Chain indices of the cost of living and the path-dependence problem: an empirical solution

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Abstract

This paper proposes a new method for estimating true cost-of-living (Konüs) indices, for large numbers of commodities, using data only on prices, aggregate budget shares and aggregate expenditure. Conventional chain indices are path-dependent unless income elasticities are (implausibly) all equal to one. The method allows this difficulty to be overcome. I show that to estimate a Konüs index, only income and not price elasticities are required. The method is applied to estimate a Konüs price index for 70 products covering nearly all the U.K.'s Retail Prices Index over 1974-2004, using the Quadratic Almost Ideal Demand System. The choice of base year for utility has a significant effect on the index.

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Key words: Index number, cost of living, Divisia, Konüs, Chain, Path-dependence, Almost Ideal Demand System

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1. Introduction[1]

The purpose of this paper is to propose an empirically feasible method for correcting what I shall call the path-dependence bias of chain indices of the cost of living. This bias (see below for a precise definition) arises because chain indices are discrete approximations to Divisia indices which, despite their other desirable properties, are known to be path-dependent except under implausible restrictions on consumer preferences. The proposed method employs the "Quadratic Almost Ideal Demand System" of Banks, Blundell, and Lewbel (1997), a flexible system that is fully consistent with economic theory.

Chain indices have become increasingly popular with national statistical agencies in recent years. In the U.K. the Retail Prices Index (RPI) is an annually chain-linked Laspeyres index (at least approximately: see Office for National Statistics, 1998, for details). The EU's Harmonised Index of Consumer Prices, which under the name Consumer Prices Index (CPI) is used as the Bank of England's inflation target, is also chained Laspeyres. In the United States, one of the CPI measures published since 2002 by the Bureau of Labor Statistics is a chained Törnqvist index; this followed criticism by the Boskin Commission (1996) of the fixed base approach. The 1993 System of National Accounts came down in favour of chain-linking for GDP (Commission of the European Communities – Eurostat *et al.*, 1993). Following that, Eurostat now requires that in all EU countries GDP and its components on both the output and expenditure sides should be annually chain-linked Laspeyres. The U.S. adopted chain-linking, using Fisher indices, in 1997 (Landefeld and Parker, 1997). Canada, Australia and Japan also use chain-linking in their national accounts.

Chain indices have the obvious intuitive justification that the weights are kept up-to-date. A more theoretical justification is that, if the aim is to measure the cost of living, then by comparison to fixed base indices they reduce substitution bias. Chain indices can also be justified as discrete approximations to continuous Divisia indices which have many desirable properties. For example, the product of a Divisia price index and a Divisia quantity index is

[1] I owe thanks to Chuck Hulten for a stimulating discussion on the path-dependence problem. I am also grateful to Simon Price for advice on econometric issues, to Andrew Leicester of the Institute for Fiscal Studies for supplying me with his data on retail prices and budget shares and for answering my queries, and to Bert Balk, John Van Reenen, Kevin Fox, Robert Hill, Hamish Low, and two anonymous referees for a number of helpful comments. This paper also benefited from the comments of seminar participants at St Andrews, the London School of Economics, the Bank of England, and the Centre for Applied Economic Analysis at the University of New South Wales.

the value index. And Divisia indices are consistent in aggregation: a one-stage Divisia index of food prices computed for (say) apples, oranges, lamb and beef is equal to a two-stage index computed first for fruit and meat and then for food as a whole from the sub-indices for fruit and meat.

But chain indices suffer from a drawback that was noted and discussed by Irving Fisher (Fisher (1927), who refers to much earlier work by Walsh and Westergaard): they fail the transitivity, or circularity, test (Diewert, 1987; Balk, 1995). Suppose that we are making comparisons over four periods and that prices and quantities return to their original values in the fourth period. Then we want the price index (and the quantity index) for the fourth period relative to the first to equal one. But simple algebra shows that none of the chain indices in common use (Laspeyres, Paasche, Törnqvist or Fisher) can satisfy the circularity test for a comparison over four or more periods. More generally, it can be shown that *no* chain index which also satisfies other properties which we require of an index number can satisfy the circularity test (see Balk (1995) for a formal proof). This failure of chain indices is well-known to practitioners as chain index drift; it is the main reason why chain-linking is not recommended for monthly price indices, where due to seasonal factors or promotions "price bounce" is quite common.

The proof that no chain index can satisfy all the desirable tests assumes that prices and quantities can vary freely and independently of each other. But if quantities are constrained to respond to prices as economic theory requires, then it is possible that the impossibility result could be avoided. However, it turns out that this is the case if and only if all income elasticities are equal to one (Hulten, 1973; Samuelson and Swamy, 1974). But this is a very unattractive assumption to make about consumer demand. After all, one of the oldest empirical findings in this area is Engel's Law: the share of the budget spent on food declines as income rises.[2]

Divisia indices suffer from a related problem to that of their discrete counterparts: path-dependence. Path-dependence means that the level of a Divisia index at some time period T, relative to its level in the base period 0, depends not just on the price relatives $p_i(T)/p_i(0)$ of the commodities in the index over this time span, but also on the path that prices have followed between the endpoints. Different paths, even if they begin and finish at the same points, produce different values of the Divisia index. So what credence can we give to

[2] Engel's Law is still apparently flourishing in the U.K. Blow et al. (2004) find that the proportion of the household budget spent on food declined from 25% to 15% between 1975 and 1999.

comparisons across countries of the average rate of inflation or the average rate of growth of real output if the results are influenced by the particular paths that the countries have followed within the period studied?

Theory gives us a standard by which to judge any real world consumer price index: the true cost-of-living or Konüs index (Konüs, 1939). The Konüs price index is defined as the ratio of the cost of buying some reference utility level at the prices of time *t* to the cost of buying the same utility level in the base period (period 0). It has been shown that a Divisia price index and the Konüs price index are equal if and only if the Konüs index is independent of the particular reference level of utility, which implies that the utility function is homothetic. Another way of putting this is that the two indices are equal if and only if the income elasticities of all goods are equal to one, a parallel result to the one for discrete index numbers; see Balk (2005) for a formal proof. So even if chain indices have alleviated one sort of bias, substitution bias, they have brought in another sort, which may be called path-dependence bias.[3]

The fact that the Konüs price index generally varies with the reference utility level can be given a simple intuitive justification. Consider a household with a very low standard of living spending 60% of its budget on food.[4] Suppose the price of food rises by 20%, with other prices constant. Then money income will probably have to rise by close to (0.60 x 20% =) 12%, to leave utility unchanged, due to the limited possibilities for substituting clothing and shelter for food. Compare this household to a modern day British one, spending 15% of its budget on food (Blow *et al*., 2004). Now the maximum rise in income required is only (0.15 x 20 =) 3% and probably a good bit less as substitution opportunities are greater (eg by reducing the order from jumbo fries to regular fries).

The solution to the path-dependence problem seems at first sight simple: estimate the expenditure function and then compute the Konüs price index. But this appears to be impossible in practice. Cost-of-living indices are usually computed from hundreds of

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[3] My term “path-dependence bias” is meant to be exactly analogous to the term “substitution bias”. Both refer to the difference between an index number and a specific Konüs index (with a particular reference level for utility). “Substitution bias” refers to the difference between a fixed base index like the Laspeyres and a Konüs index. Path dependence bias refers to the difference between a Divisia index (and so also its chain index approximations) and a Konüs index. So “bias” only has meaning with reference to this standard and other standards are conceivable.

[4] Quoting the work of Engel, Marshall (1920, chapter IV) reports that in Saxony in 1857 households headed by a “workman with an income of £45 to £60 a year” spent 62% of their income on food.

components. For example, the U.K.'s RPI contains around 650 "items". And usually statistical agencies have only aggregate data on budget shares, prices and expenditure. A demand system which is consistent with economic theory and is sufficiently flexible to be a good fit to the data, such as the Quadratic Almost Ideal Demand System (QAIDS) of Banks *et al*. (1997), contains $(N^2 + 5N - 6)/2$ independent parameters, where N is the number of commodities. If this were estimated for the RPI on annual data,[5] with all the cross-equation restrictions imposed, at a minimum over 328 years of data would be required for each product! Clearly this is out of the question. It is true that household data are also available, eg in the U.K. the Family Expenditure Survey, and these sorts of data are used in practice to estimate systems like the QAIDS. Even so, such systems are usually estimated for only half a dozen or so commodity groups. One reason for this is that the expenditure shares derived from these surveys are at a relatively high level of aggregation, and are really for commodity groups, not individual commodities.[6] The corresponding prices are therefore themselves index numbers. Also, these surveys yield less data than at first appears. The households in the sample keep expenditure diaries for only two weeks. So for many products a household's expenditure is recorded as zero (eg expenditure on summer holidays if the household's diary is kept for two weeks in winter). The upshot is that any attempt to correct the path-dependence bias in conventional consumer price indices using the theory of demand, at the level of detail at which these indices are constructed, seems impossible in practice.

The purpose of this paper is to show that this last conclusion is wrong. We can in fact estimate a true cost-of-living index using only the aggregate data commonly available to national statistical agencies — the same data as these agencies use to estimate conventional index numbers. Testing the economic theory of demand requires a huge amount of data. But this is not the point of the present exercise. Here we accept that some demand system like the QAIDS is a good approximation to consumer behaviour. Then subject to this assumption we can compute a correction to a conventional chain index of the cost-of-living. It turns out that this requires not very much data at all. The reason is that the correction involves estimating only the parameters relating to income elasticities and this can be done quite parsimoniously provided one does not attempt to recover all the other parameters (ie those relating to price elasticities).

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[5] The prices in the RPI are collected monthly but the budget shares are only available annually.

[6] The Family Expenditure divides household expenditure into 14 major categories and 77 sub-categories.

The plan of the paper is as follows. In section 2 I consider the path-dependence problem for Divisia indices in more detail and characterise the difference between the growth rates of a Konüs index and of a Divisia index. Section 3 shows how this difference between the two indices can be estimated in practice, using the QAIDS as a maintained hypothesis. Section 4 then applies this method to actual data, the U.K.'s Retail Prices Index (RPI), at the level of 70 commodities over the period 1974-2004. Finally, section 5 concludes.

2. The Path-Dependence Problem for Cost-of-Living Indices

The Konüs price index is defined as the ratio of the cost of buying the reference utility level at the prices of time *t* to the cost of buying the same utility level at the prices prevailing in the base period. Without loss of generality we can number time periods so that the base period is period 0. Then the Konüs price index, or true cost of living index, is defined as:

$$\frac{P^K(t,0)}{P^K(0,0)} = \frac{E[\mathbf{p}(t),u(0)]}{E[\mathbf{p}(0),u(0)]} \tag{1}$$

where $E[\cdot,\cdot]$ is the expenditure function and $\mathbf{p}(t) = (p_1, p_2,..., p_N)$ is the price vector at time *t*. (Usually, we would normalise so that $P^K(0,0) = 1$). At least for economists, the Konüs index is the theoretical ideal, to which real world price indices aspire. The connection between Konüs and Divisia price indices can be seen by differentiating equation (1) logarithmically with respect to time and applying Shephard's Lemma:[7]

$$\begin{aligned} \frac{d\ln P^K(t,0)}{dt} &= \frac{d\ln E[\mathbf{p}(t),u(0)]}{dt} = \sum_i \frac{\partial \ln E[\mathbf{p}(t),u(0)]}{\partial \ln p_i(t)} \frac{d\ln p_i(t)}{dt} \\ &= \sum_i s_i^0(t) \frac{d\ln p_i(t)}{dt} \end{aligned} \tag{2}$$

where

[7] Shephard's Lemma states that $\partial E / \partial p_i = q_i$, where q_i is the quantity demanded of the ith commodity at prices $\mathbf{p}$ and utility level *u*: $q_i = q_i(\mathbf{p}, u)$, the Hicksian demand function.

$$s_i^0(t) = \frac{\partial \ln E[\mathbf{p}(t), u(0)]}{\partial \ln p_i} = \frac{p_i(t) q_i^0(t)}{\sum_i p_i(t) q_i^0(t)}, \qquad i = 1, ..., N$$

and $q_i^0(t)$ is the quantity of the *i*th commodity that would be demanded at prices $\mathbf{p}(t)$ if utility were held constant at the base period level. The s_i^0 are the *hypothetical* shares in total expenditure, if utility were held constant at the base period level but prices were at their actual, observed levels. These shares could also be called *compensated* shares, by analogy with compensated (or Hicksian) elasticities.

By contrast, a Divisia price index[8] is defined by:

$$\frac{d \ln P^D(t)}{dt} = \sum_i s_i(t) \left[\frac{d \ln p_i(t)}{dt} \right] \qquad (3)$$

where $s_i(t) = \frac{p_i(t) q_i(t)}{\sum_i p_i(t) q_i(t)} = \frac{\partial \ln E[\mathbf{p}(t), u(t)]}{\partial \ln p_i(t)}, \qquad i = 1, ..., N$,

applying again Shephard's Lemma. The weights in the Divisia index are observed, actual shares, as opposed to the unobserved, compensated shares of the Konüs index (see Balk (2005) for this way of characterising the two indices).[9] So, intuitively, for the two price indices to be equal, the two sets of shares have to be equal: $s_i^0(t) = s_i(t)$. This means that the

[8] For a general discussion of Divisia indices, including the path-dependence problem, see Hulten (1973).

[9] Previous results on the relationship between chain, Divisia and Konüs indices include Diewert (1981), Feenstra and Reinsdorf (2000) and Balk (2004). Suppose a utility function exists which rationalises the data but may be non-homothetic. Diewert (1981) showed that there exists a utility level which is intermediate between the levels at the endpoints of the interval under study such that a Konüs price index over this interval, with utility fixed at the intermediate level, is bounded below by the Paasche and above by the Laspeyres. Balk (2004) showed that when the growth of prices is piecewise log linear a chained Fisher price index approximates a Konüs price index over an interval when the reference utility level is fixed at that of some intermediate point in the interval. A somewhat more precise result for the Almost Ideal demand system is due to Feenstra and Reinsdorf (2000). If prices are growing at constant rates, they show that the Divisia index between two time periods equals the Konüs price index when the reference utility level is a weighted average of utility levels along the path.

value shares have to be independent of the utility level, which implies that the utility function is homothetic, ie all income elasticities are equal to one.[10]

If income elasticities are not all equal to one, then in general compensated and actual shares differ. But there is more to it than that, for in this case the Divisia index is *path dependent*. This means that its level at some time period *T*, relative to its level in the base period 0, depends not just on the price relatives $p_i(T)/p_i(0)$, but also on the path that prices have followed between the endpoints. So different paths, even if they begin and finish at the same points, produce different values of the Divisia index. In comparing the cost of living in period *T* with the cost in period 0, only prices at 0 and *T* would seem to be relevant. Any prices strictly within the interval [0,*T*] would seem irrelevant. And the path between the endpoints should not influence the comparison between the situations at the two endpoints. Suppose that prices in periods 0 and *T* are *identical*, and also that quantities are identical. Then we should certainly want the Divisia price index $P^D(T)/P^D(0)$ (and also the Divisia quantity index) to equal one for this period. But this is not guaranteed unless all income elasticities equal one.

Despite its apparent similarity to the Divisia index, the Konüs index is *not* path-dependent. That is to say, we can recover the level of the price index by integration:

$$\int_0^T \frac{d\ln P^K(t,0)}{dt}dt = \int_0^T \sum_i \left[\frac{\partial \ln E[\mathbf{p}(t),u(0)]}{\partial \ln p_i}\right]\left(\frac{d\ln p_i}{dt}\right)dt$$

$$= \int_0^T \left[\frac{d\ln E[\mathbf{p}(t),u(0)]}{dt}\right]dt$$

$$= \ln E[\mathbf{p}(T),u(0)] - \ln E[\mathbf{p}(0),u(0)]$$

$$= \ln P^K(T,0) - \ln P^K(0,0)$$

The reason this works is that the compensated shares depend only on prices, since by definition utility is being held constant. The actual shares on the other hand depend not only

[10] Here I ignore the trivial case where all prices grow at the same rate, in which case any pattern of weights which sum to one will produce the same value for the price index. Balk (2005) provides a formal proof that the two indices are equal if and only if the utility function is homothetic (his Theorem 1).

on prices but also on the level of utility which varies over the path, unless utility is homothetic, in which case shares depend only on prices.

Path-dependence or -independence is a mathematical concept, a property of line integrals like equations (2) and (3) above. The main mathematical result is that a line integral like (3) is path-independent if and only if there exists a potential function $\phi(\mathbf{p})$ such that

$$s_i(t) = \frac{\partial \ln \phi(\mathbf{p}(t))}{\partial \ln p_i(t)}, \qquad i = 1,\ldots,N$$

ie shares depend only on prices.[11] But this is equivalent to requiring the utility function to be homothetic. If this condition holds, the expenditure function can be written as $E[\mathbf{p}(t), u(t)] = E[\mathbf{p}(t), 1]u(t)$ and then the potential function $\phi(\mathbf{p})$ is in fact the expenditure function, since now $s_i(t) = \partial \ln E(\mathbf{p}(t), 1) / \partial \ln p_i(t)$.

The size of the path-dependence bias in a Divisia index is a function of the gap between actual and compensated budget shares, $s_i(t) - s_i^0(t)$. But it is also and primarily an empirical question. After all, if all prices rise at the same rate, then the bias would be zero, since the Konüs and Divisia price indices would be equal, whatever the difference between compensated and actual budget shares. To estimate the bias, we need a theory of consumer demand that fits the facts empirically, the topic of the next section.[12]

3. The Quadratic Almost Ideal Demand System

A good starting point is the almost ideal (AI) demand system of Deaton and Muellbauer (1980a) and (1980b, chapter 3). This is fully consistent with economic theory and also possesses the property of exact aggregation. Most importantly in the present context, income elasticities can differ from one. Applying Shephard's Lemma to the AI expenditure function leads to a set of share equations:

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[11] See Hulten (1973), with references to the mathematical literature, eg Apostol (1957), chapter 10.

[12] An alternative approach, complementary to the present one, has been proposed by Hill (1999) and (2004). Roughly speaking, he suggests using a chain index (for intertemporal, cross-country or panel comparisons) but choosing the links in the chain so that on average the growth of prices between any two links is as close to proportional as possible.

$$s_i(t) = \alpha_i + \sum_j \gamma_{ij} \ln p_j + \beta_i \ln z(t), \qquad i = 1,\ldots,N \tag{4}$$

Here z is deflated expenditure: $z(t) = x(t)/P(t)$, where $x(t)$ is total expenditure, $x(t) = \sum_i p_i(t) q_i(t)$, and P is a price index defined by:

$$\ln P(t) = \alpha_0 + \sum_i \alpha_i \ln p_i(t) + \tfrac{1}{2} \sum_i \sum_j \gamma_{ij} \ln p_i(t) \ln p_j(t) \tag{5}$$

We normalise by setting $p_i(0) = 1,$ all i. Consistency with economic theory requires that the parameters of the system satisfy the following adding-up and symmetry restrictions:

$$\sum_i \alpha_i = 1; \sum_j \gamma_{ij} = 0, \forall i; \sum_i \beta_i = 0; \; \gamma_{ij} = \gamma_{ji}, \forall i \neq j$$

The "Quadratic Almost Ideal Demand System" of Banks, Blundell and Lewbel (1997), or QAIDS, is a generalisation of the AI system. It too is consistent with economic theory and exactly aggregable. Empirically, its strength is that it allows Engel curves to be quadratic which provides a much better fit to the data; it turns out that for many commodities Engel curves are not linear in the log of deflated expenditure as would be required by the AI system (Banks *et al*., 1997; Blow *et al*., 2004). The share equations corresponding to the QAIDS are similar to those of the AI system, except that there is an additional term in the square of deflated expenditure:

$$s_i(t) = \alpha_i + \sum_j \gamma_{ij} \ln p_j(t) + \beta_i \ln z(t) + \frac{\lambda_i}{\prod_i p_i^{\beta_i}(t)} \left[\ln z(t)\right]^2, \qquad i = 1,\ldots,N \tag{6}$$

In addition to the previous parameter restrictions, consistency with economic theory also requires that

$$\sum_i \lambda_i = 0.$$

The indirect utility function for the QAIDS is given by Banks *et al*. (1997) as:

$$\ln u(t) = \frac{\ln z(t)}{\prod_i p_i^{\beta_i}(t) + \ln z(t) \sum_i \lambda_i \ln p_i(t)} \tag{7}$$

(applying my notation to their equation (3)).[13] Let us pick a particular year as the base year for utility, say year *R*. Now, by appropriate choice of currency and quantity units, we can normalise so that $\ln z(R) = 0$. Then $\ln u(R) = 0$ also. Equivalently, we can choose utility units so that $\ln u(R) = 0$; then $\ln z(R) = 0$. So by setting the left hand side of equation (7) equal to 0, we can solve for the level of deflated expenditure required to keep utility at the base year level (zero), when the prices of year *t* prevail; this is denoted by $z^R(t)$:

$$0 = \frac{\ln z^R(t)}{\prod_i p_i^{\beta_i}(t) + \ln z^R(t) \sum_i \lambda_i \ln p_i(t)} \Rightarrow \ln z^R(t) = 0 \tag{8}$$

For the QAIDS, the *compensated* shares (the budget shares that would prevail if utility were held constant at its base year level) are obtained from equation (6):

$$\begin{aligned} s_i^R(t) &= \alpha_i^R + \sum_j \gamma_{ij} \ln p_j(t) + \beta_i \ln z^R(t) + \frac{\lambda_i}{\prod_i p_i^{\beta_i}(t)} \left[\ln z^R(t)\right]^2 \\ &= \alpha_i^R + \sum_j \gamma_{ij} \ln p_j(t), \qquad i = 1,\ldots,N \end{aligned} \tag{9}$$

using (8). I now write α_i^R rather than simply α_i since the values of the α_i depend on the normalisation adopted for $\ln z(R)$: see the next sub-section below. Plugging (9) into (6) and solving,

$$s_i^R(t) = s_i(t) - \beta_i \ln z(t) - \frac{\lambda_i}{\prod_i p_i^{\beta_i}} \left[\ln z(t)\right]^2, \qquad i = 1,\ldots,N \tag{10}$$

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[13] By setting all the λ_i equal to zero, we obtain the indirect utility function and the expenditure function of the AI system.

This is the relationship between compensated and actual shares that we are seeking and on which we shall rely for the empirical analysis. Also, by setting $t = R$ in (10), we obtain:

$$s_i^R(R) = s_i(R), \qquad i = 1, \ldots, N \tag{11}$$

since $\ln z(R) = \ln z^R(R) = 0$ by normalisation. That is, compensated and actual shares are equal in the base year. Actually, this result is independent of any normalisations as follows from consideration of the expenditure function:

$$s_i^R(R) = \partial \ln E[\mathbf{p}(R), u(R)] / \partial \ln p_i = s_i(R)$$

In the empirical work reported below it proves convenient to make the base year for utility the same as the reference year for the price index, ie we set $R = 0$.[14] So (10) and (11) then become

$$\begin{aligned} s_i^0(t) &= s_i(t) - \beta_i \ln z(t) - \frac{\lambda_i}{\prod_i p_i^{\beta_i}} [\ln z(t)]^2, \\ s_i^0(0) &= s_i(0), \qquad i = 1, \ldots, N \end{aligned} \tag{12}$$

since $\ln z(0) = 0$ by normalisation.

3.1 A new interpretation of the AI and QAIDS price index *P*

The AI price index P defined by equation (5) has never till now been given a clear interpretation. The fact that it depends only on prices and not on utility or deflated expenditure suggests that it might have some connection with the Konüs price index. The following argument shows that the connection is a very close one: the price index P is in fact the Konüs price index for the AI system and the QAIDS.

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[14] A further natural normalisation is to choose currency and quantity units so that $x(0) = 1$. Together with the other normalisations this then implies that $P(0) = 1$, which fixes α_0: $\alpha_0 = 0$. However, this is not required for any of the results in the present paper.

As before choose year R as the base year. Then from (8) we have that $\ln z^R(t) = 0$. Corresponding to deflated expenditure $z^R(t)$ there is also a level of nominal expenditure required to keep utility at its level in year R which we can write as $x^R(t)$. So from the definition of $z(t)$ we have:

$$0 = \ln z^R(t) = \ln[x^R(t) / P(t,R)], \text{ ie } x^R(t) / P(t,R) = 1 \tag{13}$$

I now write the AI price index P as $P(t,R)$ since as I am about to show it depends on the base year R. Now $x^R(t)$ is the cost of purchasing the period R level of utility at the prices of period t, so $x^R(t)/x^R(0)$ is an index of the cost of purchasing the period R level of utility, at the prices prevailing in year t, relative to the expenditure required at period 0 prices. In other words:

$$x^R(t)/x^R(0) = E[\mathbf{p}(t), u(R)] / E[\mathbf{p}(0), u(R)] = P^K(t,R)/P^K(0,R)$$

Since equation (13) holds for $t = 0$ too, we have by division that

$$x^R(t) / x^R(0) = P(t,R) / P(0,R)$$

So the last result shows that the price index P of the AI system and the QAIDS, given by equation (5), is not just any old price index but is identical to the Konüs price index with base period R:

$$P^K(t,R) / P^K(0,R) = P(t,R) / P(0,R) \tag{14}$$

Surprisingly, as far as I know this result has not been pointed out before. The result may appear puzzling at first sight since we know that there is a different Konüs price index for each choice of base year for utility (unless tastes are homothetic), while there seems to be only one AI system price index, given by equation (5). The paradox is resolved by noting that the α_i parameters which partly determine the growth rate of P depend on the choice of base year for utility. By setting $t = R$ in (9) and using (11):

$$\alpha_i^R = s_i(R) - \sum_j \gamma_{ij} \ln p_j(R), \quad i = 1, ..., N \tag{15}$$

(When $R = 0$, $\alpha_i^0 = s_i(0)$). However the values of the other parameters in the definition of the price index in equation (5), the γ_{ij}, are invariant to the choice of base year since these are semi-elasticities of budget shares with respect to prices, with utility held constant. Thus, as asserted, the AI system (and QAIDS) price index *P* is identical to the Konüs price index if demand is correctly described by this system. And the growth rate of this price index varies with the choice of base year, through the α_i parameters.

3.2 Estimating the compensated shares empirically

An empirical counterpart to the share equations can be written by transforming equations (6) to discrete time and adding an error term:

$$s_{it} = \alpha_i^0 + \sum_{j=2}^N \gamma_{ij} \ln(p_{jt} / p_{1t}) + \beta_i \ln z_t + \lambda_i \ln y_t + \varepsilon_{it}, \qquad i = 1, ..., N;\ t = 0, ..., T-1 \tag{16}$$

Here ε_{it} is the error term, we have put $\ln y_t = [\ln z_t]^2 / \prod_i p_{it}^{\beta_i}$, and we have used the fact that $\sum_{j=1}^N \gamma_{ij} = 0, \forall i$, to express the share equations in terms of relative prices (taking the first good as the numeraire).

In the QAIDS, there are $(N^2 - N)/2$ independent γ_{ij} parameters and the α_i, β_i, and λ_i number a further $3(N-1)$ independent parameters, for a total of $(N^2 + 5N - 6)/2$. So if the number of commodities is at all large, it is quite impractical to estimate such a system, because the number of γ_{ij} parameters explodes. But for the purpose of estimating a cost-of-living (Konüs) index, we don't need to! The trick is to find a parsimonious way of estimating $\sum_{j=2}^N \gamma_{ij} \ln(p_{jt} / p_{1t})$ as a linear combination of parameters and variables, without trying to recover the individual parameters. There are two ways to do this. The first way comes from noting that according to equation (10) we only need estimates of the parameters relating to income elasticities (the β_i and λ_i) in order to derive compensated shares from actual shares;

we do *not* need estimates of the parameters relating to price elasticities.[15] If we had access to household survey data for one or more periods, we could estimate these income elasticity parameters econometrically, assuming that all households face the same prices in a given period. Here however we follow a second approach which only requires aggregate data on budget shares and prices.

First we can note a special case in which the effects of relative prices can be exactly captured by just one variable. Suppose that all relative prices are growing at constant but possibly different rates:

$$\ln(p_{jt} / p_{1t}) = \mu_j t, \quad j = 2,...,N$$

where the μ_j are the growth rates of the relative prices and the first product is taken as the numeraire. Then $\sum_{j=2}^{N} \gamma_{ij} \ln(p_{jt} / p_{1t}) = t\left[\sum_{j=2}^{N} \gamma_{ij}\mu_j\right] = \delta_i t$, say. In this case the effect of relative prices is captured entirely by a time trend, with a different coefficient in each share equation (subject to the cross-equation restriction that $\sum_i \delta_i = 0$). A more general case can be treated by using principal components as a data reduction technique.[16] We can collapse the relative price data into M principal components, where $M < N-1$ is to be chosen empirically. Then the share equations (16) can be written as:

$$s_{it} = \alpha_i^0 + \sum_{k=1}^{M} \theta_{ik} PC_{kt} + \beta_i \ln z_t + \lambda_i \ln y_t + \varepsilon_{it}, \qquad i = 1,...,N;\ t = 0,...,T-1 \tag{17}$$

where PC_{kt} is the kth principal component of the $N-1$ relative prices and the θ_{ik} are coefficients subject to the cross-equation restrictions $\sum_i \theta_{ik} = 0, \ \forall k$.[17] The success of this strategy will depend on whether the variation in relative prices can be captured by a fairly small number of principal components — small that is in relation to the number of time series observations, *T*. We have now reduced the problem to estimating a system of equations, each

[15] I ignore here the fact that the AI system price index *P* used to estimate deflated expenditure is a function of these parameters. The iterative procedure explained below gets round this difficulty.

[16] For a textbook exposition of principal components, see Johnson and Wichern (2003, chapter 8).

[17] The special case just discussed is where the whole variation in relative prices can be captured by one principal component, a time trend.

of which contains only $M+3$ coefficients (the $\theta_{ik}, \alpha_i, \beta_i$, and λ_i). We must also take account of the cross-equation restrictions:

$$\sum_i \alpha_i^0 = 1;\ \sum_i \beta_i = 0;\ \sum_i \lambda_i = 0;\ \sum_i \theta_{ik} = 0,\ \forall k$$

These adding-up restrictions are automatically satisfied if the system is estimated equation-by-equation using OLS, though if the regressors are endogenous or the errors are non-normal this might not be the best method. There is one loss from using principal components: we can no longer impose the symmetry restrictions.[18] Finally, we must recognise that z_t and y_t are measured with error, since both involve initially unknown parameters. We can proceed here in the same way as do empirical workers who are seeking to estimate all the parameters of the QAIDS (eg Banks *et al*., 1997): that is, use iteration. Start with an initial estimate of z_t and y_t, and then estimate the system of (17). Use the regression estimates of the unknown coefficients to obtain updated estimates of z_t and y_t. Then re-estimate the system (17) using these updated estimates, and continue to iterate till convergence is reached.

In more detail, we can generate an initial estimate of the price index P using a chained Törnqvist index (a chained Laspeyres or chained Fisher index would serve as well):

$$\Delta \ln P_t^{(1)} = \tfrac{1}{2}\sum_i (s_{it} + s_{it-1})\Delta \ln p_{it},\ \ P_0^{(1)} = 1$$

where a superscript number in parentheses denotes the number of the estimation round; in this case (1) denotes the first round. Initially, we can set our estimates of the β_i to zero. Then

$$\ln z_t^{(1)} = \ln[x_t / P_t^{(1)}] \text{ and } \ln y_t^{(1)} = \{\ln[x_t / P_t^{(1)}]\}^2$$

In the next round, we can update the price index by

[18] For example, suppose that $N=3$ and that the special case of all relative prices changing at constant rates applies. Then, dropping the third equation, taking the first product as the numeraire, and imposing all the constraints, the relationship between the δ_i and the γ_{ij} is as follows: $\delta_1 = \gamma_{12}\mu_2 - (\gamma_{11} + \gamma_{12})\mu_3$, $\delta_2 = \gamma_{22}\mu_2 - (\gamma_{12} + \gamma_{22})\mu_3$. These relationships imply no further restrictions on δ_1 and δ_2.

$$\Delta \ln P_t^{(2)} = \tfrac{1}{2}\sum_i (s_{it}^{0(1)} + s_{it-1}^{0(1)})\Delta \ln p_{it}, \quad P_0^{(2)} = 1$$

Here the $s_{it}^{0(1)}$ are the first round predictions of the compensated shares from (17), derived as:

$$s_{it}^{0(1)} = \hat{\alpha}_i^{(1)} + \sum_{k=1}^{M} \hat{\theta}_{ik}^{(1)} PC_{kt}, \qquad i = 1,...,N;\ t = 0,...,T-1$$

where a hat (^) denotes a regression estimate. Then we can update z_t and y_t by

$$\ln z_t^{(2)} = \ln[x_t / P_t^{(2)}] \ \text{ and } \ \ln y_t^{(2)} = \{\ln[x_t / P_t^{(2)}]\}^2 \Big/ \prod_i p_{it}^{\hat{\beta}_i^{(1)}}$$

Assuming convergence, the final estimate of P will be (an approximation to) the Konüs price index, when period 0 is the base year for utility. It will be a chain index number, but it will be path-independent. This method could be applied by national statistical agencies, using exactly the same data as they employ to construct real world consumer price indices. Though I do not pursue this point in the present paper, the same method could also be applied to the construction of cross-country price indices (international comparisons of purchasing power).[19]

3.3 Changing the base year

Once we have estimated a Konüs price index with zero as the base, we can use the following relationships to estimate a Konüs price index with some other period, say R, as the base. Taking first differences in equation (9), we obtain:

$$\Delta s_i^R(t) = \sum_j \gamma_{ij} \Delta \ln p_j(t) = \Delta s_i^0(t), \qquad i = 1,...,N \tag{18}$$

ie the changes in compensated share are independent of the choice of base year. Also, according to (11), $s_i^R(R) = s_i(R),\ i = 1,...,N$, ie the compensated and actual shares are equal in the base year. Hence from (18)

[19] Neary (2004) uses the QAIDS to estimate Konüs price indices and real income for 60 countries in 1980, based on 11 commodity groups.

$$s_i^R(t) = s_i(R) + \sum\nolimits_{\tau=R+1}^{t} \Delta s_i^0(\tau), \qquad T-1 \geq t > R; \;\; i = 1,...,N$$
$$= s_i(R) - \sum\nolimits_{\tau=t+1}^{R} \Delta s_i^0(\tau), \qquad 0 \leq t < R; \;\; i = 1,...,N \tag{19}$$

Once we have estimated the compensated share changes with 0 as the base year (from the estimated s_{it}^0, see above), we can then recover the levels of the compensated shares when *R* is the base year from (19). These compensated shares can then be used to estimate a Konüs price index with period *R* rather than period 0 as the base. So we only need to estimate the parameters of interest once, for one base year. Then we can calculate a Konüs price index with any other year as the base; there is no need for any further econometric estimation.

4. The Method in Practice

The results reported here are based on a dataset of prices and budget shares for 70 products in the U.K.'s Retail Prices Index (RPI) over the period 1974-2004. The dataset was originally put together by the Institute for Fiscal Studies: see the Data Appendix in Oulton (2007) for more detail and for descriptive statistics.[20] These 70 products account for virtually 100% of the items in the RPI in the earlier years though the coverage gradually falls after 1992 to reach 91% in 2004. Total expenditure (*x*) is measured on a per capita basis. It is estimated as total final consumers' expenditure by U.K. households in the U.K. and abroad in current prices (ONS code: ABJQ), multiplied by the proportion of total expenditure on the "All items" RPI (ie the overall index of retail prices) that is covered by the prices included in the present study, divided by the population. The mean inflation rate as measured by a number of conventional price indices is shown in Table 1. All the chained measures are similar to each other; indeed, the chained Fisher and Törnqvist indices are identical to two decimal places. However the fixed weight indices, which use either the first year (1974) or the last (2004), differ more markedly. Interestingly, the Paasche index grew more rapidly than the Laspeyres, contrary to the normal expectation.

[20] The data on U.K. retail prices and budget shares were kindly supplied by Andrew Leicester of the Institute for Fiscal Studies (IFS). A very similar dataset underlies Blow *et al.* (2004).

The first step was to estimate the principal components of the 69 log relative prices.[21] The proportion of the variation explained by successive components is given in Table 2. The first six principal components account for 97.8% of the variation. With 13 components the cumulative proportion rises to 99.7%. On this basis, it was decided to employ six principal components (see below for the effect of including more or fewer principal components). However a potential problem for the empirical analysis is the high degree of multicollinearity between the principal components, deflated expenditure ($\ln(z)$) and deflated expenditure squared ($\ln(y)$). The multiple correlation coefficient between the six principal components and $\ln(z)$ is 0.995 and that between the six principal components and $\ln(y)$ is 0.994; the simple correlation coefficients between the first principal component and $\ln(z)$ and $\ln(y)$ are respectively 0.982 and 0.955.

Next, equation (17) was estimated for each of the 70 products over the period 1975-2004 by OLS.[22] Convergence of the estimates of equation (17) was rapid. There was very little change in the estimated growth rate of the Konüs price index (with 1974 as the base) after three iterations; nevertheless a further five iterations were carried out by which time the mean growth rate was stable up to the 7th decimal place. Once convergence was reached, Konüs price indices were then constructed with each of the years 1974 to 2004 in turn serving as the base, using equations (19) to generate the compensated shares for the reference years 1975-2004.[23]

The results of the eighth and final round of estimates appear in Table 3.[24] On the whole the model fits quite well, as measured by R^2 (last column). The Breusch-Godfrey LM test for serial correlation suggests that at the 5% level first-order serial correlation is present in only 15 of the 70 equations. Still, as a precaution, the *t* ratios are based on Newey-West standard errors which are robust to serial correlation. On the basis of a Wald test, the six principal components of log relative prices are jointly significant (non-zero) at the 5% level in 64 out of 70 equations.

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[21] Since the variables to be summarised are log relative prices and so have the same units, the principal components were based on the covariance matrix, not the correlation matrix, ie the variables were not standardised. Estimation was done by Stata's *pca* command, with the covariance option.

[22] Since the regressors are the same in each share equation, estimation by SUR would lead to identical results.

[23] These Konüs price indices were estimated using the Törnqvist formula.

[24] In a small number of cases, affecting nine products, the estimated compensated shares were negative in a few years. In these cases the estimates of the compensated shares were set to zero and the sum of the shares was constrained to equal one.

A Wald test shows that the coefficients on ln(z) and ln(y), the β_i and λ_i, are jointly non-zero at the 5% level for 37 products; that is, for each of these 37 products the income elasticity differs significantly from one, while for the other 33 products it does not (Table 3). (On the basis of t tests, ln(z) is individually significant at the 5% level for 25 products, ln(y) for 36 products). So despite the high multicollinearity we have already noted between the principal components, ln(z) and ln(y), income elasticities are still found to differ significantly from one in the majority of cases. The importance of the λ_i parameters is apparent in the Engel curves. These show budget shares as a function of log real income per capita with prices held constant. Only in 14 out of 70 cases are the Engel curves approximately linear even when attention is confined to the range of real income observed over the study period 1974-2004.

As explained in the previous section, these regression results can be used to generate 31 different Konüs price indices, one for each possible base year for utility over 1974-2004. The average growth rates of these 31 Konüs price indices over three intervals, 1974-1990, 1990-2004 and the whole period 1974-2004, appear in the left hand panel of Table 4. The mean of the average growth rates of these 31 indices is close to the conventional chained Laspeyres index of the 70 component prices. In fact, over the whole period the mean of the 31 average growth rates, 6.20 per cent per annum, is almost exactly the same as that of the chained Laspeyres, 6.21 per cent per annum (Table 1); at 6.15 per cent per annum, the values for the chained Fisher and chained Törnqvist indices are also close. This is in accordance with the theoretical predictions of Balk (2004) and Feenstra and Reinsdorf (2000), even though these were established under more restrictive conditions than apply here.

Nevertheless there is quite a lot of variation between indices with different base years. Thus over the whole 31 year period the minimum average growth rate is 5.91 per cent per annum (using 2004 as the base)[25] while the maximum is 6.39 per cent per annum (with 1978 as the base), a difference of 0.48 percentage points. Surprisingly, there is almost as much variation between indices with different base years in the more recent, low-inflation period 1990-2004 as there is in the high-inflation period 1974-1990. In the low-inflation period, the maximum average growth rate (2.97 per cent per annum) is found when 1993, 1995 or 1996 are used as the base year, the minimum (2.34 per cent per annum) when 2004 is the base; the

[25] Recall that "5.91 per cent per annum" is the answer to the following question: given the money income of the average household in 2004, what is the change in its money income between 1974 and 2004 (expressed as an annual percentage rate) which would have allowed the household to enjoy its 2004 utility level in 1974?

difference between minimum and maximum is 0.63 percentage points. By contrast the difference between minimum and maximum average growth rates in the high-inflation period is 0.69 percentage points (comparing 2002 with 1978 or 1979 as the base).

Table 4 in conjunction with Table 1 also reveals the size of the path-dependence bias of a conventional index number like the chained Laspeyres. Depending on the base year for utility and the time period, this can be as large as +0.45 or -0.43 per cent per annum (Table 5).

The relationship between the average growth rates of the Konüs indices and the base year can be seen more clearly in Figures 1-3. Over the first sub-period, 1979-1990, and over the whole period, 1974-2004, the relationship between the average rate of inflation and the base year is roughly linear and negative. Over the second sub-period, 1990-2004, there is still a negative relationship but only for base years later than 1996. Since on average real income rose steadily over 1974-2004, a negative relationship means that the less well off were more adversely affected by inflation than were the more prosperous. This is an interesting empirical finding though not one that would necessarily generalise to other periods and other countries.

4.1 IV estimates

As mentioned earlier, estimation by OLS might be questioned since the real expenditure variables (z and y) are measured with error (given that the QAIDS price index P is itself an estimate) and also may be endogenous: a rise in the price of some good may lead households to draw down their liquid reserves. The share equations were therefore also estimated by IV (2SLS), using as instruments one lag of $\ln(z)$ and of $\ln(y)$ and the chained Laspeyres measure of the overall inflation rate.[26] The rationale for using the latter as an instrument is that households are more likely to suffer unwelcome surprises and so are more likely to draw down their savings when inflation is high. In fact, the partial R^2 statistics of Shea (1997) and Bound *et al.* (1995) are both quite high when all three instruments are included. But when only the lags of $\ln(z)$ and $\ln(y)$ are included the same statistics suggest that these two instruments by themselves are weak. With all three instruments included, Hansen's J statistic suggests that we can reject the null of no correlation between the instruments and the errors at the 5% level in only 9 out of 70 cases.

[26] The use of GMM is likely not justified due to the small number of time series observations on each share (Baum et al., 2003).

The resulting IV estimates of the 31 Konüs price indices were remarkably similar to the OLS ones as the right hand panel of Table 4 shows.[27] The correlation coefficient between the OLS and IV estimates of the mean growth rates of the 31 indices over 1974-2004 is 0.991. However, the problem noted earlier — the tendency for some of the estimated compensated shares to be negative after 1990 — is a bit more serious now. After the negative shares are set to zero (but prior to the sum of the shares being constrained to sum to one), the sum of the IV estimates of the compensated shares rises steadily to reach 1.103 in 2004; the same sum is only 1.050 for the OLS estimates. So in this case nothing much seems to be gained by moving from OLS to IV. But at any rate it is reassuring that the OLS and IV estimates of the Konüs price indices are so similar.

4.2 Varying the number of principal components

The decision to include just six principal components in the estimation of equation (17) might be criticised as arbitrary. So I also estimated this equation and the resulting Konüs indices using alternately one through nine principal components (ie setting M equal to successively $1, 2, ..., 9$ in equation (17)). The effect on the mean inflation rate over 1974-2004 (using either 1974 or 2004 as the base year for utility) is illustrated in Table 6. Here a different kind of convergence is apparent. For example, with just one principal component included, the mean inflation rate with 2004 as the base year is 6.20 % p.a., while with six included it is 5.91%. In this case the inflation rate falls as the number of principal components is increased, but with little further effect once four are included. A similar comment applies when 1974 is the base: there is again little further effect on the estimated inflation rate once four principal components have been included (though now the inflation rate rises as the number of principal components is increased). Hence the use of six principal components as in Table 3 can be defended as a reasonable compromise between the desire to account for as much of the variation of relative prices as possible and the need to conserve degrees of freedom.

[27] The IV estimates were produced within Stata by the *ivreg2* command written by Baum et al. (2003).

5. Conclusions

I have argued that chain indices typically suffer from path-dependence bias by comparison with true cost-of-living indices. I have proposed a method of removing this bias, using a flexible model of consumer demand that is known to fit the data well (at least at a high level of aggregation), the QAIDS. Although *testing* the QAIDS requires a large amount of data, *using* it to remove the bias requires much less data.

I have applied this method to estimate Konüs price indices for 70 products covering most of the U.K.'s Retail Prices Index over 1974-2004, with each year in turn as the base. In this case it turns out that annual data sufficient to estimate nine coefficients for each commodity are all that is required. The choice of base year is found to have a significant effect on the index, even in the low inflation period since 1990. For example, with 1993 as the base year for utility, the average growth rate of the Konüs price index over 1990-2004 is 2.97 per cent per annum; with 2004 as the base year it is 2.34 per cent per annum, a difference of 0.63 percentage points. The path-dependence bias of a conventional index number like the chained Laspeyres index of the RPI can be as large as +0.45 or -0.43 per cent per annum (depending on the base year for utility and the time period).

Judging by these results, the method proposed here could be implemented in practice on the consumer price index at a detailed level by any statistical agency possessing 30 years or more of annual data on prices and budget shares. It could also be used to improve the measurement of cross-country differences in living standards, using the kind of data generated by the World Bank's International Comparison Program.

These results do however point to an issue that as far as I am aware is unresolved: what is the "best" base year for utility? Fixed base indices, where it is the weights (prices or quantities) that are fixed, are no longer popular with statistical agencies. But now the problem of a fixed base for the weights seems to reappear in the guise of a fixed base for utility. A closely related issue has been discussed extensively in the literature on international comparisons of purchasing power and real income (see eg Caves *et al.*, 1982; Diewert, 1987; Hill, 1999 and 2004; Neary, 2004). In the cross-country context, the base country plays the same role as does the base year in the time series context. It is widely held that the index number for real income should be invariant to the choice of base country. One could take the purchasing pattern of a single country, eg the United States, as the base but it is far from clear that this is appropriate if we want to compare the real incomes of Albania and Zambia. So the index numbers commonly employed for international comparisons (Geary, EKS, or the measure proposed by Caves *et al.* (1982)) represent some sort of average of the bilateral indices based on each country in turn as the base. However, it is not clear that the arguments for base-country invariance carry over to the time series context, the concern of the present paper. Here the choice of base would seem to depend on the purpose at hand. The choice for a central bank targeting inflation might be different from the choice of a statistical agency or an economic historian measuring real GDP.

Table 1

Conventional price indices: average annual growth rates, % p.a.

	1974-1990	*1990-2004*	*1974-2004*
Chained Törnqvist	9.38	2.46	6.15
Chained Fisher	9.38	2.46	6.15
Chained Laspeyres	9.42	2.54	6.21
Chained Paasche	9.34	2.38	6.10
Laspeyres (base 1974)	9.68	3.21	6.66
Paasche (base 2004)	10.11	3.40	6.98

Source Office for National Statistics, Institute for Fiscal Studies and own calculations. All indices are for 70 products covering most of the items in the RPI.

Table 2
Principal component analysis of 69 log relative prices:
U.K. Retail Prices Index, 1974-2004

	Eigenvalue	Difference	Proportion	Cumulative
Component 1	2.8454	2.6303	0.8482	0.8482
Component 2	0.2150	0.1421	0.0641	0.9122
Component 3	0.0729	0.0028	0.0217	0.9340
Component 4	0.0701	0.0212	0.0209	0.9549
Component 5	0.0489	0.0215	0.0146	0.9694
Component 6	0.0273	0.0082	0.0082	0.9776
Component 7	0.0192	0.0011	0.0057	0.9833
Component 8	0.0180	0.0085	0.0054	0.9887
Component 9	0.0096	0.0025	0.0029	0.9915
Component 10	0.0071	0.0011	0.0021	0.9937
Component 11	0.0060	0.0028	0.0018	0.9954
Component 12	0.0032	0.0010	0.0009	0.9964
Component 13	0.0022	0.0003	0.0006	0.9970

Note The principal components were estimated from the logs of the prices for the 70 products within the U.K. RPI listed in Table 3, with the first product ("Bread") taken as the numeraire; each relative price takes the value 1 in 1974 (0 in logs).

Table 3
Regression results for the QAIDS with 70 products in the U.K. RPI, 1974-2004: dependent variables are budget shares

Product		ln(z)		ln(y)		Probability that coefficients on both ln(z) and ln(y) are zero [(a)]	Probability that coefficients on all six principal components are zero [(b)]	Probability of no serial correlation [(c)]	Durbin-Watson	R^2
Number	*Name*	Coeff.	*t* ratio	Coeff.	*t* ratio					
1	Bread	0.0009	0.26	-0.0123	1.70	0.237	0.000	0.410	2.09	0.9817
2	Cereals	0.0136	3.43	-0.0170	3.18	0.005	0.004	0.812	2.07	0.6284
3	Biscuits	0.0029	0.64	-0.0235	2.92	0.020	0.000	0.377	2.25	0.9403
4	Beef	-0.0217	2.48	0.0240	1.94	0.067	0.000	0.043	2.65	0.9812
5	Lamb	-0.0083	1.91	-0.0096	1.40	0.008	0.001	0.032	2.62	0.9683
6	Pork	0.0064	1.56	-0.0169	3.27	0.009	0.000	0.001	2.98	0.9640
7	Bacon	0.0015	0.37	-0.0215	3.18	0.009	0.000	0.155	2.40	0.9663
8	Other poultry	0.0119	1.20	-0.0496	3.57	0.005	0.000	0.209	2.25	0.9637
9	Fish	-0.0052	1.28	-0.0137	2.00	0.010	0.007	0.046	2.62	0.9192
10	Butter	-0.0179	4.22	0.0240	4.77	0.000	0.000	0.045	2.67	0.9745
11	Oils & fats	0.0051	1.73	-0.0009	0.26	0.071	0.000	0.101	2.41	0.9581
12	Cheese	0.0107	2.88	-0.0141	2.51	0.027	0.027	0.294	1.61	0.7775
13	Eggs	0.0008	0.12	-0.0331	2.75	0.032	0.043	0.665	1.89	0.9354
14	Milk, fresh	-0.0164	2.58	0.0140	1.26	0.053	0.000	0.000	3.11	0.9798
15	Milk products	0.0001	0.02	-0.0017	0.42	0.806	0.168	0.775	1.90	0.7460
16	Tea	0.0046	1.10	-0.0098	1.68	0.253	0.000	0.192	2.36	0.8876
17	Coffee	0.0067	1.27	-0.0160	2.70	0.032	0.002	0.704	2.11	0.7605
18	Soft drinks	-0.0224	2.78	0.0371	3.96	0.001	0.000	0.851	1.98	0.9522
19	Sugar & preserves	0.0001	0.01	0.0059	0.78	0.669	0.001	0.017	2.62	0.9377
20	Sweets & chocolates	0.0180	2.50	-0.0310	1.98	0.044	0.014	0.158	2.34	0.6664
21	Potatoes	-0.0384	3.16	0.0678	3.21	0.010	0.005	0.023	2.77	0.7637
22	Other vegetables	-0.0286	2.03	0.0240	1.44	0.151	0.030	0.003	2.97	0.8286
23	Fruit	-0.0107	1.53	-0.0052	0.55	0.018	0.001	0.832	1.92	0.8919

Table 3, continued

24	Other food	-0.0229	1.93	0.0302	2.12	0.090	0.000	0.157	1.44	0.6223
25	Canteen meals	0.0196	1.31	-0.0170	1.12	0.427	0.000	0.809	2.03	0.9065
26	Other snacks	0.0438	3.25	-0.0022	0.12	0.003	0.007	0.460	2.22	0.9469
27	Beer	-0.0024	0.10	-0.0151	0.43	0.712	0.000	0.159	1.51	0.7090
28	Wine & spirits	-0.0190	0.98	0.0936	3.04	0.011	0.123	0.316	1.59	0.4908
29	Cigarettes	0.0033	0.28	0.0046	0.38	0.674	0.000	0.032	2.59	0.9278
30	Other tobacco	0.0020	0.31	0.0117	1.26	0.131	0.016	0.149	1.53	0.4759
31	Rent	0.0268	1.44	-0.0747	2.19	0.113	0.000	0.129	1.40	0.9541
32	Mortgage interest payments	-0.1069	1.57	0.0153	0.23	0.274	0.000	0.413	1.60	0.8940
33	Rates	0.0140	0.59	0.1029	3.58	0.000	0.000	0.015	2.78	0.7733
34	Water	0.0189	2.31	-0.0589	6.52	0.000	0.000	0.078	2.57	0.9663
35	Repairs	0.0064	0.76	-0.0138	1.28	0.449	0.005	0.297	1.57	0.7871
36	DIY materials	0.0116	0.71	0.0193	1.03	0.227	0.169	0.867	2.06	0.4739
37	Coal	0.0000	0.01	-0.0121	2.12	0.019	0.000	0.897	2.00	0.9916
38	Electricity	0.0187	2.66	-0.0236	3.54	0.006	0.000	0.017	2.84	0.9843
39	Gas	0.0523	3.81	-0.0600	2.65	0.004	0.000	0.296	1.51	0.8915
40	Oil & other fuels	-0.0100	2.55	0.0158	2.41	0.029	0.000	0.001	3.08	0.8771
41	Furniture	-0.0265	5.17	0.0066	0.85	0.000	0.000	0.694	1.85	0.9826
42	Furnishings	-0.0048	0.60	-0.0060	0.51	0.401	0.000	0.832	2.04	0.8657
43	Electrical appliances	0.0229	1.51	0.0152	0.90	0.003	0.000	0.042	2.70	0.8734
44	Other household appliances	0.0089	1.31	-0.0040	0.56	0.368	0.000	0.761	2.08	0.8514
45	Household consumables	0.0026	0.32	-0.0257	2.27	0.078	0.021	0.772	1.85	0.5879
46	Pet care	0.0072	1.25	-0.0136	1.86	0.160	0.000	0.021	2.81	0.8423
47	Postage	-0.0032	1.22	0.0065	1.59	0.299	0.000	0.136	2.31	0.8795
48	Telephone	-0.0068	0.77	0.0354	3.07	0.003	0.000	0.462	2.24	0.9693
49	Domestic services	0.0073	1.35	-0.0069	0.99	0.408	0.000	0.059	2.61	0.9456
50	Fees & subscriptions	-0.0018	0.07	0.0025	0.07	0.997	0.000	0.298	2.32	0.9299
51	Men's outerwear	-0.0078	0.70	0.0312	2.83	0.011	0.000	0.758	1.87	0.6751
52	Women's outerwear	0.0119	1.25	-0.0111	0.82	0.460	0.000	0.125	2.50	0.7465
53	Children's outerwear	0.0146	3.43	-0.0042	0.66	0.006	0.000	0.546	2.16	0.8607
54	Other clothing	0.0069	0.97	-0.0350	2.97	0.025	0.000	0.083	2.56	0.9702

Table 3, continued

55	Footwear	0.0192	2.90	-0.0198	2.48	0.020	0.000	0.982	1.98	0.9525
56	Chemists' goods	0.0198	2.89	-0.0438	4.10	0.002	0.000	0.490	2.23	0.9570
57	Personal services	-0.0041	0.54	0.0255	2.36	0.041	0.002	0.946	1.98	0.9254
58	Motor vehicles	-0.0477	1.09	0.0751	1.38	0.387	0.019	0.541	1.76	0.6765
59	Maintenance of motor vehicles	0.0034	0.33	-0.0117	0.69	0.763	0.003	0.650	2.10	0.9332
60	Petrol & oil	0.0392	1.66	-0.1086	3.25	0.013	0.000	0.008	2.90	0.8196
61	Vehicle tax & insurance	-0.0074	0.66	0.0169	1.06	0.577	0.388	0.333	1.57	0.9200
62	Rail fares	-0.0044	0.76	0.0276	3.20	0.004	0.001	0.476	2.21	0.6252
63	Bus & coach fares	-0.0160	2.67	0.0188	2.59	0.029	0.000	0.105	2.43	0.9734
64	Other travel	0.0100	0.95	0.0080	0.55	0.283	0.006	0.209	2.41	0.9370
65	Audio-visual equipment	0.0198	2.80	0.0288	2.37	0.000	0.001	0.817	1.92	0.6455
66	Records, tapes, CDs, etc	-0.0263	1.84	0.0458	2.29	0.070	0.042	0.936	1.87	0.8462
67	Books & newspapers	0.0116	2.12	-0.0070	0.73	0.038	0.002	0.708	2.10	0.8165
68	Garden products	-0.0027	0.40	-0.0002	0.02	0.682	0.008	0.052	1.27	0.8160
69	TV licence	0.0148	1.95	-0.0019	0.19	0.090	0.000	0.000	3.08	0.7508
70	Entertainment	-0.0304	1.91	0.0181	0.90	0.185	0.000	0.662	2.13	0.9267

Note Estimates of equation (17). Each regression includes a constant and six principal components of the 69 logs of relative prices; coefficients on these variables are not shown. 31 time series observations. Estimation method is OLS. Results reported are after 8 iterations (see text). *t*-ratios are based on Newey-West standard errors.

(a) Wald test of the null hypothesis that the coefficients on both $\ln(z)$ and $\ln(y)$ are insignificantly different from zero, $F(2, 22)$. A value less than 0.05 indicates that the null cannot be accepted at the 5% level.
(b) Wald test of the null hypothesis that the coefficients on the six principal components of relative prices are not significantly different from zero, $F(6, 22)$. A value less than 0.05 indicates that the null cannot be accepted at the 5% level.
(c) Probability of Breusch-Godfrey LM test with null of no first-order serial correlation, $\chi^2(1)$. A value less than 0.05 indicates that the null cannot be accepted at the 5% level.

Table 4
Comparison of mean growth rates of Konüs price indices with different base years: 1974-1990, 1990-2004 and 1974-2004 (per cent per annum)

	OLS estimates			*IV estimates*		
Base year	*1974-1990*	*1990-2004*	*1974-2004*	*1974-1990*	*1990-2004*	*1974-2004*
1974	9.57	2.61	6.32	9.58	2.61	6.33
1975	9.52	2.62	6.30	9.52	2.62	6.30
1976	9.53	2.55	6.27	9.52	2.55	6.27
1977	9.61	2.58	6.33	9.58	2.58	6.31
1978	9.66	2.66	6.39	9.63	2.66	6.38
1979	9.66	2.59	6.36	9.68	2.59	6.37
1980	9.59	2.56	6.31	9.64	2.56	6.34
1981	9.62	2.56	6.32	9.63	2.56	6.33
1982	9.59	2.59	6.32	9.55	2.59	6.30
1983	9.54	2.62	6.31	9.46	2.62	6.27
1984	9.54	2.56	6.28	9.47	2.56	6.25
1985	9.53	2.56	6.28	9.47	2.56	6.25
1986	9.51	2.54	6.26	9.46	2.54	6.23
1987	9.51	2.64	6.30	9.45	2.64	6.27
1988	9.36	2.61	6.21	9.32	2.61	6.19
1989	9.30	2.65	6.20	9.28	2.65	6.19
1990	9.21	2.63	6.14	9.19	2.62	6.12
1991	9.24	2.72	6.20	9.22	2.70	6.18
1992	9.12	2.83	6.18	9.10	2.81	6.16
1993	9.06	2.97	6.22	9.04	2.93	6.18
1994	9.03	2.94	6.19	9.01	2.89	6.15
1995	9.08	2.97	6.23	9.06	2.92	6.20
1996	9.11	2.97	6.24	9.09	2.91	6.20
1997	9.03	2.84	6.14	9.01	2.78	6.10
1998	9.06	2.67	6.08	9.04	2.61	6.04
1999	9.04	2.55	6.01	9.02	2.50	5.98
2000	9.02	2.49	5.97	9.00	2.44	5.94
2001	9.06	2.47	5.99	9.04	2.45	5.97
2002	8.97	2.48	5.94	8.95	2.52	5.95
2003	9.04	2.47	5.98	9.02	2.52	5.99
2004	9.02	2.34	5.91	9.00	2.36	5.90
Mean	9.31	2.64	6.20	9.29	2.63	6.18
Minimum	8.97	2.34	5.91	8.95	2.36	5.90
Maximum	9.66	2.97	6.39	9.68	2.93	6.38

Source Office for National Statistics and Institute for Fiscal Studies; own calculations.
Note The Konüs price indices are aggregates over 70 U.K. retail prices, using the Törnqvist formula; the weights are the estimated compensated shares, derived from estimates of equation (17). The IV estimates use three instruments: one lag of $\ln(z)$, one lag of $\ln(y)$, and the chained Laspeyres measure of the inflation rate. See text for further explanation.

Table 5
Path dependence bias of a chained Laspeyres price index (the RPI)

Greatest absolute bias	*1974-1990*	*1990-2004*	*1974-2004*
Positive bias	+0.45	+0.20	+0.30
Negative bias	-0.24	-0.43	-0.18

Source Tables 1 and 4. The bias is the difference between the growth rate of the chained Laspeyres index and either the maximum or the minimum growth rate of the Konüs indices over the same period.

Table 6
Average inflation rates of Konüs price indices over 1974-2004, % p.a.: effect of including different numbers of principal components

	Base year	
Number of principal components	*1974*	*2004*
1	6.12	6.20
2	6.06	6.25
3	6.09	6.18
4	6.30	5.96
5	6.30	5.94
6	6.32	5.91
7	6.31	5.90
8	6.33	5.88
9	6.30	5.90

Note Each Konüs price index is estimated from equation (17), but using a different number of principal components, from one through nine.

Figure 1

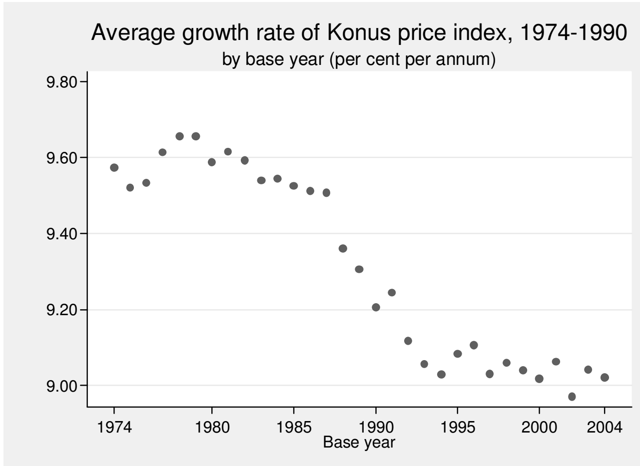


Figure 2

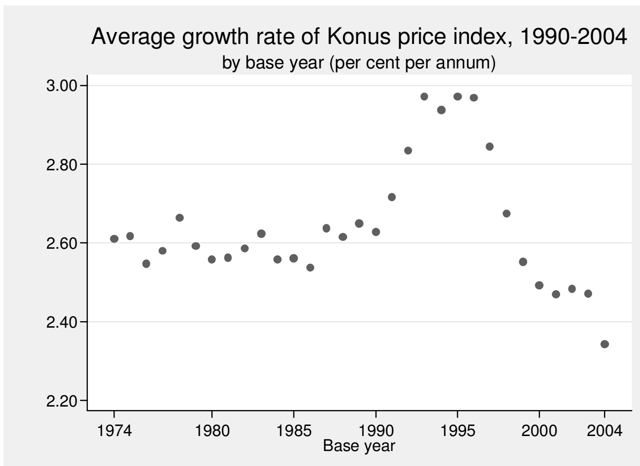
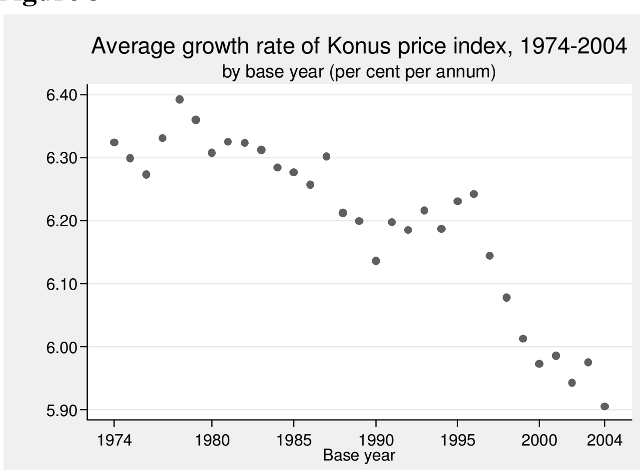


Figure 3



Note The Konüs price indices are aggregates over 70 U.K. retail prices, using the Törnqvist formula; the weights are the estimated compensated shares, derived from OLS estimates of equation (17): see Table 3.

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