A Term Structure Estimation with Money, Habit, and Asset Market Segmentation*

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Abstract

In this paper, we empirically evaluate the relationships between money, inflation, output growth, and the interest rates of different maturities using a monetary model featuring inflation targeting behavior, asset market segmentation, and external habit extended for nominal economy. This model can generate liquidity effect, average upward sloping yield curve, and time-varying bond risk premia for bearing inflation and real shocks. Exploiting the term structure equations derived from the model, we identify deep parameters describing risk preference, inflation targeting behavior, and market segmentation between bond traders and non-traders. We estimate the model under alternative data specifications: latent factors; macroeconomic factors; and both latent and macroeconomic factors. We find that all of the methods give consistent estimates of the parameters and show that asset market segmentation, inflation targeting, and time-varying risk aversion are significant to account for term structure dynamics. Our empirical findings suggest that monetary factors and monetary policy are important to understand both short-run and long-run behaviors of bond prices.

JEL Classification: G12, E43, E44

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1 Introduction

Important stylized facts about money, inflation, and interest rates can be summarized as follows:

- (Fisher effect) Nominal interest rates and inflation have almost one to one relationship.
- (Liquidity effect) Nominal interest rates and money growth show negative correlations in the short-run.
- (Quantity theory) Nominal interest rates and money growth move together in the long-run. In addition, money growth and inflation move together in the long-run, but not in the short-run.
- (Long-run Neutrality) Inflation and output growth have almost zero correlation in the longrun. But in the short-run inflation or money growth appear to have positive relationship with output growth. This pattern is clearer for richer households and countries.
- (Time-varying risk bond risk premia) Excess bond returns fluctuate over time and they do not look i.i.d. Expected excess bond returns seem to move in a counter-cyclical fashion.
- (Failure of the expectations hypothesis) Related to the above, the expectations hypothesis test fails, and average yield curve has a positive slope.

Several models have been developed to account for some or all of these stylized facts and common features of these theoretical models are the existence of some frictions either in goods market or financial market, and an explicit modeling of risk preferences. Empirical evaluation of these models using both macroeconomic and financial data can shed light on the significance of these channels.

In this paper, we empirically assess the relationships between money, inflation, output growth, and both real and nominal interest rates of different maturities via a monetary DSGE model featuring inflation targeting behavior, asset market segmentation, and external habit extended for nominal economy. Kim and Moon (2007) theoretically show that the model can generate liquidity effect, average upward sloping yield curve, and time-varying bond risk premia for bearing inflation and real shocks.

Exploiting the nominal and real yield curve equations derived from the model, we identify important parameters describing risk preference, inflation targeting behavior, and the extent to which bond market is segmented between market participants and non-participants. In so doing, we experiment with alternative specifications in terms of using data. Traditional method of estimating term structure dynamics is only using the yield data. That is, this is close to the idea of factor analysis. Meanwhile, macroeconomic approach can be understood as VAR, including all the variables to study the interactions. Nowadays, a hybrid approach of employing both yields and macroeconomic variables is popular. Our paper uses all three methods to analyze how interest rates, money, and inflation as well as output interact with each other. Since we estimate the same model, we can crosscheck the validity of these estimations through macroeconomic and/or financial variables.

Our empirical findings are as follows: First, we find that asset market segmentation channel is quite significant. This means that liquidity effect channel matters despite the strong existence of the Fisher effect. Second, there exists an active inflation targeting for the US during post-war period. Third, bond market investors have time-varying risk aversion depending upon business cycles. They dislike inflation uncertainty as well as consumption uncertainty and the extent to which they are averse to these sources of aggregate risk vary depending upon the level of these variables.

The remainder of the paper begins with describing the economic model, then deriving short-term rates, the nominal and real term structures of interest rates and excess holding period returns. Section 3 further analyzes the model by estimations. Then we conclude.

2 Monetary affine term structure model

2.1 Economic model

The economy consists of a continuum of households, indexed in the $[0, 1]$ interval, a goods market and a government bond market. The government bond market opens first, and then the goods market follows. All the households in the economy attend the goods market every period. Each household is endowed with Y_t unit of consumption good each period t. However, we assume a segmented asset market. That is, only a fraction λ of all the households $(0 < \lambda < 1)$ called traders trade government bonds in the asset market while the remaining $(1-\lambda)$ agents, non-traders, never attend the asset market. It is assumed that no one changes the status between being a trader and a non-trader for simplicity. Now let us assume that the traders have the following preferences[1] defined over a sequence $\{C_t^T\}$ of a single, perishable consumption good and X_t is an external habit process.

$$\sum_{t=0}^{\infty} \beta^t \frac{(C_t^T - X_t)^{1-\gamma}}{1-\gamma}$$

We define the surplus consumption process S_t as $(C_t - X_t)/C_t$ following Campbell and Cochrane (1999). An external habit model is usually about specifying $s_t (= \log S_t)$.[2] We denote Δc_{t+1}^T and Δs_{t+1} as growth rates of consumption for traders and surplus consumption respectively. Then we can define an unexpected innovations of consumption growth and inflation as $\epsilon_{ct+1}^T = \Delta c_{t+1}^T - E_t \Delta c_{t+1}^T$, $\epsilon_{\pi t+1} = \pi_{t+1} - E_t \pi_{t+1}$. Now we write down our surplus consumption process as

$$\Delta s_{t+1} = \xi_c(\log C_t^T/Y_t)\epsilon_{ct+1}^T - \xi_\pi(\pi_t)\epsilon_{\pi t+1} + \frac{\gamma}{2} Var_t(K_{t+1}), \tag{1}$$

where $K_{t+1} = (\phi_{c0} - \phi_c \log C_t^T/Y_t)\epsilon_{ct+1}^T - (\phi_{\pi 0} + \phi_\pi \pi_t)\epsilon_{\pi t+1}$, $\xi_c(\log C_t^T/Y_t) = \phi_{c0} - 1 - \phi_c \log C_t^T/Y_t$, $\xi_\pi(\pi_t) = \phi_{\pi 0} + \phi_\pi \pi_t$.[3] Our external habit is from Kim and Moon (2007) which extends the Campbell-Cochrane model to incorporate inflation uncertainty in a limited participation model.

Note that this formation allows relative risk aversion (RRA) to be stochastic as

$$RRA_t = \frac{\gamma}{S_t}, \tag{2}$$
$$\ln RRA_t = \ln \gamma - s_t.$$

An unexpected decrease in consumption growth or an unexpected increase in inflation will increase s_t. This, in turn, will increase the relative risk aversion. s_t is conditionally heteroskedastic and the sensitivity functions, ξ_c and ξ_π, are assumed to be linear in current consumption-output ratio and inflation respectively.

Next, we describe how inflation is determined in this model. We follow Alvarez, Lucas, and Weber (2001). Specifically, money is introduced via cash-in-advance constraint and money is injected or withdrawn through open market operations. We write down the basic

[1] We can define utility functions for the household who does not partake in the bond market. Due to the market segmentation assumption, however, stochastic discount factor is determined only by the participant's utility function. Thus, for brevity we will focus on the preference of the bond market participants.

[2] For expositional convenience, we will use lower case letters as logarithms of capital letter variables, unless they are specified separately.

[3] We impose some sign restrictions as $(\phi_{c0} - 1 - \phi_c \log C_t^T/Y_t) > 0$, $\phi_\pi > \phi_{\pi 0}$, ϕ_{c0}, ϕ_c, $\phi_{\pi 0}$, $\phi_\pi > 0$.

equations in the below without derivations. (See Alvarez, Lucas, Weber (2001) or Kim and Moon (2007) for details). First, a version of the quantity equation holds as follows:

$$\pi_t = \mu_t - g_t + \varpi_t$$

where π_t is inflation, μ_t is money growth, g_t is output growth, and ϖ_t is velocity growth. Then, we specify the shock processes as follows:

$$\begin{aligned} g_{t+1} &= (1-\phi_g)g + \phi_g g_t + \sigma_g \varepsilon_{gt+1}, \\ \varepsilon_{gt+1} &\sim NIID(0,1), \\ \varpi_{t+1} &= \phi_\varpi \varpi_t + \sigma_\varpi \varepsilon_{\varpi t+1}, \\ \varepsilon_{\varpi t+1} &\sim NIID(0,1). \end{aligned}$$

Monetary policy is set as a rule describing inflation targeting.

$$\begin{aligned} \mu_t &= \bar{\mu} - a_1(\pi_t - \bar{\pi}) + \varepsilon_{\mu t} \\ &= a_0 - a_1\pi_t + \varepsilon_{\mu t} \end{aligned} \tag{3}$$

where

$$\begin{aligned} a_0 &= \bar{\mu} + a_1\bar{\pi}, \\ \varepsilon_{\mu t+1} &= \phi_\mu \varepsilon_{\mu t} + \sigma_\mu \hat{\varepsilon}_{\mu t+1}, \\ \hat{\varepsilon}_{\mu t+1} &\sim NIID(0,1). \end{aligned}$$

2.2 Nominal term structure dynamics

Now we derive the nominal stochastic discount factor as follows:

Proposition 1 *The nominal stochastic discount factor at time t is denoted as SDF_t and derived as*

$$-\log SDF_{t+1} = r_t + \frac{1}{2}\Lambda(x_t)'\Sigma\Lambda(x_t) + \Lambda(x_t)'\Sigma^{1/2}\Xi_{t+1}, \tag{4}$$

where $\Lambda(x_t) = \Lambda_0 + \Lambda_1 x_t$ with

$$\Lambda_0 = \begin{bmatrix} \zeta_0 - \gamma(\phi_{c0}\zeta_1 + \phi_{\pi 0}\zeta_0) + \gamma a_0(\phi_c\zeta_1\zeta_3 - \phi_\pi\zeta_0^2) \\ -\zeta_0 + \gamma\phi_{c0}\zeta_2 + \gamma\phi_{\pi 0}\zeta_0 + \gamma a_0(\phi_\pi\zeta_0^2 - \phi_c\zeta_1\zeta_3) - \gamma\phi_c a_0\zeta_3 \\ \zeta_0 + \gamma\phi_{c0}\zeta_1 - \gamma\phi_{\pi 0}\zeta_0 - \gamma a_0(\phi_\pi\zeta_0^2 + \phi_c\zeta_1\zeta_3) \end{bmatrix}, \tag{5}$$

$$\Lambda_1 = (\gamma\phi_c) \times \begin{bmatrix} -(\zeta_1)^2 & (\zeta_1)^2 & \zeta_1\zeta_3 \\ \zeta_1\zeta_2 & -\zeta_1\zeta_2 & -\zeta_2\zeta_3 \\ (\zeta_1)^2 & -(\zeta_1)^2 & -\zeta_1\zeta_3 \end{bmatrix} \tag{6}$$

$$+ (\gamma\phi_\pi) \times \begin{bmatrix} -(\zeta_0)^2 & (\zeta_0)^2 & -(\zeta_0)^2 \\ (\zeta_0)^2 & -(\zeta_0)^2 & (\zeta_0)^2 \\ -(\zeta_0)^2 & (\zeta_0)^2 & -(\zeta_0)^2 \end{bmatrix},$$

$$\zeta_0 = \frac{1}{1+a_1},$$
$$\zeta_1 = \frac{\varphi a_1}{1+a_1},$$
$$\zeta_2 = 1 + \frac{\varphi a_1}{1+a_1},$$
$$\zeta_3 = \frac{\varphi}{1+a_1}.$$

$$x_{t+1} = \kappa + \Phi x_t + \Sigma^{1/2}\Xi_{t+1}, \tag{7}$$

where

$$\kappa = [0, (1-\phi_g)g, 0]',$$
$$diag(\Phi) = [\phi_\varpi, \phi_g, \phi_\mu]',$$

$$diag(\Sigma^{1/2}) = [\sigma_\varpi, \sigma_g, \sigma_\mu]',$$
$$\Sigma^{1/2}(i,j) = 0 \textit{ for all } i \neq j,$$

$$\Xi_t = [\varepsilon_{\varpi t}, \varepsilon_{gt}, \hat{\varepsilon}_{\mu t}]',$$

where $[\varepsilon_{\varpi t}, \varepsilon_{gt}, \hat{\varepsilon}_{\mu t}]' \sim NIID(0, I)$.

Proof. See Appendix. ■

The stochastic discount factor (4) of our model resembles that of the essentially affine term structure model (E-ATSM) following Duffee (2002). Specifically, the factor structure is Gaussian as in (7), but the process for market price of risk, $\Lambda(x_t)$ is time-varying and affine. (5) and (6) show that factor loadings for the market price of risk process are determined by risk preference terms ϕ_c and ϕ_π, market segmentation φ, and inflation targeting a_1. Thus, unlike latent factor models or hybrid factor models, we endogenously determine the market price of risk process $\Lambda(x_t)$ from the model.

Using (4), now we solve for bond prices.

Proposition 2 *The price of a zero-coupon, nominal bond with maturity* n *periods denoted as* $Q_t^{(n)}$ *can be found by the recursive formula*

$$Q_t^{(n)} = \exp\left[\mathcal{A}(n) + \mathcal{B}(n)' x_t\right],$$

where

$$\begin{aligned} \mathcal{A}(n+1) &= \mathcal{A}(n) - \Theta_0 + \mathcal{B}(n)'[\kappa - \Sigma\Lambda_0] + \frac{1}{2}\mathcal{B}(n)'\Sigma\mathcal{B}(n), \qquad (8) \\ \mathcal{B}(n+1)' &= -\Theta_1' + \mathcal{B}(n)'[\Phi - \Sigma\Lambda_1], \end{aligned}$$

where $\mathcal{A}(0) = \mathcal{B}(0) = 0$.

Proof. See Appendix. ∎

Yield to maturity is then defined as

$$y_t^{(n)} = -\frac{\log Q_t^{(n)}}{n} = a(n) + b(n)' x_t,$$

where we denote $a(n) = -\mathcal{A}(n)/n$ and $b(n) = -\mathcal{B}(n)/n$.

In a similar fashion, the expected excess holding period returns can be derived as follows:

Proposition 3 *Conditional expectation of the holding period returns of* $n-$*period bonds over the one period interest rate is time varying and has the following form:*

$$\begin{aligned} E_t\left[hprx_{t+1}^{(n)}\right] = &-(n-1)\Lambda_0'\Sigma b(n-1) + (n-1)\left[\psi_\varpi \varpi_t + \psi_g g_t + \psi_\mu \varepsilon_{\mu_t}\right] \\ &- \frac{(n-1)^2}{2} b(n-1)'\Sigma b(n-1), \end{aligned}$$

$$\psi_\varpi = \left[\begin{array}{c} \gamma\phi_c\left\{(\zeta_1)^2\left(\sigma_\varpi^2 b(n-1;\varpi) - \sigma_\mu^2 b(n-1;\mu)\right) - \zeta_1\zeta_2\sigma_g^2 b(n-1;g)\right\} \\ +\gamma\phi_\pi(\zeta_0)^2\left\{\sigma_\varpi^2 b(n-1;\varpi) + \sigma_\mu^2 b(n-1;\mu)) - \sigma_g^2 b(n-1;g)\right\} \end{array}\right],$$

$$\psi_g = \left[\begin{array}{c} \gamma\phi_c\left\{-(\zeta_1)^2\left(\sigma_\varpi^2 b(n-1;\varpi) - \sigma_\mu^2 b(n-1;\mu)\right) + \zeta_1\zeta_2\sigma_g^2 b(n-1;g)\right\} \\ -\gamma\phi_\pi(\zeta_0)^2\left\{\sigma_\varpi^2 b(n-1;\varpi) + \sigma_\mu^2 b(n-1;\mu)) - \sigma_g^2 b(n-1;g)\right\} \end{array}\right],$$

$$\psi_\mu = \left[\begin{array}{c} \gamma\phi_c\left\{-\zeta_1\zeta_3(\sigma_\varpi^2 b(n-1;\varpi) - \sigma_\mu^2 b(n-1;\mu)) + \zeta_2\zeta_3\sigma_g^2 b(n-1;g)\right\} \\ +\gamma\phi_\pi(\zeta_0)^2\left\{\sigma_\varpi^2 b(n-1;\varpi) + \sigma_\mu^2 b(n-1;\mu)) - \sigma_g^2 b(n-1;g)\right\} \end{array}\right],$$

where

$$b(n-1) = [b(n-1;\varpi), b(n-1;g), b(n-1;\mu)]'.$$

Proof. See Appendix. ∎

2.3 Real term structure dynamics

We can also derive the real term structure of interest rates. The Euler equation for the short-term real interest rate is as follows;

$$Q_{Rt} = E_t \left[\beta \left(\frac{C_{t+1}^T}{C_t^T} \right)^{-\gamma} \left(\frac{S_{t+1}}{S_t} \right)^{-\gamma} \right]$$

We can also write the price of zero-coupon bond at maturity n denoted as $Q_{Rt}^{(n)}$ in real terms

$$Q_{Rt}^{(n)} = E_t \left[\beta^n \left(\frac{C_{t+n}^T}{C_t^T} \right)^{-\gamma} \left(\frac{S_{t+n}}{S_t} \right)^{-\gamma} \right]$$

Using the equation for Q_{Rt} above, we can solve for the short term real interest rate.

Proposition 4 *Denote R_t as the short-term real interest rate. Then R_t is expressed as*

$$R_t = \Theta_{R0} + \Theta_{R1} x_t,$$

where x_t is again defined as the state vector

$$\begin{aligned} x_t &= \left[\begin{array}{ccc} \varpi_t, & g_t, & \varepsilon_{\mu t} \end{array} \right]', \\ \Theta_{R1} &= \left[\begin{array}{ccc} \theta_{R1}, & \theta_{R2}, & \theta_{R3} \end{array} \right], \end{aligned}$$

$$\Theta_{R0} = \delta + \gamma \left(\frac{\varphi a_1}{1 + a_1} + 1 \right) g \left(1 - \phi_g \right),$$

$$\theta_{R1} = \frac{\gamma \varphi a_1 \left(1 - \phi_\varpi \right)}{1 + a_1},$$

$$\theta_{R2} = \gamma \left(\phi_g - \left(1 - \phi_g \right) \frac{\varphi a_1}{1 + a_1} \right),$$

$$\theta_{R3} = \frac{\gamma \varphi \left(\phi_\mu - 1 \right)}{1 + a_1}.$$

Proof. *See Appendix.* ■

We can also derive the real stochastic discount factor by using the preceding proposition.

Proposition 5 *The real stochastic discount factor at time t is denoted as $RSDF_t$ and derived as*

$$-\log RSDF_{t+1} = R_t + \frac{1}{2}\Lambda_R(x_t)'\,\Sigma\Lambda_R(x_t) + \Lambda_R(x_t)'\,\Sigma^{1/2}\Xi_{t+1},$$

where $\Lambda_R(x_t) = \Lambda_{R0} + \Lambda_{R1}x_t$ with

$$\Lambda_{R0} = \begin{bmatrix} -\gamma\left[\zeta_1\left(\phi_{c0} - \phi_c\varphi\hat{a}_0\right) + \zeta_0\left(\phi_{\pi 0} + \phi_\pi\hat{a}_0\right)\right] \\ \gamma\left[\zeta_2\left(\phi_{c0} - \phi_c\varphi\hat{a}_0\right) + \zeta_0\left(\phi_{\pi 0} + \phi_\pi\hat{a}_0\right)\right] \\ \gamma\left[\zeta_3\left(\phi_{c0} - \phi_c\varphi\hat{a}_0\right) - \zeta_0\left(\phi_{\pi 0} + \phi_\pi\hat{a}_0\right)\right] \end{bmatrix},$$

$$\begin{aligned} \Lambda_{R1} &= (\gamma\phi_c)\times\begin{bmatrix} -(\zeta_1)^2 & (\zeta_1)^2 & \zeta_1\zeta_3 \\ \zeta_1\zeta_2 & -\zeta_1\zeta_2 & -\zeta_2\zeta_3 \\ \zeta_1\zeta_3 & -\zeta_1\zeta_3 & -(\zeta_3)^2 \end{bmatrix} \\ &\quad + (\gamma\phi_\pi)\times\begin{bmatrix} -(\zeta_0)^2 & (\zeta_0)^2 & -(\zeta_0)^2 \\ (\zeta_0)^2 & -(\zeta_0)^2 & (\zeta_0)^2 \\ -(\zeta_0)^2 & (\zeta_0)^2 & -(\zeta_0)^2 \end{bmatrix} \end{aligned}$$

Proof. *See Appendix.* ■

Using the definition of the real stochastic discount factor, we can solve for real bond prices.

Proposition 6 *Price of zero coupon real bond with maturity n periods denoted as $Q_{Rt}^{(n)}$ can be found by the following recursive formula*

$$Q_{Rt}^{(n)} = \exp\left[\mathcal{A}_R(n) + \mathcal{B}_R(n)'\,x_t\right],$$

where

$$\begin{aligned} \mathcal{A}_R(n+1) &= \mathcal{A}_R(n) - \Theta_{R0} + \mathcal{B}_R(n)'\left(\kappa - \Sigma\Lambda_{R0}\right) + \frac{1}{2}\mathcal{B}_R(n)'\,\Sigma\mathcal{B}_R(n), \\ \mathcal{B}_R(n+1)' &= -\Theta_{R1}' + \mathcal{B}_R(n)'\left(\Phi - \Sigma\Lambda_{R1}\right), \end{aligned}$$

where $\mathcal{A}_R(0) = \mathcal{B}_R(0) = 0$.

Proof. *See Appendix.* ■

Yield to maturity in real terms can then be written as

$$y_{Rt}^{(n)} = -\frac{\log Q_{Rt}^{(n)}}{n} = a_R(n) + b_R(n)'\,x_t,$$

where we denote $a_R(n) = -\mathcal{A}_R(n)/n$ and $b_R(n) = -\mathcal{B}_R(n)/n$.

3 Empirical Results

In this section, we use four distinct methods to estimate the model's parameters. All four methods employed here give consistent estimates of the parameters. The first method makes use of the nominal bond yields data together with the data on macroeconomic variables to estimate the parameters of the model presented above. The second method uses the data on nominal yields of different maturities, and extracts the latent variables from this data set in order to estimate the model's underlying parameters. The third method employs the real bond yields of different maturities and the macroeconomic variables to obtain the estimates of the parameters. Finally, the fourth method uses data on nominal yields and some of the latent variables, and obtains the rest of the latent variables from this data set and equation system to estimate the parameters.

3.1 Method 1: Yields and Macroeconomic variables

We start our estimation with the yields and macroeconomic variables model. Data used in this model is in a quarterly frequency covering from 1964 to 2000. The macroeconomic data set is obtained from the St. Louis Federal Reserve Bank's web page (FRED). We use real gross domestic product (GDP) to calculate output growth g_t. We use consumer price index to compute inflation π_t. We use M1 money stock as monetary aggregate. In order to construct data for the monetary policy shock $\varepsilon_{\mu t}$, we first obtain the money growth rate μ_t and estimate (3) by ordinary least squares. By using the ordinary least square estimates and data on money growth rate and inflation rate, we extract the residuals that is the monetary policy shock $\varepsilon_{\mu t}$. Finally, velocity changes ϖ_t can be constructed by using M1 money stock, real GDP and CPI following the definition previously given in the model. We compute bond yields using the data from the Center for Research in Security Prices (CRSP). Particularly, we use data on zero coupon yields of maturities 4, 12 and 20 quarters from 1st quarter of 1964 to the 4th quarter of 2000.

The yields are derived in the previous sections as affine functions of the underlying macroeconomic variables. In other words, yields are affine functions of latent variables, and latent variables are autoregressive with a lag length of one. Thus, the yields together with the latent factors make up a first order Gaussian vector autoregression (VAR). We estimate this first order VAR's parameters using maximum likelihood, and derive the likelihood function in detail. For convenience, the following equations are rewritten,

$$r_t = \Theta_0 + \Theta_1' x_t,$$

$$\Lambda(x_t) = \Lambda_0 + \Lambda_1 x_t$$

$$x_{t+1} = \kappa + \Phi x_t + \Sigma^{1/2}\Xi_{t+1} \tag{9}$$

$$\begin{aligned} \mathcal{A}(n+1) &= \mathcal{A}(n) - \Theta_0 + \mathcal{B}(n)'[\kappa - \Sigma\Lambda_0] + \frac{1}{2}\mathcal{B}(n)'\Sigma\mathcal{B}(n), \\ \mathcal{B}(n+1)' &= -\Theta_1' + \mathcal{B}(n)'[\Phi - \Sigma\Lambda_1], \end{aligned}$$

$$y_t^{(n)} = -\frac{\log Q_t^{(n)}}{n} = -\frac{\mathcal{A}(n)}{n} - \frac{\mathcal{B}(n)'}{n} x_t \tag{10}$$

where $x_t = \begin{bmatrix} \varpi_t, & g_t, & \varepsilon_{\mu t} \end{bmatrix}'$

One-period nominal interest rate, the process for the market price of risk and latent factor process (9) are all used in the recursive formulas to obtain the yield curves. Finally, (9) and (10) are used directly as inputs into a VAR.

To summarize, we estimate the following VAR model. z_t is a 6×1 vector that contains the yields of maturities 4, 12 and 20 quarters, as well as the change in velocity ϖ_t, output growth g_t and monetary policy shock $\varepsilon_{\mu t}$, respectively. The dynamics of z_t is governed by a first-order VAR,

$$z_t = \Psi_0 + \Psi_1 z_{t-1} + \Omega^{1/2}\epsilon_t \tag{11}$$

where $\epsilon_t \sim NIID(0,\ 1)$ and.$\epsilon_t = [\epsilon_{4t}, \epsilon_{12t}, \epsilon_{20t}, \varepsilon_{\varpi t}, \varepsilon_{gt}, \hat{\varepsilon}_{\mu t}]'$. ϵ_{4t}, ϵ_{12t} and ϵ_{20t} are assumed to be the measurement errors associated with yields of maturities 4, 12 and 20 quarters, respectively. Ψ_0 and Ψ_1 are unconstrained and $\Omega^{1/2}$ is a 6×6 diagonal matrix.

We have observed each of these six variables in z_t for $T = 148$ periods. We can now write the density of the t^{th} observation conditioned on first $(t-1)$ observations as follows:

$$\begin{aligned} & f_{Z_t|Z_{t-1},\ldots,Z_1}(z_t \mid z_{t-1}, z_{t-2}, \ldots, z_1; \theta) \\ = \;& (2\pi)^{-m/2}|\Omega|^{-1/2}\exp\left[-1/2\,(z_t - \Psi_0 - \Psi_1 z_{t-1})'\,\Omega^{-1}(z_t - \Psi_0 - \Psi_1 z_{t-1})\right] \end{aligned} \tag{12}$$

where m is the number of variables in z_t.and θ is a vector that contains the parameter to be estimated We can easily write the joint density of observations 2 through t conditioned on the first,

$$
\begin{aligned}
& f_{Z_t,Z_{t-1},\ldots,Z_2|Z_1}\left(z_t, z_{t-1}, z_{t-2}, \ldots z_2 \mid z_1; \theta\right) \\
= \; & f_{Z_{t-1}\ldots Z_2|Z_1}\left(z_{t-1}, z_{t-2}, \ldots z_2 \mid z_1; \theta\right) \times f_{Z_t|Z_{t-1},\ldots,Z_1}\left(z_t \mid z_{t-1}, z_{t-2}, \ldots, z_1; \theta\right)
\end{aligned}
$$

Applying this formula recursively, we can obtain the likelihood function for the full sample conditioned on the first observation as,

$$
\begin{aligned}
& f_{Z_t,Z_{t-1},\ldots,Z_2|Z_1}\left(z_t, z_{t-1}, z_{t-2}, \ldots z_2 \mid z_1; \theta\right) \\
= \; & \prod_{t=2}^{T} f_{Z_t|Z_{t-1},\ldots,Z_1}\left(z_t \mid z_{t-1}, z_{t-2}, \ldots, z_1; \theta\right)
\end{aligned}
\tag{13}
$$

Then, we can write the sample log likelihood function by substituting (12) into (13), and taking logarithms.

$$
\begin{aligned}
\log\left(\mathcal{L}\left(\theta\right)\right) = \; & -\frac{\left(T-1\right)m}{2}\log\left(2\pi\right) + \frac{\left(T-1\right)}{2}\log\left|\Omega^{-1}\right| \\
& -\frac{1}{2}\sum_{t=2}^{T}\left[\left(z_t - \Psi_0 - \Psi_1 z_{t-1}\right)'\Omega^{-1}\left(z_t - \Psi_0 - \Psi_1 z_{t-1}\right)\right]
\end{aligned}
\tag{14}
$$

Finally, in order to estimate the parameters of our model, we first maximize (14) numerically with respect to the unknown parameters in the matrices Ψ_0 and Ψ_1. To estimate the parameters in matrix Ω, we again maximize (14) numerically with respect to the parameters in matrix Ω by taking the parameters estimated in the first step fixed. We repeat this procedure to obtain converged estimates.

The second column of Table 1 displays the parameter estimates that are obtained by Method 1. The third column gives the corresponding standard errors that are calculated by taking the square roots of the diagonal elements of -1 times the inverted matrix of second derivatives of the maximized log likelihood function. The AR(1) coefficient of output growth, ϕ_g, is equal to 0.0465, showing that the aggregate consumption have a stable volatility over time. Another important parameter is the monetary policy shock persistence, ϕ_μ. An estimate of 0.5203 for this parameter is quite consistent with the existing literature. The estimate $\phi_\varpi = 0.7816$ indicates that the velocity changes exhibit a considerable amount of persistence. This, in turn, implies that the interest rates are persistent as data suggest. If we turn our attention to the parameters related to time-varying risk aversion, the estimates γ, ϕ_c, ϕ_{c0}, ϕ_π, $\phi_{\pi 0}$, φ, and a_1 are very consistent. The estimate $a_1 = 1.3225$ implies that the Fed follows an active inflation targeting. The responsiveness of the Fed to high inflation rates affects the consumption growth uncertainty greatly. The utility curvature estimate, $\gamma = 3.2742$, although a little high, is in the acceptable range. The market segmentation parameter φ, which is very important in determining the consumption innovation, is equal to 0.2465. It shows the degree of market segmentation present in the model, and the estimate matches the expectations. The estimates $\phi_c = 120.09$, $\phi_{c0} = 19.147$, $\phi_\pi = 118.77$, and $\phi_{\pi 0} = 0.2503$ are very robust to changes in initial values. Further proving that the surplus consumption process is correctly identified in the model. The time discount parameter β calculated from the estimate $\delta = 0.0180$ is very close to 1, and consistent with those reported in literature. The estimate $\bar{\pi} = 0.0023$ implies an annual steady state inflation level of 1%. Finally, the estimates $\sigma_g = 0.0110$, $\sigma_\mu = 0.0883$ and $\sigma_\varpi = 0.0030$ are quite sizeable implying that they are very important in explaining the fluctuations in the data.

Figure 1 displays the impulse responses of the nominal yields with maturities of 4, 12 and 20 quarters to the nominal yields and macroeconomic variables model's three shocks: monetary policy shock, $\hat{\varepsilon}_{\mu t}$, output growth shock, ε_{gt}, and velocity change shock, $\varepsilon_{\varpi t}$. An expansionary monetary policy decreases the nominal yields of all three maturities, showing that the liquidity effect prevails. The segmented market effect together with expectation effect will decrease the interest rates in response to a money injection. In addition, an increase in bond traders' consumption resulting from money growth decreases risk aversion, lowering the nominal yields even further. The inflation risk premium effect works in opposite direction by increasing inflationary pressures due to the increase in money growth. However, a more active inflation targeting, consistent with the estimation results for a_1, defuses the inflationary pressures caused by the money injection. A one standard deviation output growth shock lowers the nominal yield with maturity of 4 quarters by 0.5%, and decreases the yields with 12 and 20 quarters maturities by 0.15% and 0.10%, respectively due to the intertemporal substitution effect. An increase in output growth will result in a lower inflation and a higher consumption causing investors to demand less risk premium. Having a high estimate of $\gamma = 3.2742$ for utility curvature makes the effect of output growth shock more significant. In response to a one standard deviation shock in changes in velocity of money, the nominal yield of 4 quarters maturity increases by 0.15%, the nominal yield with 12 quarters to maturity increases by almost 0.08% while the 20 quarters to maturity yield rises by 0.05%. The expectation effect together with the inflation risk premium effect increase the interest rates.

Figure 2 presents the impulse responses of the real yields of maturities 4, 12 and 20 quarters to the macroeconomic shocks in the yields and macro variables model. The parameters estimated by the yields and macro variables model are used in the plotting of the figure. An expansionary monetary policy lowers the real yield with 4 quarters to maturity by 0.25%, which is much more than the corresponding nominal yield curve's impulse response. This result implies that the strong liquidity effect occurs due to the segmented market effect, which is the only effect present here. The expectation effect and the inflation risk premium effects are not a part of the short term real interest rates derived under the real term structure. Without the inflation risk premiums, a money injection could only result in lower interest rates. One standard deviation in output growth shock decreases the real yields of maturities 4, 12 and 20 quarters by 0.4%, 0.14% and 0.08%, respectively. The absence of inflation risk premium from the real term structure of interest rates changes the magnitude of the output growth shock on real yields. The inflation risk premium effect, which is present in nominal term structure, lowers the nominal yields but not the real yields. Therefore, the output growth shock causes nominal yields to decrease more than the real yields. A one standard deviation shock in velocity changes increases the real yield of 4 quarters maturity by 0.04% while increasing the nominal yield of the same maturity by 0.15%. The sizeable difference between the changes in the nominal and real yields comes from the inflation risk premium effect present in the nominal term structure.

3.2 Method 2: Yields Only Model

This model uses data on zero coupon yields of maturities 4, 8, 12, 16 and 20 quarters from 1st quarter of 1964 to the 4th quarter of 2000. Again, we compute bond yields using the data from the Center for Research in Security Prices (CRSP). We assume that yields of maturities 4, 12, and 20 quarters are measured without error whilst the rest of the yields are measured with error. By using data on these yields, we first try to obtain data for the unobserved latent factors by inverting the yield equations for the maturities of 4, 12, and 20 quarters. After obtaining data on latent factors, we invert the equations for the yields of maturities 8, and 16 quarters to obtain data on measurement errors. Finally, we estimate the parameters of this model by maximizing the joint log likelihood function, which is derived below.

The yields of maturities 4, 12, and 20 quarters are assumed to be measured without error, and they are affine functions of unobservable latent variables. For convenience, the following equations are rewritten,

$$r_t = \Theta_0 + \Theta_1' x_t,$$

$$\Lambda(x_t) = \Lambda_0 + \Lambda_1 x_t$$

$$x_{t+1} = \kappa + \Phi x_t + \Sigma^{1/2}\Xi_{t+1}$$

$$\begin{aligned}
\mathcal{A}(n+1) &= \mathcal{A}(n) - \Theta_0 + \mathcal{B}(n)'[\kappa - \Sigma\Lambda_0] + \frac{1}{2}\mathcal{B}(n)'\Sigma\mathcal{B}(n), \\
\mathcal{B}(n+1)' &= -\Theta_1' + \mathcal{B}(n)'[\Phi - \Sigma\Lambda_1],
\end{aligned}$$

$$y_t^{(n)} = -\frac{\mathcal{A}(n)}{n} - \frac{\mathcal{B}(n)'}{n} x_t$$

where $x_t = \begin{bmatrix} \varpi_t, & g_t, & \varepsilon_{\mu t} \end{bmatrix}'$ and $n = 4, 12$ and 20. For convenience, let us write the last equation as

$$Y_t^{wo} = A^{wo} + B^{wo} x_t^u$$

where superscript wo refers to the yield equations that are measured without error, and superscript u refers to unobserved latent factors. Since all the latent factors are unobservable, it is also true that $x_t = x_t^u$. For a given parameter vector θ, we can easily infer the values of the latent variables, x_t by using inversion as follows:

$$x_t = (B^{wo})^{-1}(Y_t^{wo} - A^{wo})$$

We also make use of the yields of maturities 8, and 16 quarters, which are assumed to be measured with error, together with the newly inferred data of x_t to obtain data on measurement errors. Again, for convenience, we write the yield equations measured with error as follows:

$$Y_t^w = A^w + B^w x_t + C^w u_t^w$$

where superscript w refers to the yield equations that are measured with error. Finally, u_t^w can be inferred as,

$$u_t^w = (C^w)^{-1}(Y_t^w - A^w - B^w x_t)$$

The joint likelihood function for this model can be written as follows:

$$\mathcal{L}(\theta) = \prod_{t=2}^{T} f\left(Y_t^{wo}, Y_t^w \mid Y_{t-1}^{wo}, Y_{t-1}^w\right)$$

The joint log likelihood is,

$$
\begin{aligned}
\log \mathcal{L}(\theta) &= \sum_{t=2}^{T} - \log |J| + \log f_x (x_t \mid x_{t-1}) + \log f_{u^w} (u_t^w) \qquad (15) \\
&= -(T-1) \log |J| - (T-1) \log \left(\left| \left(\Sigma^{1/2}\right) \left(\Sigma^{1/2}\right)' \right| \right) \\
&\quad - \frac{1}{2} \sum_{t=2}^{T} (x_t - \kappa - \Phi x_{t-1})' \left(\left(\Sigma^{1/2}\right) \left(\Sigma^{1/2}\right)' \right)^{-1} (x_t - \kappa - \Phi x_{t-1}) \\
&\quad - \frac{(T-1)}{2} \sum_{i=1}^{k} \log \left(\sigma_i^2\right) - \frac{1}{2} \sum_{t=2}^{T} \sum_{i=1}^{k} \frac{\left(u_{t,i}^w\right)^2}{\sigma_i^2}
\end{aligned}
$$

where σ_i is the standard deviation of the i-th measurement error. $i = 1$ corresponds to the measurement error of the zero coupon yield with maturity of 8 quarters, and $i = 2$ corresponds to the measurement error of the zero coupon yield with maturity of 16 quarters. k refers to the number of yields that are measured with error, and it is equal to 2. The Jacobian matrix is as follows:

$$
J = \begin{pmatrix} I_{5\times5} & 0_{5\times3} & 0_{5\times2} \\ 0_{3\times5} & B^{wo} & 0_{3\times2} \\ 0_{2\times5} & B^w & C^w \end{pmatrix}.
$$

First, we maximize the log-likelihood function above numerically with respect to the unknown parameters vector θ, which includes all the parameters except for the parameters in the Σ matrix. Second, we estimate the parameters in the Σ matrix by maximizing the log-likelihood function keeping the parameters estimated in the first step fixed.

The fourth column of Table 1 displays the parameter estimates that are obtained using Method 2. The fifth column gives the corresponding standard errors. The estimate $\phi_g = 0.1028$ implies that the output growth process follows a stable path over time. This ensures a low aggregate consumption volatility for the model as always assumed by consumption-based asset pricing models. The AR(1) coefficient of monetary policy shock, ϕ_μ, is equal to 0.6365, showing that the monetary policy shock is persistent making the expectation effect from money growth changes more pronounced. The estimate $\phi_\varpi = 0.9919$ implies that the velocity changes exhibit a considerable amount of persistence, almost all of the effect caused by a shock is transmitted to the next quarter. This again reflects the findings of the data for the interest rates. Regarding the estimate $a_1 = 1.5076$, which is one of the key parameters of time-varying risk aversion, indicates that more active inflation targeting is pursued by the Fed reducing the effect of the inflationary pressures on the yield curves in response to different shocks. Another parameter that is related to time-varying risk aversion is the utility curvature, γ. The estimate $\gamma = 2.6499$ is consistent with the values reported in the literature. The market segmentation parameter estimate $\varphi = 0.1169$ shows that bond traders' consumption will be affected in a significant way in the face of a change in money growth. The estimates for the rest of the parameters that determine the coefficients of time-varying risk aversion are $\phi_c = 120.05$, $\phi_{c0} = 19.140$, $\phi_\pi = 118.78$, and $\phi_{\pi 0} = 25.00$ are again very robust to changes in initial parameter values. The estimate $\delta = 0.0330$ is used to calculate the time discount factor, β, and it is consistent with the literature. The estimate for steady state inflation is $\bar{\pi} = -0.027$. Even though the estimate is negative, when we take the standard error of the estimate into account, it is in an acceptable range. The estimate $\bar{\mu} = 0.0019$ shows that the steady state level for money growth is 0.8% annually. The standard deviation estimates for output growth, monetary policy shock and velocity change are $\sigma_g = 0.0750$, $\sigma_\mu = 0.1052$ and $\sigma_\varpi = 0.0028$ implying that they play an important role in explaining the data.

Figure 3 displays the impulse responses of the nominal yields to the macroeconomic shocks for the yields only model estimated here. A one standard deviation shock in monetary policy lowers the nominal yields, indicating that the liquidity effect occurs. A money injection causes the segmented market effect and the expectation effect to lower the interest rates. Also, the money growth results in an increase in bond traders' consumption, decreasing risk aversion, and further lowering the nominal yields. The inflation risk premium effect increases inflationary pressures due to the increase in money growth, this in turn causes investors to demand higher returns for holding nominal assets. However, a high estimate for a_1 makes the inflationary pressures caused by the money injection less pronounced. A one standard deviation output growth shock lowers the nominal yield with a maturity of 4 quarters by 0.9%. An increase in output growth lowers inflation and increases consumption causing investors to demand less risk premium. A one standard deviation shock in velocity changes increases the nominal yield of 4 quarters maturity by 0.15%. The effect of the shock on the nominal yields persists more than 16 quarters since the estimate of the AR(1) coefficient for velocity change is very close to 1. The nominal yield with 12 quarters to maturity increases by almost 0.30% while the 20 quarters to maturity yield rises by 0.45%.

Figure 4 shows the impulse responses of the real yields of maturities 4, 12 and 20 quarters to the macroeconomic shocks in the yields only model. The parameters estimated by the yields only model are used when plotting the figure. An expansionary monetary policy lowers the real yield with 4 quarters to maturity by 2%, decreases the real yields of 12 and 20 quarters to maturity by 1.5% and 1% respectively. The segmented market effect causes a strong liquidity effect to prevail. Without the inflation risk premiums, a money injection could only result in lower real interest rates and lower real yields. An output growth shock decreases the real yields of maturities 4, 12 and 20 quarters by 0.9%, 0.4% and 0.25%, respectively. A one standard deviation shock in velocity changes increases the real yield of 4 quarters maturity by 0.09% while increasing the nominal yield of the same maturity by 0.30%. The sizeable difference between the changes in the nominal and real yields again comes from the inflation risk premium effect present in the nominal term structure.

3.3 Method 3: Real yields and Macroeconomic Variables

In this method, we use data on real bond yields of maturities 20, 40 and 120 quarters, and on macroeconomic variables. The macro variables are not directly used in the estimation of the parameters but used indirectly through the latent factor equations. Data used in the model is in a quarterly frequency covering from 2002 to 2007. The macroeconomic data set together with the real bond yields data are obtained from the St. Louis Federal Reserve Bank's web page (FRED). We construct the data for velocity changes ϖ_t, output growth g_t, and monetary policy shock $\varepsilon_{\mu t}$ by using real gross domestic product (GDP), M1 money stock, consumer price index (CPI) as explained in Section 5.1. Data on real bond yields of maturities 20, 40 and 120 quarters runs from the 2nd quarter of 2002 to the 3rd quarter of 2007. The real bond yields are affine functions of the macroeconomic variables, which are output growth, monetary policy shock, and velocity changes. These macroeconomic variables as mentioned in previous sections are autoregressive processes with a lag length of one. The real bond yields together with the macroeconomic variables make up a first order Gaussian vector autoregression (VAR) as in Section 5.1. We estimate the parameters of this first order VAR using maximum likelihood estimation. Again, for convenience, the following equations are rewritten here,

$$R_t = \Theta_{R0} + \Theta_{R1} x_t,$$

$$\Lambda_R(x_t) = \Lambda_{R0} + \Lambda_{R1} x_t$$

$$x_{t+1} = \kappa + \Phi x_t + \Sigma^{1/2} \Xi_{t+1}$$

$$\mathcal{A}_R(n+1) = \mathcal{A}_R(n) - \Theta_{R0} + \mathcal{B}_R(n)'(\kappa - \Sigma\Lambda_{R0}) + \frac{1}{2}\mathcal{B}_R(n)'\Sigma\mathcal{B}_R(n),$$
$$\mathcal{B}_R(n+1)' = -\Theta'_{R1} + \mathcal{B}_R(n)'(\Phi - \Sigma\Lambda_{R1}),$$
$$y_{Rt}^{(n)} = -\frac{\log Q_{Rt}^{(n)}}{n} = -\frac{\mathcal{A}_R(n)}{n} - \frac{\mathcal{B}_R(n)'}{n}x_t$$

where $x_t = \begin{bmatrix} \varpi_t, & g_t, & \varepsilon_{\mu t} \end{bmatrix}'$

One-period real interest rate, the process for the market price of risk and latent factor process are all used in the recursive formulas to obtain the yield curves. The latent factor processes and the real yield curve are the equations that are used directly as inputs into the first order VAR.

We estimate the following VAR model. z_t is a 6×1 vector that contains the real yields of maturities 20, 40 and 120 quarters, the change in velocity of money ϖ_t, output growth g_t and monetary policy shock $\varepsilon_{\mu t}$, respectively. The dynamics of z_t is governed by a first-order VAR,

$$z_t = \Psi_0 + \Psi_1 z_{t-1} + \Omega^{1/2}\epsilon_t$$

where $\epsilon_t \sim NIID(0,\ 1)$ and.$\epsilon_t = [\epsilon_{20t}, \epsilon_{40t}, \epsilon_{120t}, \varepsilon_{\varpi t}, \varepsilon_{gt}, \hat{\varepsilon}_{\mu t}]'$. ϵ_{20t}, ϵ_{40t} and ϵ_{120t} are assumed to be the measurement errors associated with the real yields of maturities 20, 40 and 120 quarters, respectively. Ψ_0 and Ψ_1 are unconstrained and $\Omega^{1/2}$ is a 6×6 diagonal matrix. The resulting sample log likelihood function for this model is the same as the sample log-likelihood function derived under Section 5.1, equation (14). The sample size, the variables that makes up the vector z_t, and the coefficient matrices are different than those in Section 5.1. In order to estimate the parameters of our model, we first maximize (14) numerically with respect to the unknown parameters in the matrices Ψ_0 and Ψ_1. To estimate the parameters in matrix Ω, we again maximize (14) numerically with respect to the parameters in matrix Ω by keeping the parameters estimated in the first step fixed.

The sixth column of Table 1 displays the parameter estimates that are obtained by Method 3. The seventh column gives the corresponding standard errors. The AR(1) coefficient of output growth, $\phi_g = 0.0015$, implies that the aggregate consumption exhibits a low and stable volatility over time. If we turn our attention to the estimate of the monetary policy shock persistence, ϕ_μ, it is very clear from the estimation result that an estimate of 0.5098 is quite consistent and robust. The estimate $\phi_\varpi = 0.9413$ indicates that the velocity changes exhibit persistence in the sense that most of the effect of a shock has on this process will be transmitted to the next period. The persistence in velocity changes translates into the persistence of interest rates. The estimates $\phi_c = 120.09$, $\phi_{c0} = 19.138$, $\phi_\pi = 118.79$, and $\phi_{\pi 0} = 25.025$ are very consistent across all the estimation methods. The estimate $\gamma = 3.4377$ gives us the utility curvature, also very important in determining time-varying risk aversion. Inflation targeting parameter estimate is $a_1 = 2.4371$, which shows how strongly the Fed tries to keep the inflation low. The estimate for the market segmentation parameter is $\varphi = 0.2549$, which is in line with the previous estimation results for this parameter. The estimate $\delta = 0.0042$ is consistent and low as expected since low positive values of δ corresponds to the values close to 1 for β. The estimate $\bar{\mu} = 0.0074$ implies that the annual steady state level of money growth is almost 3%. The standard deviation estimates for output growth, monetary policy shock and velocity change are $\sigma_g = 0.0079$, $\sigma_\mu = 0.0250$ and $\sigma_\varpi = 0.0015$, respectively.

Figure 5 displays the impulse responses of the real yields of maturities 20, 40 and 120 quarters to the macroeconomic shocks in the real yields and macro variables model. An expansionary monetary policy lowers the real yields with 20, 40 and 120 quarters to maturity by 0.24%, 0.14% and 0.045% respectively. The strong liquidity effect is present in the model due to the segmented market effect. We can easily come to the conclusion that the segmented market effect without the distractions of the inflation risk premium could only result in lower real yields when money injection occurs. One standard deviation in output growth shock decreases the real yields of maturities 20, 40 and 120 quarters by 0.07%, 0.035% and 0.0011%, respectively. The absence of inflation risk premium from the real term structure of interest rates changes the magnitude of the output growth shock on real yields. An increase in output growth causes the real yields to fall. A one standard deviation shock in velocity changes increases the real yield with 20 quarters to maturity by 0.08%. Finally, it also lowers the real yields with 40 and 120 quarters to maturity by 0.055% and 0.02%. The estimate $\phi_\varpi = 0.9413$ justifies the persistence of the shock.

3.4 Method 4: A hybrid approach

This method uses data on zero coupon yields of maturities 4, 8, 12, 16 and 20 quarters and two of the macroeconomic variables, namely output growth g_t, and monetary policy shock $\varepsilon_{\mu t}$ from 1st quarter of 1964 to the 4th quarter of 2000. The bond yields are computed using the data from the Center for Research in Security Prices (CRSP). The data for output growth g_t, and monetary policy shock $\varepsilon_{\mu t}$ is constructed using real gross domestic product (GDP), M1 money stock, consumer price index (CPI) as explained in Section 5.1.We assume that the yield with 4 quarters to maturity is measured without error while the yields of maturities 8, 12, 16 and 20 quarters are measured with error. By using data for 4 quarter to maturity yield and for output growth g_t, and monetary policy shock $\varepsilon_{\mu t}$, we first try to obtain data for the unobserved latent factor, velocity changes ϖ_t, by inverting the 4 quarter to maturity yield equation. After obtaining data on unobserved latent factor, ϖ_t, we invert the equations for the yields of maturities 8, 12, 16 and 20 quarters to obtain data on measurement errors. Finally, we estimate the parameters of this model by maximizing the joint log likelihood function.

The yield with 4 quarter to maturity is assumed to be measured without error, and it is affine function of observable and unobservable latent factors. The yields of maturities 8, 12, 16 and 20 quarters are also affine functions of observable and unobservable latent factors, and they are assumed to be measured with error. For convenience, let us write the yield equation that is measured without error as

$$Y_t^{wo} = A^{wo} + B^{wo,o} x_t^o + B^{wo,u} x_t^u$$

where superscript wo refers to the yield equation that is measured without error, and subscripts o and u refer to observable and unobservable latent factors respectively. Latent factors matrix, x, is partitioned into $\begin{bmatrix} x_t^u, & x_t^{o\prime} \end{bmatrix}'$. Observable latent factors are $x_t^o = \begin{bmatrix} g_t, & \varepsilon_{\mu t} \end{bmatrix}'$, and unobservable latent factor is $x_t^u = [\varpi_t]'$. Given the parameter vector θ and data on Y_t^{wo} and x_t^o, we can now easily infer the data for the unobservable latent factor, x_t^u by using inversion as follows,

$$x_t^u = (B^{wo,u})^{-1} (Y_t^{wo} - A^{wo} - B^{wo,o} x_t^o)$$

By using the data on the yields of maturities 8, 12, 16 and 20 quarters together with the newly inferred data of x_t^u, we can obtain data on measurement errors. Again, for convenience, we write the yield equations measured with error as follows:

$$Y_t^w = A^w + B^{w,o} x_t^o + B^{w,u} x_t^u + C^w u_t^w$$

where superscript w refers to the yield equations that are measured with error. Finally, u_t^w can be inferred as,

$$u_t^w = (C^w)^{-1} (Y_t^w - A^w - B^{w,o} x_t^o - B^{w,u} x_t^u)$$

The joint likelihood function for this model can be written as follows:

$$\mathcal{L}(\theta) = \prod_{t=2}^{T} f\left(Y_t^{wo}, Y_t^{w}, x_t^{o} \mid Y_{t-1}^{wo}, Y_{t-1}^{w}, x_{t-1}^{o}\right)$$

The joint log likelihood is the same as the joint log likelihood of Section 5.2, 15. In this method of estimation, the number of yields that are measured with error is equal to 4, hence $k = 4$. The Jacobian matrix is given by

$$J = \begin{pmatrix} I_{2x2} & 0_{2x1} & 0_{2x4} \\ B^{wo,o} & B^{wo,u} & 0_{1\times 4} \\ B^{w,o} & B^{w,u} & C^{w} \end{pmatrix}.$$

First, we maximize the log-likelihood function, 15, numerically with respect to the unknown parameters vector θ, which includes all the parameters except for the parameters in the Σ matrix. Second, we estimate the parameters in the Σ matrix by maximizing the loglikelihood function keeping the parameters estimated in the first step fixed.

The eighth column of Table 1 displays the parameter estimates that are obtained using Method 4. The ninth column gives the standard errors of the estimates. The estimate $\phi_g = 0.0986$ corresponds to a stable output growth path and a low aggregate consumption volatility. The estimate $\phi_\mu = 0.4986$ shows that the monetary policy shock is persistent, and consistent with the previous estimates presented above. The AR(1) coefficient of velocity change ϕ_ϖ is equal to 0.9919 implying that the velocity changes exhibit a considerable amount of persistence, in turn, the effect of velocity change on the interest rates would be quite substantial while high value ϕ_ϖ will reduce the impact of market segmentation on the interest rates. In regards to the estimate $a_1 = 1.3573$, it is very clear that an active inflation targeting is pursued by the Fed in order to decrease the inflationary pressures on the yield curves. The utility curvature estimate, $\gamma = 2.6317$ is very consistent with the estimates of the previous three methods employed here. The parameter estimate for market segmentation $\varphi = 0.5118$ shows that bond traders' consumption will increase when money is injected into the economy, reducing the risk aversion temporarily. In the mean time, having a higher value for φ will increase the liquidity effect while not affecting the pure inflation risk premium. As a result, liquidity effect will prevail in the face of an expansionary monetary policy. The rest of the parameters that determine the coefficients of time-varying risk aversion are $\phi_c = 120.05$, $\phi_{c0} = 19.138$, $\phi_\pi = 118.79$, and $\phi_{\pi 0} = 25.002$. The estimate for steady state inflation is $\bar{\pi} = 0.0436$. The estimate for the steady state level for money growth is 0.0509. The standard deviation estimates for output growth, monetary policy shock and velocity change are $\sigma_g = 0.0016$, $\sigma_\mu = 0.0237$ and $\sigma_\varpi = 0.0019$.

Figure 6 displays the impulse responses of the nominal yields to the macroeconomic shocks for the model estimated here. A one standard deviation shock in monetary policy lowers the nominal yields of maturities 4, 12 and 20 quarters by 0.008%, 0.003% and 0.0015% respectively, indicating that the liquidity effect prevails even though the pure inflation risk premium works in the opposite direction. A money injection causes the segmented market effect and the expectation effect together with inflation risk premium effect due to covariation between traders' consumption and inflation to lower the nominal yields. A one standard deviation output growth shock lowers the nominal yield with a maturity of 4 quarters by 0.015%. An increase in output growth lowers inflationary expectations and increases consumption causing investors to demand less risk premium resulting in a decrease in nominal yield rates. A one standard deviation shock in velocity changes increases the nominal yield of 4 quarters maturity by 0.08% while increasing the nominal yield with 12 quarters to maturity by 0.07%. The effect of the shock on the nominal yields persists more than 16 quarters since the estimate of the AR(1) coefficient for velocity change is very close to 1. The nominal yield with 20 quarters to maturity yield rises by 0.65%.

Figure 7 shows the impulse responses of the real yields of maturities 4, 12 and 20 quarters to the macroeconomic shocks for this model. The parameters estimated by Method 4 here are used when plotting the figure. An expansionary monetary policy lowers the real yield with 4 quarters to maturity by 0.25%, decreases the real yields of 12 and 20 quarters to maturity by 0.10% and 0.06% respectively. Since there is not any inflation risk premium effect and expectation effect present in real yields, the segmented market effect causes a strong liquidity effect to prevail. Without the inflation risk premiums, a money injection could only result in lower real interest rates and lower real yields. An output growth shock decreases the real yields of maturities 4, 12 and 20 quarters by 0.013%, 0.004% and 0.0025%, respectively. Even in the absence of inflation risk premium effects and expectation effect, an increase in output growth still results in a decrease in real yields due to the segmented market effect. The reason is that all this effects work in the same direction to lower the interest rates when an increase occurs in output growth. A one standard deviation shock in velocity changes increases the real yield of 4 quarters maturity by 0.0014% while increasing the nominal yield of the same maturity by 0.08%. The sizeable difference between the changes in the nominal and real yields again comes from the inflation risk premium effects and expectation effect present in the nominal term structure but not present in the real term structure. A one standard deviation increase in velocity changes also causes the real yields with maturities 12 and 20 quarters to rise by 0.0006% and 0.0014% respectively.

Overall, the parameter estimates obtained by all four methods are quite consistent and robust regardless of the data employed. The impulse responses of the nominal and real bond yields of different maturities are very reasonable in the sense that they reflect and emphasize the expected and established results in the literature.

4 Conclusion

Term structure of interest rates is said to entail much useful information for current and future state of an economy. To decipher the informational contents from the set of asset prices requires certain economic models in which preferences of investors, mechanisms of pricing assets, allocations of resources are clearly specified. In addition, the theoretical model should be able to generate testable implications which contain stylized facts of both forcing variables and endogenous variables. In studying term structure dynamics, we believe that the relationships between money (monetary policy), inflation, and interest rates are the most important dimensions to be matched. In this vein, we adopt and estimate a dynamic general equilibrium model which is consistent with long-run stylized facts, while allowing short-term deviations. This model has three main features: asset market segmentation, inflation targeting, and time-varying risk aversion. According to our estimations, all three channels matter to capture dynamic variations of a subset of default-free bond prices. For robustness, we estimate our model using different sets of macroeconomic and asset prices data. Interestingly, all the specifications we try show very consistent results. One caveat is that we did not specify firm behaviors which can be lead to more realistic short-term behaviors of inflation. Thus, it would be an interesting task to extend this framework so that goods market frictions can be embedded. We leave this to a future work.

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TABLE 1

MAXIMUM LIKELIHOOD ESTIMATES AND STANDARD ERRORS

	yields and		yields only		real yields and		hybrid	
Parameter	macro model	standard error	model	standard error	macro model	standard error	model	standard error
γ	3.2742	0.0000	2.6499	0.0001	3.4377	0.0289	2.6317	0.0110
ϕ_{ϖ}	0.7816	0.006	0.9919	0.0578	0.9413	0.0000	0.9789	0.0001
ϕ_{μ}	0.5203	0.0012	0.6365	0.0012	0.5098	0.0001	0.4986	0.0008
ϕ_{g}	0.0465	0.0290	0.1028	0.0200	0.0015	0.0068	0.0986	0.0023
ϕ_{c}	120.09	0.0000	120.05	0.0000	120.09	1.9646	120.05	12.784
ϕ_{c0}	19.147	52.364	19.140	19.112	19.138	0.0000	19.138	13.381
ϕ_{π}	118.77	0.0000	118.78	0.0000	118.79	1.439	118.79	2.9322
$\phi_{\pi 0}$	25.023	0.0000	25.000	0.0003	25.025	0.0000	25.002	34.640
g	0.0082	0.0003	0.0313	0.0002	0.0026	0.0167	0.0007	0.0003
$\bar{\mu}$	0.0380	0.1551	0.0019	0.0001	0.0074	0.0222	0.0509	0.0547
$\bar{\pi}$	0.0023	0.1080	−0.027	0.0655	0.0179	0.0000	0.0436	0.0242
$\bar{v}$	0.8815	0.0096	0.6752	0.0079	0.4319	0.0024	0.5605	0.0015
φ	0.2465	0.0001	0.1169	0.0009	0.2549	0.0002	0.5118	0.0004
a_1	1.3225	0.0013	1.5076	0.1875	2.4371	0.0000	1.3573	0.0020
δ	0.0180	0.0739	0.0330	0.0013	0.0042	0.0426	0.0499	0.0199
σ_{g}	0.0110	0.0005	0.0750	0.00012	0.0079	0.00022	0.0016	0.00001
σ_{μ}	0.0883	0.00009	0.1052	0.00014	0.0250	0.00008	0.0237	0.00002
σ_{ϖ}	0.0030	0.0001	0.0028	0.00014	0.0015	0.00012	0.0019	0.00001

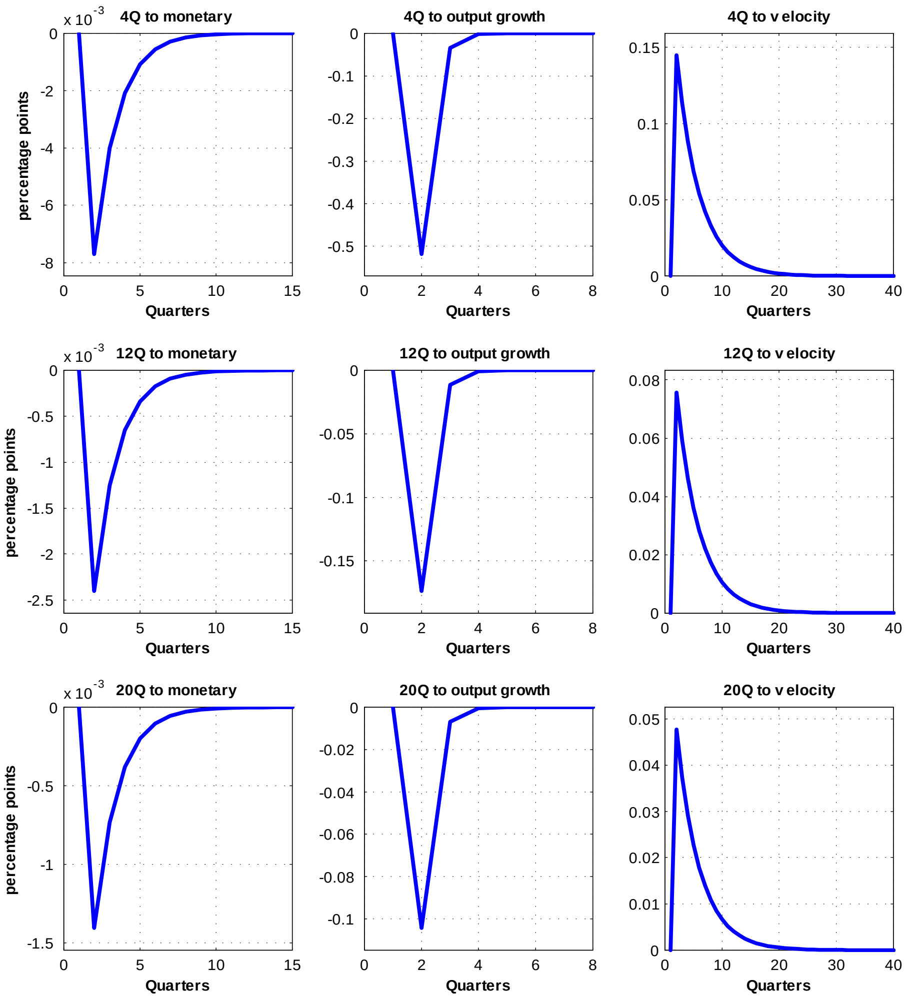

Figure 1. Impulse responses of the nominal yields to macroeconomic shocks for the nominal yields and macroeconomic variables model

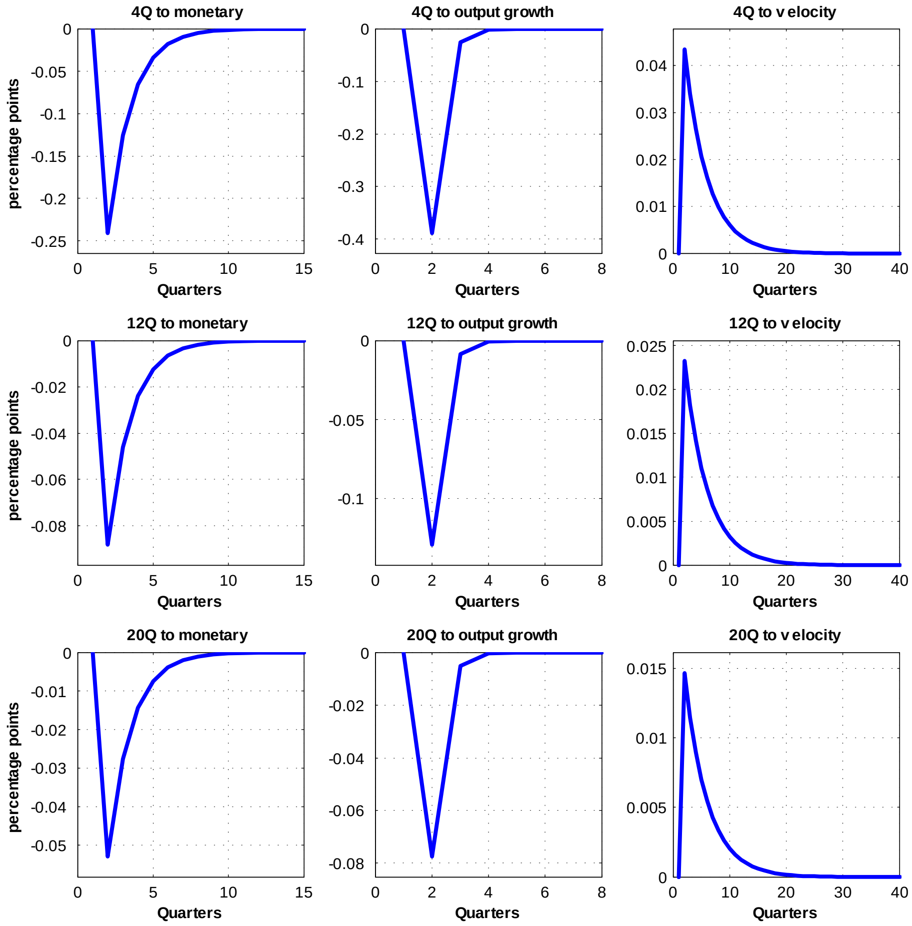

Figure 2. Impulse responses of the real yields to macroeconomic shocks for the nominal yields and macroeconomic variables model

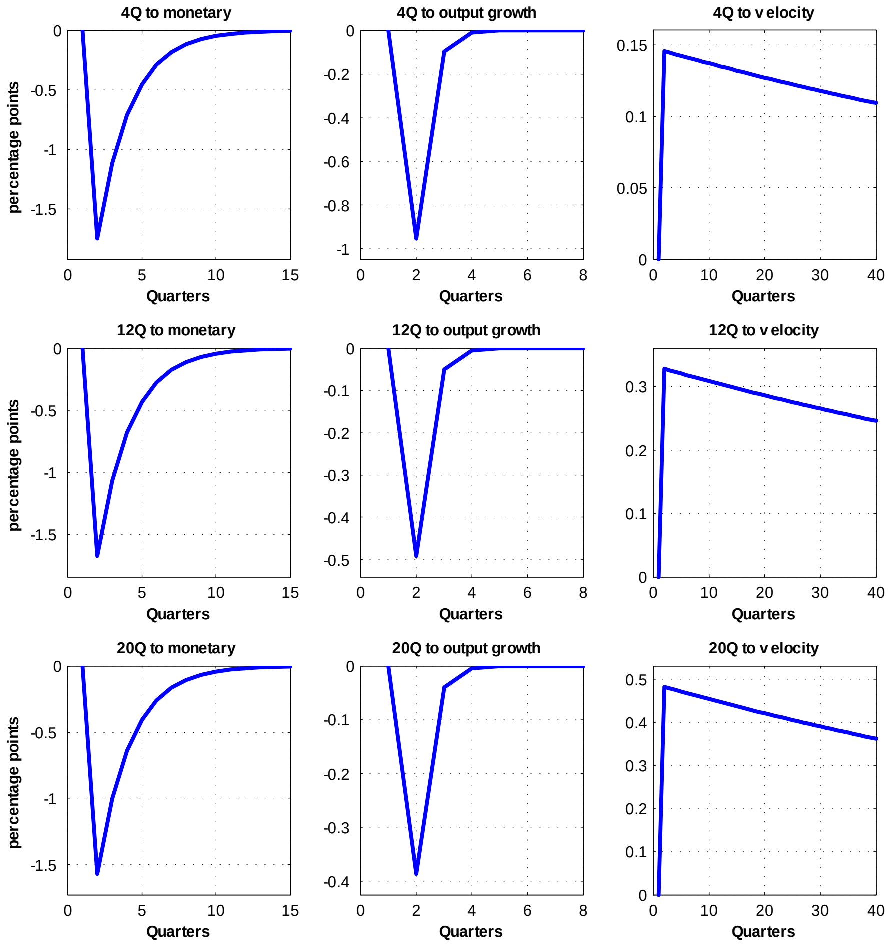

Figure 3. Impulse responses of the yields in yields only model with nominal term structure

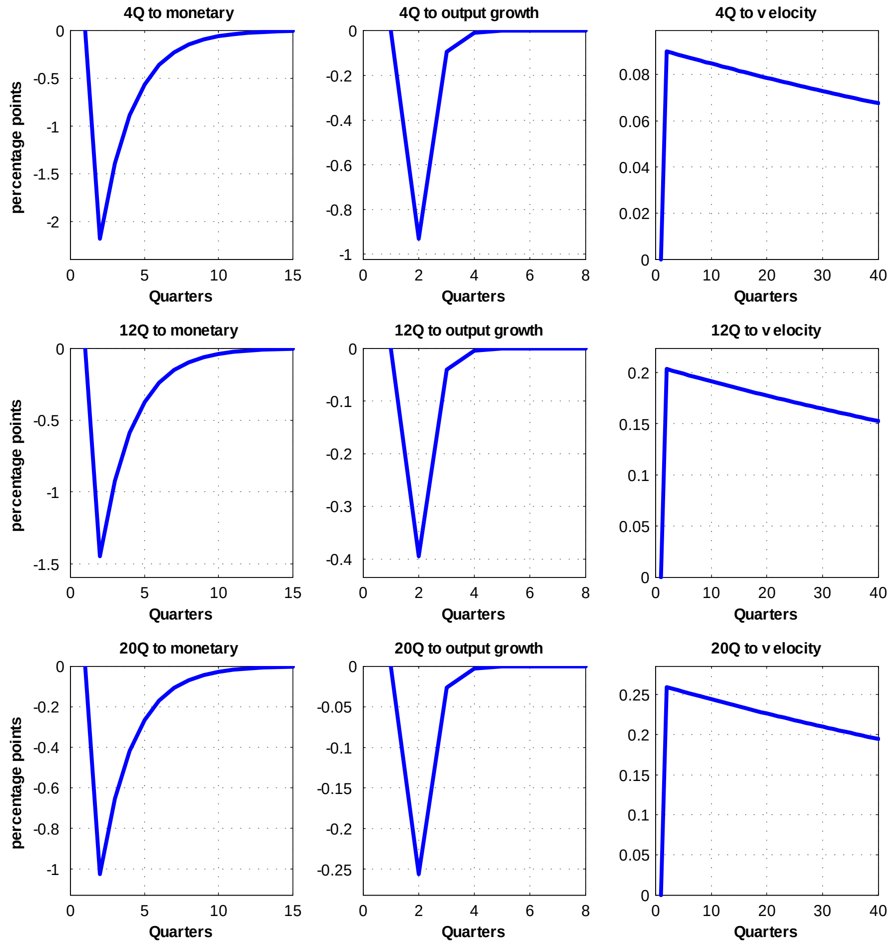

Figure 4. Impulse responses of the real yields in yields only model with nominal yields.

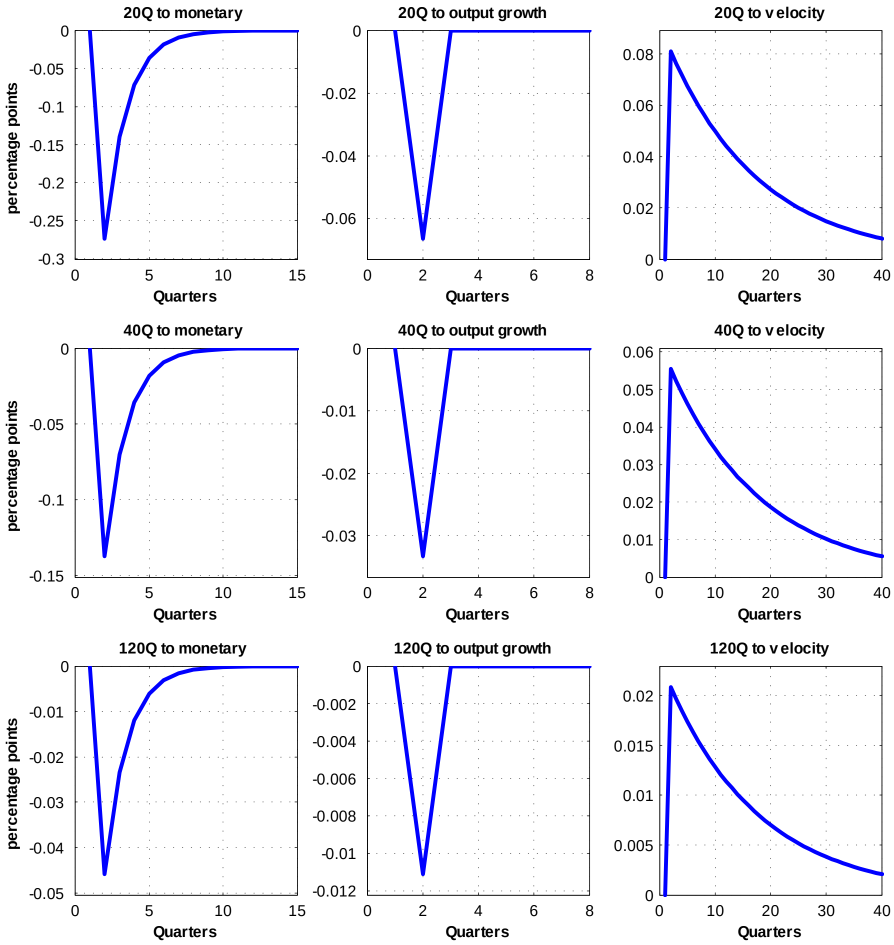

Figure 5. Impulse responses of the real yields to macroeconomic shocks in the model with real yields and macroeconomic variables model

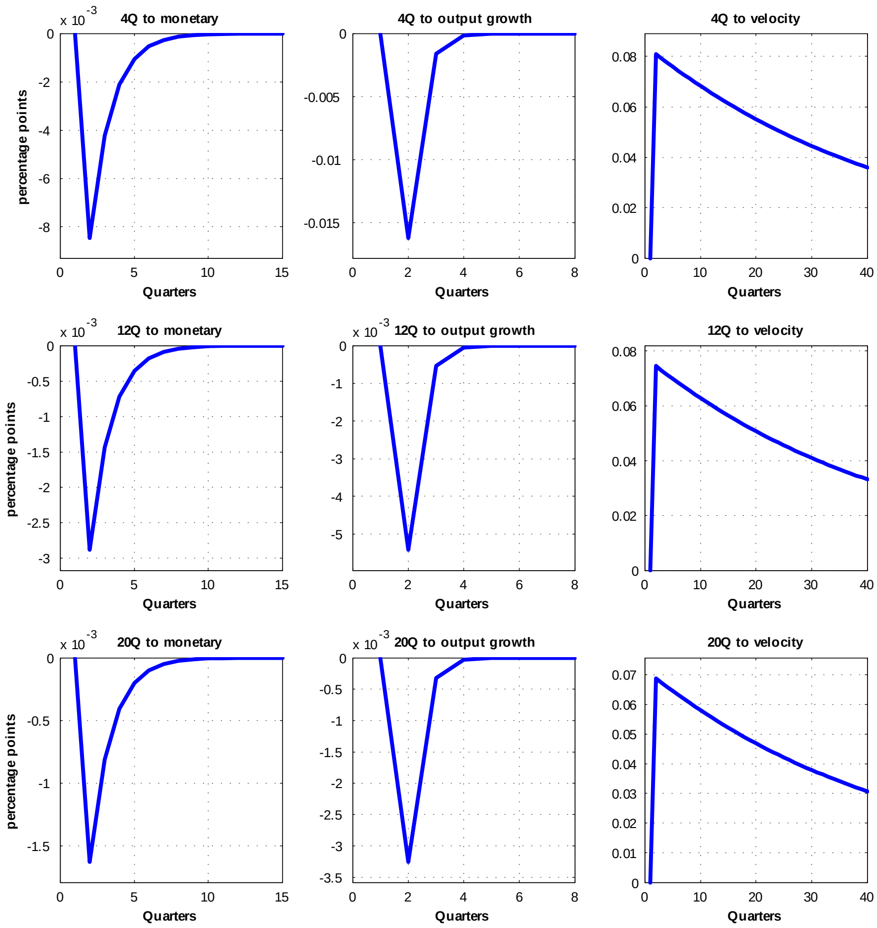

Figure 6. Impulse responses of the yields to macroeconomic shocks for the mixed model

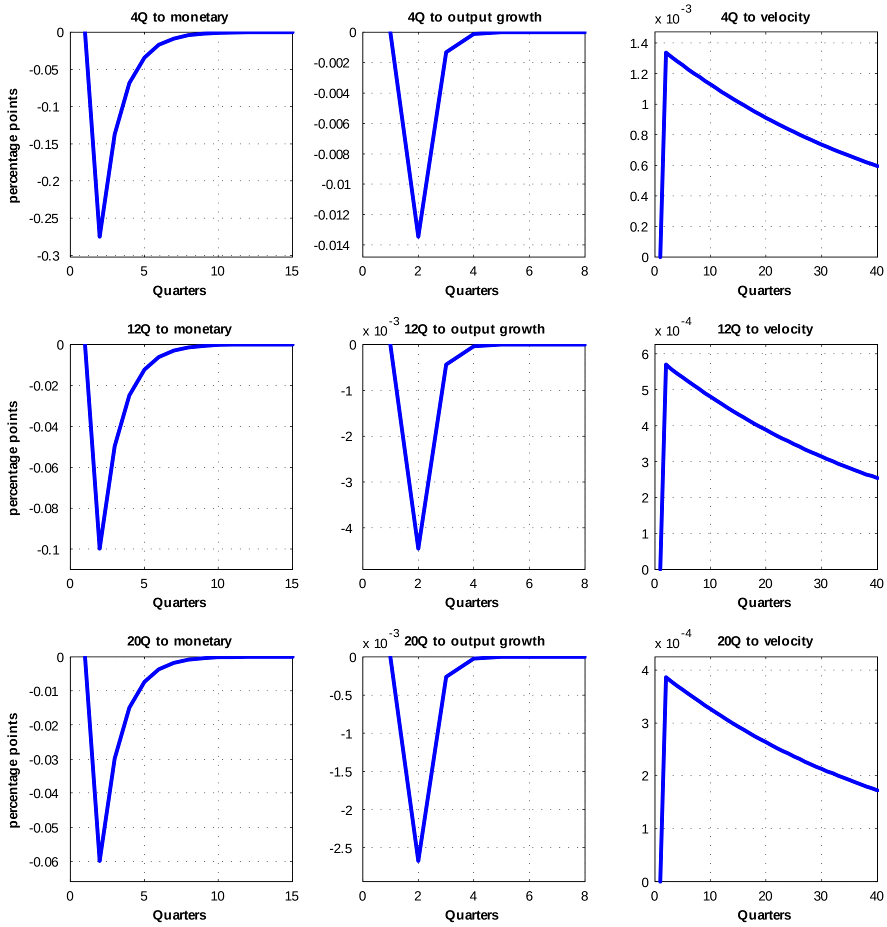

Figure 7. Impulse responses of the real yields to macroeconomic shocks for the mixed model

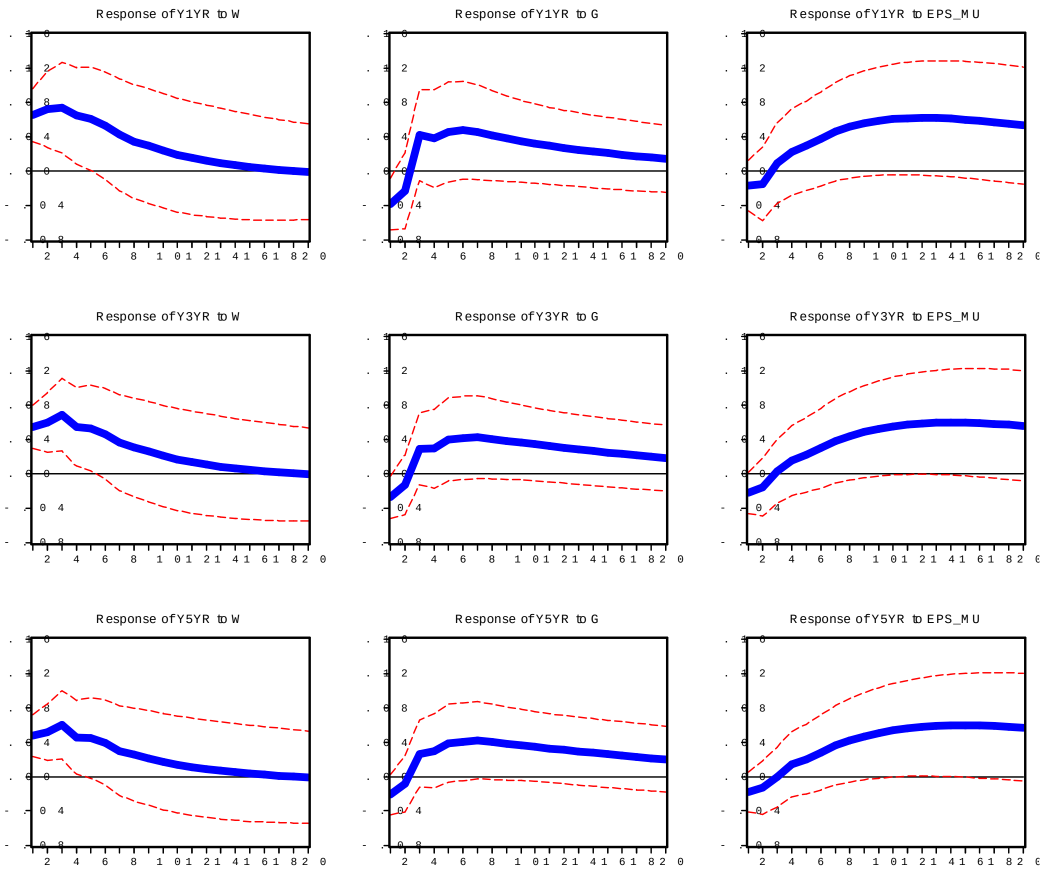

Figure 8. Impulse responses of nominal yields to macroeconomic shocks. VAR(2) with Cholesky decompostion