A Monetary Explanation of the Term Structure of Interest Rates and Bond Risk Premia

Hagen Kim and Aggie J. Moon*

Abstract

This paper presents a monetary model of the term structure of interest rates. Specifically, we study the roles of asset market segmentation, stochastic risk aversion, and inflation targeting in a cash-in-advance model to generate stylized facts regarding interest rates and bond returns.

Our model is consistent with both long-run and short-run monetary facts in that the quantity theory of money holds while there exist short-term deviations due to asset market frictions. We also let the relative risk aversion of bond market participants fluctuate depending on the uncertainties in the macroeconomic variables that determine consumption, inflation, and monetary policy. This allows us to investigate the nature of time-varying bond risk premia in reference to the macroeconomic variables.

We derive an affine macroeconomic term structure of yields from the model, and then analyze how macroeconomic shocks affect the yield curve and expected holding period returns. Quantitative results show that our model can account for many of the observed behaviors of bond prices, including the liquidity effect, expectations puzzle, and upward sloping average yield curve. In addition, our model can produce the level, slope, and curvature factors when we perform a factor analysis using simulated yields.

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Keywords: Segmented asset market, Stochastic risk aversion, Habit formation, Inflation targeting, Expectations hypothesis, Bond risk premia.

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1 Introduction

In macroeconomics, the Fisher equation states that nominal interest rate can be approximated by the sum of real interest rate and expected inflation, and the quantity theory says that money growth and inflation have a one-to-one relationship. These predictions are quite well matched with data in the long run. (See McCandless and Weber, 1995; Lucas, 1996). In the short run, empirical evidences suggest that an increase in money growth leads to decreases in interest rates, referred to as liquidity effect, and it appears that many central banks rely on this mechanism to control short-term fluctuations of inflation via open market operations. That is, it is important to take into account 'money' when we model dynamic behaviors of interest rates, their relationships in the short-run and the long-run show stark contrast. However, recent works analyzing interest rates and macroeconomic variables assign a minimal role to money, and reject the quantity theory. Explaining the short-term behaviors of interest rate is important, but we believe that doing it with correct long-run restrictions is critical, because the interest rates of longer maturities reflect expectations about future states of an economy and premium for bearing the related risk. For instance, suppose that there exists persistent and high growth of money in an economy. It can be consistent with low levels of short-term interest rates according to the liquidity effect channel. However, persistent and high money growth implies that future inflation can be high and therefore long-maturity bond holders will request higher premium for inflation risk. In this case, interpreting yield curve using only current inflation is misleading because forward looking bond market participants can assess future inflation by looking at money growth.

In order to bring together short-term and long-term aspects of money behavior, we present a monetary model of the term structure of interest rates which is consistent with both short-run and long-run stylized facts linking money growth, inflation, and interest rates of various maturities. Our monetary model is based on three assumptions: (1) the bond market is segmented in that only a fraction of households access the bond market, (2) the households have time-varying preferences for taking macroeconomic risk, and (3) the monetary authority uses an inflation targeting rule. The first one can generate liquidity effect while maintaining the quantity theory, the second one is essential in explaining time-varying bond returns, and the last assumption is about modeling monetary policy.[1]

[1]Not all the models using asset market segmentation are consistent with the quantity theory. For example, Alvarez, Atkeson, and Kehoe (2002) and Alvarez, Lucas, and Weber (2001) have this feature, but Grossman and Weiss (1983) does not. New Keynesian models can generate

We adopted the segmented asset market view partly because of its versatility of explaining both short and long-run monetary facts, but mainly because it is an empirically plausible assumption. A salient feature of the bond market is that most of households have zero amount of bond holdings. That is, the asset market is segmented between bond holders and non-holders. According to the Survey of Consumer Finances 2001 and 2004, the percentages of families holding bonds and savings bonds are merely 19.7 and 18.4 respectively.[2] Yet, government securities such as the US treasury bonds play a crucial role in implementing monetary policy. These facts imply that money supply changes in the bond market will have greater effects on asset market participants than non-participants and hence bond prices can be affected by this asset market segmentation.[3]

One drawback of this existing literature on asset market segmentation is the fact that the models generate constant risk premium for holding bonds are based on the expectations hypothesis. However, most of empirical studies report that the expectations hypothesis is rejected in favor of time-varying bond risk premium. For instance, Campbell and Shiller (1991) suggest a metric using yield spreads to document the failure of the expectations hypothesis. They find that positive term spread predicts lower future long rates, whereas the expectations hypothesis states that future rates will increase over time in this case. Under the expectations hypothesis, excess returns for holding bonds are unpredictible. However, Fama and Bliss (1987) find that the forward premiums can forecast the term premiums in the 1-year return on the n-year bond. Cochrane and Piazzesi (2005) report bond return predictability by a return-forecasting factor based on a tent-shaped linear combination of forward rates. Kim and Moon (2006) form a single factor with macroeconomic variables to explain time-varying bond risk premia. In addition, the expectations hypothesis cannot explain the positive average slope of the yield curve. All the empirical results suggest that models without proper adjustments for the term premia can lead to wrong predictions on the future short-term interest rates.

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liquidity effect as well, but its effect is usually small as reported by Christiano, Eichenbaum, and Evans (1997) and most models have no explicit role for money.

[2]Including certificates of deposit does not significantly change this trend: 35.4 percentage in 2001 and 32.1 percentage in 2004 respectively. For comparison, the percentages of families holding stock are also quite low: 21.3 and 20.7 percentages. For more, see Bucks, Kennickell, and Moore (2006)

[3]Holding a checking account that pays interest might be regarded as an indirect way of participating in bond markets. However, when money is injected or withdrawn via open market operations, these accounts have no or very little effect. Thus, we believe that it is more appropriate to define bond holders as actual holders of the treasury bonds.

We also assume a time-varying risk aversion to ascertain the driving force of time-varying bond risk premia, since bond traders will request higher risk premia when they become more risk averse in certain states.[4] Stochastic risk aversion can be understood in a similar fashion to habit formation which is widely accepted in the literature. Constantinides (1990), Ferson and Constantinides (1991), and Campbell and Cochrane (1999) adopt the habit formation to resolve the equity premium puzzle. Wachter (2006) shows that a model adding an exogenous process for expected inflation to the external habit can reproduce the expectations puzzle. Brandt and Wang (2003) focus more on the dynamics of aggregate risk aversion which is responsive to consumption growth and inflation uncertainty.

Exploiting the idea that investing in a nominal bond is about betting on future inflation and real shocks, we adopt a similar mechanism to Brandt and Wang (2003), but we make several extensions. First, the terms inducing time-varying risk premia in our model are fundamental macroeconomic variables rather than an exogenous variable describing external habit. In so doing, we model explicitly how bond investors transform nominal variables into real ones to embed them in their utility function. Second, whereas Wachter (2006) and Brandt and Wang (2003) added an exogenous inflation process to an external habit model, we derive both consumption and inflation endogenously as the functions of underlying macroeconomic variables to theoretically clarify their relationships.

Many economists attempt to link monetary policy to the term structure of interest rates in a New Keynesian framework. For instance, Wu (2004), Gallmeyer, Hollifield and Zin (2005) and Ravenna and Seppala (2005) use a sticky price assumption with an interest rate policy rule to study the term structure of interest rates. However, these models do not use money explicitly, hence it is not appropriate to study liquidity effect and risk premium effect due to changes in money supply. In addition, these types of models cannot impose long-run restrictions a priori.

In order to generate impulse responses of bond yields to a monetary policy shock, Evans and Marshall (1998) used asymmetric portfolio adjustment costs between and within periods for short-run non-neutrality and persistent liquidity effects whereas we use the segmented asset market model. The differences are subtle, but as Dotsey and Ireland (1995) argue, the ability of the former type of the model to explain the liquidity effect is substantially reduced when fitted to

[4] An alternative route is to model time-varying risk. Stochastic volatility models belong to this class. It is not difficult to include it in the fundamental shock process. Here we focus on the time-varying risk aversion to generate countercyclical variation of expected excess returns.

data. In addition, we explicitly derive bond risk premia and consider multiple sources of macroeconomic shocks.

Lastly, there exist a few continuous-time monetary models of term structure such as Bakshi and Chen (1998), Buraschi and Jiltsov (2005, 2007), and Kim (2005) which extend Cox-Ingersoll-Ross type of models into a monetary economy. But most models in this class introduce money via a money-in-the-utility setup and liquidity effect is very difficult to generate.

From our economic model, we derive an affine macroeconomic term structure, and theoretically analyze how macroeconomic shocks affect the yield curve and expected holding period returns. In doing so, we derive the conditions under which the liquidity effect and time-varying risk premia for inflation and output risks are generated.

The quantitative results with extensive robustness checks show that our model can account for many of the observed behaviors of bond prices, including the liquidity effect, expectations puzzle, time-varying bond risk premia, upward sloping average yield curve, and downward to flat yield volatility curve. We can produce the level, slope, and curvature factors when we perform a factor analysis using simulated yields. These characteristics in conjunction with our affine structure shed light on interpreting the results from yields-only or latent factors term structure models. Latent multi-factor approaches have been extensively studied since the work of Litterman and Scheinkman (1991). Duffie and Kan (1996), and Dai and Singleton (2002) explore the affine class of term structure models with no-arbitrage restriction in both sides of market price of risk and volatility. Duffee (2002) uses the term, "essentially affine" to indicate the affine class of model with a flexible market price of risk setup, and shows that the homoskedastic volatility model in this class works better than stochastic volatility. However, the factors in models are formed using interest rates, and therefore interpreting those factors is ambiguous.[5] Since our model is derived entirely from a monetary general equilibrium model and satisfies the feature of the essentially affine class supported by yields data, we believe that our model can account for dynamic evolution of interest rates due to changes in macroeconomic conditions.

The remainder of the paper organized as follows. Section 2 formulates the model. Section 3 then derives short-term rates, nominal term structure of interest rates and excess holding period returns. Section 4 further analyzes the model by

[5] Ang and Piazzesi (2004) include both macroeconomic variables and latent yield factors to estimate an affine term structure model, but still latent factors are the most important factors and the interactions between the macro factors and latent factors are highly limited.

simulation. Then section 5 concludes the study.

2 The model

Economy is populated by a continuum of households, indexed in the $[0, 1]$ interval. There exist a goods market and a government bond market. In terms of sequence, the government bond market opens first, and then the goods market opens up. All the agents attend the goods market every period. A fraction λ of the overall households $(0 < \lambda < 1)$ trade government bonds in the asset market as well. The remaining $(1 - \lambda)$ agents never attend the asset market. We assume that no one changes the status between being a trader and a non-trader for simplicity. In respect of the endowment process, traders and non-traders will have the same income components Y_t. Specifically, we use the following income process. We define g_t as the log growth rate of Y_t. The economy's resource constraint is then

$$Y_t = \lambda C_t^T + (1 - \lambda) C_t^N, \text{ for } t = 0, 1, 2, \cdots, \tag{1}$$

where C_t^T and C_t^N are the consumptions of each type of individual in period t.

Money is held in equilibrium because of the assumption that nobody consumes her own endowment. In the beginning of period t, the government bond market opens up and one-period government bonds as well as money will be traded. This market is open only to traders. Then the goods market opens up. Each consumer consists of a shopper-seller pair, where the seller sells the endowment for cash in the goods market and the shopper makes use of cash to purchase consumption goods for the family in the goods market. Money is introduced through a cash-in-advance constraint. To allow for stochastic velocity in the model, we assume that there exists a conversion delay shock v_t between cash and consumption goods. That is, the shopper is able to use only v_t fraction of the income for purchasing consumption goods. A non-trader carries his unspent receipts from period $(t - 1)$ sales into period-t trading. Adding period t balances to this, in conjunction with the assumption that every nontrader household spends all available cash, yields

$$P_t C_t^N = (1 - v_{t-1}) P_{t-1} Y_{t-1} + v_t P_t Y_t \tag{2}$$

in period t. In case of the households for traders, $\hat{m}_{t+1}$ is defined as the money at the beginning of the period $t + 1$, i.e. before the asset market opens up at $t + 1$,

$$\hat{m}_{t+1} = (\tilde{m}_t - P_t C_t^T) + (1 - v_t) P_t Y_t,$$

where $\tilde{m}_t$ is the amount of cash that the shopper brings to the goods market. Then m_{t+1} is defined as the money stock after the asset market closes but before the goods market opens up, and is written as

$$m_{t+1} = \hat{m}_{t+1} + b_t - Q_{t+1}b_{t+1} - \tau_{t+1}, \tag{3}$$

where b_t refers to the amount received from holding the one-period bond; $Q_{t+1}b_{t+1}$ is the value of bonds purchased at $t+1$; and τ_t is the lump-sum tax or transfer levied only to traders. Now we are ready to figure out the law of motion for $\tilde{m}_{t+1}$ as follows:

$$\begin{aligned} \tilde{m}_{t+1} &= m_{t+1} + v_{t+1}P_{t+1}Y_{t+1} \\ &= (\tilde{m}_t - P_tC_t^T) + (1-v_t)P_tY_t + b_t - Q_{t+1}b_{t+1} + v_{t+1}P_{t+1}Y_{t+1} - \tau_{t+1}. \end{aligned} \tag{4}$$

The cash-in-advance constraint for the trader is then

$$P_tC_t^T \leq \tilde{m}_t. \tag{5}$$

Suppose that the cash-in-advance constraint is binding.[6] Then (4) and (5) yield

$$\begin{aligned} &P_tC_t^T = (1-v_{t-1})P_{t-1}Y_{t-1} + v_tP_tY_t + b_{t-1} - Q_tb_t - \tau_t, \\ &\text{for } t = 0, 1, 2, \cdots. \end{aligned} \tag{6}$$

Monetary policy is implemented via open market operations in this economy. If we denote M_t as the aggregate money supply provided by the central bank, the following must hold in every period,

$$M_t - M_{t-1} = \lambda\left(b_{t-1} - Q_tb_t - \tau_t\right). \tag{7}$$

Since m_t is the definition of money consistent with monetary policy in this model, the money market equilibrium condition is $M_t = \lambda m_t^T + (1-\lambda)m_t^N$, where m_t^N is $(1-v_t)P_tY_t$ by applying (3) to the non-trader case. Then one can show that

$$P_tY_t(1-v_t) = M_t \tag{8}$$

will hold for each period t. That is, a quantity theoretic relationship holds in this model. (8) can be approximated as an equation determining inflation, $\pi_t = \ln P_t - \ln P_{t-1}$:

$$\pi_t \approx \mu_t + \varpi_t - g_t, \tag{9}$$

[6] With this assumption, one can show that the nominal interest rate is strictly positive in this model.

where

$$
\begin{aligned}
g_t &= \log Y_t - \log Y_{t-1}, \\
\varpi_t &\approx v_t - v_{t-1}, \\
\mu_t &\approx (M_t - M_{t-1})/M_{t-1}.
\end{aligned}
$$

Suppose that shock processes follow

$$
\begin{aligned}
g_{t+1} &= (1-\phi_g)g + \phi_g g_t + \sigma_g \varepsilon_{gt+1}, \\
\varepsilon_{gt+1} &\sim NIID(0,1), \\
\varpi_{t+1} &= \phi_\varpi \varpi_t + \sigma_\varpi \varepsilon_{\varpi t+1}, \\
\varepsilon_{\varpi t+1} &\sim NIID(0,1).
\end{aligned}
$$

Regarding monetary policy, we assume that the monetary authority sets the money growth rate in the following way:[7]

$$
\begin{aligned}
\mu_t &= \bar{\mu} - a_1(\pi_t - \bar{\pi}) + \varepsilon_{\mu t} \\
&= a_0 - a_1 \pi_t + \varepsilon_{\mu t}
\end{aligned}
\tag{10}
$$

where

$$
\begin{aligned}
a_0 &= \bar{\mu} + a_1 \bar{\pi}, \\
\varepsilon_{\mu t+1} &= \phi_\mu \varepsilon_{\mu t} + \sigma_\mu \hat{\varepsilon}_{\mu t+1}, \\
\hat{\varepsilon}_{\mu t+1} &\sim NIID(0,1).
\end{aligned}
$$

Clearly, a_1 in (10) measures how actively the monetary authority tries to control inflation.[8] $\varepsilon_{\mu t}$ describes monetary policy shock not captured by inflation targeting behavior. Alvarez, Lucas, and Weber (2001) show that this rule is similar to the Taylor rule in principle. Buraschi and Jiltsov (2005) also use a money growth rule

[7] This is a somewhat strong assumption in that the central bank can react to current inflation contemporaneously. We can relax the assumption by replacing it with the conditional expectation of inflation next period or past inflation at the expense of more complicated model structure, but we do not pursue here for maintaining tractability.

[8] (10) can be relaxed to include lags of money growth or inflation terms. This makes, however, coefficient terms functions of lag operators. This is certainly a realistic and interesting extension, though we do not pursue here for parsimony.

to describe monetary policy. Since our model implies neutrality of money over the aggregate output in (8), we did not include an output measure term usually found in the responsive monetary policy literature.

From (9) and (10), we have

$$\pi_t = \hat{a}_0 + (\frac{1}{1+a_1})(\varepsilon_{\mu t} + \varpi_t - g_t),$$

where

$$\hat{a}_0 = \frac{a_0}{(1+a_1)}.$$

Similarly, from (9) and the above equation, we derive money growth rate:

$$\begin{aligned}\mu_t &= \pi_t - \varpi_t + g_t \\ &= \hat{a}_0 + (\frac{1}{1+a_1})\varepsilon_{\mu t} - (\frac{a_1}{1+a_1})(\varpi_t - g_t).\end{aligned}$$

The consumption for the asset traders is then derived using (6), (7), (8):

$$C_t^T = \left[\frac{1 + \mu_t v_t + \mu_t(1-v_t)/\lambda}{1+\mu_t}\right] Y_t \tag{11}$$

for each t. Consumption process for the non-trader can be easily inferred from (11) and the equilibrium condition (1). Following Alvarez, Lucas, and Weber (2001), we use log-linearization around $(\mu_t, v_t) = (0, \bar{v})$ for the consumption process of the traders:

$$c_t^T = \log C_t^T \approx \varphi \mu_t + \log Y_t \tag{12}$$

where

$$\varphi = \frac{(1-\bar{v})(1-\lambda)}{\lambda}.$$

(12) shows that a decrease in money growth will reduce consumption for the trader. This non-neutrality is due to the assumption of the asset market segmentation and money is neutral in the aggregate level. Thus, the effect is purely distributional.[9] Notice that the asset market segmentation is represented by the term φ. That is,

[9] McCandless and Weber (1995) in their international comparison find out that non-neutrality prevails in the sample of OECD countries, while the whole sample shows monetary neutrality. Given that the bond markets are better established in the OECD countries, this transmission channel could be an important one to analyze international bond markets.

there will be a non-neutrality effect as long as there exists a limited participation ($0 < \lambda < 1$).

Now we assume that the households participating in the bond market have the following preferences[10]

$$\sum_{t=0}^{\infty} \beta^t \frac{(C_t^T - X_t)^{1-\gamma}}{1-\gamma}$$

which is defined over a sequence $\{C_t^T\}$ of a single, perishable consumption good and X_t is an external habit process. We define the surplus consumption process S_t as $(C_t - X_t)/C_t$ and assume that

$$\begin{aligned} \Delta s_{t+1} &= \frac{\gamma}{2} Var_t(K_{t+1}) + \xi_c(\log C_t^T/Y_t)\epsilon_{ct+1}^T - \xi_\pi(\pi_t)\epsilon_{\pi t+1}, \\ K_{t+1} &= (\phi_{c0} - \phi_c \log C_t^T/Y_t)\epsilon_{ct+1}^T - (\phi_{\pi 0} + \phi_\pi \pi_t)\epsilon_{\pi t+1}, \\ \xi_c(\log C_t^T/Y_t) &= \phi_{c0} - 1 - \phi_c \log C_t^T/Y_t, \\ \xi_\pi(\pi_t) &= \phi_{\pi 0} + \phi_\pi \pi_t, \end{aligned} \tag{13}$$

where

$$\begin{aligned} (\phi_{c0} - 1 - \phi_c \log C_t^T/Y_t) &> 0, \\ \phi_\pi &> \phi_{\pi 0}, \\ \phi_{c0},\ \phi_c,\ \phi_{\pi 0},\ \phi_\pi &> 0, \end{aligned}$$

$\epsilon_{ct+1}^T = \Delta c_{t+1}^T - E_t \Delta c_{t+1}^T$, $\epsilon_{\pi t+1} = \pi_{t+1} - E_t \pi_{t+1}$, $\Delta s_{t+1} = \log S_{t+1}/S_t$, and $\Delta c_{t+1}^T = \log C_{t+1}^T/C_t^T$.

We explain our setup and assumptions in detail. First, this formation allows relative risk aversion (RRA) to be stochastic as

$$\begin{aligned} RRA_t &= \frac{\gamma}{S_t}, \\ \ln RRA_t &= \ln \gamma - s_t. \end{aligned} \tag{14}$$

The process for s_t indicates that either an unexpected decrease in consumption growth or an unexpected increase in inflation will increase risk aversion. s_t is

[10] We can define utility functions for the household who does not partake in the bond market. Due to the market segmentation assumption, however, stochastic discount factor is determined only by the participant's utility function. Thus, for brevity we will focus on the preference of the bond market participants.

conditionally heteroskedastic and the sensitivity functions, ξ_c and ξ_π, are assumed to be linear in current consumption-output ratio and inflation respectively.

Our basic setup is similar to Campbell and Cochrane (1999), Brandt and Wang (2003), and Gallmeyer et al. (2005) which exploit the idea of time-varying risk aversion of the consumer. One difference of our model is that the aggregate consumption of the traders generates the habit process, not the aggregate consumption of the economy as a whole. That is, an individual household participating in the government bond market uses consumption in the group of participants as the reference point. Given the segmentation of asset market, we believe that this is closer to the spirit of 'Catching up with the Joneses' following Abel (1990).

To account for our surplus consumption process, we derive the forecasting error in consumption growth ϵ_{ct+1}^T as

$$\epsilon_{ct+1}^T = -(\frac{\varphi a_1}{1+a_1})\sigma_\varpi \varepsilon_{\varpi t+1} + (1 + \frac{\varphi a_1}{1+a_1})\sigma_g \varepsilon_{gt+1} + \frac{\varphi}{1+a_1}\sigma_\mu \hat{\varepsilon}_{\mu t+1}. \tag{15}$$

As clearly seen, the asset market segmentation term φ plays an important role to form ϵ_{ct+1}^T. Without this term, the output growth shock (ε_{gt+1}) solely determines the consumption innovation (ϵ_{ct+1}^T). The inflation targeting (a_1) also affects the consumption growth uncertainty in a non-trivial manner. More active inflation targeting (higher a_1) will increase the contribution by the volatility of the aggregate output growth shock (ε_{gt+1}) to the consumption growth shock while decreasing it by the velocity changes ($\varepsilon_{\varpi t+1}$) and the monetary policy shock ($\hat{\varepsilon}_{\mu t+1}$).

We can similarly write down the innovations in inflation:

$$\begin{aligned} \epsilon_{\pi t+1} &= \pi_{t+1} - E_t \pi_{t+1} \\ &= (\frac{1}{1+a_1})(\sigma_\varpi \varepsilon_{\varpi t+1} - \sigma_g \varepsilon_{gt+1} + \sigma_\mu \hat{\varepsilon}_{\mu t+1}). \end{aligned} \tag{16}$$

This term in (13) can be understood as the anxiety that bond market investors have because they hold nominal assets. (15) and (16) show how fundamental macro shocks determine stochastic risk aversion of the bond investor over time. For more intuitive explanation, we rewrite (13) in a simplified way:

$$\begin{aligned} \ln S_t - \ln S_{t-1} &= \xi_c(\ln C_t^T / C_{t-1}^T) - \xi_\pi(\ln P_t / P_{t-1}) \\ &= \xi_c \left(\ln \frac{C_t^T}{C_{t-1}^T} - \ln \frac{P_t^\chi}{P_{t-1}^\chi} \right) \\ &= \xi_c \ln \left[\left(\frac{P_t C_t^T}{P_{t-1} C_{t-1}^T} \right) / \left(\frac{P_t^{\chi+1} C_t^T}{P_{t-1}^{\chi+1} C_t^T} \right) \right], \end{aligned}$$

where $\chi = \xi_\pi/\xi_c$. The denominator inside the log is an implicit price deflator and the numerator is change in nominal consumption. That is, this ratio measures change in real consumption. Therefore, our surplus consumption process is an extension of the external habit formation incorporating nominal indexation.

If the nominal price increases by a sizable amount, surplus consumption can decrease even if there is a net growth in consumption for bond market participants, which in turn increases risk aversion. On the other hand, our model implies that bond traders' consumption could increase when there is a money injection. Then, this channel of liquidity effect could lower relative risk aversion temporarily. Short-term increase in consumption for the market participant due to open market purchase is important for generating the liquidity effect. However, in light of long-term expectation for future inflation, money injections mean high inflation uncertainty is more likely, which could make investors more concerned about their real value of long-term nominal bonds. Given the homoskedastic law of motions for the fundamental forcing variables, however, this expected inflation channel is not allowed to dominate non-neutral effect onto the consumption of bond traders as long as the liquidity effect prevails. Thus, this mechanism implies that high and persistent money growth will lead to lower risk aversion, which is not consistent with long-run stylized facts. In this sense, our model with an additional apparatus of inflation anxiety can prevent short-term volatility of money growth from transmitting itself to long-run risk aversion in a counterfactual way. Although it is only a reduced form approach, adjusting external habit or stochastic risk aversion with respect to inflation uncertainty appears to be better reconciled under limited market participation framework.

Lastly, we want to mention about our sign restrictions. A notable feature in the surplus consumption process is that the sensitivity function depends on macroeconomic variables in lieu of s_t process. This enables us to identify shocks and understand how monetary transmission mechanisms affect risk premium required by bond investors. Furthermore, we chose the affine sensitivity functions so that we could maintain affine term structure model which is prevalent in the term structure literature. Finally, one more restriction is worth mentioning: $\phi_\pi > \phi_{\pi 0} > 0$. When inflation is positive, this simply implies that inflation will increase risk aversion temporarily. However, in conjunction with no further restriction on the sign of $\xi_\pi(\pi_t)$, this implies that if inflation term π_t becomes a sufficiently large negative value (i.e. a severe deflation), an unexpected deflation shock ($\epsilon_{\pi t+1} < 0$) will actually increase risk aversion temporarily. This appears to be appropriate given the empirical evidence of the Great Depression era argued by Friedman and Schwartz (1963).

3 Term structure of interest rates

Now we are ready to derive the nominal term structure of interest rates. Individual optimization results in the usual Euler equation for the short-term interest rate

$$Q_t = E_t\left[\beta\left(\frac{C_{t+1}^T}{C_t^T}\right)^{-\gamma}\left(\frac{S_{t+1}}{S_t}\right)^{-\gamma}\left(\frac{P_t}{P_{t+1}}\right)\right]. \tag{17}$$

Similarly, the price of zero-coupon nominal bond with maturity n at t denoted as $Q_t^{(n)}$ is

$$Q_t^{(n)} = E_t\left[\beta^n\left(\frac{C_{t+n}^T}{C_t^T}\right)^{-\gamma}\left(\frac{S_{t+n}}{S_t}\right)^{-\gamma}\left(\frac{P_t}{P_{t+n}}\right)\right]. \tag{18}$$

Euler Equation says that this is isomorphic to the external habit formation. However, note that the consumption is only for traders, not the aggregate economy. In addition, surplus consumption term is also that of traders. Later, we show that habit is necessary but not sufficient for generating time-varying risk premia for bonds.

Using (17), we now solve for the short-term interest rate.

Proposition 1 *Denote the short-term (one-period) nominal interest rate by r_t:*

$$r_t = \Theta_0 + \Theta_1' x_t,$$

where x_t is defined as the state vector

$$\begin{aligned} x_t &= [\varpi_t, g_t, \varepsilon_{\mu t}]', \\ \Theta_1 &= [\theta_1, \theta_2, \theta_3]', \end{aligned}$$

$$\begin{aligned} \Theta_0 = {} & \delta + \gamma\left(\frac{\varphi a_1}{1+a_1} + 1\right) g(1-\phi_g) + \frac{a_0 - (1-\phi_g)g}{1+a_1} \\ & + \frac{\gamma(\sigma_\mu^2 + \sigma_\varpi^2 + \sigma_g^2)}{(1+a_1)^2}\left(\phi_{\pi 0} - \frac{1}{2\gamma} + \frac{\phi_\pi a_0}{1+a_1}\right) \\ & + \frac{\gamma(\phi_{c0} - \phi_c \varphi \hat{a}_0)}{(1+a_1)^2}\left[\varphi a_1 \sigma_\varpi^2 + (1 + a_1(1+\varphi))\,\sigma_g^2 - \varphi\sigma_\mu^2\right], \end{aligned}$$

$$
\begin{aligned}
\theta_1 &= \left[\begin{array}{c} \frac{\gamma\varphi a_1(1-\phi_\varpi)+\phi_\varpi}{1+a_1} + \frac{\gamma\phi_\pi(\sigma_\mu^2+\sigma_\varpi^2+\sigma_g^2)}{(1+a_1)^3} \\ +\frac{\gamma\phi_c\varphi a_1}{(1+a_1)^3}\left[\varphi a_1\sigma_\varpi^2 + (1+a_1(1+\varphi))\,\sigma_g^2 - \varphi\sigma_\mu^2\right] \end{array} \right], \\
\theta_2 &= \left[\begin{array}{c} \gamma\left(\phi_g - (1-\phi_g)\frac{\varphi a_1}{1+a_1}\right) - \frac{\phi_g}{1+a_1} - \frac{\gamma(\sigma_\mu^2+\sigma_\varpi^2+\sigma_g^2)}{(1+a_1)^2}\frac{\phi_\pi}{1+a_1} \\ -\frac{\gamma\phi_c\varphi a_1}{(1+a_1)^3}\left[\varphi a_1\sigma_\varpi^2 + (1+a_1(1+\varphi))\,\sigma_g^2 - \varphi\sigma_\mu^2\right] \end{array} \right], \\
\theta_3 &= \left[\begin{array}{c} -\frac{\gamma\varphi(1-\phi_\mu)-\phi_\mu}{1+a_1} + \frac{\gamma\phi_\pi(\sigma_\mu^2+\sigma_\varpi^2+\sigma_g^2)}{(1+a_1)^3} \\ -\frac{\gamma\phi_c\varphi}{(1+a_1)^3}\left[\varphi a_1\sigma_\varpi^2 + (1+a_1(1+\varphi))\,\sigma_g^2 - \varphi\sigma_\mu^2\right] \end{array} \right].
\end{aligned}
$$

Proof. See Appendix. ■

A few remarks are in order. The liquidity effect of money injection occurs if and only if $\theta_3 < 0$. The first term in θ_3 represents segmented market effect $-\left(\frac{\gamma\varphi(1-\phi_\mu)}{1+a_1}\right)$ and expectation effect $\left(\frac{\phi_\mu}{1+a_1}\right)$ from money growth changes. Therefore, a money injection will lower the interest rate by $\left(\frac{\gamma\varphi(1-\phi_\mu)-\phi_\mu}{1+a_1}\right)$ provided that this term is strictly positive. Existing works employing the segmented asset market assumption emphasize this effect. In our model, there exist more terms because of bond risk premia. Particularly, the second term in θ_3 is the inflation risk premium resulting from increases in money growth. Since a money injection brings about inflationary pressure, this makes investors demand a higher return for holding nominal assets. Consistent with this fact, this term gets less significant as a_1 increases (i.e. more active inflation targeting). Thus, it operates in the opposite direction of the liquidity effect. The third term in θ_3 also describes inflation risk premium due to the covariation between consumption for bond market traders and inflation. The sign of this term impinges on $Cov_t(\epsilon_{ct+1}, \epsilon_{\pi t+1})$,which is reported to be negative according to Barr and Campbell (1997) and Wachter (2006). In our model, we can show that this holds if and only if $\varphi a_1\sigma_\varpi^2 + (1+a_1(1+\varphi))\,\sigma_g^2 > \varphi\sigma_\mu^2$. In words, since our model provides that velocity changes of money and output growth affect bond traders' consumption growth and inflation in the opposite direction while money growth affects those in the same direction, the former should be larger than the latter to be consistent with the empirical facts. Then, why does this third term in θ_3 lower the short-term interest rate in response to an increase in money growth? It is because an increase in bond traders' consumption due to money injection reduces risk aversion temporarily as shown in (13) and (14). Now one can see the importance of the second term in θ_3, pure inflation risk premium as explained in the previous section. Without this term, the liquidity effect can be too pronounced even for long-term bond prices.

Secondly, $\theta_1 > 0$ holds again if and only if $\varphi a_1\sigma_\varpi^2 + (1+a_1(1+\varphi))\,\sigma_g^2 > \varphi\sigma_\mu^2$, which is the same condition as the one for negative covariance between consumption and inflation. Notice that the way ϖ_t (changes in velocity of money) affects interest rate is different than that of the money growth rate. In this case, both expectation and risk premium channels work in the same direction.

Regarding θ_2, we show that this is negative if $\phi_g < \varphi a_1/(a_1+1)/(1+\varphi a_1/(a_1+1))$. The intertemporal substitution effect says that the sign is negative. In addition, since the model implies that an increase in output lowers inflation, and increases consumption, investors will demand less risk premium and interest rate will get lower. All of these effects point to $\theta_2 < 0$.

With the interest rate derived in proposition 1, we derive the nominal stochastic discount factor as follows:

Proposition 2 *The nominal stochastic discount factor at time t is denoted as SDF_t and derived as*

$$-\log SDF_{t+1} = r_t + \frac{1}{2}\Lambda(x_t)'\Sigma\Lambda(x_t) + \Lambda(x_t)'\Sigma^{1/2}\Xi_{t+1}, \tag{19}$$

where $\Lambda(x_t) = \Lambda_0 + \Lambda_1 x_t$ with

$$\Lambda_0 = \begin{bmatrix} \zeta_0 - \gamma(\phi_{c0}\zeta_1 + \phi_{\pi 0}\zeta_0) + \gamma a_0(\phi_c\zeta_1\zeta_3 - \phi_\pi\zeta_0^2) \\ -\zeta_0 + \gamma\phi_{c0}\zeta_2 + \gamma\phi_{\pi 0}\zeta_0 + \gamma a_0(\phi_\pi\zeta_0^2 - \phi_c\zeta_1\zeta_3) - \gamma\phi_c a_0\zeta_3 \\ \zeta_0 + \gamma\phi_{c0}\zeta_1 - \gamma\phi_{\pi 0}\zeta_0 - \gamma a_0(\phi_\pi\zeta_0^2 + \phi_c\zeta_1\zeta_3) \end{bmatrix}, \tag{20}$$

$$\begin{aligned}\Lambda_1 = (\gamma\phi_c) \times &\begin{bmatrix} -(\zeta_1)^2 & (\zeta_1)^2 & \zeta_1\zeta_3 \\ \zeta_1\zeta_2 & -\zeta_1\zeta_2 & -\zeta_2\zeta_3 \\ (\zeta_1)^2 & -(\zeta_1)^2 & -\zeta_1\zeta_3 \end{bmatrix} \\ + (\gamma\phi_\pi) \times &\begin{bmatrix} -(\zeta_0)^2 & (\zeta_0)^2 & -(\zeta_0)^2 \\ (\zeta_0)^2 & -(\zeta_0)^2 & (\zeta_0)^2 \\ -(\zeta_0)^2 & (\zeta_0)^2 & -(\zeta_0)^2 \end{bmatrix},\end{aligned} \tag{21}$$

$$\begin{aligned}\zeta_0 &= \frac{1}{1+a_1}, \\ \zeta_1 &= \frac{\varphi a_1}{1+a_1}, \\ \zeta_2 &= 1 + \frac{\varphi a_1}{1+a_1}, \\ \zeta_3 &= \frac{\varphi}{1+a_1}.\end{aligned}$$

$$x_{t+1} = \kappa + \Phi x_t + \Sigma^{1/2}\Xi_{t+1}, \tag{22}$$

where

$$\kappa = [0, (1-\phi_g)g, 0]',$$
$$diag(\Phi) = [\phi_\varpi, \phi_g, \phi_\mu]',$$

$$diag(\Sigma^{1/2}) = [\sigma_\varpi, \sigma_g, \sigma_\mu]',$$
$$\Sigma^{1/2}(i,j) = 0 \text{ for all } i \neq j,$$

$$\Xi_t = [\varepsilon_{\varpi t}, \varepsilon_{gt}, \hat{\varepsilon}_{\mu t}]',$$

where $[\varepsilon_{\varpi t}, \varepsilon_{gt}, \hat{\varepsilon}_{\mu t}]' \sim NIID(0, I)$.

Proof. See Appendix. ■

In comparison with yields-only factor models, the stochastic discount factor (19) of our model resembles that of the essentially affine term structure model (E-ATSM) following Duffee (2002). Specifically, the factor structure is Gaussian as in (22), but the process for market price of risk, $\Lambda(x_t)$ is time-varying and affine. (20) and (21) show that factor loadings for the market price of risk process are determined by risk preference terms ϕ_c and ϕ_π, market segmentation φ, and inflation targeting a_1. Thus, unlike latent factor models or hybrid factor models, we endogenously determine the market price of risk process $\Lambda(x_t)$ from the model.

Using (19), now we solve for bond prices.

Proposition 3 *The price of a zero-coupon, nominal bond with maturity* n *periods denoted as* $Q_t^{(n)}$ *can be found by the recursive formula*

$$Q_t^{(n)} = \exp\left[\mathcal{A}(n) + \mathcal{B}(n)'x_t\right],$$

where

$$\begin{aligned} \mathcal{A}(n+1) &= \mathcal{A}(n) - \Theta_0 + \mathcal{B}(n)'[\kappa - \Sigma\Lambda_0] + \frac{1}{2}\mathcal{B}(n)'\Sigma\mathcal{B}(n), \\ \mathcal{B}(n+1)' &= -\Theta_1' + \mathcal{B}(n)'[\Phi - \Sigma\Lambda_1], \end{aligned} \tag{23}$$

where $\mathcal{A}(0) = \mathcal{B}(0) = 0$.

Proof. See Appendix. ■

Yield to maturity is then defined as

$$y_t^{(n)} = -\frac{\log Q_t^{(n)}}{n} = a(n) + b(n)' x_t,$$

where we denote $a(n) = -\mathcal{A}(n)/n$ and $b(n) = -\mathcal{B}(n)/n$.

In a similar fashion, we can also derive expected excess holding period returns as follows:

Proposition 4 *Conditional expectation of the holding period returns of $n-$period bonds over the one period interest rate is time varying and has the following form:*

$$E_t\left[hprx_{t+1}^{(n)}\right] = -(n-1)\Lambda_0'\Sigma b(n-1) + (n-1)\left[\psi_\varpi \varpi_t + \psi_g g_t + \psi_\mu \varepsilon_{\mu t}\right] - \frac{(n-1)^2}{2} b(n-1)'\Sigma b(n-1),$$

$$\psi_\varpi = \left[\begin{array}{c} \gamma\phi_c\left\{(\zeta_1)^2\left(\sigma_\varpi^2 b(n-1;\varpi) - \sigma_\mu^2 b(n-1;\mu)\right) - \zeta_1\zeta_2\sigma_g^2 b(n-1;g)\right\} \\ +\gamma\phi_\pi(\zeta_0)^2\left\{\sigma_\varpi^2 b(n-1;\varpi) + \sigma_\mu^2 b(n-1;\mu)) - \sigma_g^2 b(n-1;g)\right\} \end{array}\right],$$

$$\psi_g = \left[\begin{array}{c} \gamma\phi_c\left\{-(\zeta_1)^2\left(\sigma_\varpi^2 b(n-1;\varpi) - \sigma_\mu^2 b(n-1;\mu)\right) + \zeta_1\zeta_2\sigma_g^2 b(n-1;g)\right\} \\ -\gamma\phi_\pi(\zeta_0)^2\left\{\sigma_\varpi^2 b(n-1;\varpi) + \sigma_\mu^2 b(n-1;\mu)) - \sigma_g^2 b(n-1;g)\right\} \end{array}\right],$$

$$\psi_\mu = \left[\begin{array}{c} \gamma\phi_c\left\{-\zeta_1\zeta_3(\sigma_\varpi^2 b(n-1;\varpi) - \sigma_\mu^2 b(n-1;\mu)) + \zeta_2\zeta_3\sigma_g^2 b(n-1;g)\right\} \\ +\gamma\phi_\pi(\zeta_0)^2\left\{\sigma_\varpi^2 b(n-1;\varpi) + \sigma_\mu^2 b(n-1;\mu)) - \sigma_g^2 b(n-1;g)\right\} \end{array}\right],$$

where

$b(n-1) = [b(n-1;\varpi), b(n-1;g), b(n-1;\mu)]'$.

Proof. See Appendix. ■

For further analysis, it is necessary to know the signs of factor loadings for yields, $b(n;x)$ where $x = \varpi$, g, ε_μ. Although we will explore more in the next section, for expositional purpose, say that $b(n;\varpi)$ are positive for all n, $b(n;g)$ are negative, and $b(n;\mu)$ are mostly negative but changing to positive as n grows according to the calibrated model. Now we could analyze how each of the macroeconomic variables affects expected excess holding period returns of bonds with maturity n. First, when there is a contractionary shock in monetary policy (i.e. a decrease in ε_{μ_t}), one can easily see that risk premia will increase due to consumption growth. For the part with inflation risk, if $\sigma_\varpi^2 b(n-1;\varpi) + \sigma_\mu^2 b(n-1;\mu)) - \sigma_g^2 b(n-1;g) > 0$, then a contractionary monetary policy shock will decrease excess holding period returns. Overall, if the former dominates the latter, we will observe that the excess holding period returns increase in response to a contractionary monetary policy shock. Similarly, a contractionary output shock will increase the consumption risk premium and increase the inflation risk premia as well. An increase in ϖ_t also increases the consumption risk premium and the inflation risk premium.

4 Quantitative results

This section explores the model using numerical methods. Despite that we explicitly derived bond yields and excess returns as affine functions of the underlying macroeconomic variables, the complexity of derived factor loadings refrains us from interpreting many of the implications generated by the model. More importantly, we need to verify if the model can account for stylized facts on bond yields, returns, and macroeconomic variables.

4.1 Parameterization

We parameterize the model in a monthly frequency and select parameters by matching certain moments of US data covering from 1964 to 2000. The macroeconomic data set is obtained from the St. Louis Fed web page (FRED). In particular, we use the sum of non-durable goods and services for aggregate consumption; consumer price index to compute inflation; and M1 as the controllable monetary aggregate. We compute bond yields and returns using the data from the Center for Research in Security Prices (CRSP). Table 1 summarizes the parameter choices. Subjective discount parameter β is chosen so that the annualized real interest rate in the steady state is consistent with the data. We estimate our simple monetary policy rule using OLS methods to identify inflation targeting parameters a_1 and $\bar{\pi}$. Following Campbell and Cochrane (1999), we assume the aggregate consumption growth follows an independently and identically distributed process. That is, $\phi_g = 0$.[11]

[Insert Table 1 and 2 about here]

The monetary policy shock persistence parameter, ϕ_μ, is set equal to 0.5. Despite our experimentation with a broad range of parameters from 0.2 to 0.8, the results are robust. One caveat to note is that higher persistence reduces the size of the

[11] As long as the condition $\phi_g < \varphi a_1/(a_1+1)/(1+\varphi a_1/(a_1+1))$ holds, our qualitative results do not change.

liquidity effect. We choose the number so that dynamic responses of monetary policy shock disappear roughly after one year. We model monetary policy as money injection or withdrawal through open market operations instead of interest rate targeting in a monthly frequency. We, therefore, expect the mean reversion to be strong since money growth rate is much less persistent than the interest rate. The change in velocity is set so that the implied interest rates can be sufficiently persistent as data suggest. Regarding other parameters related to time-varying risk aversion (γ, ϕ_c, ϕ_{c0}, ϕ_π, $\phi_{\pi 0}$) and asset market segmentation (λ), we compare monthly mean excess holding period returns of maturities of two to twelve months derived from the model with the data. That is, we choose six parameters altogether so that six of the theoretical mean excess returns are fitted to average return historical returns by numerically solving the system of equations. It is interesting to note that the estimated parameter (λ) is close to the percentage of bond market participants in the Survey of Consumer Finance data. For robustness, we checked extensively how sensitive results are when we change λ. If λ is over 0.4, given other parameters constant, we find that liquidity effect disappears.[12] Table 2 reports the risk aversion measured by (14) using the simulated data sets. Its mean is less than 3 with standard deviation of roughly 2. This suggests that the selected preference parameters are compatible with the business cycle literature. Now we analyze both qualitative and quantitative properties of the model based on these parameters.

4.2 Yield curves and impulse responses

Empirical evidence clearly indicates that a yield curve is usually upward sloping; slopes are steeper when short-term rates are low, and flatter or even inverted when short-term rates are higher. Yield volatility slopes downward or flat over maturities. To check if our model could generate the stylized facts, we generate yields of 500 months for 2,000 times with the parameter values in table 1. We display the results in figure 1 and table 3.

[Insert Figure 1 and Table 3 about here]

Our parameterization procedure implies that we can compare the theoretical yield curves with the actual average yield curve. We can easily observe that the model generates an upward sloping yield curve and a downward sloping yield volatility. The theoretical average yield curve matches quite well with the data, if not perfect.

[12] We also check many different combinations of other parameters and impulse response results are available upon request.

Model-implied yield volatilities are smaller than those from the data. However, it depicts a downward to flat sloping yield curve successfully.

We explained theoretically directions of short-term interest rate in response to shocks in macroeconomic variables in the previous section. We expect similar movements for longer maturity yields. To verify this, we display factor loadings implied by the model with calibrated parameters in figure 2.

[Insert Figure 2 about here]

The figure suggests that the velocity factor affects all the bond yields by approximately the same amount. Of course, this is not very surprising because we set the velocity change to be a very persistent process. However, money demand literature shows that velocity of money is positively correlated to nominal interest rate and our model is consistent with this view.

Factor loadings for monetary policy shock are reminiscent of the slope factor. Monetary policy shock affects the shorter-end of a yield curve negatively and more heavily, while it affects the longer-end less negatively or even positively depending on parameter choices. We can explain this behavior as follows: market segmentation generates the liquidity effect which lowers interest rates in case of money injection. At the same time, the money injection will increase expected inflation and therefore related inflation risk premium. Since the liquidity effect tapers off as the maturities of bonds get longer, it is possible that the factor loadings of the short-end of a yield curve have bigger negative numbers compared to those of the longer-end. Thus, monetary contraction could flatten the yield curve or even invert it depending upon how persistently monetary contraction is pursued.

As for the aggregate output or consumption growth shocks, it also affects short-term yields more heavily than long-term yields consistent with the direction predicted by our theory.

Litterman and Scheinkman (1991) identify three factors (level, slope, and curvature) in their factor analysis on interest rates. To verify if our model has the ability to generate these factors, we do the following exercise. We simulate yields using the model and perform a static factor analysis. We then display the factor loadings in figure 3.

[Insert Figure 3 about here]

The figure shows a nice emergence of level, slope, and curvature factor. That is, our model can not only match the upward sloping average yield curve and downward to flat sloping yield volatility, but it can also explain the patterns of yield movements across different maturities, consistent with the empirical evidence.

Figure 4 displays theoretical impulse responses of yields following a contractionary monetary policy shock, a contractionary output growth and a decrease in velocity changes.

[Insert Figure 4 about here]

The dynamic responses from a contractionary monetary policy shock are especially in line with the findings of Evans and Marshall (1998): it increases the level of the yield curve, decreases the slope, and affects the curvature positively. Thus, according to the model, the yield curve flattens when there exists a contractionary monetary policy shock. Velocity changes mainly work through the level factor and the output growth factor appears to influence the curvature and the slope factor.

[Insert Figure 5 about here]

Figure 5 displays the impulse responses of (annualized) monthly holding period returns over one month interest rate following macroeconomic shocks in the same way as the figure 4. Both contractionary monetary policy and contractionary output growth shock increase excess holding period returns temporarily due to increases in risk premia. The responses get bigger as the maturity increases. To verify if time-varying risk premia implied by the model is consistent with the data, we simulate the model and compare the results with the historical data in the next section.

4.3 Bond risk premia and expectations hypothesis

Our model, as shown in (20) and (21), generates time variation in risk premia that could contribute to explaining longer-term bond prices. The model, sans this term, boils down to the one with expectations hypothesis (EH), according to which risk premia is a constant. Many empirical studies have tested the hypothesis and found that the EH is not well supported by postwar data. Campbell and Shiller (1991) ran following regressions

$$y_{t+m}^{(n-m)} - y_t^{(n)} = const + \beta_n \left(\frac{m}{n-m} \right) (y_t^{(n)} - y_t^{(m)}) + error \qquad (24)$$

If the EH holds, β_n should equal one. They found instead that it is negative at most of maturities and significantly different from one. Table 4 reports the regression results using the simulated data from the model. The regression coefficients in all cases but the quarterly holding period are significantly below zero and getting smaller. This matches quantitatively the pattern shown in the data. With standard errors corrected for heteroskedasticity and autocorrelation, most cases reject the EH comfortably. The price of risk process, (20) and (21), is a function of the parameters dictating stochastic risk aversion (ϕ_c, ϕ_{c0}, ϕ_π, $\phi_{\pi 0}$), asset market segmentation (λ), and inflation targeting (a_1). To check what drives the failure of the EH, we run simulations with each of the parameters set to be ineffective. Although we do not report here for brevity, we find that the model needs all of the channels to be active in order to match the expectations puzzle both qualitatively and quantitatively.

5 Conclusion

We have shown that our monetary term structure model is able to generate many of the stylized facts in bond prices, while maintaining the quantity theory of money. All of our results are produced by the trio of time-varying risk aversion, segmented asset market, and inflation targeting behavior in an interactive way. Simulated data from the fitted model can replicate three factors compatible with existing latent factor models and reject the expectations hypothesis in a very similar way that the empirical studies have reported. The level factor is mainly captured by the stochastic velocity of money, the slope by the monetary policy shock, and the curvature by both output growth shock and monetary policy shock. The model has an upward sloping yield curve with downward or flat sloping yield volatility on average. Thus, a contribution of this paper in terms of finance literature would be that our model provides a monetary general equilibrium justification of the essentially affine class of the term structure model. From a macroeconomic perspective, our paper delivers a tractable equilibrium model employing both short-run and long-run monetary facts, which accounts for historic bond price behaviors very well.

One caveat is that our model abstracts from labor market decisions and firms' choices. Adding them can shed light on many other issues including corporate bond pricing, equity returns via establishing more microeconomic foundations on the relationship between the asset market segmentation and the stochastic risk aversion. We leave these tasks to future works.

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Table 1
Parameter choices

Variables	Values	Parameters
γ	1.1	Utility curvature
β	0.999	Time discount
ϕ_{ϖ}	0.99	Velocity changes: AR(1) coefficient
ϕ_{μ}	0.5	Monetary policy shock: AR(1) coefficient
ϕ_{g}	0	Aggregate consumption growth: AR(1) coefficient
ϕ_{c}	120.1	Time varying price of risk for consumption
ϕ_{c0}	19.14	Constant price of risk for consumption
ϕ_{π}	118.78	Time varying price of risk for inflation
$\phi_{\pi 0}$	25	Constant price of risk for inflation
g	0.015*	Mean consumption growth
σ_{g}	0.012*	Standard deviation of consumption growth
σ_{μ}	0.0102*	Standard deviation of monetary policy shock
σ_{ϖ}	0.0097*	Standard deviation of velocity changes
$\bar{\mu}$	0.01*	Steady state money growth
$\bar{\pi}$	0.02*	Steady state inflation
$\bar{v}$	0.45	Steady state velocity of money $\equiv \frac{1}{1-\bar{v}}$
λ	0.1892	Market segmentation
a_1	0.40	Inflation targeting

Note: $*$ refers to annualized values, $12g$, $\sqrt{12}\sigma$, $12\bar{\mu}$, $12\bar{\pi}$, because the model is simulated in a monthly frequency

Table 2
Implied stochastic risk aversion

$$RRA_t = \gamma / S_t$$

mean	standard deviation
2.743	1.974

Note: We simulate the data series of the same length as that of the reference data set, then average over 2,000 runs using the model fitted to the parameters in Table 1. S_t is defined in (13) and (14).

Table 3
Means and standard deviations of data and model:
Zero coupon bond yields

Maturity	Mean		Stand. dev.	
(mos.)	Data	Model	Data	Model
3	5.12	4.29	2.45	1.34
6	6.5	4.69	2.48	0.75
12	6.75	5.32	2.45	0.71
24	6.90	6.35	2.43	0.71
36	7.10	7.12	2.36	0.71
48	7.37	7.66	2.33	0.70
60	7.43	7.98	2.31	0.69

Note: First column indicates yield to maturity in months.
Data is monthly and covers from May, 1964 to December, 2000.
Data source: CRSP
Model refers to the model parameterized using Table 1.

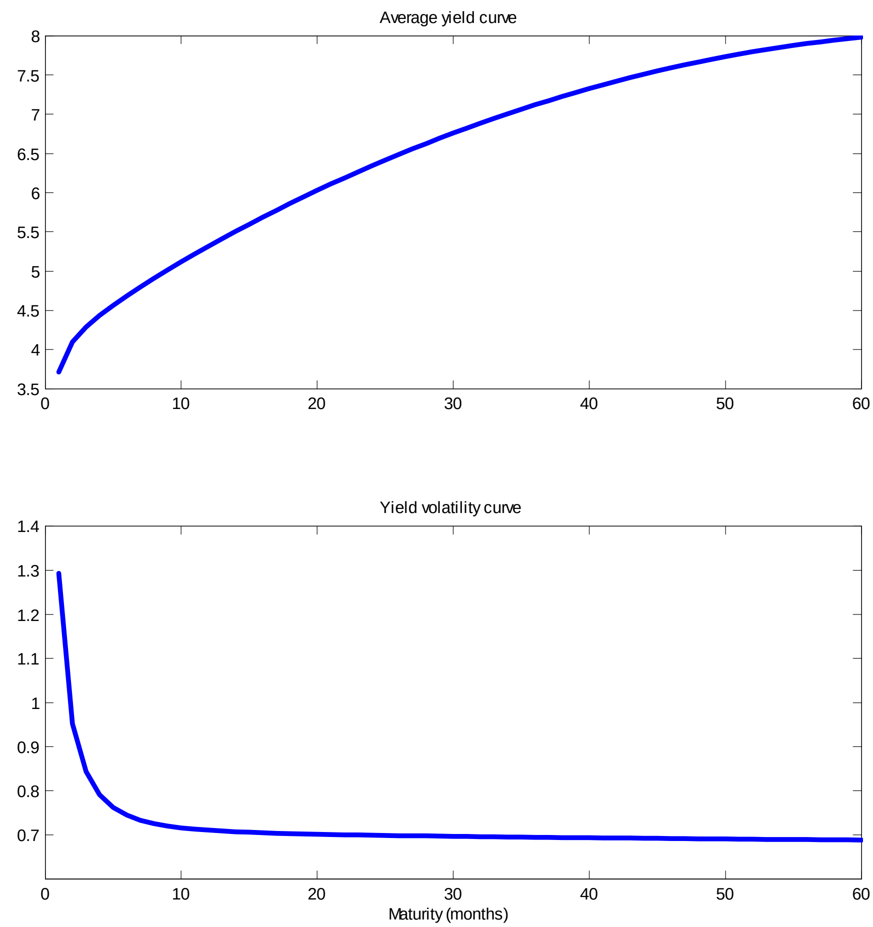
Table 4
Testing the expectations hypothesis:
Campbell and Shiller regressions in the model and in the data:

$$y_{t+m}^{(n-m)} - y_t^{(n)} = const. + \beta_n \left(\frac{m}{n-m}\right)(y_t^{(n)} - y_t^{(m)}) + error$$

	m					
	3		6		12	
n	Data	Simul.	Data	Simul.	Data	Simul.
6	0.270 (0.111)	0.710 (0.191)				
12	-0.278 (0.110)	0.276 (0.158)	-0.233 (0.302)	-0.110 (0.604)		
24	-0.310* (0.074)	-0.472 (0.140)	-0.775* (0.151)	-0.957 (0.397)	-0.915 (0.750)	-1.328 (1.334)
36	-0.332* (0.057)	-1.074 (0.131)	-1.187* (0.106)	-1.601 (0.337)	–1.339 (0.873)	-1.663 (0.981)
48	-0.541* (0.049)	-1.511 (0.123)	-1.492* (0.090)	-2.010 (0.309)	-1.749 (1.021)	-1.812 (0.844)
60	-0.681* (0.043)	-1.846 (0.121)	-1.579* (0.082)	-2.320 (0.297)	-1.899 (1.204)	-1.830 (0.793)

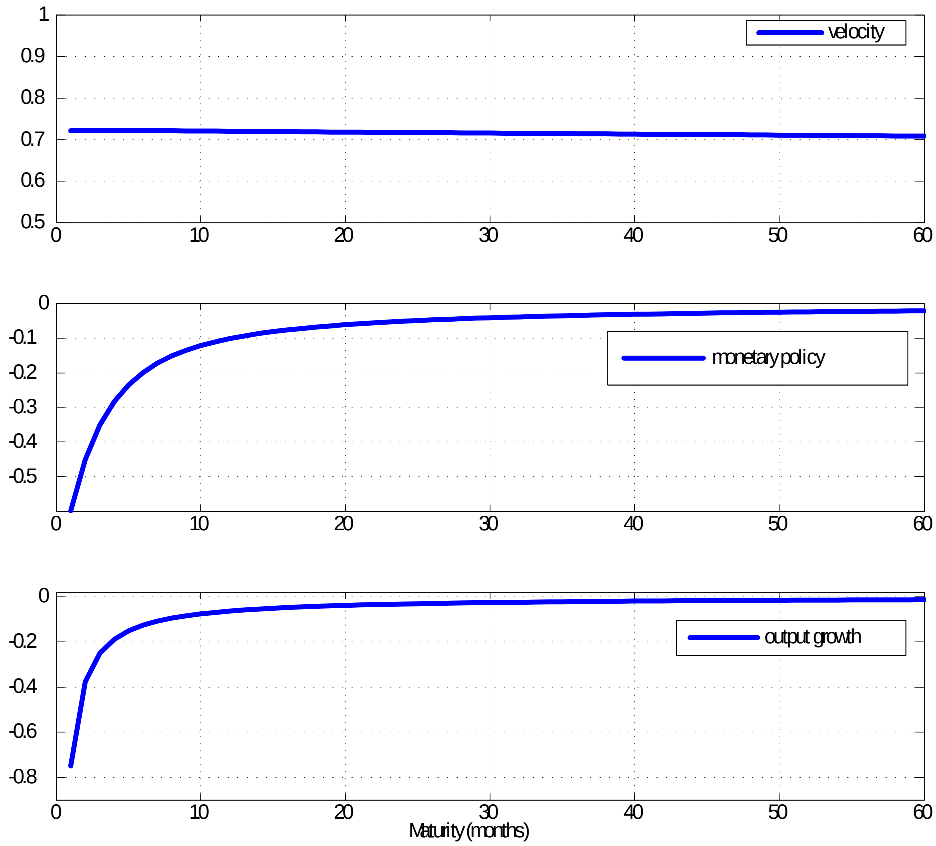
Note: According to the expectations hypothesis of interest rates, β_n should be one. Constant terms are included in all regressions. (Not shown to conserve space) Newey-West standard errors are in parentheses below estimated coefficients. Data source is CRSP and covers from May, 1964 to December, 2000. Simul. refers to estimated coefficients from simulated data of the same length averaged over 2,000 runs using the model fitted to the parameters in Table 1.
* Approximated $y_{t+m}^{(n-m)}$as $y_t^{(n)}$ due to lack of data.

Figure 1
Theoretical average yield curve



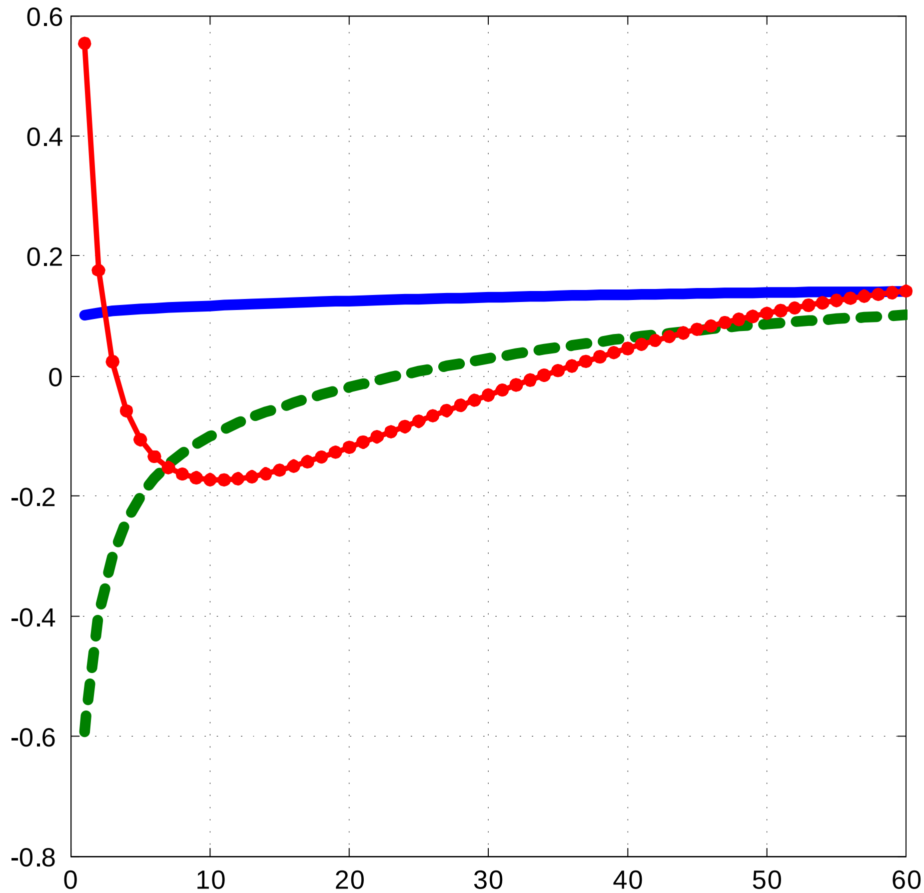
Note:
Top panel displays average yield curve measured as unconditional expectation of yield curve implied by the model. Bottom panel displays yield volatility curve measured as unconditional standard deviation of yields derived from the model. Parameters in table 1 are used. Horizontal axis is maturity in months and the vertical axis represents annualized yield in percentage.

Figure 2
Factor loadings on macro variables implied by the model



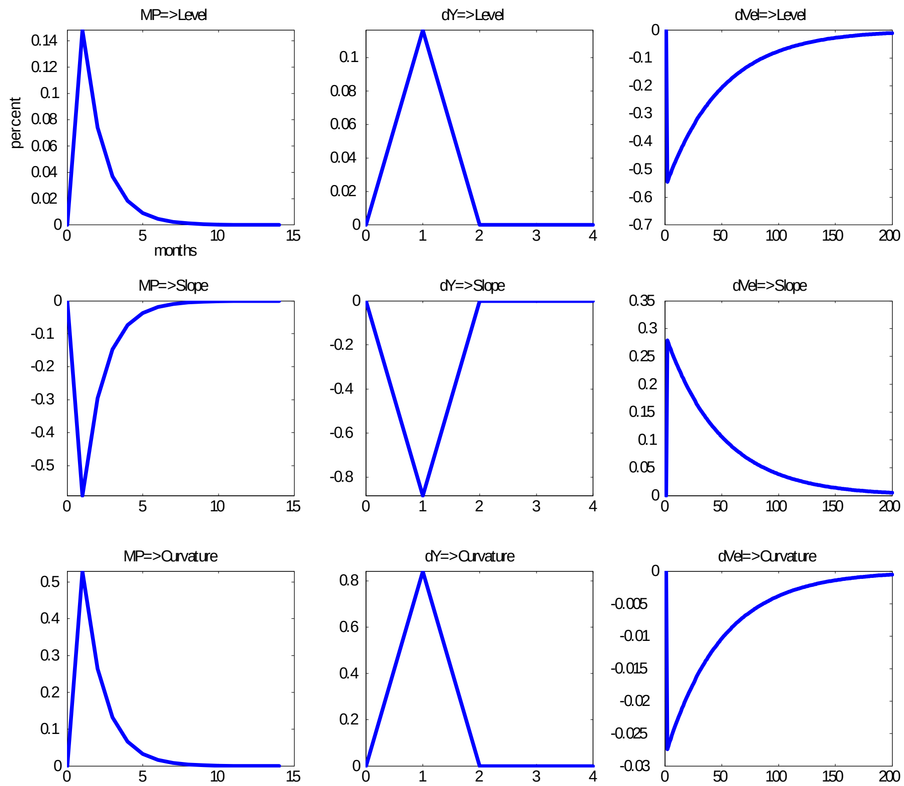
Note:
This figure displays the model-implied factor loadings for each macroeconomic shocks determining equilibrium dynamics of bond yields. 'Velocity' refers to the coefficients of bond yields for the velocity changes (ϖ); 'monetary policy' for the monetary policy shock (ε_μ); and 'output growth' for the aggregate output growth shock (g).

Figure 3
Factor analysis on simulated yields



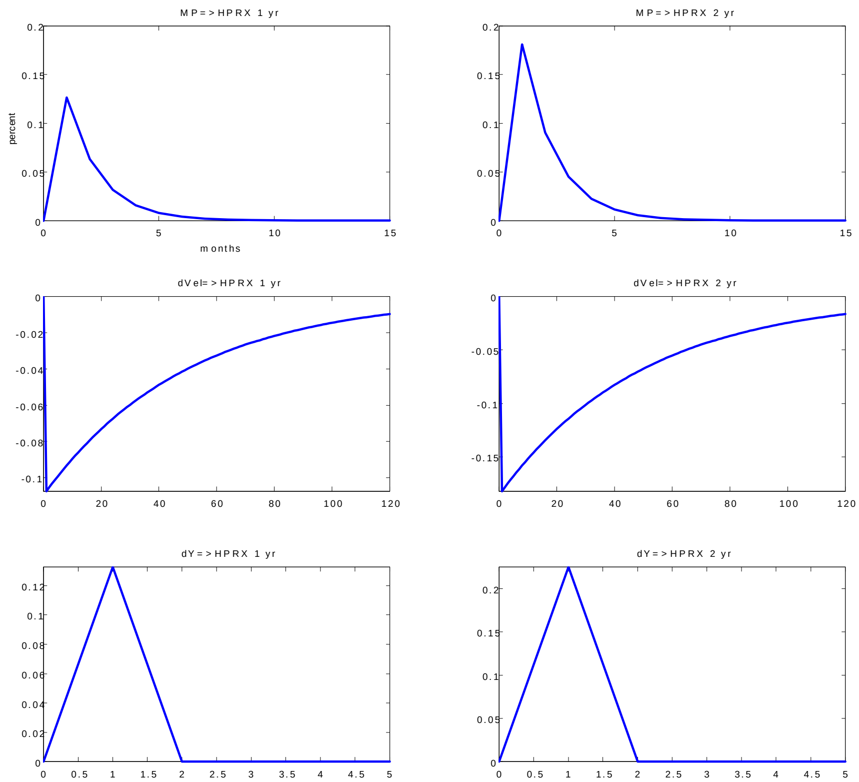
Note:
The figure displays standardized factor loadings of the first three factors of simulated yields averaged over 2,000 runs using the model with the parameters in table 1.
Horizontal axis is maturity in months. The first factor (straight line) represents the level; the second (dashed) the slope; and the third (dotted line) the curvature factor.

Figure 4
Theoretical impulse responses of yields



Note:
This figure represents impulse responses of macroeconomic shocks to three yield factors.
MP refers to one standard deviation decrease in monetary policy shock ($\varepsilon_{\mu t}$);
dY means one standard deviation decrease in output growth shock (ε_{gt});
and dVel is for one standard deviation decrease in shock of velocity changes ($\varepsilon_{\omega t}$).
Each row represents the responses of level, slope and curvature factor.

Figure 5
Theoretical impulse responses of expected excess holding period returns



Note: This figure displays impulse responses of macro shocks to monthly excess holding period returns.
First column consists of the responses of excess holding period returns for one-year bond and the second column is for two-year bond.
Each row represents alternative shocks: decreases of one standard deviation shock in monetary policy, output growth, and velocity changes, respectively.