | 1.02573 | 1.03997 | 0.99770 | 1.00581 | 0.98061 | 1.02767 | 1.01430 | 0.98631 |
| 1979 | 1.04986 | 1.03562 | 0.98577 | 1.02766 | 0.98647 | 1.03732 | 1.01276 | 1.01376 |
| 1980 | 1.01446 | 1.01163 | 0.97414 | 1.01793 | 0.98513 | 1.02171 | 1.01642 | 1.00279 |
| 1981 | 1.01796 | 1.04267 | 1.01012 | 0.99075 | 0.98542 | 1.01749 | 1.01447 | 0.97630 |
| 1982 | 0.94834 | 0.95913 | 0.97436 | 0.97756 | 1.01145 | 0.96754 | 1.01739 | 0.98875 |
| 1983 | 1.04054 | 1.03388 | 1.02215 | 0.98429 | 1.02251 | 1.00449 | 1.00696 | 1.00645 |
| 1984 | 1.06510 | 1.06845 | 1.03799 | 0.99928 | 0.99759 | 1.02334 | 1.00587 | 0.99687 |
| 1985 | 1.04607 | 1.04782 | 1.01328 | 0.99593 | 1.00241 | 1.02544 | 1.00843 | 0.99833 |
| 1986 | 1.01646 | 1.02457 | 0.98748 | 0.98943 | 1.00267 | 1.02744 | 1.00986 | 0.99208 |
| 1987 | 1.06840 | 1.05282 | 1.01140 | 0.99659 | 1.01827 | 1.03119 | 1.00947 | 1.01480 |
| 1988 | 1.05710 | 1.04510 | 1.00360 | 0.99031 | 1.02138 | 1.02923 | 1.01177 | 1.01149 |
| 1989 | 1.03365 | 1.02434 | 0.99231 | 0.99682 | 1.01231 | 1.01717 | 1.01486 | 1.00909 |
| 1990 | 0.97653 | 0.98464 | 0.96976 | 0.98160 | 1.01035 | 1.00073 | 1.01460 | 0.99177 |
| 1991 | 0.95042 | 0.95633 | 0.96826 | 0.97185 | 1.02260 | 0.97848 | 1.00939 | 0.99382 |
| 1992 | 0.99411 | 1.00071 | 1.00139 | 1.00750 | 0.98601 | 0.99300 | 1.00636 | 0.99341 |
| 1993 | 1.02293 | 1.02823 | 1.01008 | 1.01186 | 0.98319 | 1.01486 | 1.00307 | 0.99484 |
| 1994 | 1.06765 | 1.06949 | 1.04006 | 1.02199 | 0.97680 | 1.02559 | 1.00264 | 0.99828 |
| 1995 | 1.04609 | 1.02603 | 1.00090 | 1.03219 | 0.98776 | 1.01867 | 1.00632 | 1.01955 |
| 1996 | 1.06697 | 1.05631 | 1.02872 | 0.99493 | 1.01524 | 1.01871 | 1.00795 | 1.01010 |
| 1997 | 1.05407 | 1.05583 | 1.02459 | 0.99597 | 1.00237 | 1.01984 | 1.01045 | 0.99833 |
| 1998 | 1.02035 | 1.04397 | 1.00576 | 0.99104 | 0.98621 | 1.01994 | 1.01770 | 0.97737 |
| 1999 | 1.05408 | 1.04425 | 1.00550 | 0.99858 | 1.01085 | 1.02250 | 1.01568 | 1.00941 |
| 2000 | 1.10106 | 1.07133 | 1.03221 | 1.02574 | 1.00196 | 1.02271 | 1.01485 | 1.02775 |
| 2001 | 1.00211 | 1.01387 | 0.98908 | 0.99299 | 0.99538 | 1.00864 | 1.01628 | 0.98840 |
| 2002 | 1.02107 | 1.03798 | 1.01427 | 0.97810 | 1.00573 | 1.01271 | 1.01054 | 0.98371 |
| 2003 | 1.01130 | 0.98263 | 0.96320 | 0.98327 | 1.04669 | 1.01188 | 1.00820 | 1.02918 |
| 2004 | 1.07409 | 1.04685 | 1.01708 | 1.00372 | 1.02222 | 1.02172 | 1.00739 | 1.02601 |
| 2005 | 1.04776 | 1.02602 | 1.00573 | 1.00150 | 1.01965 | 1.00862 | 1.01146 | 1.02119 |
| 2006 | 1.03109 | 1.02701 | 1.00214 | 0.98843 | 1.01572 | 1.01018 | 1.01449 | 1.00397 |
| A62-06 | 1.0416 | 1.0391 | 1.0114 | 1.0002 | 1.0023 | 1.0160 | 1.0111 | 1.0024 |
| A62-73 | 1.0620 | 1.0595 | 1.0273 | 0.99999 | 1.0024 | 1.0210 | 1.0101 | 1.0023 |
| A74-91 | 1.0285 | 1.0286 | 1.0025 | 0.99888 | 1.0011 | 1.0133 | 1.0126 | 0.99982 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| A92-99 | 1.0408 | 1.0406 | 1.0146 | 1.0068 | 0.99355 | 1.0166 | 1.0088 | 1.0002 |
| A00-06 | 1.0412 | 1.0294 | 1.0034 | 0.99625 | 1.0153 | 1.0138 | 1.0119 | 1.0115 |

Comparing the above Table with Table 4 in the main text, it can be seen that there are two differences in the Tables:

- The above Table 1 has dropped the domestic price growth factor, $\alpha_D{}^t$, due to the fact that it is identically equal to one when we divide nominal income by the price of deliveries to domestic demanders, $P_D{}^t$, and
- We have added (one plus) the rate of growth of real value added produced by the business sector, $Q_Y{}^t/Q_Y{}^{t-1}$, as an additional column for Table 1 in this Appendix.

The last five rows of the above Table 1 report the average growth factor for the entire sample period (1962-2006) and for the four subperiods considered in the main text: 1962-1973, 1974-1991, 1992-1999 and 2000-2006. Comparing the entire sample results in the above Table with the corresponding averages in Table 4 in the main text, it can be seen that there are a few small changes:

- Kohli type real income growth averaged 4.16 percent per year whereas our real income growth averaged 4.10 percent per year;
- Declining real domestic prices subtracted 0.07 percentage points per year in our framework whereas this factor is neutral in the Kohli framework;
- Declining real export prices subtracted 0.03 percentage points per year in our framework whereas this factor made a positive contribution of 0.02 percentage points per year in the Kohli framework;
- Decreasing real import prices contributed 0.28 percentage points per year in our framework and 0.23 percentage points in Kohli's framework and
- The contribution factors for TFP growth, labour input growth and capital input growth were exactly the same in both frameworks.[108]

The last column in Table 1 above gives the product of the real export and real import price contribution factors, $\alpha_{XM}{}^t$, defined in the usual way as follows:

$$(4)\ \alpha_{XM}{}^t \equiv \alpha_X{}^t \alpha_M{}^t .$$

As noted in section 3 of the main text, $\alpha_{XM}{}^t$ is our *terms of trade contribution factor*; it gives the contribution to real income growth of the combined effects of real changes in the international prices facing the Canadian business sector. Comparing the sample average results for $\alpha_{XM}{}^t$ using our approach (see Table 4 in the main text) and using Kohli's approach (dividing nominal income by $P_D$ instead of $P_C$), it can be seen that the differences are small: using our approach, $\alpha_{XM}{}^t$ averaged 0.23 percent per year and using the Kohli approach, $\alpha_{XM}{}^t$ averaged 0.24 percent per year. The reason why there is little difference is due to the fact that for the Canadian data, the movements in the price of

[108] It can be shown that this will always happen.

consumption $P_C^t$ are very similar to the movements in the price of deliveries to domestic final demand, $P_D^t$. This will not necessarily be the case for other countries.

There is one additional task left to do in this Appendix and that is to reconcile Kohli's trading gains factor with our terms of trade contribution factor, $\alpha_{XM}^t$ defined above by (4).

Kohli (2006; 50) defines Gross Domestic Income in year t as nominal income in year t, $P_Y^tQ_Y^t$, divided by the nontraded goods price index for year t which is $P_D^t$ using our notation; i.e., he defines real income in year t, $\rho^t$, by (2) above, which we rewrite in a slightly different way as follows:

$$(5)\ \rho^t \equiv P_Y^tQ_Y^t/P_D^t \qquad t = 1961, ..., 2006$$
$$= [P_Y^t/P_D^t]Q_Y^t .$$

Thus real income decomposes into a price factor, $[P_Y^t/P_D^t]$, times real output, $Q_Y^t$. Kohli (2006; 50) defines this price factor as his *trading gains factor*; i.e., we have

$$(6)\ TGF^t \equiv P_Y^t/P_D^t\ ; \qquad t = 1961, ..., 2006.$$

We will show that the rate of growth of Kohli's trading gains factor is equal to our terms of trade contribution factor, which is already expressed as a rate of growth. We will require two additional results in order to do this. We have already noted that when we choose to define real income by deflating nominal income by the domestic price index $P_D^t$, then using the translog methodology explained in Appendix 1, we will find that the domestic price contribution factor, $\alpha_D^t$, will be identically unity; i.e., we have

$$(7)\ \alpha_D^t = 1\ ; \qquad t = 1962, ..., 2006.$$

Again using the translog methodology explained in Appendix 1, it can be shown that the product of the labour and capital input growth factors is equal to the rate of growth of aggregate input; i.e., we have

$$(8)\ \beta_L^t\ \beta_K^t = Q_Z^t/Q_Z^{t-1}\ ; \qquad t = 1962, ..., 2006.$$

Now start with the translog identity given by (3):

$$(9)\ \rho^t/\rho^{t-1} = \alpha_D^t\ \alpha_X^t\ \alpha_M^t\ \beta_L^t\ \beta_K^t\ \tau^t\ ; \qquad t = 1962, ...,2006$$
$$= \alpha_X^t\ \alpha_M^t\ [Q_Z^t/Q_Z^{t-1}]\ \tau^t \qquad \text{using (7) and (8)}$$
$$= \alpha_X^t\ \alpha_M^t\ [Q_Z^t/Q_Z^{t-1}]\ [Q_Y^t/Q_Y^{t-1}]/[Q_Z^t/Q_Z^{t-1}] \qquad \text{using (1)}$$
$$= \alpha_X^t\ \alpha_M^t\ [Q_Y^t/Q_Y^{t-1}] \qquad \text{canceling terms}$$
$$= \alpha_{XM}^t\ [Q_Y^t/Q_Y^{t-1}] \qquad \text{using definition (4).}$$

Using (5), (6) and (9), it can be seen that we have the following equality between $\alpha_{XM}^t$ and the rate of growth of Kohli's Trading Gain Factor:

(10) $TGF^{t}/TGF^{t-1} = \alpha_{XM}^{t}$ ; $t = 1962, ...,2006.$

Thus Kohli's basic methodology is equivalent to our methodology, provided that we divide nominal income by the relevant domestic deliveries price deflator, $P_D^t$, in place of the consumption price deflator, $P_C^t$.[109] The equivalence of the two approaches should not be a big surprise since both approaches rely on the same translog decomposition analysis originally developed by Diewert and Morrison (1986) and independently by Kohli (1990).

A final implication of the analysis in this Appendix is that we can use the decomposition of real income growth given by (9) above to rewrite our basic decomposition of real income growth given by (3) in the following alternative form:

(11) $\rho^t/\rho^{t-1} = \alpha_D^t\, \alpha_X^t\, \alpha_M^t\, [Q_Y^t/Q_Y^{t-1}]$.

Thus if we go back to deflating year t nominal income by the price of consumption, $P_C^t$, instead of by $P_D^t$, then the domestic real price change term $\alpha_D^t$ again makes its appearance in (11).

## Appendix 4: The Statistics Canada KLEMS Estimates of Business Sector Multifactor Productivity Growth

### 1. Introduction

As was mentioned in the main text, the Statistics Canada KLEMS program has recently provided estimates of the multifactor productivity growth for the Canadian business sector; see Baldwin, Gu and Yan (2007) and Baldwin and Gu (2007) for a description of the methods used in this program. In section 2 of the main text, we explained that our level of business sector Total Factor Productivity using our user cost framework ended up at 1.65 in 2006 from its starting value of 1 in 1961 whereas the KLEMS Multifactor business sector productivity ended up at 1.20 in 2006.[110] In this Appendix, we will try to determine why our estimates are so different from the corresponding KLEMS program estimates.

---

[109] Kohli (2006; 52) lists his estimates for his trading gains growth factor for the years 1982-2005 and his estimates can be compared with our estimates for $\alpha_{XM}^t$ listed in Table 1 in this Appendix. Our average terms of trade growth factor for this period (using $P_D$ as our nominal income deflator) was 1.0034 whereas Kohli's reported average rate of growth for his trading gains factor was 1.0011. Our average growth factor for real income using our $P_D$ as the deflator over this period 1982-2005 was 1.0328 compared to Kohli's 1.0286 and our average growth factor for business sector real value added was 1.0292 over this period compared to Kohli's 1.0273 average growth factor for economy wide real GDP. The differences in our results are not methodological; rather they are due to the factor that business sector value added is different from economy wide value added; i.e., these two value aggregates have very different price and quantity growth rates.

[110] See CANSIM II series V41712881, Canada, Multifactor Productivity, Business Sector, table 3830021, Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub-Sectors.

Since our measure of business sector labour input is identical to the measure used by the KLEMS program, the differences between the two sets of results must be due to either differences in the outputs or in the capital services inputs. We will address each source of potential difference in turn.

**2. Differences in the Output Concepts**

As was mentioned in section 2 of the main text, our business sector output concept differs from the corresponding KLEMS concept in two ways:

- We exclude the services of both owned and rented residential housing from our output concept whereas the KLEMS program excludes only owned residential housing services and
- We measure real inventory change as a difference in real inventory stocks whereas the KLEMS program follows national income accounting conventions and measures inventory change in a rather different manner.

We will now attempt to adjust our output measure to make it closer to the corresponding KLEMS measure. Before making any adjustments, our average rate of business sector real output growth over the years 1962-2006 was 3.91% per year compared to the corresponding KLEMS average rate of 3.86%[111] which is not a large difference.

From Table 1 in Appendix 2, we have listed series for the price and quantity of paid residential rents in Canada, $P_{PR}^{t}$ and $Q_{PR}^{t}$ respectively. From Tables 6 and 7 in the main text, we have listed the price and quantity of business sector output, $P_{Y}^{t}$ and $Q_{Y}^{t}$ respectively. We can construct an adjusted measure of Canadian business sector output that will be closer to the KLEMS measure by taking a chained Fisher index of these two series. The average rate of growth of the resulting quantity aggregate turned out to be 3.89% per year which is very close to the KLEMS average growth rate of 3.86% per year.

We now attempt to adjust our output measure for our different treatment of inventory change. Information on the nominal value of inventory change can be obtained from CANSIM II series V498100, Canada; Current Prices; Seasonally Adjusted at Annual Rates; Business Investment in Inventories, table 3800002, Gross Domestic Product (GDP), Expenditure Based National Income and Product Accounts. Information on the corresponding real value of inventory change can be obtained from the same table, CANSIM II series V1992057, Canada; Chained 2002 Dollars; Seasonally Adjusted at Annual Rates; Business Investment in Inventories and this series can be used as an "official" quantity series for inventory change. Dividing the former series by the latter series gives us an "official" price series for inventory change.[112] Now take the price and

[111] The source for the KLEMS real output series is CANSIM II series V41712932, Canada; Real Gross Domestic Product (GDP); Business Sector, table 3830021, Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub-Sectors.

[112] This price series was very erratic and not very believable in general, which explains why we used the stock price of inventories from the National Balance Sheet Accounts as our price concept and the

quantity series for business sector adjusted output that was described in the previous paragraph, add the "official" price and quantity series for inventory change to it and subtract[113] our price and quantity series for inventory change, $P_{II}^t$ and $Q_{II}^t$, that are listed in Tables 2 and 3 of Appendix 2 using chained Fisher indexes. The resulting adjusted quantity series for business sector output should be closer to the official KLEMS output series. The average rate of growth of the resulting quantity aggregate turned out to be 3.82% per year which is now a bit lower than the KLEMS average growth rate of 3.86% per year.[114]

Thus adjusting our output aggregate for our different treatment of paid residential rents and inventory change appears to make very little overall difference on average to our rate of growth of business sector output: making these adjustments does not change our overall average growth rate very much and thus our average business sector output growth rate is quite comparable to the corresponding KLEMS average growth rate.[115]

**3. Differences in Labour Input Concepts**

Since we used the KLEMS program estimates for the price and quantity of the three types of labour that are available on CANSIM, there are no differences in our estimates of TFP growth for the Canadian business sector and the KLEMS Multifactor growth rates due to differing measures of labour input.[116]

The KLEMS data base has however, provided us with revised measures of hours worked for the Canadian business sector; see CANSIM II series, Canada; Hours Worked; Business Sector, table 3830021, Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub-Sectors. It is of some interest to use hours worked as our measure of labour input instead of quality adjusted labour input. Thus Table 1 below is a counterpart to Table 4 in the main text, where we have replaced the quality adjusted labour quantity series for business sector input by the revised KLEMS program hours worked series, keeping the value of labour compensation the same. It should be noted that the quality adjusted price of labour increased 10.26 fold over the period 1961-2006 while the price of an hour of work (without quality adjustment) increased 14.12 fold over the same period.

---

difference in real stocks as our quantity concept for inventory change in place of the official series. The implied official price for 1981 was negative so we replaced this official price by the average price of the adjacent observations. The 1981 official quantity was obtained by dividing the official value for 1981 by the interpolated price for 1981.

[113] Thus we change the sign of $Q_{II}^t$ from plus to minus in the Fisher formula.

[114] Note that our treatment of inventory change appears to add about 0.07 percentage points per year to output growth as compared to the official treatment of inventory change.

[115] The remaining differences between the average growth rates can be explained by (i) index number aggregation errors; (ii) the fact that our treatment of indirect tax wedges is only approximately correct and (iii) we have not adjusted our output measure for any intermediate inputs that may be used by the rental of residential housing industry.

[116] However, since our output measure excludes the provision of residential rental services, we should also exclude the labour input associated with these services. We did not do this because it is difficult to find a breakdown of the rental of structures industry into residential, commercial and industrial components.

**Table 1: Business Sector Year to Year Growth in Real Income and Year to Year Contribution Factors Using Hours Worked as Labour Input**

| Year t | $\rho^t/\rho^{t-1}$ | $\tau^t$ | $\alpha_D{}^t$ | $\alpha_X{}^t$ | $\alpha_M{}^t$ | $\beta_L{}^t$ | $\beta_K{}^t$ | $\alpha_{XM}{}^t$ |
|---|---|---|---|---|---|---|---|---|
| 1962 | 1.07821 | 1.05859 | 1.00067 | 1.00920 | 0.98609 | 1.01968 | 1.00306 | 0.99516 |
| 1963 | 1.05029 | 1.04018 | 0.99925 | 0.99807 | 0.99863 | 1.00830 | 1.00547 | 0.99670 |
| 1964 | 1.10006 | 1.05937 | 1.00605 | 1.00481 | 1.00035 | 1.02029 | 1.00643 | 1.00517 |
| 1965 | 1.08182 | 1.03751 | 1.00762 | 1.00131 | 1.00466 | 1.01788 | 1.01060 | 1.00597 |
| 1966 | 1.07661 | 1.02884 | 0.99973 | 0.99736 | 1.00544 | 1.02872 | 1.01465 | 1.00279 |
| 1967 | 1.02153 | 0.99423 | 0.99956 | 0.99578 | 1.00536 | 1.00890 | 1.01772 | 1.00112 |
| 1968 | 1.04777 | 1.04243 | 0.99609 | 0.99423 | 1.00554 | 0.99641 | 1.01297 | 0.99974 |
| 1969 | 1.05614 | 1.03332 | 1.00239 | 0.99584 | 1.00164 | 1.01215 | 1.00996 | 0.99747 |
| 1970 | 1.04903 | 1.03316 | 1.00427 | 1.00211 | 1.00051 | 0.99707 | 1.01137 | 1.00262 |
| 1971 | 1.05583 | 1.02956 | 1.01294 | 0.99357 | 0.99891 | 1.00983 | 1.01014 | 0.99249 |
| 1972 | 1.05576 | 1.02167 | 1.00516 | 0.99960 | 1.00599 | 1.01285 | 1.00938 | 1.00559 |
| 1973 | 1.11836 | 1.03812 | 1.01037 | 1.02234 | 1.00073 | 1.03246 | 1.00940 | 1.02309 |
| 1974 | 1.04771 | 0.99195 | 1.00448 | 1.03927 | 0.97762 | 1.02116 | 1.01349 | 1.01601 |
| 1975 | 1.00214 | 1.00649 | 0.98587 | 0.99710 | 0.99965 | 0.99720 | 1.01609 | 0.99675 |
| 1976 | 1.09970 | 1.05925 | 1.00996 | 1.00729 | 1.00748 | 0.99736 | 1.01562 | 1.01482 |
| 1977 | 1.05105 | 1.04971 | 0.99720 | 1.00947 | 0.97745 | 1.00255 | 1.01503 | 0.98670 |
| 1978 | 1.02348 | 1.00043 | 0.99775 | 1.00501 | 0.98144 | 1.02486 | 1.01430 | 0.98636 |
| 1979 | 1.04906 | 0.98742 | 0.99922 | 1.02735 | 0.98678 | 1.03559 | 1.01276 | 1.01377 |
| 1980 | 1.00906 | 0.97929 | 0.99464 | 1.01572 | 0.98731 | 1.01634 | 1.01642 | 1.00283 |
| 1981 | 1.02555 | 1.01250 | 1.00753 | 0.99378 | 0.98234 | 1.01510 | 1.01447 | 0.97623 |
| 1982 | 0.94088 | 0.98050 | 0.99216 | 0.97446 | 1.01464 | 0.96147 | 1.01739 | 0.98873 |
| 1983 | 1.02708 | 1.02721 | 0.98739 | 0.97925 | 1.02743 | 0.99954 | 1.00696 | 1.00611 |
| 1984 | 1.05903 | 1.04074 | 0.99443 | 0.99694 | 0.99980 | 1.02063 | 1.00587 | 0.99674 |
| 1985 | 1.04639 | 1.01245 | 1.00031 | 0.99607 | 1.00228 | 1.02628 | 1.00843 | 0.99834 |
| 1986 | 1.01904 | 0.99444 | 1.00254 | 0.99051 | 1.00158 | 1.02024 | 1.00986 | 0.99208 |
| 1987 | 1.07256 | 1.0157 | 1.00392 | 0.99822 | 1.01658 | 1.02682 | 1.00947 | 1.01477 |
| 1988 | 1.05736 | 1.00911 | 1.00024 | 0.99041 | 1.02128 | 1.02361 | 1.01177 | 1.01149 |
| 1989 | 1.03403 | 0.99570 | 1.00037 | 0.99697 | 1.01215 | 1.01370 | 1.01486 | 1.00909 |
| 1990 | 0.95991 | 0.97373 | 0.98263 | 0.97480 | 1.01777 | 0.99665 | 1.01460 | 0.99212 |
| 1991 | 0.93069 | 0.97379 | 0.97873 | 0.96354 | 1.03198 | 0.97293 | 1.00939 | 0.99435 |
| 1992 | 0.99159 | 1.00661 | 0.99739 | 1.00641 | 0.98716 | 0.98785 | 1.00636 | 0.99348 |
| 1993 | 1.02225 | 1.01668 | 0.99932 | 1.01154 | 0.98351 | 1.00827 | 1.00307 | 0.99486 |
| 1994 | 1.07823 | 1.04327 | 1.01001 | 1.02729 | 0.97166 | 1.02244 | 1.00264 | 0.99817 |
| 1995 | 1.04840 | 1.00613 | 1.00217 | 1.03350 | 0.98654 | 1.01337 | 1.00632 | 1.01959 |
| 1996 | 1.06056 | 1.03059 | 0.99422 | 0.99136 | 1.01865 | 1.01687 | 1.00795 | 1.00986 |
| 1997 | 1.04673 | 1.02924 | 0.99324 | 0.99182 | 1.00635 | 1.01523 | 1.01045 | 0.99812 |
| 1998 | 1.01833 | 1.00915 | 0.99805 | 0.98983 | 0.98739 | 1.01651 | 1.01770 | 0.97734 |
| 1999 | 1.04942 | 1.00819 | 0.99570 | 0.99570 | 1.01364 | 1.01977 | 1.01568 | 1.00929 |
| 2000 | 1.09734 | 1.03940 | 0.99682 | 1.02340 | 1.00405 | 1.01563 | 1.01485 | 1.02755 |
| 2001 | 0.99648 | 0.99471 | 0.99479 | 0.98925 | 0.99873 | 1.00293 | 1.01628 | 0.98800 |
| 2002 | 1.02449 | 1.01731 | 1.00314 | 0.98020 | 1.00379 | 1.00969 | 1.01054 | 0.98391 |
| 2003 | 1.00153 | 0.96673 | 0.99081 | 0.97747 | 1.05240 | 1.00818 | 1.00820 | 1.02869 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 2004 | 1.07580 | 1.01954 | 1.00152 | 1.00465 | 1.02134 | 1.01926 | 1.00739 | 1.02609 |
| 2005 | 1.05285 | 1.01016 | 1.00462 | 1.00429 | 1.01706 | 1.00419 | 1.01146 | 1.02142 |
| 2006 | 1.03631 | 1.00291 | 1.00489 | 0.99120 | 1.01306 | 1.00941 | 1.01449 | 1.00415 |
| A62-06 | 1.0410 | 1.0162 | 0.99934 | 0.99974 | 1.0028 | 1.0112 | 1.0111 | 1.0023 |
| A62-73 | 1.0660 | 1.0347 | 1.0037 | 1.0012 | 1.0012 | 1.0137 | 1.0101 | 1.0023 |
| A74-91 | 1.0253 | 1.0061 | 0.99663 | 0.99756 | 1.0025 | 1.0096 | 1.0126 | 0.99985 |
| A92-99 | 1.0394 | 1.0187 | 0.99876 | 1.0059 | 0.99436 | 1.0125 | 1.0088 | 1.0001 |
| A00-06 | 1.0407 | 1.0073 | 0.99951 | 0.99578 | 1.0158 | 1.0099 | 1.0119 | 1.0114 |

Comparing Table 1 above with the corresponding entries in Table 4 in the main text, it can be seen that all of the entries are the same except that the entries for TFP growth, $\tau^t$, have increased by an average of 0.48 percentage points per year going from the labour quality adjusted results to the unadjusted case while the contribution of labour input growth, $\beta_L{}^t$, has decreased by an average of 0.48 percentage points per year. Thus if we (mistakenly) do not quality adjust labour, TFP growth increases from an average of 1.14% per year to 1.62% per year. This illustrates the importance of quality adjusting labour input.

**4. Differences in Capital Input Concepts**

The Statistics Canada KLEMS program has developed new estimates of capital services input over the period 1961-2006 for the Canadian business sector, $Q_{KO}{}^t$; see CANSIM II series V41713051, Canada; Capital Input; Business Sector, in table 3830021, Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub-Sectors. Using this series, we can compute the KLEMS average growth rate for capital services for the years 1962-2006 and this average growth rate turns out to be 4.96% per year. This rate can be compared to the average growth rate for our estimates for the quantity of capital services implied by the $Q_K{}^t$ series listed in Table 2 of the main text, which turned out to be 3.31% per year. This is an enormous difference in average growth rates and explains why our estimates of business sector TFP growth are so much larger than the corresponding estimates from the KLEMS program. Our series of capital services (one plus) growth rates, $Q_K{}^t/Q_K{}^{t-1}$, are listed in Table 2 below along with the corresponding official KLEMS series, $Q_{KO}{}^t/Q_{KO}{}^{t-1}$.

The Statistics Canada KLEMS program has also developed new estimates of the stock of capital used by the Canadian business sector over the period 1961-2006, $QK_O{}^t$; see CANSIM II series V41713068, Canada; Capital Stock; Business Sector, in table 3830021, Multifactor Productivity, Value Added, Capital Input and Labour Input in the Aggregate Business Sector and Major Sub-Sectors. Using this series, we can compute the KLEMS average growth rate for the business sector capital stock for the years 1962-2006 and this average growth rate turns out to be 3.24% per year. This rate can be compared to the average growth rate for our estimates for the quantity of capital used by our Canadian business sector which turned out to be 2.82% per year. Our capital stock aggregate for year t, $QK^t$ (with corresponding stock price $PK^t$) is defined as a chained Fisher aggregate of:

- The smoothed data quantity series for machinery and equipment stocks and nonresidential structures, $QK_{ME}^t$ and $QK_{NR}^t$ listed in Table 9 of Appendix 2 (with prices $P_{IME}^t$ and $P_{INR}^t$ listed in Table 2 of Appendix 2) and
- The quantity series for the stock of inventories, $QK_{BI}^t$, the stock of business land, $QK_{BL}^t$, and the stock of agricultural land, $QK_{AL}^t$, listed in Table 11 of Appendix 2 (with the corresponding stock price series $P_{BI}^t$, $P_{BL}^t$ and $P_{AL}^t$ from Table 10 in Appendix 2).

Our series of aggregate capital stock (one plus) growth rates, $QK^t/QK^{t-1}$, is listed in Table 2 below along with the counterpart official KLEMS capital stock growth rate series, $QK_O^t/QK_O^{t-1}$.

**Table 2: Unofficial and KLEMS Capital Services and Capital Stock Growth Rates**

| Year | $Q_K^t/Q_K^{t-1}$ | $Q_{KO}^t/Q_{KO}^{t-1}$ | $QK^t/QK^{t-1}$ | $QK_O^t/QK_O^{t-1}$ |
|---|---|---|---|---|
| 1962 | 1.01000 | 1.03731 | 1.00928 | 1.02622 |
| 1963 | 1.01768 | 1.04317 | 1.02195 | 1.03285 |
| 1964 | 1.02022 | 1.06897 | 1.02059 | 1.05300 |
| 1965 | 1.03348 | 1.08387 | 1.03102 | 1.05369 |
| 1966 | 1.04740 | 1.10714 | 1.04502 | 1.07006 |
| 1967 | 1.05932 | 1.06989 | 1.05267 | 1.05357 |
| 1968 | 1.04353 | 1.05025 | 1.03457 | 1.03955 |
| 1969 | 1.03290 | 1.05742 | 1.02992 | 1.04602 |
| 1970 | 1.03735 | 1.05882 | 1.03742 | 1.04416 |
| 1971 | 1.03327 | 1.04701 | 1.02849 | 1.03731 |
| 1972 | 1.03172 | 1.06122 | 1.02693 | 1.04317 |
| 1973 | 1.03107 | 1.08077 | 1.02470 | 1.04828 |
| 1974 | 1.04303 | 1.07829 | 1.03485 | 1.05702 |
| 1975 | 1.05115 | 1.06931 | 1.04483 | 1.05394 |
| 1976 | 1.04931 | 1.06173 | 1.04027 | 1.05118 |
| 1977 | 1.04652 | 1.05233 | 1.04181 | 1.04307 |
| 1978 | 1.04283 | 1.05525 | 1.04114 | 1.03770 |
| 1979 | 1.03698 | 1.06283 | 1.03320 | 1.04671 |
| 1980 | 1.04679 | 1.07389 | 1.04170 | 1.04298 |
| 1981 | 1.04176 | 1.07339 | 1.03018 | 1.04596 |
| 1982 | 1.05178 | 1.02564 | 1.04063 | 1.01667 |
| 1983 | 1.01975 | 1.02500 | 1.00856 | 1.01043 |
| 1984 | 1.01571 | 1.03252 | 1.01282 | 1.01032 |
| 1985 | 1.02260 | 1.03937 | 1.02347 | 1.02044 |
| 1986 | 1.02743 | 1.03598 | 1.02588 | 1.02289 |
| 1987 | 1.02671 | 1.05302 | 1.02316 | 1.03217 |
| 1988 | 1.03371 | 1.05556 | 1.02837 | 1.04065 |
| 1989 | 1.04400 | 1.05757 | 1.03524 | 1.03776 |
| 1990 | 1.04457 | 1.03577 | 1.03641 | 1.02886 |
| 1991 | 1.03009 | 1.01952 | 1.02133 | 1.01585 |
| 1992 | 1.02102 | 1.01767 | 1.01349 | 1.00000 |

| | | | | |
|---|---|---|---|---|
| 1993 | 1.00988 | 1.01447 | 1.00254 | 1.00480 |
| 1994 | 1.00789 | 1.03138 | 1.00514 | 1.00836 |
| 1995 | 1.01798 | 1.03873 | 1.01628 | 1.00948 |
| 1996 | 1.02177 | 1.03595 | 1.02007 | 1.01291 |
| 1997 | 1.02774 | 1.05656 | 1.02878 | 1.03360 |
| 1998 | 1.04841 | 1.05109 | 1.04496 | 1.02803 |
| 1999 | 1.04376 | 1.05787 | 1.03694 | 1.02944 |
| 2000 | 1.04008 | 1.04814 | 1.03018 | 1.03072 |
| 2001 | 1.04333 | 1.02714 | 1.03748 | 1.01542 |
| 2002 | 1.02823 | 1.01626 | 1.02034 | 1.01215 |
| 2003 | 1.02208 | 1.03400 | 1.01886 | 1.02200 |
| 2004 | 1.01961 | 1.03772 | 1.01207 | 1.02446 |
| 2005 | 1.02940 | 1.04567 | 1.02461 | 1.03056 |
| 2006 | 1.03684 | 1.04635 | 1.03171 | 1.03522 |
| Average | 1.0331 | 1.0496 | 1.0282 | 1.0324 |

Looking at the entries in Table 2, it can be seen that there is little correspondence between the growth rates for our aggregate capital services series $Q_K^t/Q_K^{t-1}$ and the corresponding KLEMS aggregate capital services series $Q_{KO}^t/Q_{KO}^{t-1}$. There is a bit more correspondence between the growth rates for our aggregate capital stock series $QK^t/QK^{t-1}$ and the corresponding KLEMS aggregate capital services series $QK_O^t/QK_O^{t-1}$ but the series are still not close.

It is possible to explain why the average growth rate of capital services should be bigger than the average growth rate of capital stock components. Using our estimates for beginning of the year capital stocks, one plus the average growth rate of machinery and equipment stocks is 1.0493 over the years 1962-2006, one plus the average growth rate of nonresidential structure stocks is 1.0318, one plus the average growth rate of business nonagricultural, nonresidential land is 1.0000, one plus the average growth rate of agricultural land is 0.99929 and one plus the average growth rate of inventory stocks is 1.0301. Note that the average growth rate for machinery and equipment is just under 5% per year, followed by nonresidential structures at 3.2% per year, followed by inventory stocks at 3.0% per year and the two land stocks are essentially constant. The average expenditure shares of these 5 capital inputs in total user cost over the years 1961-2006 are 0.34938 (M&E), 0.43458 (NR), 0.08571 (BL), 0.05128 (AL) and 0.07905 (Business Inventories). An approximation to the overall average year to year (one plus) growth of capital services can be obtained by multiplying each of the 5 component average (one plus) growth rates by the corresponding average cost shares. When this computation is carried out, we obtain an average growth rate for capital services of 1.0334, which is very close to the average of our top down capital service growth rates, $Q_K^t/Q_K^{t-1}$, listed in the last row of column 2 of Table 2 above, which was 1.0331. Now carry out the same type of approximate calculation for capital stocks. The average growth rates for the components of the capital stock remain the same but now the shares of each asset in the total asset value of all capital stocks will change. The average shares of these 5 capital assets in the total asset value of capital used by our top down business sector over the years 1961-2006 are 0.22370 (M&E), 0.41706 (NR), 0.13361 (BL), 0.08236 (AL) and

0.15539 (Business Inventories). An approximation to the overall average year to year (one plus) growth of capital stocks can be obtained by multiplying each of the 5 component average (one plus) growth rates by the corresponding average stock shares. When this computation is carried out, we obtain an average growth rate for the aggregate business sector capital stock of 1.0286, which is very close to the average of our top down capital stock growth rates, $QK^t/QK^{t-1}$, listed in the last row of column 4 of Table 2 above, which was 1.0282. Thus what is happening when we shift from the growth of capital services to the growth of capital stocks is that the weight for the machinery and equipment growth rate (the fastest growing component) drops from 0.35 to 0.22 and the weight for the land components (which do not grow at all) increases from 0.137 to 0.216 and thus the overall growth rate drops when we shift from capital services to capital stocks.

We cannot carry out the same type of exercise for the consistency of the KLEMS capital stock and service flow growth rates because information on the 30 types of asset that the KLEMS program considers has not yet been released. However, it seems unlikely that the capital *services* average growth rate could be close to 5% per year and yet the corresponding aggregate *stocks* could grow at only 3.24% per year.

Recall that the KLEMS business sector includes the services of residential land and structures that are rented whereas our business sector excludes these capital inputs. In what follows, we will make some rough estimates for these excluded capital services and add them to our other capital services in order to determine whether our omission of these residential housing capital services could materially affect our overall rate of growth of capital services.

Recall that in Appendix 2, we made estimates for the total stocks of residential land and residential structures that were used in the Canadian economy in both the Owner Occupied Housing (OOH) sector and the rental sector. Recall that using investment and balance sheet information, we found that an appropriate reconciling depreciation rate for residential structures in year t, $\delta_{RS}^t$, was 0.04 or 4% per year. Using this depreciation rate, a starting value for the stock and the investment information on residential structures (see the series for $Q_{IR}^t$ which is listed in Table 3 of Appendix 2), we formed estimates for beginning of the year t stock of residential structures, $QK_{RS}^t$, which are listed in Table 11 of Appendix 2. The corresponding year t prices $P_{RS}^t$ are listed in Table 10 of Appendix 2, which is also equal to the corresponding year t investment price for residential structures, $P_{INR}^t$, which was listed in Table 2 of Appendix 2. Recall also that in Appendix 2, we used National Balance Sheet information in order to obtain series for the price of residential land, $P_{RL}^t$ listed in Table 10, and for the corresponding quantity, $QK_{RL}^t$ listed in Table 11 of Appendix 2. There is one more table to recall from Appendix 2 and that is Table 1, which listed the final demand value, quantity and price series for imputed residential rents, $V_{IMR}^t$, $Q_{IMR}^t$ and $P_{IMR}^t$ respectively, and for paid residential rents, $V_{PR}^t$, $Q_{PR}^t$ and $P_{PR}^t$ respectively.

The user cost for residential land in year t should be equal to a real rate of return $r^t$ times the stock price of residential land, $P_{RL}^t$, and the user cost for residential structures in year t should be equal to the same real rate of return $r^t$ plus the depreciation rate $\delta_{RS}^t$ times the stock price of residential structures, $P_{RS}^t$.[117] Multiplying these user costs by the corresponding stocks, $QK_{RL}^t$ and $QK_{RS}^t$ respectively, should be approximated equal to the value of imputed rents in year t, $V_{IMR}^t$, plus the value of paid residential rents, $V_{PR}^t$; i.e., the following equation should hold for each year t:

(1) $V_{IMR}^t + V_{PR}^t = r^t P_{RL}^t QK_{RL}^t + (r^t + \delta_{RS}^t) P_{RS}^t QK_{RS}^t$ ; t = 1961,...,2006.

For each year t, the above equation can be solved for a balancing real rate of return, $r^t$. The sample average for these real rates was 0.06045 (with a minimum of 0.043 and a maximum of 0.071), which is very reasonable considering the fact that $r^t$ also has to include property taxes. Once the balancing real rates of return $r^t$ have been determined, we can postulate that the value of imputed rents $V_{IMR}^t$ is equal to a fraction $f_1$ of the aggregate value of land rents $r^t P_{RL}^t QK_{RL}^t$ plus another fraction $f_2$ of the aggregate value of residential structures rents $(r^t + \delta_{RS}^t)P_{RS}^t QK_{RS}^t$. Thus the value of paid rents $V_{PR}^t$ should be equal to the fraction $1 - f_1$ of the aggregate value of land rents $r^t P_{RL}^t QK_{RL}^t$ plus the fraction $1 - f_2$ of the aggregate value of residential structures rents $(r^t + \delta_{RS}^t)P_{RS}^t QK_{RS}^t$. Thus the following two equations should hold, at least approximately:

(3) $V_{IMR}^t = f_1 r^t P_{RL}^t QK_{RL}^t + f_2 (r^t + \delta_{RS}^t)P_{RS}^t QK_{RS}^t$ ; t = 1961,...,2006;
(4) $V_{PR}^t = (1 - f_1) r^t P_{RL}^t QK_{RL}^t + (1 - f_2)(r^t + \delta_{RS}^t)P_{RS}^t QK_{RS}^t$ ; t = 1961,...,2006.

The parameters $f_1$ and $f_2$ in the above two equations were estimated using the Nonlinear option in Shazam; see White (2004). The estimated values for these parameters turned out to be $f_1 = 1.0333$ and $f_2 = 0.64876$. This tells us that the OOH sector contains about 65% of the residential structures and 103% of the residential land, which of course, is not sensible.[118] In what follows, we will assume that the OOH sector uses 65% of the stock

[117] We include property taxes in the rate of return $r^t$ here as opposed to what was done in Appendix 2.
[118] This result indicates the difficulties in forming accurate estimates for the amounts of land and structures used in the rental housing market and explains why we excluded rental housing from our business sector value aggregate. Baldwin, Gu and Yan (2007; 43) also reported difficulties in obtaining accurate information on the housing market: "As the output of the lessors of real estate includes the paid rents of rental residential buildings, capital input to the lessors of real estate industry needs to be adjusted to include investment in rental buildings. Data on investment in rental residential buildings are not available. For the annual MFP programs, we divide the total investment in residential building into rental building and owner-occupied dwelling using paid rents for rental buildings and imputed rents for owner occupied dwelling as the split ratios. The investment in residential buildings and paid and imputed rents are available from the Income and Expenditure Accounts. On average, we find that about 30% of total rents are paid rents and the remaining 70% are imputed rents." On the problems associated with obtaining estimates for residential rented land and land by sector in general, Baldwin, Gu and Yan (2007; 43-44) make the following observations: "In the past, the MFP programs assumed that there was little change in the real value of land in the business sector and estimated the real value of land at the industry level, based on the industry distribution of property taxes. We have now adopted the BLS methodology for estimating land stock in the MFP programs of Statistics Canada. The overall effect of adopting the BLS methodology on the business sector MFP growth is small. ... Data on the value of land at the industry level are scarce. We assume that land stock is proportional to the structures stock. The land–structure ratios are derived from the corporate balance sheets by sector which provide data on book values of land and structures by industry for the 1972-to-1987 period (CANSIM Table 180-0002). The real value of land at the industry level is estimated by deflating the nominal value of land using the structure capital's deflators. The final estimates

of residential structures and 90% of the residential land. In Table 3 below, we list the quantity of residential land that is used in the residential housing rental sector, $QK_{RRL}^t$ (which is equal to 0.1 times the total quantity of residential land $QK_{RL}^t$), the quantity of residential structures used in the rental housing sector (which is equal to 0.25 times the total quantity of residential structures $QK_{RL}^t$), the stock price of residential land $P_{RL}^t$, the stock price of residential structures $P_{RS}^t$, the user cost of residential rental housing land $U_{RL}^t$ (which is equal to $r^t P_{RL}^t$) and the user cost of residential rental housing structures $U_{RS}^t$ (which is equal to $(r^t + \delta_{RS}^t)P_{RS}^t$).

**Table 3: Rented Residential Housing Quantities, Prices and User Costs of Land and Structures**

| Year | $QK_{RRL}^t$ | $QK_{RRS}^t$ | $P_{RL}^t$ | $P_{RS}^t$ | $U_{RL}^t$ | $U_{RS}^t$ |
|---|---|---|---|---|---|---|
| 1961 | 1068 | 10049 | 1.00000 | 1.00000 | 0.06470 | 0.10470 |
| 1962 | 1068 | 10421 | 1.06956 | 1.00504 | 0.07005 | 0.10602 |
| 1963 | 1068 | 10799 | 1.13134 | 1.02769 | 0.07713 | 0.11118 |
| 1964 | 1068 | 11190 | 1.17487 | 1.07312 | 0.07962 | 0.11565 |
| 1965 | 1068 | 11693 | 1.28037 | 1.13368 | 0.08211 | 0.11805 |
| 1966 | 1068 | 12214 | 1.40187 | 1.20765 | 0.08762 | 0.12378 |
| 1967 | 1068 | 12670 | 1.53198 | 1.28518 | 0.09711 | 0.13287 |
| 1968 | 1068 | 13127 | 1.65845 | 1.31431 | 0.11287 | 0.14202 |
| 1969 | 1068 | 13699 | 1.85587 | 1.38118 | 0.12787 | 0.15041 |
| 1970 | 1068 | 14393 | 2.07191 | 1.42615 | 0.14785 | 0.15882 |
| 1971 | 1068 | 14957 | 2.29311 | 1.53179 | 0.16015 | 0.16825 |
| 1972 | 1068 | 15663 | 2.62645 | 1.67349 | 0.16967 | 0.17505 |
| 1973 | 1068 | 16460 | 3.07361 | 1.97123 | 0.16708 | 0.18601 |
| 1974 | 1068 | 17331 | 3.74740 | 2.36134 | 0.16931 | 0.20114 |
| 1975 | 1068 | 18200 | 4.45217 | 2.56072 | 0.20476 | 0.22020 |
| 1976 | 1068 | 19007 | 4.99931 | 2.76853 | 0.24687 | 0.24745 |
| 1977 | 1068 | 20057 | 5.54073 | 2.87768 | 0.30491 | 0.27347 |
| 1978 | 1068 | 21090 | 6.21911 | 3.04069 | 0.35922 | 0.29726 |
| 1979 | 1068 | 22098 | 6.89280 | 3.28046 | 0.39385 | 0.31866 |
| 1980 | 1068 | 23052 | 7.82598 | 3.55455 | 0.44679 | 0.34511 |
| 1981 | 1068 | 23872 | 9.46028 | 3.99273 | 0.52505 | 0.38131 |
| 1982 | 1068 | 24765 | 11.12266 | 4.08226 | 0.66425 | 0.40708 |
| 1983 | 1068 | 25293 | 10.97429 | 4.25350 | 0.71838 | 0.44857 |
| 1984 | 1068 | 26059 | 11.89202 | 4.41785 | 0.78820 | 0.46953 |
| 1985 | 1068 | 26812 | 13.04218 | 4.55564 | 0.87265 | 0.48704 |
| 1986 | 1068 | 27692 | 13.36634 | 4.90827 | 0.88352 | 0.52077 |
| 1987 | 1068 | 28778 | 15.51183 | 5.40819 | 0.94700 | 0.54650 |
| 1988 | 1068 | 30143 | 18.32549 | 5.78293 | 1.06691 | 0.56800 |
| 1989 | 1068 | 31507 | 20.90485 | 6.13195 | 1.20746 | 0.59946 |
| 1990 | 1068 | 32920 | 24.58101 | 6.11231 | 1.45274 | 0.60573 |
| 1991 | 1068 | 33996 | 24.21398 | 6.32257 | 1.47245 | 0.63738 |

of land stocks in both current and constant dollars at the industry level are benchmarked to the aggregate land stock in the total non-farm business sector."

| | | | | | | |
|---|---|---|---|---|---|---|
| 1992 | 1068 | 34674 | 26.87384 | 6.39701 | 1.65973 | 0.65097 |
| 1993 | 1068 | 35470 | 28.74160 | 6.58445 | 1.74548 | 0.66325 |
| 1994 | 1068 | 36160 | 30.93333 | 6.76485 | 1.88137 | 0.68203 |
| 1995 | 1068 | 36909 | 33.01685 | 6.76717 | 2.03648 | 0.68809 |
| 1996 | 1068 | 37301 | 32.16502 | 6.75581 | 2.07432 | 0.70591 |
| 1997 | 1068 | 37857 | 33.03810 | 6.87512 | 2.13195 | 0.71866 |
| 1998 | 1068 | 38558 | 35.06845 | 6.95993 | 2.25210 | 0.72536 |
| 1999 | 1068 | 39153 | 36.91589 | 7.13210 | 2.34121 | 0.73760 |
| 2000 | 1068 | 39800 | 39.80725 | 7.29782 | 2.48795 | 0.74803 |
| 2001 | 1068 | 40538 | 42.38558 | 7.48766 | 2.61434 | 0.76135 |
| 2002 | 1068 | 41494 | 47.02347 | 7.81242 | 2.78990 | 0.77601 |
| 2003 | 1068 | 42775 | 53.71548 | 8.21290 | 2.96724 | 0.78220 |
| 2004 | 1068 | 44163 | 59.28730 | 8.71303 | 3.05448 | 0.79742 |
| 2005 | 1068 | 45727 | 68.59715 | 9.11300 | 3.26660 | 0.79848 |
| 2006 | 1068 | 47347 | 78.10301 | 9.77854 | 3.40529 | 0.81749 |

Using the above information on the user costs of rental land and structures, $U_{RL}{}^t$ and $U_{RS}{}^t$, and their corresponding quantities, $QK_{RRL}{}^t$ and $QK_{RRS}{}^t$, we formed chained Fisher indexes of these two price and quantity series along with our earlier price and quantity series for aggregate business sector capital services, $P_K{}^t$ from Table 1 in the main text and $Q_K{}^t$ from Table 2 in the main text. Denote the new aggregate capital services quantity index by $Q_{KN}{}^t$ for year t. We then formed (one plus) the growth rates for this augmented capital services aggregate, $Q_{KN}{}^t/Q_{KN}{}^{t-1}$, for the years 1962-2006. We found that the sample average of these growth rates was 1.0331, which is exactly the same average growth rate that we obtained for our capital services aggregate that excluded residential rental housing from the business sector.[119] Hence it seems unlikely that the fact that the KLEMS definition of the Canadian business sector includes rented residential housing whereas our definition excludes this sector could explain the large divergence on our rates of growth for capital services, which averaged 3.31% per year, compared to the KLEMS average growth rate of 4.96% per year.

Recall that we defined our capital stock aggregate for year t as $QK^t$ and the corresponding official KLEMS aggregate stock as $QK_O{}^t$ and the rates of growth for these stock aggregates can be found in Table 2 above. Using the information on the stock prices of rental land and structures, $P_{RL}{}^t$ and $P_{RS}{}^t$, and their corresponding quantities, $QK_{RRL}{}^t$ and $QK_{RRS}{}^t$, which is listed in Table 3 above, we formed chained Fisher indexes of these two price and quantity series along with our earlier price and quantity series for aggregate business sector capital services, $PK^t$ and $QK^t$, whose construction is described above Table 2 in this Appendix. Denote the new aggregate capital services quantity index by $QK_N{}^t$ for year t. We then formed (one plus) the growth rates for this augmented capital stock aggregate, $QK_N{}^t/QK_N{}^{t-1}$, for the years 1962-2006. We found that the sample average of these growth rates was 1.0287, which is a bit higher than our old average growth rate for the aggregate capital stock used by our business sector, 1.0282.[120] Thus the addition of rented residential property to our old business sector capital stock

[119] See the sample average at the bottom of column 2 in Table 2 in this Appendix.
[120] See the last row in column 4 of Table 2 in this Appendix.

increases the average growth rate from 2.82% per year to 2.87% per year, which brings us a bit closer to the KLEMS average growth rate for business sector capital stocks of 3.24% per year.

Our conclusion at this point is that the differences in coverage between our definition of the aggregate Canadian business sector cannot explain the differences in the average rate of growth of capital services that we obtain for our gross capital model (3.31% per year) and the corresponding KLEMS average rate of growth (4.96% per year).

There is one additional experiment that we can undertake to try and explain our much smaller rate of growth of capital services: since our rental prices for capital do not have asset specific capital gains terms in them, inserting these terms into our user costs should increase the shares of machinery and equipment services in the capital services aggregate and hence lead to a higher average rate of growth of capital services. Thus changing our user cost formula to include ex post asset specific rates of price change should bring us closer to the user cost concept used in the Statistics Canada KLEMS program, which evidently includes some form of asset price appreciation terms in their user costs.[121] We will now explore how much difference adding ex post capital gains terms to the user costs will affect our capital services aggregate growth rate.[122]

We first define the ex post rates of price change for the five assets that we have data; i.e., recalling that $P_{IME}{}^{t}$, $P_{INR}{}^{t}$, $P_{BL}{}^{t}$, $P_{AL}{}^{t}$ and $P_{BI}{}^{t}$ are our year t asset prices for machinery and equipment, nonresidential structures, nonagricultural business land, agricultural land and business inventories respectively, the *ex post rates of price change* for these assets are defined as follows:

(5) $\kappa_{ME}{}^{t} \equiv (P_{IME}{}^{t+1}/P_{IME}{}^{t}) - 1$;
(6) $\kappa_{NR}{}^{t} \equiv (P_{INR}{}^{t+1}/P_{INR}{}^{t}) - 1$;

---

[121] It is not that easy to determine exactly how the KLEMS program user costs were constructed. Baldwin, Gu and Yan (2007; 24) describe the KLEMS capital service measures as follows. "The asset detail for capital services estimates in the MFP programs consists of 15 types of equipment, and 13 types of structures, and land and inventories for a total of 30 types of assets. The methodology for estimating capital services is documented in Baldwin and Gu (2007a) and Harchaoui and Tarkhani (2002). Here we mention two main features of capital services measures in Canada. First, the capital services measure for Statistics Canada's MFP programs is based on the bottom up approach. This bottom-up approach involves the estimation of capital stock by asset, the aggregation of capital stock of various asset types within each industry to estimate industry capital services, and the aggregation of capital services across industries to derive capital services in the business sector and in the aggregate industry sectors. Second, investment is benchmarked on the estimates of investment included in the input–output tables in order to ensure consistency between capital input measures and output measures. Recent studies by Statistics Canada provide new empirical evidence on the depreciation rate for various types of assets (Statistics Canada 2007). As a result, we have incorporated these new estimates of depreciation rates in the capital service estimates." However, this general introduction to the KLEMS capital services measurement program does not provide us with the details on the exact form of the user cost formula that was used except to refer the reader to Baldwin and Gu (2007). But this latter study contains many user cost variants and none of them appear to match up exactly with what actually appears in the most recent CANSIM tables on Multifactor productivity.

[122] For other studies that explore empirically the differences between various user cost formulae, see Harper, Berndt and Wood (1989), Diewert (2005a) and Baldwin and Gu (2007).

(7) $\kappa_{BL}{}^t \equiv (P_{BL}{}^{t+1}/P_{BL}{}^t) - 1$;
(8) $\kappa_{AL}{}^t \equiv (P_{AL}{}^{t+1}/P_{AL}{}^t) - 1$;
(9) $\kappa_{BI}{}^t \equiv (P_{BI}{}^{t+1}/P_{BI}{}^t) - 1$.

The above ex post asset specific rates of price change are listed in Table 4 below for the years 1961-2005. Jorgenson and his coworkers have long maintained that user costs of capital should include the above asset specific rates of price inflation in the formula as a negative contribution term.[123] Thus using this Jorgensonian methodological approach, the old user costs defined by equations (8)-(12) in Appendix 2 should be replaced by the following user costs for our five assets:

(10) $U_{ME}{}^t \equiv [r^t + \tau_B{}^t + \delta_{ME}{}^t - \kappa_{ME}{}^t]\, P_{IME}{}^t$ ;
(11) $U_{NR}{}^t \equiv [r^t + \tau_B{}^t + \tau_P{}^t + \delta_{NR}{}^t - \kappa_{NR}{}^t]\, P_{INR}{}^t$ ;
(12) $U_{BL}{}^t \equiv [r^t + \tau_B{}^t + \tau_P{}^t - \kappa_{BL}{}^t]\, P_{BL}{}^t$ ;
(13) $U_{AL}{}^t \equiv [r^t + \tau_B{}^t + \tau_P{}^t - \kappa_{AL}{}^t]\, P_{AL}{}^t$ ;
(14) $U_{BI}{}^t \equiv [r^t + \tau_B{}^t - \kappa_{BI}{}^t]\, P_{BI}{}^t$.

However, now that the asset specific price change terms have been included in the above user costs, the rate of return $r^t$ which appears in those user costs are no longer real rates of return but are *nominal rates* of return; i.e., they have the amount of general inflation which occurred during year t imbedded in them.

A balancing or endogenous nominal rate of return for the Canadian business sector can be determined by solving the following counterpart to equation (13) in Appendix 2:

$$
\begin{aligned}
(15)\quad & P_C{}^tQ_C{}^t + P_{IG}{}^tQ_{IG}{}^t + P_{IR}{}^tQ_{IR}{}^t + P_{INR}{}^tQ_{INR}{}^t + P_{IME}{}^tQ_{IME}{}^t + P_{II}{}^tQ_{II}{}^t + P_{GN}{}^tQ_{GN}{}^t \\
& \quad + P_{XG}{}^tQ_{XG}{}^t + P_{XS}{}^tQ_{XS}{}^t + P_{MG}{}^tQ_{MG}{}^t + P_{MS}{}^tQ_{MS}{}^t \\
& = P_{L1}{}^tQ_{L1}{}^t + P_{L2}{}^tQ_{L2}{}^t + P_{L3}{}^tQ_{L3}{}^t + [r^t + \tau_B{}^t + \delta_{ME}{}^t - \kappa_{ME}{}^t]\, P_{IME}{}^tQK_{ME}{}^t \\
& \quad + [r^t + \tau_B{}^t + \tau_P{}^t + \delta_{NR}{}^t - \kappa_{NR}{}^t]\, P_{INR}{}^tQK_{NR}{}^t + [r^t + \tau_B{}^t + \tau_P{}^t - \kappa_{BL}{}^t]\, P_{BL}{}^tQK_{BL}{}^t \\
& \quad + [r^t + \tau_B{}^t + \tau_P{}^t - \kappa_{AL}{}^t]\, P_{AL}{}^tQK_{AL}{}^t + [r^t + \tau_B{}^t - \kappa_{BI}{}^t]\, P_{BI}{}^tQK_{BI}{}^t\ ; \quad t = 1961, ..., 2005.
\end{aligned}
$$

Define the before business taxes year t nominal rate of return for the Canadian business sector, $r_G{}^t$, as the after tax nominal rate of return $r^t$ defined by solving (15) above plus the rate of general business taxation, $\tau_B{}^t$. The gross nominal balancing rates of return $r_G{}^t$ and the after tax nominal rates of return $r^t$ are reported in Table 4 below.

**Table 4: Before and After Tax Nominal Balancing Rates of Return and Asset Specific Inflation Rates**

| Year | $r_G{}^t$ | $r^t$ | $\kappa_{ME}{}^t$ | $\kappa_{NR}{}^t$ | $\kappa_{BL}{}^t$ | $\kappa_{AL}{}^t$ | $\kappa_{BI}{}^t$ |
|---|---|---|---|---|---|---|---|
| 1961 | 1.07338 | 1.04331 | 1.01973 | 1.00592 | 1.06956 | 1.04000 | 1.01546 |
| 1962 | 1.08306 | 1.05213 | 1.01352 | 1.02643 | 1.06760 | 1.05769 | 0.98661 |
| 1963 | 1.10993 | 1.07810 | 1.03372 | 1.02815 | 1.07678 | 1.10909 | 1.04277 |

[123] This choice of user cost formula with ex post asset price changes imbedded in the formula dates back to the pioneering work of Jorgenson and Griliches (1967) (1972) and Christensen and Jorgenson (1969).

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 1964 | 1.12620 | 1.09283 | 1.02299 | 1.05768 | 1.08890 | 1.11475 | 1.01584 |
| 1965 | 1.13124 | 1.09860 | 1.02401 | 1.06272 | 1.12698 | 1.11765 | 1.02555 |
| 1966 | 1.12317 | 1.09130 | 1.01178 | 1.04077 | 1.14031 | 1.13158 | 1.02351 |
| 1967 | 1.09141 | 1.06126 | 1.01541 | 1.00837 | 1.10637 | 1.10465 | 1.01915 |
| 1968 | 1.11371 | 1.08102 | 1.03968 | 1.05804 | 1.08428 | 1.02105 | 1.01753 |
| 1969 | 1.11233 | 1.07829 | 1.04060 | 1.04953 | 1.11823 | 1.01031 | 1.02128 |
| 1970 | 1.11305 | 1.08159 | 1.03014 | 1.05576 | 1.11578 | 1.02041 | 1.00172 |
| 1971 | 1.12012 | 1.08803 | 1.03271 | 1.05644 | 1.13355 | 1.08000 | 1.01649 |
| 1972 | 1.16718 | 1.13331 | 1.03642 | 1.10816 | 1.15682 | 1.21296 | 1.09670 |
| 1973 | 1.26400 | 1.22629 | 1.10884 | 1.18354 | 1.21288 | 1.30534 | 1.18854 |
| 1974 | 1.24413 | 1.20288 | 1.12009 | 1.11758 | 1.25432 | 1.26901 | 1.22049 |
| 1975 | 1.15735 | 1.12058 | 1.06942 | 1.05611 | 1.18911 | 1.18433 | 1.02498 |
| 1976 | 1.14849 | 1.11515 | 1.07283 | 1.05368 | 1.12345 | 1.15175 | 1.01317 |
| 1977 | 1.17440 | 1.14302 | 1.09125 | 1.07180 | 1.12693 | 1.18581 | 1.05173 |
| 1978 | 1.19708 | 1.16590 | 1.08551 | 1.09282 | 1.13090 | 1.23647 | 1.11087 |
| 1979 | 1.22775 | 1.19540 | 1.07879 | 1.12220 | 1.15748 | 1.26037 | 1.16081 |
| 1980 | 1.20793 | 1.17529 | 1.12002 | 1.10873 | 1.16523 | 1.12431 | 1.13457 |
| 1981 | 1.15216 | 1.12079 | 1.07752 | 1.07442 | 1.15171 | 0.99837 | 1.09550 |
| 1982 | 1.07043 | 1.04279 | 1.02435 | 0.99237 | 1.07960 | 0.97781 | 1.01502 |
| 1983 | 1.09757 | 1.06900 | 1.00100 | 1.03829 | 1.02661 | 0.95562 | 1.03035 |
| 1984 | 1.11380 | 1.08187 | 1.01229 | 1.03236 | 1.05641 | 0.93867 | 1.02476 |
| 1985 | 1.10283 | 1.07105 | 1.01106 | 1.01464 | 1.04296 | 0.92688 | 1.00920 |
| 1986 | 1.10907 | 1.07831 | 0.98323 | 1.04631 | 1.07128 | 0.94021 | 1.01316 |
| 1987 | 1.13305 | 1.09998 | 0.98638 | 1.05705 | 1.08126 | 0.98263 | 1.02900 |
| 1988 | 1.13468 | 1.10153 | 1.00325 | 1.04162 | 1.09520 | 1.06537 | 1.04180 |
| 1989 | 1.12109 | 1.08794 | 1.00477 | 1.03316 | 1.08310 | 1.08121 | 1.00651 |
| 1990 | 1.06470 | 1.03237 | 0.95015 | 0.98321 | 1.06719 | 1.02693 | 0.99723 |
| 1991 | 1.05662 | 1.02619 | 1.00543 | 0.99446 | 1.02552 | 0.98049 | 0.96653 |
| 1992 | 1.07416 | 1.04387 | 1.02083 | 1.01250 | 1.01884 | 1.00515 | 1.03944 |
| 1993 | 1.09967 | 1.06733 | 1.03274 | 1.03323 | 1.04689 | 1.03894 | 1.03421 |
| 1994 | 1.10619 | 1.07149 | 1.00413 | 1.01313 | 1.05512 | 1.07690 | 1.01885 |
| 1995 | 1.11706 | 1.08038 | 0.99199 | 1.02978 | 1.03380 | 1.08211 | 1.05836 |
| 1996 | 1.11805 | 1.07841 | 1.00022 | 1.02515 | 1.04751 | 1.08129 | 0.91586 |
| 1997 | 1.12509 | 1.08167 | 1.01316 | 1.02651 | 1.04345 | 1.06095 | 0.92441 |
| 1998 | 1.10573 | 1.06652 | 0.96955 | 1.02020 | 1.04387 | 1.03782 | 1.00000 |
| 1999 | 1.12622 | 1.08175 | 0.99147 | 1.03393 | 1.04886 | 1.03122 | 1.04207 |
| 2000 | 1.13907 | 1.08973 | 1.01254 | 1.00856 | 1.05737 | 1.03294 | 1.01033 |
| 2001 | 1.12603 | 1.08656 | 1.00919 | 1.01685 | 1.03980 | 1.02703 | 1.00000 |
| 2002 | 1.10741 | 1.06955 | 0.93512 | 1.01995 | 1.04482 | 1.02323 | 0.98699 |
| 2003 | 1.13899 | 1.09920 | 0.95576 | 1.06241 | 1.06154 | 1.02268 | 1.03107 |
| 2004 | 1.16026 | 1.11756 | 0.96885 | 1.05651 | 1.08120 | 1.02352 | 1.01827 |
| 2005 | 1.16058 | 1.11742 | 0.96385 | 1.04449 | 1.05927 | 1.02551 | 1.02063 |
| Average | 1.1299 | 1.0953 | 1.0221 | 1.0461 | 1.0913 | 1.0707 | 1.0337 |

The average nominal ex post before tax rate of return earned by the Canadian business sector was a whopping 12.99% on average according to the above computations and the after tax average nominal rate of return was 9.53%. These rates of return seem to be too high, particularly in recent years. Again, this could be a reflection of depreciation rates that are too high, investment quantities that are too low or asset valuations for real property that are too low. The average ex post rate of price change for machinery and equipment over the period was 2.21% per year, followed by business inventories at 3.37%, nonresidential structures at 4.61%, agricultural land at 7.07% and as might be expected, the rate of price inflation for nonagricultural, nonresidential business land was the highest at 9.13% per year.

Using the new ex post Jorgensonian user costs defined by (10)-(14) above, we can recompute our capital services aggregate as a direct Törnqvist index of the five capital quantities weighted by their new user costs.[124] As expected, the new capital services aggregate grows more rapidly; the average growth rate over the 44 years running from 1962-2005 turned out to be 3.62% per year, which is a ten percent increase over our old average rate of 3.31% per year (recall the last row of column 2 of Table 2 above. Recall our earlier analysis in this section where we attempted to explain why the growth of capital services would be greater than the growth of capital stocks. Recall that our old expenditure shares for capital services in total user cost over the years 1961-2006 were 0.34938 (M&E), 0.43458 (NR), 0.08571 (BL), 0.05128 (AL) and 0.07905 (Business Inventories). Using the new user costs that include ex post capital gains, the new cost shares are 0.38516 (M&E), 0.43795 (NR), 0.04658 (BL), 0.03731 (AL) and 0.09300 (Business Inventories). Thus the share of the rapidly growing machinery and equipment component of capital services has increased and the shares of the no growth land components has decreased as expected when we include ex post capital gains in our user cost formulae. An approximation to the overall average year to year (one plus) growth of capital services using the new user costs can be obtained by multiplying each of the 5 component average (one plus) growth rates by the corresponding new average cost shares. When this computation is carried out, we obtain an average growth rate for

[124] The new ex post user cost for nonagricultural business land turned out to be negative for the years 1966 and 1975 and negative for agricultural land for 1972-1975 and for 1979-1979. When forming the capital services aggregate, we did not remove these negative user costs because we wanted to insure that the value of business sector input was equal to the value of business sector output. Baldwin, Gu and Yan (2007; 25) explained how the KLEMS program dealt with negative user costs as follows: "The second empirical issue involves the way in which we have dealt with negative capital service prices during the estimation procedure. This arises from negative capital income in some periods in a few industries. Capital income is calculated from the input–output system as a residual, and is the difference between nominal value added and labour compensation of paid workers and self-employed workers. Negative capital income and negative capital service prices make aggregation difficult. More importantly, it is not clear that they are in keeping with the spirit of the estimation procedure for capital services. Enterprises are assumed to hire factors to bring the marginal product into equality with these prices. In the case of labour contracts, it is clear what the relevant price is for short-term decisions on hiring. But in the case of capital, the expected long-run capital cost is the relevant concept and short-run fluctuations in return are not likely to heavily influence expectations of long-run rates of returns. Therefore, to construct aggregate capital service input from asset-level capital stock and service prices, we have made adjustments for those assets whose user costs turn negative in the short run. We have set the user costs of the assets with negative user costs equal to the average user costs of the assets across all industries for those assets that are then adjusted for inter-industry differences in the user cost of capital." Unfortunately, these adjustments for negative user costs appear to upset the old value of outputs equals value of inputs identity that the KLEMS program used and it is not clear from the above explanation how balance was restored.

capital services of 1.0356, or 3.56% per year, which is reasonably close to our more accurate index number estimate of the average growth rate for Jorgensonian capital services of 3.62% per year.

The fact that capital services grow more rapidly when we move to Jorgensonian ex post user costs means that total factor productivity growth estimates are correspondingly reduced to 1.06% per year on average over the years 1962-2005 from our previous estimate of 1.14% per year over the years 1962-2006. Table 5 below is a counterpart to Table 4 in the main text where we repeat our decomposition analysis for real (gross) income growth using Jorgensonian user costs in place of our user costs which do not include ex post capital gains terms. The differences in the results are relatively small.

**Table 5: Business Sector Year to Year Growth in (Gross) Real Income and Year to Year Contribution Factors Using Jorgensonian User Costs**

| Year t | $\rho^t/\rho^{t-1}$ | $\tau^t$ | $\alpha_D{}^t$ | $\alpha_X{}^t$ | $\alpha_M{}^t$ | $\beta_L{}^t$ | $\beta_K{}^t$ | $\alpha_{XM}{}^t$ |
|---|---|---|---|---|---|---|---|---|
| 1962 | 1.07821 | 1.04888 | 1.00067 | 1.0092 | 0.98609 | 1.02905 | 1.00312 | 0.99516 |
| 1963 | 1.05029 | 1.02950 | 0.99925 | 0.99807 | 0.99863 | 1.01817 | 1.00606 | 0.99670 |
| 1964 | 1.10006 | 1.04926 | 1.00605 | 1.00481 | 1.00035 | 1.02964 | 1.00691 | 1.00517 |
| 1965 | 1.08182 | 1.02485 | 1.00762 | 1.00131 | 1.00466 | 1.02955 | 1.01149 | 1.00597 |
| 1966 | 1.07661 | 1.02010 | 0.99973 | 0.99736 | 1.00544 | 1.03541 | 1.01674 | 1.00279 |
| 1967 | 1.02153 | 0.98679 | 0.99956 | 0.99578 | 1.00536 | 1.01396 | 1.02028 | 1.00112 |
| 1968 | 1.04777 | 1.03485 | 0.99609 | 0.99423 | 1.00554 | 1.00293 | 1.01377 | 0.99974 |
| 1969 | 1.05614 | 1.02874 | 1.00239 | 0.99584 | 1.00164 | 1.01642 | 1.01020 | 0.99747 |
| 1970 | 1.04903 | 1.02667 | 1.00427 | 1.00211 | 1.00051 | 1.00235 | 1.01241 | 1.00262 |
| 1971 | 1.05583 | 1.02428 | 1.01294 | 0.99357 | 0.99891 | 1.01456 | 1.01061 | 0.99249 |
| 1972 | 1.05576 | 1.01315 | 1.00516 | 0.99960 | 1.00599 | 1.02036 | 1.01037 | 1.00559 |
| 1973 | 1.11836 | 1.02875 | 1.01037 | 1.02234 | 1.00073 | 1.04019 | 1.01103 | 1.02309 |
| 1974 | 1.04771 | 0.98684 | 1.00448 | 1.03927 | 0.97762 | 1.02367 | 1.01623 | 1.01601 |
| 1975 | 1.00214 | 1.00141 | 0.98587 | 0.9971 | 0.99965 | 0.99952 | 1.01887 | 0.99675 |
| 1976 | 1.0997 | 1.05479 | 1.00996 | 1.00729 | 1.00748 | 0.99967 | 1.01756 | 1.01482 |
| 1977 | 1.05105 | 1.04351 | 0.99720 | 1.00947 | 0.97745 | 1.00676 | 1.01679 | 0.98670 |
| 1978 | 1.02348 | 0.99602 | 0.99775 | 1.00501 | 0.98144 | 1.02767 | 1.01600 | 0.98636 |
| 1979 | 1.04906 | 0.98432 | 0.99922 | 1.02735 | 0.98678 | 1.03732 | 1.01426 | 1.01377 |
| 1980 | 1.00906 | 0.97282 | 0.99464 | 1.01572 | 0.98731 | 1.02171 | 1.01780 | 1.00283 |
| 1981 | 1.02555 | 1.00929 | 1.00753 | 0.99378 | 0.98234 | 1.01749 | 1.01531 | 0.97623 |
| 1982 | 0.94088 | 0.97381 | 0.99216 | 0.97446 | 1.01464 | 0.96754 | 1.01797 | 0.98873 |
| 1983 | 1.02708 | 1.02176 | 0.98739 | 0.97925 | 1.02743 | 1.00449 | 1.00734 | 1.00611 |
| 1984 | 1.05903 | 1.03827 | 0.99443 | 0.99694 | 0.99980 | 1.02334 | 1.00560 | 0.99674 |
| 1985 | 1.04639 | 1.01343 | 1.00031 | 0.99607 | 1.00228 | 1.02544 | 1.00829 | 0.99834 |
| 1986 | 1.01904 | 0.98745 | 1.00254 | 0.99051 | 1.00158 | 1.02744 | 1.00988 | 0.99208 |
| 1987 | 1.07256 | 1.01082 | 1.00392 | 0.99822 | 1.01658 | 1.03119 | 1.01005 | 1.01477 |
| 1988 | 1.05736 | 1.00251 | 1.00024 | 0.99041 | 1.02128 | 1.02923 | 1.01288 | 1.01149 |
| 1989 | 1.03403 | 0.99086 | 1.00037 | 0.99697 | 1.01215 | 1.01717 | 1.01634 | 1.00909 |
| 1990 | 0.95991 | 0.96813 | 0.98263 | 0.97480 | 1.01777 | 1.00073 | 1.01631 | 0.99212 |
| 1991 | 0.93069 | 0.96775 | 0.97873 | 0.96354 | 1.03198 | 0.97848 | 1.00993 | 0.99435 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 1992 | 0.99159 | 1.00144 | 0.99739 | 1.00641 | 0.98716 | 0.99300 | 1.00631 | 0.99348 |
| 1993 | 1.02225 | 1.00988 | 0.99932 | 1.01154 | 0.98351 | 1.01486 | 1.00326 | 0.99486 |
| 1994 | 1.07823 | 1.03995 | 1.01001 | 1.02729 | 0.97166 | 1.02559 | 1.00275 | 0.99817 |
| 1995 | 1.04840 | 1.00061 | 1.00217 | 1.03350 | 0.98654 | 1.01867 | 1.00662 | 1.01959 |
| 1996 | 1.06056 | 1.02785 | 0.99422 | 0.99136 | 1.01865 | 1.01871 | 1.00881 | 1.00986 |
| 1997 | 1.04673 | 1.02162 | 0.99324 | 0.99182 | 1.00635 | 1.01984 | 1.01339 | 0.99812 |
| 1998 | 1.01833 | 1.00337 | 0.99805 | 0.98983 | 0.98739 | 1.01994 | 1.02011 | 0.97734 |
| 1999 | 1.04942 | 1.00418 | 0.99570 | 0.99570 | 1.01364 | 1.02250 | 1.01702 | 1.00929 |
| 2000 | 1.09734 | 1.03130 | 0.99682 | 1.02340 | 1.00405 | 1.02271 | 1.01575 | 1.02755 |
| 2001 | 0.99648 | 0.98849 | 0.99479 | 0.98925 | 0.99873 | 1.00864 | 1.01690 | 0.98800 |
| 2002 | 1.02449 | 1.01362 | 1.00314 | 0.98020 | 1.00379 | 1.01271 | 1.01118 | 0.98391 |
| 2003 | 1.00153 | 0.96242 | 0.99081 | 0.97747 | 1.05240 | 1.01188 | 1.00901 | 1.02869 |
| 2004 | 1.07580 | 1.01629 | 1.00152 | 1.00465 | 1.02134 | 1.02172 | 1.00817 | 1.02609 |
| 2005 | 1.05285 | 1.00474 | 1.00462 | 1.00429 | 1.01706 | 1.00862 | 1.01246 | 1.02142 |
| A62-06 | 1.0411 | 1.0106 | 0.99921 | 0.99993 | 1.0025 | 1.0162 | 1.0121 | 1.0023 |
| A62-73 | 1.0660 | 1.0263 | 1.0037 | 1.0012 | 1.0012 | 1.0210 | 1.0111 | 1.0023 |
| A74-91 | 1.0253 | 1.0013 | 0.99663 | 0.99756 | 1.0025 | 1.0133 | 1.0137 | 0.99985 |
| A92-99 | 1.0394 | 1.0136 | 0.99876 | 1.0059 | 0.99436 | 1.0166 | 1.0098 | 1.0001 |
| A00-05 | 1.0414 | 1.0028 | 0.99861 | 0.99654 | 1.0162 | 1.0144 | 1.0122 | 1.0126 |

The above results show that moving to user costs that include ex post capital gains in the formula does tend to increase the rate of growth of capital services and so this factor does explain some portion of the big differences in our rates of capital services growth and the corresponding rates of KLEMS capital services growth. It is likely that moving from the five asset universe for which data are readily available to the 30 asset framework that is used by the KLEMS program would further narrow the gap between our capital and TFP growth rates and the corresponding KLEMS growth rates but it seems unlikely that aggregation errors in our computations can be the entire explanation for the huge differences in our results compared to the KLEMS results.[125]

It seems appropriate to raise the following issue at this point: is it better to *include* capital gains terms in the user cost formula or to *exclude* them as we have done in this study with the exception of the present Appendix? This is a rather deep question and deserves a lengthy discussion.[126] Suffice it to say here that the present author favors the inclusion of smoothed or anticipated capital gains terms in the user cost formula[127] but that this question is not completely resolved in the existing user cost literature.

[125] Another possible explanation for the discrepancy between our rates of growth for capital services and the corresponding KLEMS estimates is the fact that our treatment of business taxes is an average approach as opposed to the KLEMS marginal approach, which is based on the treatment of tax distortions pioneered by Hall and Jorgenson (1967). Again, it seems unlikely that this factor alone could explain the differences.
[126] For preliminary discussions of this issue, see Diewert (1980; 475-476) (2005a; 492-502) (2006a) and Schreyer (2007).
[127] Given this preference, the reader may well ask: why was this approach not implemented in the present study? The practical problem is to decide *exactly* how to form anticipated asset specific inflation rates in an *objective* and *reproducible* manner. Thus to avoid controversy about the choice of the method for estimating anticipated capital gains terms, we decided to use real interest rates and omit anticipated capital gains terms from our user cost formulae in the main text.

There is one additional factor which could help to explain the differences in our rates of capital services growth as compared to the corresponding Statistics Canada rates of growth: we have only one aggregate business sector in our model (due to data limitations) and hence only one aggregate rate of return for each year whereas the KLEMS program calculates balancing rates of return for each industry in the Statistics Canada list of industries in the input output tables:

"First, aggregate capital services in the business sector are constructed using the so-called 'bottom-up approach'. Baldwin and Gu (2007a) find that there is a large variation in the endogenous rate of return across industries and the endogenous rate of return is positively correlated with capital stock growth across industries. This suggests that the difference in the rate of return across industries is real, and capital tends to move toward those industries that earn relatively high rates of return." J.R. Baldwin, W. Gu and B. Yan (2007; 25).

Thus given that high rates of return are associated with rapid rates of capital accumulation, the disaggregated treatment of business sector industries used by the KLEMS program will lead to higher rates of growth of capital services as compared to our estimates.[128]

## 5. Recommendations for the Statistics Canada Productivity Program

There are substantial difficulties in accessing data on the prices and quantities of primary inputs used by the business and nonbusiness sectors from CANSIM. Also it is evident that the coverage of primary input usage by industry by Statistics Canada is not nearly as

[128] We are not able to estimate how significant this factor is due to the lack of published disaggregated data on capital stock and flow components at the industry level. However, Baldwin and Gu (2007; 47) address this issue in Table 11 of their paper. They provide alternative estimates of the average rate of growth of Multifactor productivity growth and for capital services growth for their Canadian business sector for two time periods and for two treatments for the balancing rates of return. The two treatments use either (i) industry specific rates or (ii) a total business sector single rate of return. For the 1961-1981 period, for treatment (i), they estimate average MFP growth at 0.59% per year and capital services growth at 6.19% per year and for treatment (ii), they estimate average MFP growth at 0.91% per year and capital services growth at 5.35% per year. For the 1981-2001 period, for treatment (i), they estimate average MFP growth at 0.12% per year and capital services growth at 3.55% per year and for treatment (ii), they estimate average MFP growth at 0.31% per year and capital services growth at 3.09% per year. Thus simply taking the arithmetic average of the above growth rates as a rough approximation to business sector performance over the period 1961-2001, using the KLEMS disaggregated balancing rate of return approach, MFP growth was about 0.355% per year which is in the neighborhood of the official KLEMS average MFP growth rate from CANSIM II series V41712881 for these years which was 0.49% per year. Taking the average of the treatment (i) capital services growth rates over the two periods gives us an average rate of growth of 4.87% per year, which is somewhat comparable to the official KLEMS average MFP growth rate from CANSIM II series V41713051 for these years which was 5.13% per year. Returning to Table 11 in Baldwin and Gu (2007) and their estimated growth rates using treatment (ii), we see that the average MFP growth rate over the period 1961-2001 was roughly 0.61% per year and the average rate of growth of capital services was 4.22% per year. Thus moving from the disaggregated balancing rates of return to the single business sector balancing rate of return increased their Table 11 KLEMS average rate of MFP growth from 0.355% to 0.61% (an increase of 0.255%) and decreased the Table 11 KLEMS average rate of growth of capital services from 4.87% to 4.22% (a decrease of 0.65% per year). Thus it appears that the KLEMS disaggregation of balancing rates of return could explain perhaps 0.25% of the difference in our much higher TFP growth rates compared to the corresponding KLEMS MFP growth rates.

extensive as the corresponding coverage of gross outputs and intermediate inputs. With the next revision of the System of National Accounts recommending a decomposition of gross operating profits into price and quantity components, it seems time for Statistics Canada to devote more effort into improving measurement with respect to primary inputs used by industries in the Canadian economy. Without accurate information on the flow of labour and capital services by industry, governments and businesses will not be able to plan ahead for Canada's future. It is important to know past trends in TFP growth by industry so that future trends can be anticipated and so that budgetary planning can be carried out on a more rational basis. Hopefully, other national departments interested in Canadian productivity growth (the Bank of Canada, the Department of Finance and Industry Canada to name a few) will support an initiative that will put more resources into the hands of Statistics Canada so that they can provide better information on productivity growth.

Important priorities for improving Statistics Canada's productivity program include the following ones:

- The National Balance Sheet accounts need to be fully integrated with the productivity program; i.e., Statistics Canada collects information on 30 classes of assets with some degree of industry breakdown but publishes only a crude four type of asset by households, corporations and governments breakdown. The household sector needs to be split into a self employed business component and a "consumer of goods and services" component and the corporate sector should be decomposed into industries *with price and quantity information for the 30 classes of asset made available* by quarter and by industry.
- The National Balance Sheet information on the value of land, residential structures and nonresidential structures needs to be greatly expanded so that more information on the *price* and *quantity* of real property by industry is made available.[129] The problems associated with finding adequate constant quality price indexes for residential and nonresidential structures are formidable[130] but given the importance of real property in the Canadian economy, it is necessary to put additional resources into this area of economic measurement.
- The KLEMS program has developed very useful price and quantity information on 56 types of labour used by the Canadian business sector but has only made this information generally available in a highly aggregated form with the information on three types of labour service used in this study being made available on CANSIM II. Evidently, the KLEMS program has developed price and quantity information for 56 types of labour by industry for the business sector and it would be extremely useful for this information to be made available to the general community. If it is felt that the disaggregated information is not

[129] We have some concerns that the National Balance Sheets are perhaps missing some growth in the value of real assets. Indirect evidence that points in this direction includes declining capital output ratios for the Canadian business sector and substantially increasing nominal and real rates of return earned by the business sector. Part of the problem may be the very high depreciation rates that are being used by the KLEMS program.

[130] For a review of these problems, see Diewert (2007a).

reliable enough to be released in this form, then it should be aggregated up and released at some level of detail that is more detailed than the present three price and quantity series that are available on CANSIM II. Furthermore, corresponding information on disaggregated labour input by type of worker should also be provided for the nonbusiness sector.[131]

- More information on the incidence of taxes needs to be provided in the input output accounts; i.e., we need to know exactly in which cell of the input output accounts various indirect and direct taxes are applied.[132] Not only is this information required to reconcile final demand indexes with production accounts indexes, it is also required in order to evaluate the efficiency of our tax system.[133]
- This study has shown that over short periods of time, changes in the real price of exports and imports can have substantial effects on living standards. The methodology used here applied only to the aggregate business sector. In Appendix 1, we showed how the methodology can be extended to the industry level but in order to implement this methodology to show the effects of changes in the terms of trade by industry, it will be necessary to expand existing input output tables to include information on exports produced and imports used by industry.[134] Government departments who have an interest in productivity measurement by industry will have to consider whether it would be worthwhile extending the production accounts in this direction. These extended accounts would enable researchers to study issues related to outsourcing and globalization in a more scientific manner.
- Baldwin and Gu (2007; 15-22) have a nice discussion about many of the unresolved issues in constructing an appropriate user cost formula in order to price capital services and note that an unambiguous “best practice” measure has not yet emerged. Given this state of affairs, we recommend that Statistics Canada provide not only the actual user costs by asset and year that they used in the KLEMS program but that they provide supplementary information on the various ingredients (interest rates, property taxes, business taxes, asset price appreciation terms and asset prices) that go into the making of the user costs so that researchers can construct their own preferred versions of user costs. Eventually, a view will form on what the “best practice” user cost is but we are not at this point yet and hence it is essential that Statistics Canada provide analysts with information on the various components of user costs.

---

[131] Statistics Canada has been a pioneer in developing and publishing very detailed information on the prices and quantities of outputs produced and intermediate inputs used by industry back to 1961 in its input output tables. What we are asking here is that these tables be extended to also cover the 56 types of labour input and 30 types of capital input that are being used in the Statistics Canada KLEMS program. Note that extending the input output tables to cover primary input allocations will also involve extensions to the corresponding final demand accounts, which in the case of inputs, will be corresponding household and government supplies of labour and capital.

[132] The reader will recall that in Appendix 2, we were forced to make guesses about the incidence of various consumption, import, property and capital taxes in order to reconcile final demand prices with producer prices. For additional material on how to accomplish this reconciliation, see Diewert (2006b) (2007b).

[133] See Diewert (2001; 97-98) for an elaboration of this point.

[134] Diewert (2007b) (2007c) explains these expanded production accounts in more detail.

**References**


Alterman, W., W.E. Diewert and R.C. Feenstra (1999), *International Trade Price Indexes and Seasonal Commodities*, Washington: U.S. Department of Labor, Bureau of Labor Statistics, (1999).

Archibald, R.B. (1977), "On the Theory of Industrial Price Measurement: Output Price Indexes", *Annals of Economic and Social Measurement* 6, 57-72.

Baldwin, J.R and W. Gu (2007), *Multifactor Productivity in Canada: An Evaluation of Alternative Methods of Estimating Capital Services*, , Catalogue No. 15-206–XIE–No. 009, Research Paper, The Canadian Productivity Review, Ottawa: Statistics Canada.

Baldwin, J.R., W. Gu and B. Yan (2007), *User Guide for Statistics Canada's Annual Multifactor Productivity Program*, Catalogue No. 15-206–XIE–No. 14, Research Paper, The Canadian Productivity Review, Ottawa: Statistics Canada.

Balk, B.M. (1998), *Industrial Price, Quantity and Productivity Indices*, Boston: Kluwer Academic Publishers.

Christensen, L.R. and D.W. Jorgenson (1969), "The Measurement of U.S. Real Capital Input, 1929-1967", *Review of Income and Wealth* 15, 293-320.

Christensen, L.R., D.W. Jorgenson and L.J. Lau (1971), "Conjugate Duality and the Transcendental Logarithmic Production Function", *Econometrica* 39, 255-256.

Denison, Edward F. (1974); *Accounting for United States Economic Growth 1929-69*, Washington D.C.: The Brookings Institution.

Diewert, W.E. (1973), "Functional Forms for Profit and Transformation Functions", *Journal of Economic Theory* 6, 284-316.

Diewert, W.E., (1974), "Applications of Duality Theory," pp. 106-171 in M.D. Intriligator and D.A. Kendrick (ed.), *Frontiers of Quantitative Economics*, Vol. II, Amsterdam: North-Holland. http://www.econ.ubc.ca/diewert/theory.pdf

Diewert, W.E. (1977), "Walras' Theory of Capital Formation and the Existence of a Temporary Equilibrium", pp. 73-126 in *Equilibrium and Disequilibrium in Economic Theory*, E. Schwödiauer (ed.), Reidel Publishing Company. http://www.econ.ubc.ca/diewert/walras.pdf

Diewert, W.E. (1978), "Superlative Index Numbers and Consistency in Aggregation", *Econometrica* 46, 883-900.

Diewert, W.E. (1980), "Aggregation Problems in the Measurement of Capital", pp. 433-528 in *The Measurement of Capital*, D. Usher (ed.), Chicago: The University of Chicago Press. http://www.econ.ubc.ca/diewert/1capital.pdf http://www.econ.ubc.ca/diewert/2capital.pdf

Diewert, W.E. (1983), "The Theory of the Output Price Index and the Measurement of Real Output Change", pp. 1049-1113 in *Price Level Measurement*, W.E. Diewert and C. Montmarquette (eds.), Ottawa: Statistics Canada. http://www.econ.ubc.ca/diewert/1output.pdf http://www.econ.ubc.ca/diewert/2capital.pdf

Diewert, W.E. (1993), "Symmetric Means and Choice Under Uncertainty", pp. 355-433 in *Essays in Index Number Theory, Volume I,* Contributions to Economic Analysis 217, W.E. Diewert and A.O. Nakamura (eds.), Amsterdam: North Holland. http://www.econ.ubc.ca/diewert/means1.pdf http://www.econ.ubc.ca/diewert/means2.pdf

Diewert, W.E. (1997), "Commentary" on Mathew D. Shapiro and David W. Wilcox, "Alternative Strategies for Aggregating Price in the CPI", *The Federal Reserve Bank of St. Louis Review*, 79:3, 127-137. http://research.stlouisfed.org/publications/review/97/05/9705ned.pdf

Diewert, W.E. (2001), "Which (Old) Ideas on Productivity Measurement are Ready to Use?", pp. 85-101 in *New Developments in Productivity Analysis*, C.R. Hulten, E.R. Dean and M.J. Harper (eds.), NBER Studies in Income and Wealth Volume 63, Chicago: University of Chicago Press. http://www.econ.ubc.ca/diewert/oldidea.pdf

Diewert, W.E. (2002), "Productivity Trends and Determinants in Canada", pp. 31-57 in *Productivity Issues in Canada*, S. Rao and A. Sharpe (eds.), Calgary: University of Calgary Press. http://www.econ.ubc.ca/diewert/trends.pdf

Diewert, W.E. (2005a), "Issues in the Measurement of Capital Services, Depreciation, Asset Price Changes and Interest Rates", pp. 479-542 in *Measuring Capital in the New Economy*, C. Corrado, J. Haltiwanger and D. Sichel (eds.), Studies in Income and Wealth Volume 65, NBER, Chicago: University of Chicago Press. http://www.econ.ubc.ca/discpapers/dp0411.pdf

Diewert, W.E. (2005b), "On Measuring Inventory Change in Current and Constant Dollars", Discussion Paper 05-12, Department of Economics, The University of British Columbia, Vancouver, Canada, V6T 1Z1. Available at http://www.econ.ubc.ca/diewert/disc.htm

Diewert, W.E. (2005c), "Welfare, Productivity and Changes in the Terms of Trade", unpublished paper presented at the Bureau of Economic Analysis, Washington D.C., November 17.

Diewert, W.E. (2006a), "The Measurement of Income", Chapter 7 in *The Measurement of Business Capital, Income and Performance*, Tutorial presented at the University Autonoma of Barcelona, Spain, September 21-22, 2005; revised June 2006. http://www.econ.ubc.ca/diewert/barc7.pdf

Diewert, W.E. (2006b), "Comment on Aggregation Issues in Integrating and Accelerating the BEA's Accounts: Improved Methods for Calculating GDP by Industry", pp. 287-307 in *A New Architecture for the U.S. National Accounts*, D.W. Jorgenson, J.S. Landefeld and W.D Nordhaus (eds.), NBER Studies in Income and Wealth Volume 66, Chicago: University of Chicago Press. http://www.econ.ubc.ca/diewert/bea.pdf or for a longer version, see http://www.econ.ubc.ca/discpapers/dp0506.pdf

Diewert, W.E. (2007a), "The Paris OECD-IMF Workshop on Real Estate Price Indexes: Conclusions and Future Directions", Discussion Paper 07-01, Department of Economics, The University of British Columbia, Vancouver, Canada, V6T 1Z1. http://www.oecd.org/dataoecd/32/21/37848333.pdf

Diewert, W.E. (2007b), "The Economic Approach", draft of Chapter 17 of the *Export Import Price Manual*, forthcoming, IMF, Washington D.C. http://www.econ.ubc.ca/diewert/chapter17.pdf

Diewert, W.E. (2007c), "Price Indices Using an Artificial Data Set", draft of Chapter 19 of the *Export Import Price Manual*, forthcoming, IMF, Washington D.C. http://www.econ.ubc.ca/diewert/chapter19.pdf

Diewert, W.E. and K.J. Fox (2001), "The Productivity Paradox and Mismeasurement of Economic Activity", pp. 175-197 in *Monetary Policy in a World Knowledge-based Growth, Quality Change, and Uncertain Measurement*, K. Okina and T. Inoue (eds.), London: MacMillan Press. http://www.econ.ubc.ca/diewert/paradox.pdf

Diewert, W.E. and D. Lawrence (2000), "Progress in Measuring the Price and Quantity of Capital", pp. 273-326 in *Econometrics Volume 2: Econometrics and the Cost of Capital: Essays in Honor of Dale W. Jorgenson*, Lawrence J. Lau (ed.), Cambridge: The MIT Press. http://www.econ.ubc.ca/diewert/progress.pdf

Diewert, W.E. and D. Lawrence (2002), "The Deadweight Costs of Capital Taxation in Australia", pp. 103-167 in *Efficiency in the Public Sector*, Kevin J. Fox (ed.), Boston: Kluwer Academic Publishers. http://www.econ.ubc.ca/diewert/capital.pdf

Diewert, W.E. and D. Lawrence (2005), *Estimating Aggregate Productivity Growth for Australia: The Role of Information and Communications Technology*, Occasional Economic Paper, Canberra: Department of Communications, Information Technology and the Arts, Australian Government, September, 83 pp.

http://www.econ.ubc.ca/diewert/roleinfo.pdf

Diewert, W.E. and D. Lawrence (2006), *Measuring the Contributions of Productivity and Terms of Trade to Australia's Economic Welfare*, Report by Meyrick and Associates to the Productivity Commission, Canberra, Australia. http://www.oecd.org/dataoecd/7/19/37503743.pdf

Diewert, W.E., H. Mizobuchi and K. Nomura (2005), "On Measuring Japan's Productivity, 1955-2003", Discussion Paper 05-22, Department of Economics, University of British Columbia, Vancouver, B.C., Canada, V6T 1Z1, December . http://www.econ.ubc.ca/discpapers/dp0522.pdf

Diewert, W.E. and C.J. Morrison (1986), "Adjusting Output and Productivity Indexes for Changes in the Terms of Trade", *The Economic Journal* 96, 659-679.

Diewert, W.E. and A.M. Smith (1994), "Productivity Measurement for a Distribution Firm", *the Journal of Productivity Analysis* 5, 335-347. http://www.econ.ubc.ca/diewert/other.htm

Edwards, E.O. and P.W. Bell (1961), *The Theory and Measurement of Business Income*, Berkeley: University of California Press.

Eurostat, International Monetary Fund, Organisation for Economic Cooperation and Development, United Nations and World Bank (1993), *System of National Accounts 1993*, Luxembourg, New York, Paris, Washington D.C.

Feenstra, R.C. (2004), *Advanced International Trade: Theory and Evidence*, Princeton N.J.: Princeton University Press.

Fisher, F.M. and K. Shell (1972), "The Pure Theory of the National Output Deflator", pp. 49-113 in *The Economic Theory of Price Indexes*, New York: Academic Press.

Fisher, I. (1922), *The Making of Index Numbers*, Houghton-Mifflin, Boston.

Fox, K.J. and U. Kohli (1998), "GDP Growth, Terms of Trade Effects and Total Factor Productivity", *Journal of International Trade and Economic Development* 7, 87-110.

Gorman, W.M. (1968), "Measuring the Quantities of Fixed Factors", pp. 141-172 in *Value, Capital and Growth: Papers in Honour of Sir John Hicks*, J.N Wolfe (ed.), Chicago: Aldine Press.

Hall, R.E. and D.W. Jorgenson (1967), "Tax Policy and Investment Behavior", *American Economic Review* 57:3, 391-414.

Harper, M.J., E.R. Berndt and D.O. Wood (1989), "Rates of Return and Capital Aggregation Using Alternative Rental Prices", pp. 331-372 in *Technology and Capital Formation*, Dale W. Jorgenson and Ralph Landau (eds.). Cambridge, Massachusetts: MIT Press.

Hicks, J.R. (1946), *Value and Capital*, Second Edition, Oxford: The Clarendon Press.

Hicks, J.R. (1961), "The Measurement of Capital in Relation to the Measurement of Other Economic Aggregates", pp. 18-31 in *The Theory of Capital*, F.A. Lutz and D.C. Hague (eds.), London: Macmillan.

Hill, R.J. (2006), "Superlative Indexes: Not All of Them are Super", *Journal of Econometrics* 130, 25-43.

Hotelling, H. (1932), "Edgeworth's Taxation Paradox and the Nature of Demand and Supply Functions", *Journal of Political Economy* 40, 577-616.

Jorgenson, D.W. (1989), "Capital as a Factor of Production", pp. 1-35 in *Technology and Capital Formation*, D.W. Jorgenson and R. Landau (eds.), Cambridge MA: The MIT Press.

Jorgenson, D.W. (1996a), "Empirical Studies of Depreciation", *Economic Inquiry* 34, 24-42.

Jorgenson, D.W. (1996b), *Investment: Volume 2; Tax Policy and the Cost of Capital*, Cambridge, Massachusetts: The MIT Press.

Jorgenson, D.W. and Z. Griliches (1967), The Explanation of Productivity Change", *The Review of Economic Studies* 34, 249-283.

Jorgenson, D.W. and Z. Griliches (1972), "Issues in Growth Accounting: A Reply to Edward F. Denison", *Survey of Current Business* 52:4, Part II (May), 65-94.

Kohli, U. (1978), "A Gross National Product Function and the Derived Demand for Imports and Supply of Exports", *Canadian Journal of Economics* 11, 167-182.

Kohli, U. (1990), "Growth Accounting in the Open Economy: Parametric and Nonparametric Estimates", *Journal of Economic and Social Measurement* 16, 125-136.

Kohli, U. (1991), *Technology, Duality and Foreign Trade: The GNP Function Approach to Modeling Imports and Exports*, Ann Arbor: University of Michigan Press.

Kohli, U. (2003), "Growth Accounting in the Open Economy: International Comparisons", *International Review of Economics and Finance* 12, 417-435.

Kohli, U. (2004a), "An Implicit Törnqvist Index of Real GDP", *Journal of Productivity Analysis* 21, 337-353.

Kohli, U. (2004b), "Real GDP, Real Domestic Income and Terms of Trade Changes", *Journal of International Economics* 62, 83-106.

Kohli, U. (2006), "Real GDP, Real GDI and Trading Gains: Canada, 1982-2005", *Productivity Monitor*, Number 13, Fall, 46-56.

Kohli, U. (2008), "Terms of Trade, Real Exchange Rates, and Trading Gains", forthcoming in *Price and Productivity Measurement*, W.E. Diewert, B.M. Balk, D. Fixler, K.J. Fox and A.O. Nakamura (eds.), Canada: Trafford Press.

Konüs, A.A. (1924), "The Problem of the True Index of the Cost of Living", translated in *Econometrica* 7, (1939), 10-29.

Lau, L. (1976), "A Characterization of the Normalized Restricted Profit Function", *Journal of Economic Theory*, 12:1, 131-163.

Leacy, F.H. (ed.) (1983), *Historical Statistics of Canada*, Second Edition, Ottawa: Statistics Canada.

Morrison, C.J. and W.E. Diewert (1990), "Productivity Growth and Changes in the Terms of Trade in Japan and the United States", pp. 201-227 in *Productivity Growth in Japan and the United States*, Chicago: University of Chicago Press. http://www.econ.ubc.ca/diewert/terms.pdf

McFadden, D. (1978), "Cost, Revenue and Profit Functions", pp. 3-109 in *Production Economics: A Dual Approach to Theory and Applications*. Volume 1, M. Fuss and D. McFadden (eds.), Amsterdam: North-Holland.

Moyer, B.C., M.B. Reinsdorf and R.E. Yuskavage (2006), "Aggregation Issues in Integrating and Accelerating the BEA's Accounts: Improved Methods for Calculating GDP by Industry", pp. 263-287 in *A New Architecture for the U.S. National Accounts*, D.W. Jorgenson, J.S. Landefeld and W.D Nordhaus (eds.), NBER Studies in Income and Wealth Volume 66, Chicago: University of Chicago Press.

O.E.C.D. (1972), *Labour Force Statistics: 1959-1970*. Paris: OECD.

O.E.C.D. (1991), *Labour Force Statistics: 1969-1989*. Paris: OECD.

Rymes, T.K. (1968), "Professor Read and the Measurement of Total Factor Productivity", *The Canadian Journal of Economics* 1, 359-367.

Rymes, T.K. (1983), "More on the Measurement of Total Factor Productivity", *The Review of Income and Wealth* 29 (September), 297-316.

Samuelson, P.A. (1953), "Prices of Factors and Goods in General Equilibrium', *Review of Economic Studies* 21, 1-20.

Samuelson, P.A. and S. Swamy (1974), "Invariant Economic Index Numbers and Canonical Duality: Survey and Synthesis", *American Economic Review* 64, 566-593.

Sato, K. (1976), "The Meaning and Measurement of the Real Value Added Index", *Review of Economics and Statistics* 58, 434-442.

Schreyer, P. (2001), *OECD Productivity Manual: A Guide to the Measurement of Industry-Level and Aggregate Productivity Growth*, Paris: OECD.

Schreyer, P. (2007), *Measuring Capital: Revised Manual*, OECD Statistics Directorate, Paper presented at the Meeting of the Working Party on National Accounts, October 3-5, Paris: OECD.

Solow, R.M. (1957), "Technical Change and the Aggregate Production Function", *Review of Economics and Statistics* 39, 312-320.

Statistics Canada (1987), *The Input-Output Structure of the Canadian Economy 1961-1981,* Ottawa: Statistics Canada.

Statistics Canada (1997), *National Balance Sheet Accounts; Annual Estimates 1996*, Ottawa: Statistics Canada.

Statistics Canada (2005), *Canadian Economic Observer*, *Historical Statistical Supplement 2004/05*, July, Ottawa: Statistics Canada.

Törnqvist, L. (1936), "The Bank of Finland's Consumption Price Index", *Bank of Finland Monthly Bulletin* 10: 1-8.

Törnqvist, L. and E. Törnqvist (1937), "Vilket är förhällandet mellan finska markens och svenska kronans köpkraft?", *Ekonomiska Samfundets Tidskrift* 39, 1-39 reprinted as pp. 121-160 in *Collected Scientific Papers of Leo Törnqvist*, Helsinki: The Research Institute of the Finnish Economy, 1981.

White, K.J. (2004), *SHAZAM Econometrics Software: User's Reference Manual Version 10,* Vancouver: Northwest Econometrics.

Woodland, A.D. (1982), *International Trade and Resource Allocation*, Amsterdam: North-Holland.