FURTHER MODEL-BASED ESTIMATES OF U.S. TOTAL MANUFACTURING PRODUCTION CAPITAL AND TECHNOLOGY, FOR 1949-2005

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OUTLINE

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1. INTRODUCTION

- Conventional estimates of capital and technology (k, τ) in U.S. industries are based on limited theoretical and sample information.

  Examples:

  ♦ k stock estimates are based mostly on depreciation schedules

  ♦ τ stock follows exogenous processes or to be Solow residuals

- These estimates ignore *dynamic interactions* over business cycles among (k, τ) and other variables, useful information for estimating (k, τ)

INTRODUCTION (cont.)

- We propose and illustrate an econometric method for jointly estimating (k, τ), which has the following advantages:

  - ♦ (k, τ) are endogenous, being determined by firms' joint decisions on investment, i, research, r, and other inputs and outputs

  - ♦ Use correlations implied by the model between

    - unobserved (k, τ) and
    - observed prices and quantities of inputs and outputs

    to estimate unobserved (k, τ)

  - ♦ Use all theoretical restrictions and sample information to estimate (k, τ)

II. HOW WE PROCEED

1. STEPS OF THE ESTIMATION PROCESS:

- Specify a structural dynamic economic model of an industry, including the representative firm's dynamic optimization problem
- Solve the firm's dynamic optimization problem
- Set up reduced-form system and use it to obtain ML estimates of structural parameters:
  - ♦ Set up the reduced form in a state-space format
  - ♦ Use Kalman filter to "correctly ignore" missing observations on (k, τ) in estimation
- Compute KF estimates of (k, τ).

HOW WE PROCEED (cont.)

- Thus, we use the KF in two ways:
  - ♦ Estimate the model
  - ♦ Use the estimated model to estimate unobserved k and τ

2. **COMPARISON WITH CONVENTIONAL APPROACH:**

a. **Conventionally:**

- ♦ k is estimated based on observed i flows and assumed or estimated depreciation schedules
- ♦ τ is assumed to be an exogenous process or is determined residually as the Solow residual
- ♦ k and τ are estimated separately, not under unifying principles

HOW WE PROCEED (cont.)

a. <u>This study</u>:

- Uses system-wide identifying restrictions, involving all variables in the model, to jointly estimate (k, τ) stocks

- <u>Two identification conditions</u>:

  - ***Parameter identification*** condition of determining unique values of the model's structural parameters, which requires a concave likelihood function at MLE

  - ***Filtering identification*** condition of estimating unobserved variables, using the estimated model and the data, which requires a ***reconstructible*** state vector

III. STRUCTURAL ECONOMIC MODEL

1. GENERAL DESCRIPTION:

- View a competitive industry in terms of a representative firm

- A sophisticated output supply side: explicitly describe and solve the representative firm's dynamic optimization problem

  - Firm uses (k, ℓ, m) to produce output, investment and research (q, i, r), as 3 "outputs" in a production function with:

    CET = constant elasticity of output transformation

    CES = constant elasticity of input substitution

  - Concave-to-origin transformation curves for (q, i, r) imply convex adjustment costs on (k, τ) through (i, r).

- A simple output demand side: static demand curve

Resulting industry equilibrium: a dynamic, stochastic, simultaneous-equations system

STRUCTURAL ECONOMIC MODEL (cont.)

2. VARIABLES IN THE MODEL:

- **Endogenous Variables:**

p_q = Price of output

ℓ = Labor input

i = Investment-in-capital output

k = Capital input

q = Produced and sold output

m = Materials input

r = Research-in-technology output

τ = Technology input

- **Exogenous Variables:**

P_i = Price of investment

p_ℓ = Price of labor

p_r = Price of research

p_m = Price of materials

- **Disturbances:**

ξ = output-demand state

ε's = structural disturbances

Note: **k, τ, ξ, and ε's are unobserved; all variables in real terms**

STRUCTURAL ECONOMIC MODEL (cont.)

3. FIRM'S DYNAMIC OPTIMIZATION PROBLEM

a. CET/CES production function:

$$[\gamma_1 \cdot q^{\rho} + \gamma_2 \cdot i^{\rho} + \gamma_3 \cdot r^{\rho}]^{1/\rho} = \tau \cdot [\alpha_1 \cdot k^{\beta} + \alpha_2 \cdot \ell^{\beta} + \alpha_3 \cdot m^{\beta}]^{1/\beta},$$

where $\gamma_i \geq 0$, $\Sigma_i \gamma_i = 1$, $\rho > 1$, $\alpha_i \geq 0$, $\Sigma_i \alpha_i = 1$, $\beta < 1$.

- $\rho > 1$ => CET $= (\rho - 1)^{-1} > 0$ => concave-to-origin output transformation curves
- $\beta < 1$ => CES $= (\beta - 1)^{-1} < 0$ => convex-to-origin input isoquants

STRUCTURAL ECONOMIC MODEL (cont.)

b. Restricted variable cost function:

- **Dual description of the production function:**

$$C(z) \;=\; \min_{\{\ell,\, m\}} \; p_{\ell}\cdot\ell \;+\; p_m\cdot m,$$

subject to the CET/CES production function, for given

$$z \;=\; (q,\; i,\; r,\; k,\; \tau,\; p_{\ell},\; p_m)^T.$$

- **Use quadratic approximation** $C(z) \cong (1/2)z^T\cdot\nabla^2 C_0\cdot z$, **where**

$$\nabla^2 C_0 = \begin{bmatrix} c_{11} & \cdots & c_{17} \\ \vdots & & \vdots \\ c_{71} & \cdots & c_{77} \end{bmatrix} = \text{Hessian of } C(z) \text{ at } z = z_0.$$

STRUCTURAL ECONOMIC MODEL (cont.)

c. Output inverse demand curve:

$$p_{qt} = -\eta \cdot q_t + d_t + \xi_t,$$

where $\eta > 0$ is the slope, $d_t \sim AR(2)$ is demand state .

d. Representative firm's REAL PROFIT FUNCTION:

$$\pi_t = -(1/2)\eta \cdot (q_t)2 + q_t \cdot \xi_t - (1/2)(z_t)^T \cdot \nabla 2 C_0 \cdot z_t - p_{it} \cdot i_t - p_{rt} \cdot r_t.$$

e. Firm maximizes its EXPECTED PRESENT VALUE:

$$\max_{\{p_{qt},\, q_t,\, \ell_t,\, m_t,\, i_t,\, r_t\}} V_t = E_t \sum_{k=0}^{\infty} \delta^k \pi_t + k_t,$$

for predetermined (k_t, τ_t) and exogenous $(p_{it}, p_{rt}, p_{\ell t}, p_{mt}, \xi_t)$ processes.

STRUCTURAL ECONOMIC MODEL (cont.)

4. SOLVE FIRM'S DYNAMIC OPTIMIZATION PROBLEM:

- **Eliminate $(p_{qt}, q_t, \ell_t, m_t)$ so that the problem reduces to:**

$$\max_{\{K\}} V_t = E_t \sum_{k=0}^{\infty} \delta^k \pi_{t+k},$$

subject to the linear decision rule

$$u_t = K \cdot x_{t-1},$$

and to exogenous output-demand and input-price processes, where

$$u_t = (i_t, r_t)^T = \text{control vector},$$

$$x_t = (k_t, \tau_t, p_{it}, p_{rt}, p_{\ell t}, p_{mt}, \xi_t)^T = \text{state vector}.$$

- **Compute K by solving an algebraic matrix Riccati equation**
- **Figure 2 illustrates optimal responses to a rise in output demand**

IV. ESTIMATION STRATEGY

1. **Assemble 13 equations in 8 endogenous and 5 exogenous variables from model solution as *dynamic simultaneous equations system*.**

- **8 endogenous variable equations:**

  - **Output inverse demand:**

$$p_{qt} = -\eta \cdot q_t + \xi_t.$$

  - **Output , labor, and materials decision rules:**

$$q_t = -c_0[c_{12}\cdot i_t + c_{13}\cdot r_t + c_{14}\cdot k_t + c_{15}\cdot \tau_t + c_{16}\cdot p_{\ell t} + c_{17}\cdot p_{mt} - \xi_t] + \varepsilon_{qt},$$

$$\ell_t = c_{16}\cdot q_t + c_{26}\cdot i_t + c_{36}\cdot r_t + c_{46}\cdot k_t + c_{56}\cdot \tau_t + c_{66}\cdot p_{\ell t} + c_{67}\cdot p_{mt} + \varepsilon_{\ell t},$$

$$m_t = c_{17}\cdot q_t + c_{27}\cdot i_t + c_{37}\cdot r_t + c_{47}\cdot k_t + c_{57}\cdot \tau_t + c_{67}\cdot p_{\ell t} + c_{77}\cdot p_{mt} + \varepsilon_{mt}.$$

ESTIMATION STRATEGY (cont.)

- **Investment and research decision rules:**

$$i_t = K_{11}\cdot k_{t-1} + K_{12}\cdot \tau_{t-1} + K_{13}\cdot p_{i,t-1} + K_{14}\cdot p_{r,t-1} + K_{15}\cdot p_{\ell,t-1} + K_{16}\cdot p_{m,t-1}$$

$$+ K_{17}\cdot \xi_{t-1} + \varepsilon_{it},$$

$$r_t = K_{21}\cdot k_{t-1} + K_{22}\cdot \tau_{t-1} + K_{23}\cdot p_{i,t-1} + K_{24}\cdot p_{r,t-1} + K_{25}\cdot p_{\ell,t-1} + K_{26}\cdot p_{m,t-1}$$

$$+ K_{27}\cdot \xi_{t-1} + \varepsilon_{rt}.$$

- **Capital and technology accumulation:**

$$k_t = \phi_k\cdot k_{t-1} + i_t + \varepsilon_{kt},$$

$$\tau_t = \phi_r\cdot \tau_{t-1} + r_t + \varepsilon_{\tau t}.$$

ESTIMATION STRATEGY (cont.)

- **5 Exogenous AR(2) processes:**

  - **Output-demand state:**

    $$\xi_t = \phi_{q1}\cdot\xi_{t-1} + \phi_{q2}\cdot\xi_{t-2} + \varepsilon_{qt}.$$

  - **Input prices:**

    $$p_{it} = \phi_{i1}\cdot p_{i,t-1} + \phi_{i2}\cdot p_{i,t-2} + \varepsilon_{it},$$

    $$p_{rt} = \phi_{r1}\cdot p_{r,t-1} + \phi_{r2}\cdot p_{r,t-2} + \varepsilon_{rt},$$

    $$p_{\ell t} = \phi_{\ell 1}\cdot p_{\ell,t-1} + \phi_{\ell 2}\cdot p_{\ell,t-2} + \varepsilon_{\ell t},$$

    $$p_{mt} = \phi_{m1}\cdot p_{m,t-1} + \phi_{m2}\cdot p_{m,t-2} + \varepsilon_{mt}.$$

ESTIMATION STRATEGY (cont.)

2. Add a VAR(1) correction for each autocorrelated disturbance and restate the 13 equations as a VAR(2):

$$y_t = B_1 \cdot y_{t-1} + B_2 \cdot y_{t-2} + \zeta_t,$$

$$y = (p_q,\ q,\ \ell,\ m,\ i,\ r,\ k,\ \tau,\ p_i,\ p_r,\ p_\ell,\ p_m,\ \xi)^T.$$

= 13×1 vector of all variables in the model,

and

$$\zeta_t = (\zeta_{pqt}, \zeta_{qt}, \zeta_{\ell t}, \zeta_{mt}, \zeta_{it}, \zeta_{rt}, \zeta_{kt}, \zeta_{\tau t}\ \zeta_{pit}, \zeta_{prt}, \zeta_{p\ell t}, \zeta_{pmt}, \zeta_{dt})$$

= 13x1 vector of disturbances (6 AR(1) and 7 iid).

Use KF to "correctly ignore" missing observations on (k, τ) during MLE.

V. ESTIMATION RESULTS

- **Parameters to be estimated using MLE:**

$$\vartheta = (\vartheta_1, \vartheta_2)$$

Where

$$\vartheta_1 = (\delta, \alpha_1, \alpha_2, \gamma_1, \gamma_2, \sigma^2_{pq}, \sigma^2_{\ell}, \sigma^2_{m})$$

$$\vartheta_2 = (\phi_{pi,1}, \phi_{pr,1}, \phi_{p\ell,1}, \phi_{pm,1}, \phi_{pi,2}, \phi_{pr,2}, \phi_{p\ell,2}, \phi_{pm,2}, \sigma^2_{pi}, \sigma^2_{pr}, \sigma^2_{p\ell}, \sigma^2_{pm},$$

$$\theta_{pq}, \theta_q, \theta_{\ell}, \theta_m, \theta_i, \theta_r, \eta, \beta, \rho, \phi_{k1}, \phi_{\tau 1}, \phi_{q1}, \phi_{q2}, \sigma^2_q, \sigma^2_i, \sigma^2_r, \sigma^2_k, \sigma^2_{\tau}, \sigma^2_d)$$

Data

- U.S. total manufacturing data on prices and quantities of output and inputs
  - Investment data and GDP deflator from BEA
  - Research data from NSF
  - All other variables from BLS

- Sample: 1949 – 2005
  - SIC DATA: 1949 – 2001
  - NAICS DATA: 1987 – 2005
  - Data from 1987 – 2001 are derived using average growth rates of the two series.

Figure 1: U.S. manufacturing prices and quantities of output and inputs, (1947-2003)

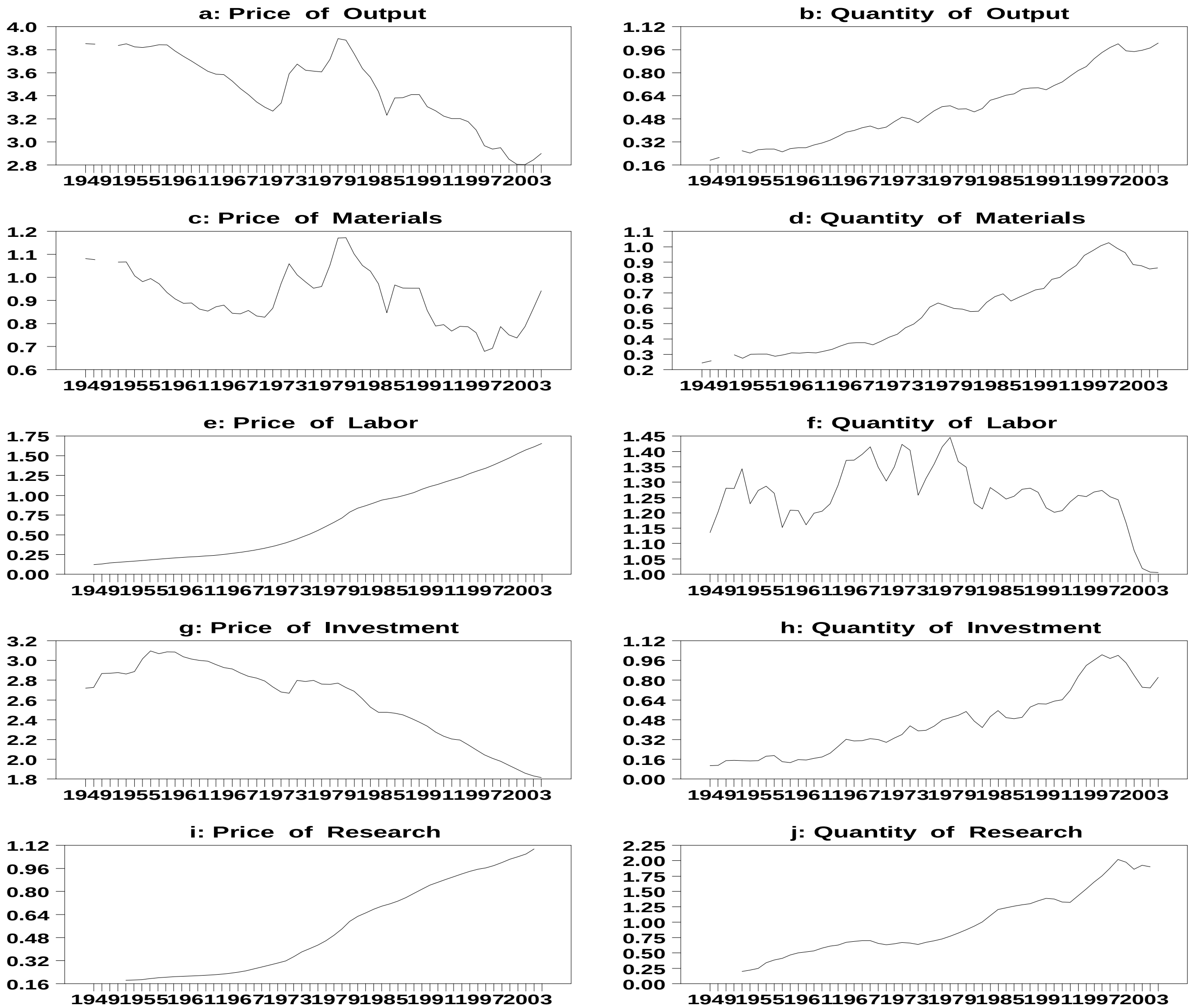


Table 1: OLS Estimates of Input-Price Process Parameters

Variables	Parameter Estimates			Fit Statistics	
	$\hat{\phi}_{\cdot,1}$	$\hat{\phi}_{\cdot,2}$	$\lvert\bar{\lambda}\rvert$	R^2	**Q**
p_i	1.37 (10.5)	-.356 (2.61)	1.02	.988	6.76 (.563)
p_r	1.90 (28.2)	-.900 (13.1)	.999	.999	17.6 (.124)
p_ℓ	1.88 (25.6)	-.874 (11.7)	1.03	.999	13.3 (.101)
p_m	1.16 (8.34)	-.319 (2.32)	.712	.812	4.40 (.820)

Table 2: ML Estimates of Remaining Structural Parameters

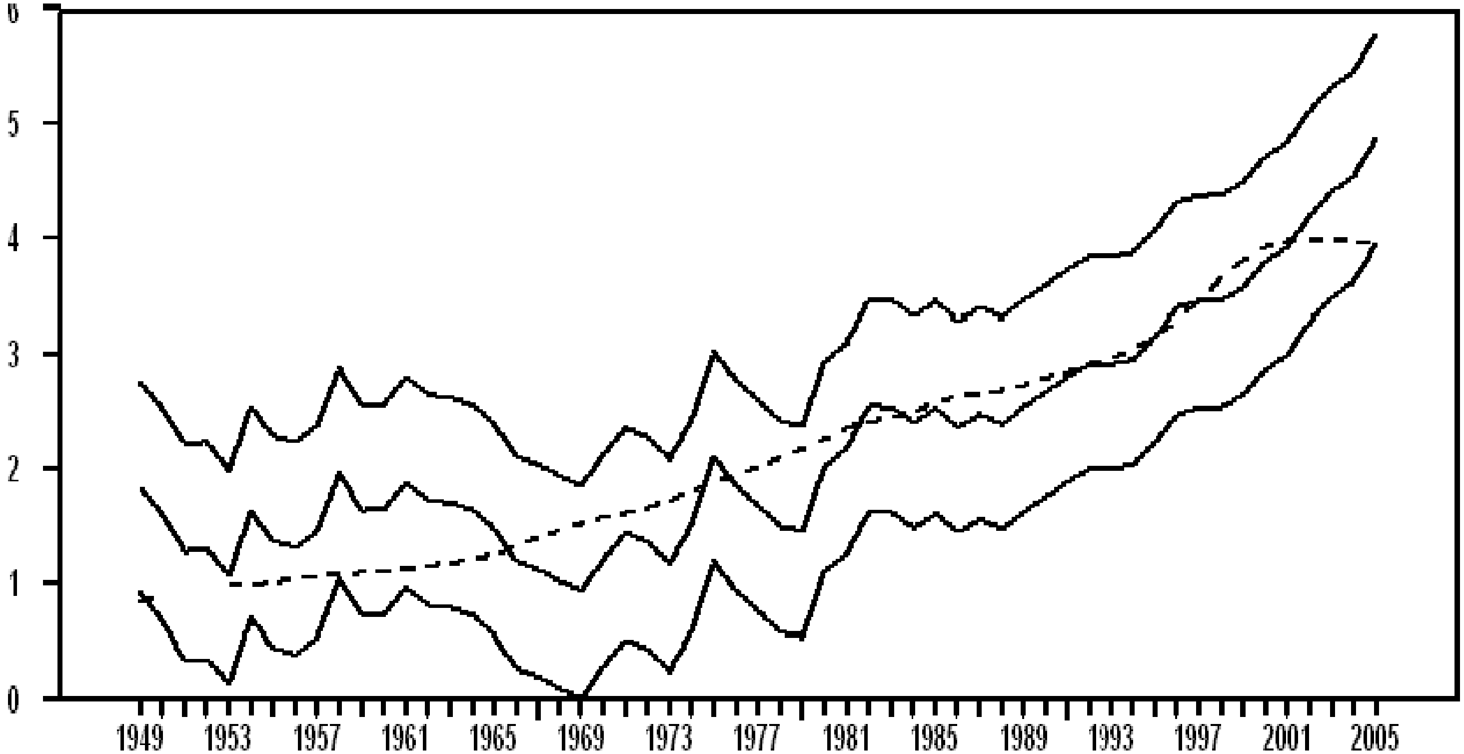
Production Function Parameters
$\hat{\beta}$ = -5.48 (CES = -.182), $\hat{\rho}$ = 275 (CET = .004)
Output-Demand Curve Parameters
$\hat{\eta}$ = .869, $\hat{\phi}_{d1}$ = 1.18, $\hat{\phi}_{d2}$ = -.367
Capital and Technology Equation Coefficients
$\hat{\phi}_{k1}$ = .610, $\hat{\phi}_{i0}$ = .789, $\hat{\phi}_{\tau 1}$ = .043, $\hat{\phi}_{r0}$ = .304
Residual Autocorrelation Coefficients
$\hat{\theta}_{pq}$ = .999, $\hat{\theta}_{q}$ = .675, $\hat{\theta}_{\ell}$ = .999, $\hat{\theta}_{m}$ = .999, $\hat{\theta}_{i}$ = .848, $\hat{\theta}_{r}$ = .982
Structural Disturbance Standard Deviations
$\hat{\sigma}_{q}$ = .144, $\hat{\sigma}_{i}$ = .246, $\hat{\sigma}_{r}$ = .106, $\hat{\sigma}_{k}$ = .995, $\hat{\sigma}_{\tau}$ = .001, $\hat{\sigma}_{d}$ = .207
Reduced-Form Equation Fit Statistics
R^2_{pq} = .932, R^2_{q} = .942, R^2_{ℓ} = .651, R^2_{i} = .938, R^2_{r} = .990
Q_{pq} = 2.52 (.989), Q_{q} = 2.55 (.990), Q_{ℓ} = 13.1 (.218), Q_{i} = 13.6 (.194), Q_{r} = 12.9 (.230)

Table 3: Structural Variance Decomposition of the Estimated Model

	σ^2_q	σ^2_i	σ^2_r	σ^2_k	σ^2_τ	σ^2_{pi}	σ^2_{pr}	$\sigma^2_{p\ell}$	σ^2_{pm}	σ^2_d
$s_{10,pq,j}$	4.5	2.8	.7	.2	.0	5.2	.1	.0	3.0	83.5
$s_{10,q,j\cdot}$	19.4	12.2	3.1	.8	.2	27.5	.7	.0	15.9	20.2
$s_{10,\ell,j}$	.9	3.9	.0	92.7	.2	.0	.0	.0	1.6	.1
$s_{10,m,j}$	.9	3.9	.0	92.7	.8	.0	.0	.0	1.6	.1
$s_{10,i,j}$	.0	44.5	.1	14.3	.1	17.5	.4	.0	11.5	11.6
$s_{10,r,j}$	.0	.0	5.4	1.1	.2	39.3	1.0	.1	25.8	27.1
$s_{10,k,j}$	.0	4.0	.0	95.3	.0	.3	.0	.0	.2	.2
$s_{10,\tau,j}$	.0	.0	1.9	1.1	1.6	39.9	1.1	.1	26.5	27.8
$\bar{S}_{10,j}$	1.3	5.2	.7	69.6	.4	7.4	.2	.0	5.4	9.8

Figure 2-a: Model-Based and Standard Capital Stock Estimates for U.S. Total Manufacturing, 1949-2005

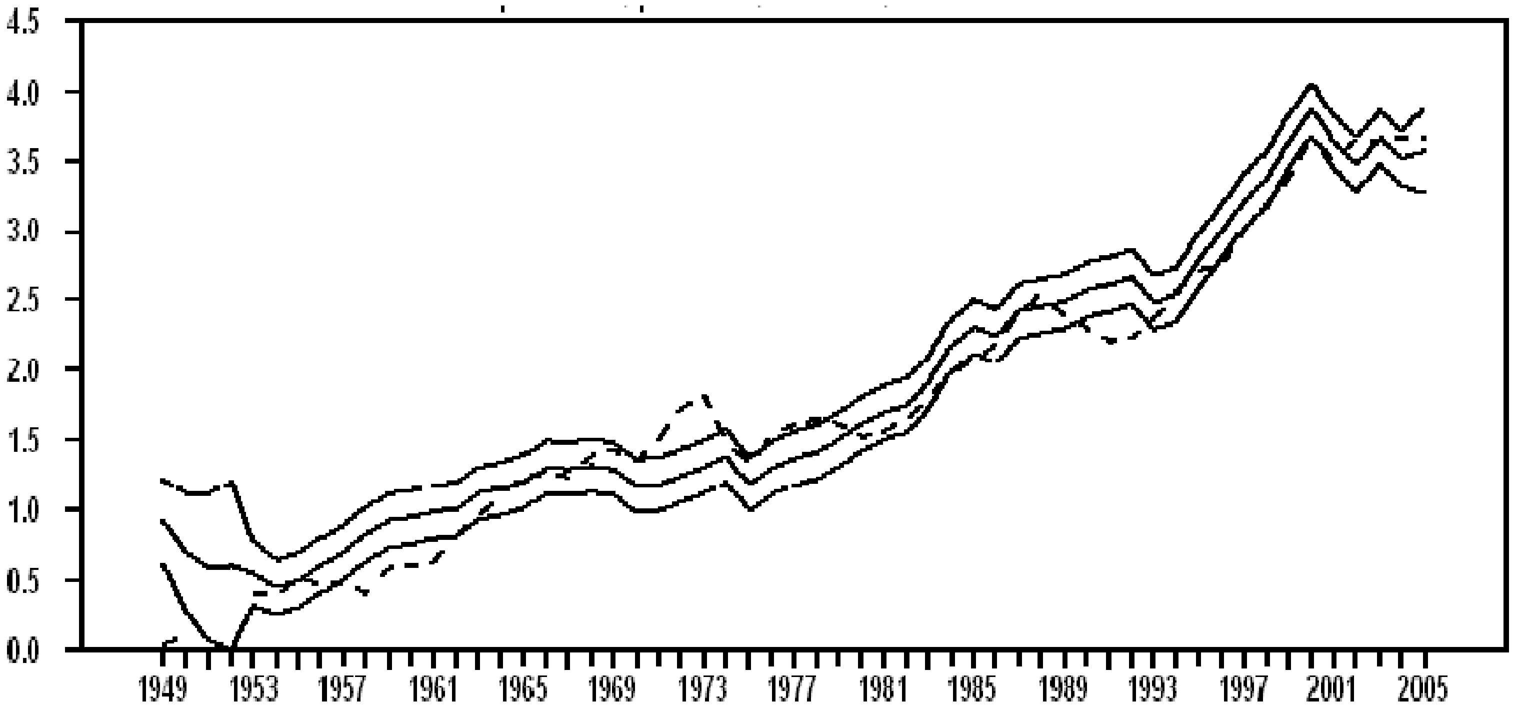
Model-Based and Standard Estimates of Capital Stock



Solid lines depict model-based capital estimates and 2-standard-error confidence bounds produced by the Kalman filter. Dashed lines depict standard capital stock estimates produced by BLS.

Figure 2-b: Model-Based and Standard Technology (TFP) Estimates for U.S. Total Manufacturing, 1949-2005

Model-Based and Standard Estimates of Technology



Solid lines depict model-based technology estimates and 2-standard-error confidence bounds produced by the Kalman filter. Dashed lines depict standard technology estimates as total factor productivity produced by BLS.

Conventional versus Filtered Estimates

1. 1949 ~ 2000

- Similar trends of model based and standard k and τ

2. 2000 ~ 2005

- Diverge paths of model-based and standard k τ
  - Model-based k continues to grow; standard k levels off
  - Model-based τ declines and levels off; standard τ continue to grow

3. 1949 ~ 2005

- Model-based k is noisy and uncertain; standard k is smooth
- Model-based τ is smooth and certain; standard τ is more noisy
- (i, k) explain growth more than (r, τ)

VI. CONCLUSION

- **Proposed filtered estimates of (k, τ) are feasible:**
    - ♦ **Results show two identification conditions of parameter identification in MLE and state reconstructibility in k and τ estimation hold numerically.**
- **Proposed method uses more information:**
    - ♦ **Use more sample information of correlations among more observed variables;**
    - ♦ **Uses theoretical information of correlations among all variables in the model, as implied by the model**
- **Good overall fit gives (k, τ) estimates credibility**
- **Extension: Includes time varying variables like interest rates and taxes**