SOME POLICY EXPERIMENTS USING A MARSHALLIAN MACROECONOMETRIC MODEL: Case of South Africa

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Jacques Kibambe Ngoie ***Arnold Zellner***

OUTLINE:

- Brief introduction;
- Key contributions of the paper;
- General descriptions;
- Model specification;
- Estimation techniques and results;
- Results and Policy shocks.

INTRODUCTION

‘(1) Use mathematics as shorthand language, rather than as an engine of inquiry.

(2) Keep to them till you have done.

(3) Translate into English.

(4) Then illustrate by examples that are important in real life

(5) Burn the mathematics.

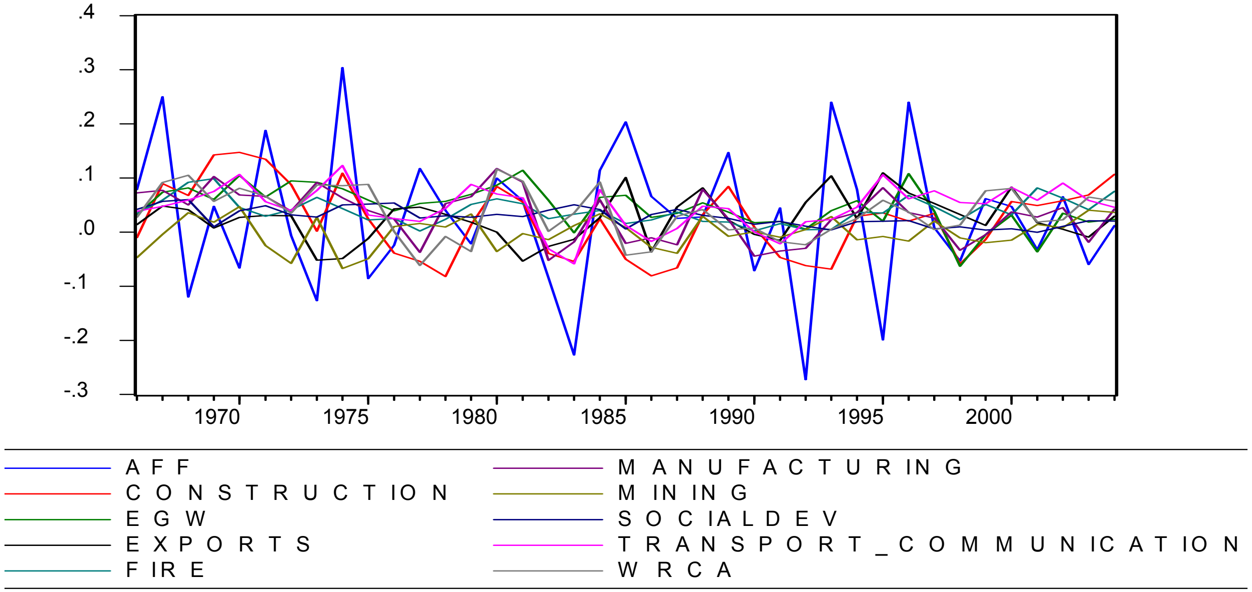
(6) If you can’t succeed in 4, burn 3. This I do often.’

Alfred Marshall to A.C. Pigou

KEY CONTRIBUTIONS TO THE LITERATURE

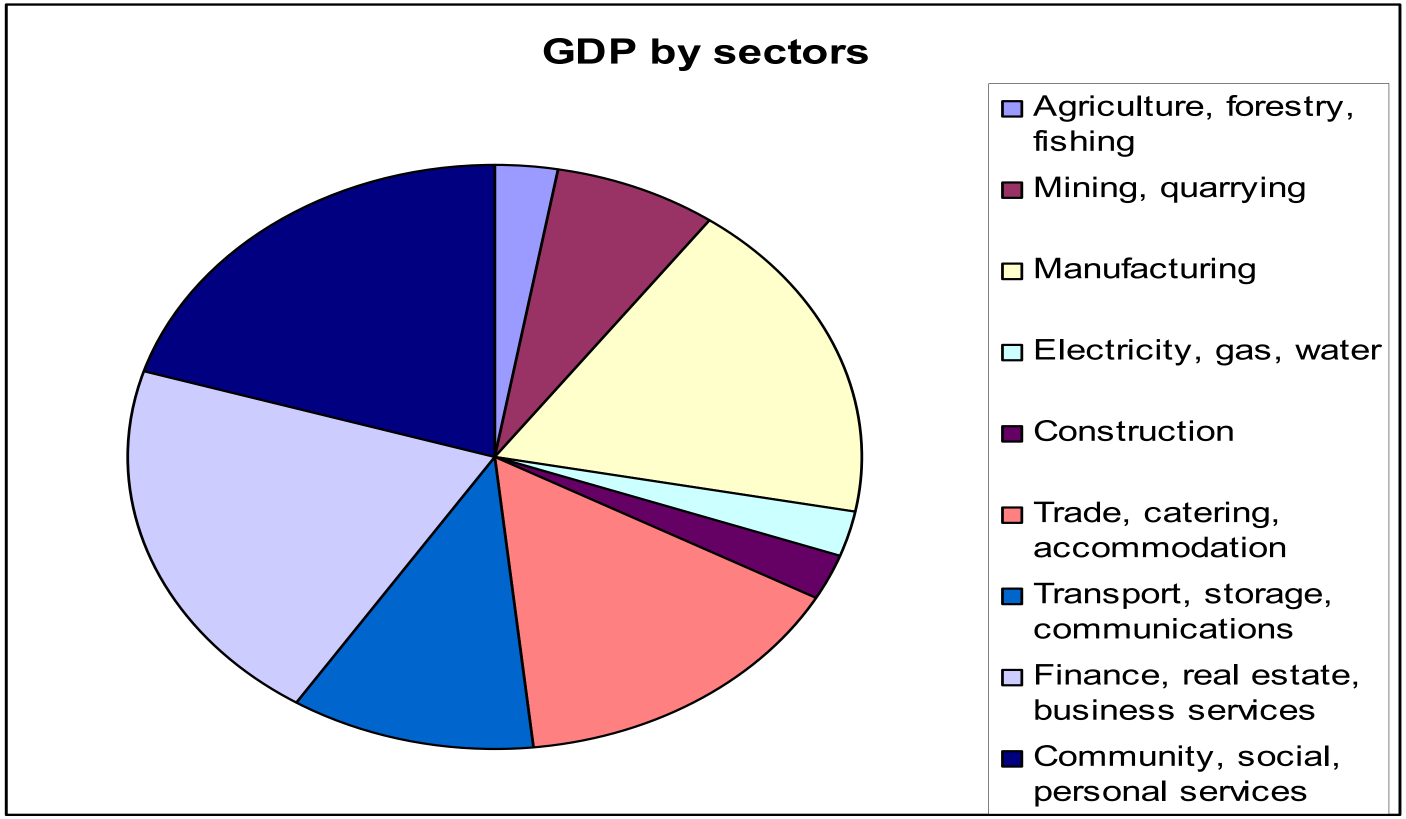
- The use of Human Capital in Marshallian Modeling;
- The use of an ‘entry cost (price)’ that helps regulating ‘Entry/Exit’;
- Expansion of the Sales Demand function;
- Introduction of the foreign sector.

Sectoral growth rates



South African annual real output growth rates per development sectors

Sectors' contribution to GDP



Thatcher-like revolution

Main points:

- Promoting free enterprises and perfect competition;
- Better quality of education;
- Reduction of trade unions influence;
- Rendering the labor market less rigid;
- Higher control of money supply;
- Tax-cut for high income groups;
- Introduction of a ‘poll’ tax.

Background of the MMM: Progress report

- Introduction of the SEMTSA to check Dynamic Econometric Models (Zellner and Palm, 1974/75, 2004);
- Building macro models: (1) to explain the past; (2) to make valid predictions; and (3) to advise policymakers. (Garcia-Ferrer et al, mid-1980s);
- Development of dynamic equations for individual variables testing them with past data for forecasting experiments;

Background of the MMM: Progress report (cont'd 1)

- Use of Autoregressive-Leading Indicator models (ARLI);
- Use of ARLI/WI (with world indicators such as: the median growth rate;
- Use of time-varying parameter state-space models versus fixed-parameter models;
- Use of Bayesian shrinkage and model-combining techniques to improve forecasting precision;

Background of the MMM: Progress report (cont'd 2)

- Rationalization of models using economic theory such as: (1) Aggregate demand – Aggregate Supply model (Zellner 2000); (2) Hicksian IS-LM model (Hong 1989); (3) Generalized Real-Business-Cycle model (Min 1992);
- Use of disaggregation to improve forecasting precision (Leontief, Stone, Orcutt, Zellner, Lutkepohl, de Alba,..)
- Origins of the Marshallian Model (Chen, Israilevich and Zellner, 2001/2005)

MODEL SPECIFICATION

I. Optimization

$$Q = A\,(z\,L\,)^{\alpha}\,K^{\beta} \tag{1}$$

$$\pi = TR - TC \tag{2}$$

$$TC = wL + rK + \Gamma \tag{3}$$

Assuming two set of prices:

- P_Q : Current prices;
- P_Q^e : Expected prices.

The optimization problem becomes:

Max: $$\pi = P_Q^e Q - wL - rK - \Gamma \quad (4)$$

Constraint: $$Q_{it} = A_{iN} (z_{it} L_{it})^{\alpha} K_{it}^{\beta} \quad (5)$$

First order conditions:

Before price adjustment mechanism:

$$K^{*} = \left[\frac{\beta A P_{Q}^{e}}{r} \left(\frac{\alpha r}{\beta w} \right)^{\alpha} \right]^{\frac{1}{1-\alpha-\beta}} \qquad (6)$$

$$z\ L^{*} = \frac{\alpha}{\beta} . \frac{r}{w} \left[\frac{\beta A . P_{Q}^{e}}{r} \left(\frac{\alpha . r}{\beta . w} \right)^{\alpha} \right]^{\frac{1}{1-\alpha-\beta}} \qquad (7)$$

Optimization (cont'd 3)

$$L^{*} = \frac{\alpha}{\beta}.\frac{r}{w}\left[\frac{\beta A.P_Q^e}{r}\left(\frac{\alpha .r}{\beta .w}\right)^{\alpha}\right]^{\frac{1}{1-\alpha-\beta}}.z^{-1} \tag{8}$$

After price adjustment mechanism:

$$K^{*} = \left[\frac{\beta AP_Q^e}{r}\left(\frac{\alpha r}{\beta w}\right)^{\alpha}\right]^{\frac{1}{1-\alpha-\beta}}.\left[\frac{P_Q}{P_Q^e}\right]^{\phi_K} \tag{9}$$

Optimization (cont'd 4)

$$L^* = \frac{\alpha}{\beta}.\frac{r}{w}\left[\frac{\beta.A\ .P_Q^e}{r}\left(\frac{\alpha r}{\beta w}\right)^{\alpha}\right]^{\frac{1}{1-\alpha-\beta}}\left[\frac{P_Q}{P_Q^e}\right]^{\phi_L}.z^{-1} \tag{10}$$

$$Q = A^{\frac{1}{1-\alpha-\beta}}.\alpha^{\frac{\alpha}{1-\alpha-\beta}}.\beta^{\frac{\beta}{1-\alpha-\beta}}.(P_Q^e)^{\frac{\alpha+\beta}{1-\alpha-\beta}}.w^{\frac{-\alpha}{1-\alpha-\beta}}.r^{\frac{-\beta}{1-\alpha-\beta}}.\left(\frac{P_Q}{P_Q^e}\right)^{\alpha\phi_L+\beta\phi_K}.z^{-\alpha} \tag{11}$$

II. The Sales Supply equation

$$S_S = A^{\frac{1}{1-\alpha-\beta}} . \alpha^{\frac{\alpha}{1-\alpha-\beta}} . \beta^{\frac{\beta}{1-\alpha-\beta}} . FN(\Gamma) . w^{\frac{-\alpha}{1-\alpha-\beta}} . r^{\frac{-\beta}{1-\alpha-\beta}} . P^{1+\alpha\phi_L+\beta} . \left(P_Q^e\right)^{-\alpha\phi_L-\beta\phi_K+\frac{\alpha+\beta}{1-\alpha-\beta}} . z^{-\alpha}$$

(12)

$$\frac{\partial N}{\partial \Gamma} \prec 0$$

Basic definition of sales:

$$S_S = (FN) . P_Q . q \qquad (13)$$

Sales Supply equation (cont'd 1)

$$F = \sum_{j=1}^{N} f_j$$

f_j : firm j^{th} share in the sector's total activities

$$F = \sum_{j=1}^{N} f_j = 1$$

Logging both side and differentiating wrt time:

$$\frac{\dot{S}_S}{S_S} = \theta_1 \frac{\dot{A}}{A} + \left(\frac{\dot{F}}{F} + \frac{\dot{N}(\Gamma)}{N(\Gamma)} \right) + \theta_2 \frac{\dot{P}}{P} + \theta_3 \frac{\dot{w}}{w} + \theta_4 \frac{\dot{r}}{r} + \sum_{l=1}^{T} \sigma_l \frac{\dot{P}_l}{P_l} + \theta_5 \frac{\dot{z}}{z} + \theta_6 \frac{\dot{\Gamma}}{\Gamma} \quad (14)$$

Sales Supply equation (cont'd 2)

$$P_{Q_{it}}^{e} = \prod_{l=1}^{T} P_{l}^{\sigma_k} \qquad (15)$$

with: - $\theta_1 = \dfrac{1}{1-\alpha-\beta}$

- $\theta_2 = 1 + \alpha\phi_L + \beta$

- $\theta_3 = \dfrac{-\alpha}{1-\alpha-\beta}$

- $\theta_4 = \dfrac{-\beta}{1-\alpha-\beta}$

- $\theta_5 = -\alpha$

- $\theta_6 = -\nu$

II. Sales demand equations

$$S_D = (\Re D)P_Q.q \tag{16}$$

where $\Re = \sum_{k=1}^{D} \upsilon_k$

with: - D being the total number of demanders of the sector’s products;

- and υ_k represents the k^{th} demander’s size (share) of the sector’s products demand.

Sales Demand equation (cont'd 1)

The demanders include:

(1) firms;

(2) private households;

(3) government;

(4) foreign entities.

$$\Re = \sum_{k=1}^{D} \upsilon_k = 1$$

Expanded sales demand function

$$S_D = P.\left[C_S (P_Q^e)^{\lambda_1} .(Y_d)^{\lambda_2} .(\Re D)^{\lambda_3} \prod_{j=1}^{m} X_j^{\chi_j} .\left(\frac{P_Q^e}{P_Q} \right)^{\Delta} \right] \quad (17)$$

$$\frac{\dot{S}_D}{S_D} = (1-\Delta)\frac{\dot{P}_Q}{P_Q} + (\lambda_1 + \Delta)\frac{\dot{P}_Q^e}{P_Q} + \lambda_2 \frac{(\dot{Y}_d)}{(Y_d)} + \lambda_3 \frac{\Re\dot{D}}{\Re D} + \chi_{j1} \frac{\dot{WY}}{WY} \quad (18)$$

where:

- Y_d : Gross National Disposable Income
- S_D :Sales Demand;
- X :Other variables affecting sales demand;
- WY : World Income.

III. Factor market

a) Labor Supply Equation

$$zL = C_L \left(\frac{w}{P_Q} \right)^{\psi_1} \left(\frac{S_S}{P_Q} \right)^{\psi_2} \left(\frac{P_Q}{P_Q^e} \right)^{\psi_3} \left(\vartheta(\rho D) \right)^{\psi_4} \qquad (19)$$

where: $\vartheta = \sum_{k=1}^{H} \vartheta_k$

ϑ_k is an index capturing the share of household '*k'* in supplying effective labor (zL) for the given sector

$$\frac{\dot{(zL)}}{(zL)} = \psi_1 \left(\frac{\dot{w}}{w} - \frac{\dot{P}}{P} \right) + \psi_2 \left(\frac{\dot{S}_S}{S_S} - \frac{\dot{P}}{P} \right) + \psi_3 \left(\frac{\dot{P}_Q}{P_Q} - \frac{\dot{P}_Q^e}{P_Q^e} \right) + \psi_4 \left(\frac{\dot{\vartheta}}{\vartheta} + \frac{\dot{\rho D}}{\rho D} \right) \qquad (20)$$

b) Labor demand equation

$$zL = \alpha . \frac{S_S}{w} . \left(\frac{P_Q^e}{P_Q} \right)^{1+\beta\phi_K + (\alpha-1)\phi_L} \tag{21}$$

$$\frac{\dot{(zL)}}{(zL)} = \frac{\dot{S}_S}{S_S} - \frac{\dot{w}}{w} - (1+\beta\phi_K + (\alpha-1)\phi_L)\frac{\dot{P}_Q}{P_Q} + (1+\beta\phi_K + (\alpha-1)\phi_L)\frac{\dot{P}_Q^e}{P_Q^e} \tag{22}$$

$$\frac{\dot{(zL)}}{(zL)} = \frac{\dot{S}_S}{S_S} - \frac{\dot{w}}{w} + (1+\beta\phi_K + (\alpha-1)\phi_L)\left[\frac{\dot{P}_Q^e}{P_Q^e} - \frac{\dot{P}_Q}{P_Q} \right] \tag{23}$$

IV. Capital

a) Capital Supply Equation

$$K = C_K \left(\frac{r}{P_Q} \right)^{\gamma_1} \left(\frac{S_S}{P_Q} \right)^{\gamma_2} \left(\frac{P_Q}{P_Q^e} \right)^{\gamma_3} (\delta D)^{\gamma_4} \qquad (24)$$

$$\delta = \sum_{k=1}^{D} \delta_k$$

$$\frac{\dot{K}}{K} = \gamma_1 \left(\frac{\dot{r}}{r} - \frac{\dot{P}_Q}{\dot{P}_Q} \right) + \gamma_2 \left(\frac{\dot{S}_S}{\dot{S}_S} - \frac{\dot{P}_Q}{\dot{P}_Q} \right) + \gamma_3 \left(\frac{\dot{P}_Q}{P_Q} - \frac{\dot{P}_Q^e}{P_Q^e} \right) + \gamma_4 \left(\frac{\dot{\delta}}{\delta} + \frac{\dot{D}}{D} \right) \qquad (25)$$

b) Capital Demand equation

$$K = \beta \frac{S_S}{r} \left(\frac{P_Q^e}{P_Q} \right)^{1+\alpha\phi_L + (\beta - 1)\phi_K} \tag{26}$$

$$\frac{\dot{K}}{K} = \frac{\dot{S}_S}{S_S} - \frac{\dot{r}}{r} - [1 + \alpha\phi_L + (\beta - 1)\phi_K] \left(\frac{\dot{P}_Q}{P_Q} \right) + [1 + \alpha\phi_L + (\beta - 1)\phi_K] \left(\frac{\dot{P}_Q^e}{P_Q^e} \right) \tag{27}$$

$$\frac{\dot{K}}{K} = \frac{\dot{S}_S}{S_S} - \frac{\dot{r}}{r} + [1 + \alpha\phi_L + (\beta - 1)\phi_K] \left(\frac{\dot{P}_Q^e}{P_Q^e} - \frac{\dot{P}_Q}{P_Q} \right) \tag{28}$$

V. The Money Market

a) Money supply equation

$$M_S = C_{M_S} . P^{\pi_1} . r^{\pi_2} \qquad (29)$$

$$\frac{\dot{M}_S}{M_S} = \pi_1 \left(\frac{\dot{P}}{P} \right) + \pi_2 \left(\frac{\dot{r}}{r} \right) \qquad (30)$$

b) Money Demand equation

$$M^d = C_{M^d}.(\Re D)^{\nabla_1}.(\Im N)^{\nabla_2}.\left(\frac{r}{P_Q^e}\right)^{\nabla_3}.\left(\frac{S_S}{P_Q^e}\right)^{\nabla_4}.\left(\frac{P_Q}{P_Q^e}\right)^{\nabla_5} \tag{31}$$

$$\frac{\dot{M}^d}{M^d} = \nabla_1\left(\frac{\dot{\Re}}{\Re}+\frac{\dot{D}}{D}\right)+\nabla_2\left(\frac{\dot{\Im}}{\Im}+\frac{\dot{N}}{N}\right)+\nabla_3\left(\frac{\dot{r}}{r}-\frac{\dot{P}_Q^e}{P_Q^e}\right)+\nabla_4\left(\frac{\dot{S}_S}{S_S}-\frac{\dot{P}_Q^e}{P_Q^e}\right)+\nabla_5\left(\frac{\dot{P}_Q}{P_Q}-\frac{\dot{P}_Q^e}{P_Q^e}\right) \tag{32}$$

VI. Entry/Exit equation

$$\frac{\dot{N}}{N} = C_E (S_S - \pi^e) \qquad (33)$$

VII. A note on 'expected price P_Q^e'

$$\ln P_{Qt}^e = \varpi \ln P_{Q(t-1)} + (1-\varpi) \ln P_{Q(t-1)}^e \qquad (34)$$

$$\ln P_{Qt}^e = \varpi \ln P_{Q(t-1)} + \varpi(1-\varpi) \ln P_{Q(t-2)} + \varpi(1-\varpi)^2 \ln P_{Q(t-3)} \qquad (35)$$

$$\ln P_{Q_t}^e = \ln\left[P_{Q_{(t-1)}}^{\varpi} . P_{Q_{(t-2)}}^{\varpi(1-\varpi)} . P_{Q_{(t-3)}}^{\varpi(1-\varpi)^2} \right] \qquad (36)$$

Expected price (cont'd)

$$P_{Q_t}^{e} = \prod_{j=1}^{n} P_{Q_{(t-j)}}^{\sigma_j}$$

where: $\sigma_j = \varpi(1-\varpi)^{j-1}$. (37)

$$\frac{\dot{P}_{Q_t}^{e}}{P_{Q_t}^{e}} = \varpi \frac{\dot{P}_{Q_{t-1}}}{P_{Q_{t-1}}} + \varpi(1-\varpi)\frac{\dot{P}_{Q_{t-1}}^{e}}{P_{Q_{t-1}}^{e}} + \varepsilon \qquad (38)$$

THE ARLI (3) MODEL

$$\ln\left(\frac{S_{St}}{S_{S(t-1)}}\right) \approx \theta_0''' + \theta_1''' S_{S(t-1)} + \theta_2''' S_{S(t-2)} + \theta_3'' S_{S(t-3)} + \theta_4'' \ln\left(\frac{SP_{Q_{(t-3)}}}{SP_{Q_{(t-4)}}}\right) + \theta_5'' \ln\left(\frac{M_{(t-1)}}{M_{(t-2)}}\right) + \varepsilon_{St}' \quad (39)$$

Note: $$\frac{\dot{S}_{St}}{S_{St}} = \frac{d \ln S_{St}}{dt} \approx \ln\left(\frac{S_{St}}{S_{S(t-1)}}\right)$$

DERIVING THE REDUCED FORM EQUATIONS

Considering the three equations:

$$\frac{\dot{S}_S}{S_S} = \theta_1 \frac{\dot{A}}{A} + \left(\frac{\dot{\Im}}{\Im} + \frac{\dot{N}}{N} \right) + \theta_2 \frac{\dot{P}_Q}{P_Q} + \theta_3 \frac{\dot{w}}{w} + \theta_4 \frac{\dot{r}}{r} + \sum_{l=1}^{T} \sigma_l \frac{\dot{P}_l}{P_l} + \theta_5 \frac{\dot{z}}{z} + \theta_6 \frac{\dot{\Gamma}}{\Gamma} \qquad (40)$$

$$\frac{\dot{S}_D}{S_D} = (1-\Delta)\frac{\dot{P}_Q}{P_Q} + \frac{(\dot{\Re D})}{(\Re D)} + (\lambda_1 + \Delta)\frac{\dot{P}_Q^e}{P_Q^e} + \chi_{j1}\frac{\dot{Y}_d}{Y_d} + \chi_{j2}\frac{\dot{WY}}{WY} \qquad (41)$$

$$\frac{\dot{N}}{N} = C_E (S_S - \pi^e) \qquad (42)$$

Reduced Form Equations (cont'd 1)

Equating 40 and 41:

$$\theta_1 \frac{\dot{A}}{A} + \left(\frac{\dot{\Im}}{\Im} + \frac{\dot{N}(\Gamma)}{N(\Gamma)} \right) + \theta_2 \frac{\dot{P}_Q}{P_Q} + \theta_3 \frac{\dot{w}}{w} + \theta_4 \frac{\dot{r}}{r} + \sum_{l=1}^{T} \sigma_l \frac{\dot{P}_l}{P_l} + \theta_5 \frac{\dot{z}}{z} = (1-\Delta) \frac{\dot{P}_Q}{P_Q} + \left(\frac{\dot{\Re}}{\Re} + \frac{\dot{D}}{D} \right) + (\lambda_1 + \Delta) \frac{\dot{P}_Q^e}{P_Q^e}$$

$$+ \chi_1 \frac{\dot{Y}_d}{Y_d} + \chi_2 \frac{\dot{W}Y}{WY} \qquad (43)$$

Replacing $\frac{\dot{N}}{N}$ in equation 43 by equation 42:

Reduced Form Equations (cont'd 2)

$$\frac{\dot{P}_Q}{P_Q} = \frac{C_E}{\theta_2 - 1 + \Delta}.\pi^e - \frac{C_E}{\theta_2 - 1 + \Delta}.S_S + \frac{1}{\theta_2 - 1 + \Delta}.\frac{\dot{D}}{D} + \frac{(\lambda_1 + \Delta)}{\theta_2 - 1 + \Delta}.\frac{\dot{P}_Q^e}{P_Q^e} - \frac{\theta_1}{\theta_2 - 1 + \Delta}.\frac{\dot{A}}{A} - \frac{\theta_3}{\theta_2 - 1 + \Delta}.\frac{\dot{w}}{w}$$

$$- \frac{\theta_4}{\theta_2 - 1 + \Delta}.\frac{\dot{r}}{r} - \frac{\sum_{l=1}^{T}\sigma_l}{\theta_2 - 1 + \Delta}.\frac{\dot{P}_l}{P_l} - \frac{\theta_5}{\theta_2 - 1 + \Delta}.\frac{\dot{z}}{z} - \frac{\theta_6}{\theta_2 - 1 + \Delta}.\frac{\dot{\Gamma}}{\Gamma} + \frac{\chi_1}{\theta_2 - 1 + \Delta}.\frac{\dot{Y}_d}{Y_d} + \frac{\chi_2}{\theta_2 - 1 + \Delta}.\frac{\dot{WY}}{WY} \qquad (44)$$

Plugging 44 into 40 we obtain the RFE-DA for price and sales supply:

Reduced Form Equations (cont'd 3)

$$\frac{\dot{S}_{Si}}{S_{Si}} = \theta_{0i}^{'} + \theta_{1i}^{'} S_{Si} + \theta_{2i}^{'} \frac{\dot{D}_i}{D_i} + \theta_{3i}^{'} \frac{\dot{A}_i}{A_i} + \theta_{4i}^{'} \frac{\dot{w}_i}{w_i} + \theta_{5i}^{'} \frac{\dot{r}}{r} + \theta_{6i}^{'} \frac{\dot{z}_i}{z_i} + \theta_{7i}^{'} \frac{\dot{Y}_d}{Y_d} + \theta_{8i}^{'} \frac{\dot{WY}}{WY} + \theta_{9i}^{'} \frac{\dot{\Gamma}}{\Gamma} + \sum_{l=1}^{T} \wp_{il} \frac{\dot{P}_{il}}{P_{il}} + \upsilon_{si} \tag{45}$$

$$\frac{\dot{P}_{Qi}}{P_{Qi}} = \sigma_{0i}^{'} + \sigma_{1i}^{'} S_{Si} + \sigma_{2i}^{'} \frac{\dot{D}_i}{D_i} + \sigma_{3i}^{'} \frac{\dot{A}_i}{A_i} + \sigma_{4i}^{'} \frac{\dot{w}_i}{w_i} + \sigma_{5i}^{'} \frac{\dot{r}}{r} + \sigma_{6i}^{'} \frac{\dot{z}_i}{z_i} + \sigma_{7i}^{'} \frac{\dot{Y}_d}{Y_d} + \sigma_{8i}^{'} \frac{\dot{WY}}{WY} + \sigma_{9i}^{'} \frac{\dot{\Gamma}}{\Gamma} + \sum_{l=1}^{T} \wp_{il} \frac{\dot{P}_{il}}{P_{il}} + \upsilon_{Pi} \tag{46}$$

With: $$\sum_{l=1}^{T} \wp_{il} \frac{\dot{P}_{il}}{P_{il}} = \frac{(\lambda_1 + \Delta)}{\theta_2 - 1 + \Delta} \cdot \frac{\dot{P}_{Q_{it}}^{e}}{P_{Q_{it}}^{e}} - \frac{\sum_{l=1}^{T} \sigma_l}{\theta_2 - 1 + \Delta} \cdot \frac{\dot{P}_l}{P_l} \tag{47}$$

DERIVING THE TRANSFER EQUATIONS

The structural equations model can be presented under matrix form:

$$\begin{bmatrix} 1 & -\lambda(L) & -1 \\ 1 & -\gamma(L) & 0 \\ 0 & 0 & 1 \end{bmatrix} . \begin{bmatrix} s_{i,t} \\ p_{i,t} \\ n_{i,t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \delta_{0,i} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \delta_{1,i} \end{bmatrix} S_{i,t-1} + \begin{bmatrix} \kappa_{1,i} \\ 0 \\ 0 \end{bmatrix} w_{i,t} + \begin{bmatrix} \kappa_{2,i} \\ 0 \\ 0 \end{bmatrix} r_t + \begin{bmatrix} \kappa_{3,i} \\ 0 \\ 0 \end{bmatrix} a_{i,t} + \begin{bmatrix} \kappa_{4,i} \\ 0 \\ 0 \end{bmatrix} z_{i,t}$$

$$+ \begin{bmatrix} \kappa_{5,i} \\ 0 \\ 0 \end{bmatrix} \Gamma_{i,t} + \begin{bmatrix} \kappa_{6,i} \\ 0 \\ 0 \end{bmatrix} X_t + \begin{bmatrix} 0 \\ \Delta_{1,i} \\ 0 \end{bmatrix} y_t + \begin{bmatrix} 0 \\ \Delta_{2,i} \\ 0 \end{bmatrix} wy_t + \begin{bmatrix} 0 \\ \Delta_{3,i} \\ 0 \end{bmatrix} d_t + \begin{bmatrix} \varepsilon_{Ti,t} \\ \mu_{Ti,t} \\ v_{Ti,t} \end{bmatrix} \qquad (48)$$

Multiplying both side of equation 48 by the matrix $\overset{*}{A}$

$$A^{*} = (\det A).A^{-1}$$

$$A = \begin{bmatrix} 1 & -\lambda(L) & -1 \\ 1 & -\gamma(L) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det A}\begin{bmatrix} -\gamma(L) & \lambda(L) & -\gamma(L) \\ -1 & 1 & -1 \\ 0 & 0 & \lambda(L)-\gamma(L) \end{bmatrix}$$

Transfer Equations (cont'd 2)

After multiplying both sides of equation 48 by A^{*} :

$$
[\lambda(L)-\gamma(L)].\begin{bmatrix} s_{i,t} \\ p_{i,t} \\ n_{i,t} \end{bmatrix} = \begin{bmatrix} -\gamma(L)\delta_{0,i} \\ -\delta_{0,i} \\ \delta_{0,i}[\lambda(L)-\gamma(L)] \end{bmatrix} + \begin{bmatrix} -\gamma(L)\delta_{1,i} \\ -\delta_{1,i} \\ \delta_{1,i}[\lambda(L)-\gamma(L)] \end{bmatrix} S_{i,t-1} + \begin{bmatrix} -\gamma(L)\kappa_{1,i} \\ -\kappa_{1,i} \\ 0 \end{bmatrix} w_{i,t}
$$

Transfer Equations (cont'd 3)

$$
+\begin{bmatrix} -\gamma(L)\kappa_{2,i} \\ -\kappa_{2,i} \\ 0 \end{bmatrix} r_t + \begin{bmatrix} -\gamma(L)\kappa_{3,i} \\ -\kappa_{3,i} \\ 0 \end{bmatrix} a_{i,t} + \begin{bmatrix} -\gamma(L)\kappa_{4,i} \\ -\kappa_{4,i} \\ 0 \end{bmatrix} z_{i,t} + \begin{bmatrix} -\gamma(L)\kappa_{5,i} \\ -\kappa_{5,i} \\ 0 \end{bmatrix} \Gamma_{i,t}
$$

$$
+\begin{bmatrix} -\gamma(L)\kappa_{6,i} \\ -\kappa_{6,i} \\ 0 \end{bmatrix} X_t + \begin{bmatrix} \lambda(L)\Delta_{1,i} \\ \Delta_{1,i} \\ 0 \end{bmatrix} y_t + \begin{bmatrix} \lambda(L)\Delta_{2,i} \\ \Delta_{2,i} \\ 0 \end{bmatrix} wy_t + \begin{bmatrix} \lambda(L)\Delta_{3,i} \\ \Delta_{3,i} \\ 0 \end{bmatrix} d_t +
$$

$$
+\begin{bmatrix} -\gamma(L)\varepsilon_{Ti,t} + \lambda(L)\mu_{Ti,t} - \gamma(L)v_{Ti,t} \\ -\varepsilon_{Ti,t} + \mu_{Ti,t} - v_{Ti,t} \\ [\lambda(L) - \gamma(L)]v_{Ti,t} \end{bmatrix} \qquad (49)
$$

Transfer Equations (cont'd 4)

Equation 49 can be transformed into a system of linear equations for both price and sales supply:

$$\left[\lambda(L)-\gamma(L)\right].s_{i,t} = -\gamma(L)\delta_{0,i} - \gamma(L)\delta_{1,i}S_{i,t-1} - \gamma(L)\kappa_{1,i}w_{i,t} - \gamma(L)\kappa_{2,i}r_t - \gamma(L)\kappa_{3,i}a_{i,t} - \gamma(L)\kappa_{4,i}z_{i,t}$$

$$-\gamma(L)\kappa_{5,i}\Gamma_{i,t} - \gamma(L)\kappa_{6,i}X_t + \lambda(L)\Delta_{1,i}y_t + \lambda(L)\Delta_{2,i}wy_t + \lambda(L)\Delta_{3,i}d_t - \gamma(L)\varepsilon_{Ti,t} + \lambda(L)\mu_{Ti,t} - \gamma(L)v_{Ti,t}$$

(50)

$$\left[\lambda(L)-\gamma(L)\right].p_{i,t} = -\delta_{0,i} - \delta_{1,i}S_{i,t-1} - \kappa_{1,i}w_{i,t} - \kappa_{2,i}r_t - \kappa_{3,i}a_{i,t} - \kappa_{4,i}z_{i,t} - \kappa_{5,i}\Gamma_{i,t} - \kappa_{6,i}X_t + \Delta_{1,i}y_t$$

$$+\Delta_{2,i}wy_t + \Delta_{3,i}d_t - \varepsilon_{Ti,t} + \mu_{Ti,t} - v_{Ti,t} \qquad (51)$$

Transfer Equations (cont'd 5)

where:

- $\lambda(L)$ and $\gamma(L)$: lag operators
- X: set of other exogenous variables obtained from the ARLI (3) model: *SP* (Stock Prices) and M (Money Supply: M2);
- $\ln\left(\dfrac{S_{i,t}}{S_{i,t-1}}\right) = s_{i,t}$;
- $\ln\left(\dfrac{N_{i,t}}{N_{i,t-1}}\right) = n_{i,t}$;

Transfer Equations (cont'd 6)

- $$\ln\left(\frac{w_{i,t}}{w_{i,t-1}}\right) = w_{i,t};$$

- $$\ln\left(\frac{r_{i,t}}{r_{i,t-1}}\right) = r_{i,t};$$

- $$\ln\left(\frac{A_{i,t}}{A_{i,t-1}}\right) = a_{i,t};$$

- $$\ln\left(\frac{z_{i,t}}{z_{i,t-1}}\right) = z_{i,t};$$

- $$\ln\left(\frac{\Gamma_{i,t}}{\Gamma_{i,t-1}}\right) = \Gamma_{i,t};$$

RESULTS USING ITERATIVE SUR and MCMC (1500 iterations)

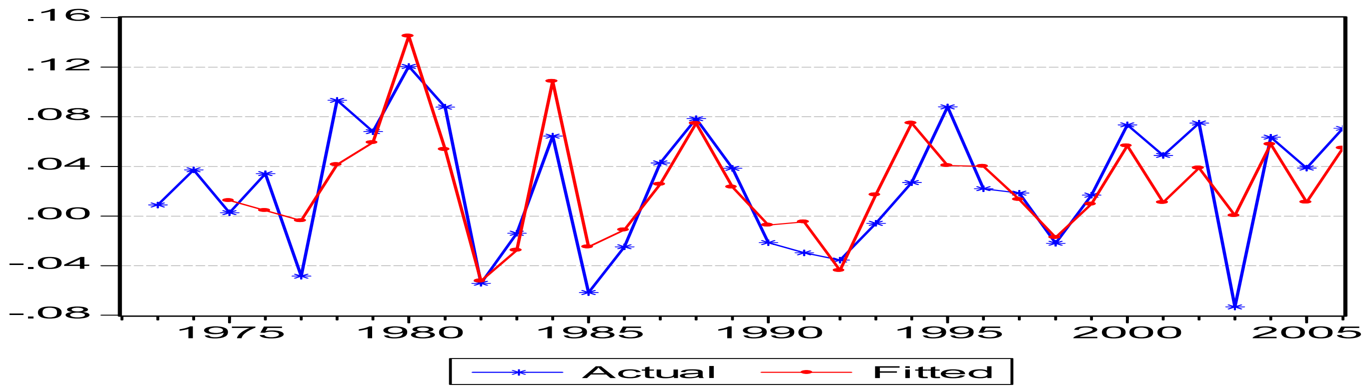
Table 1- RMSE and MAE

	ARLI (3)	MMM-DA (no shrinkage)	MMM-DA (shrinkage)	MCMC
RMSE	2.75	1.61	1.72	1.51
MAE	2.17	1.28	1.31	1.26

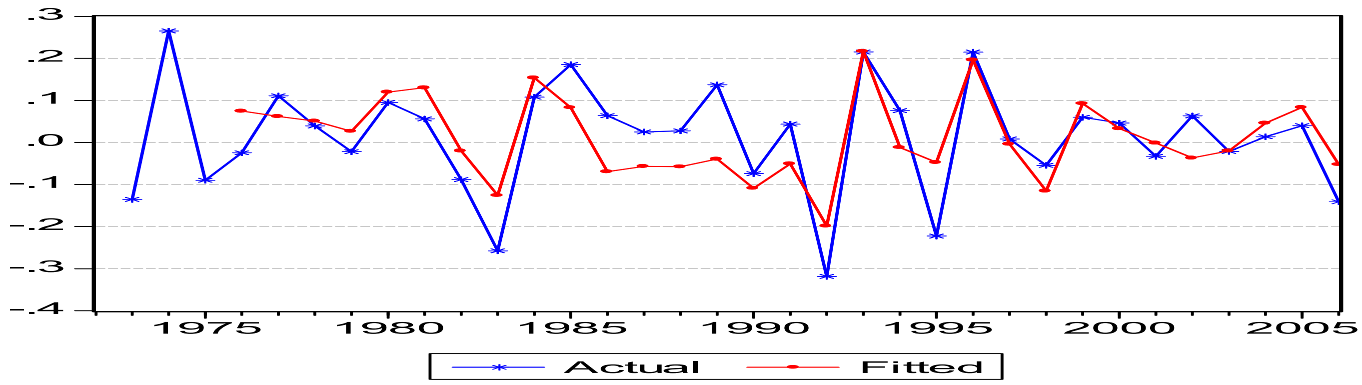
Formulas:

- $MAE = \frac{1}{T}\sum_{t}^{T}\left|\hat{y}_t - y_t\right|$
- $RMSE = \sqrt{\frac{1}{T}\sum_{t}^{T}(\hat{y}_t - y_t)^2}$

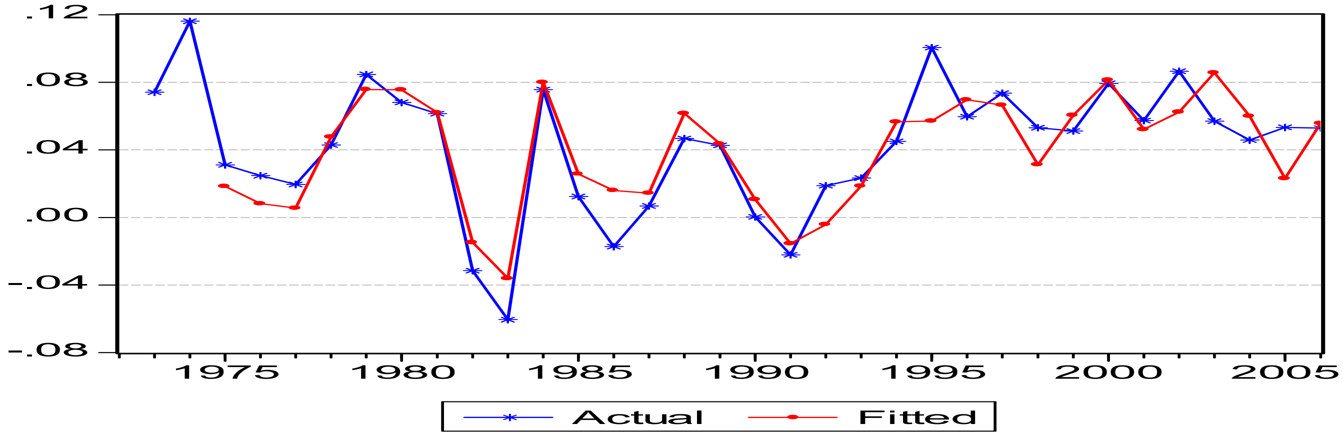
Model fitness (per sector)



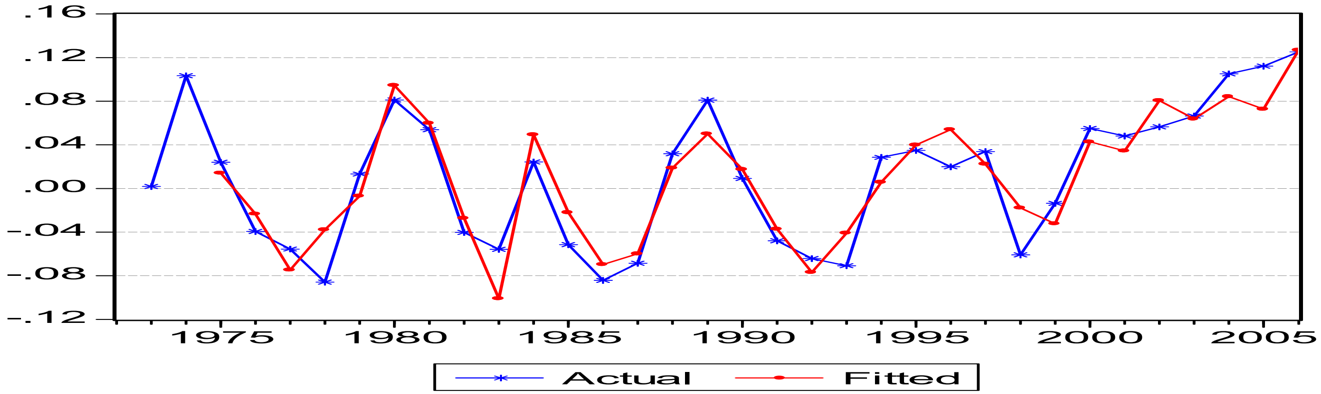
Manufacturing



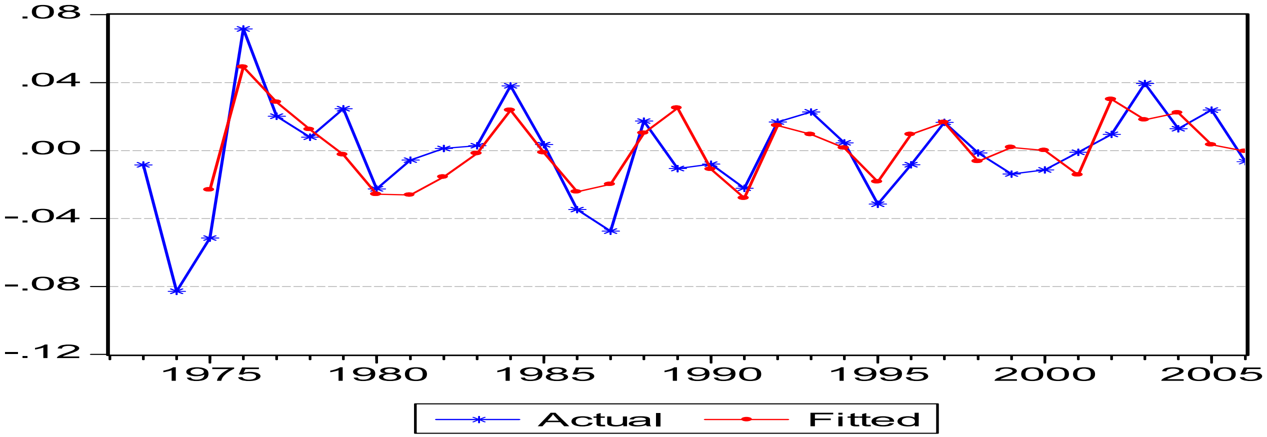
Agriculture



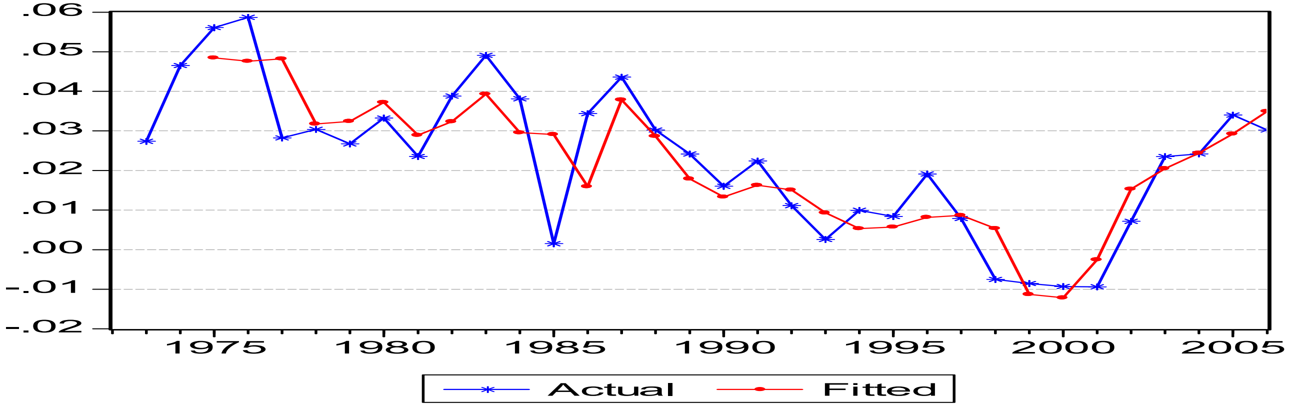
Transport



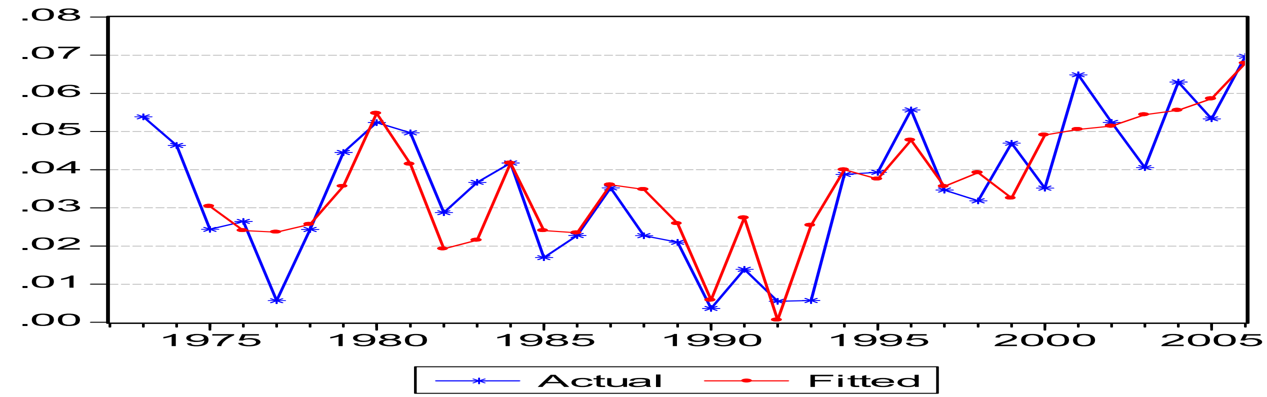
Construction



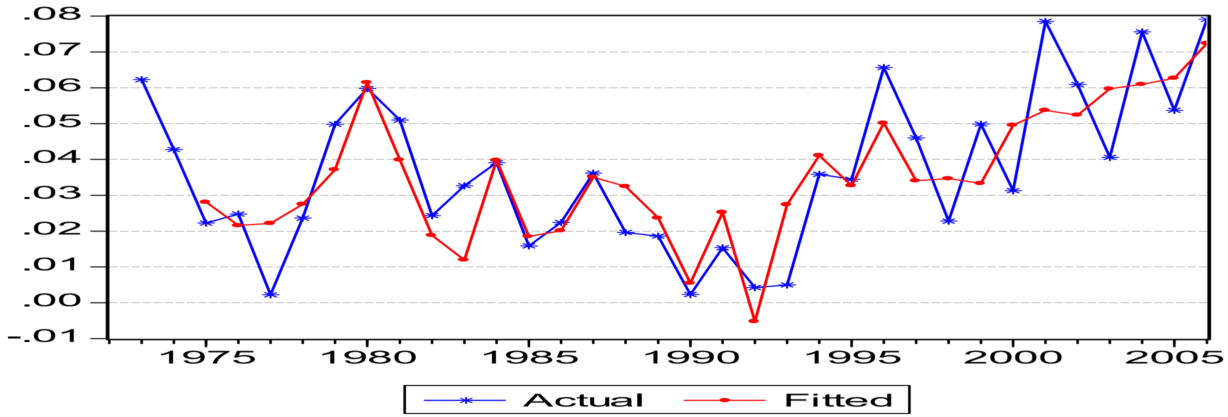
Mining



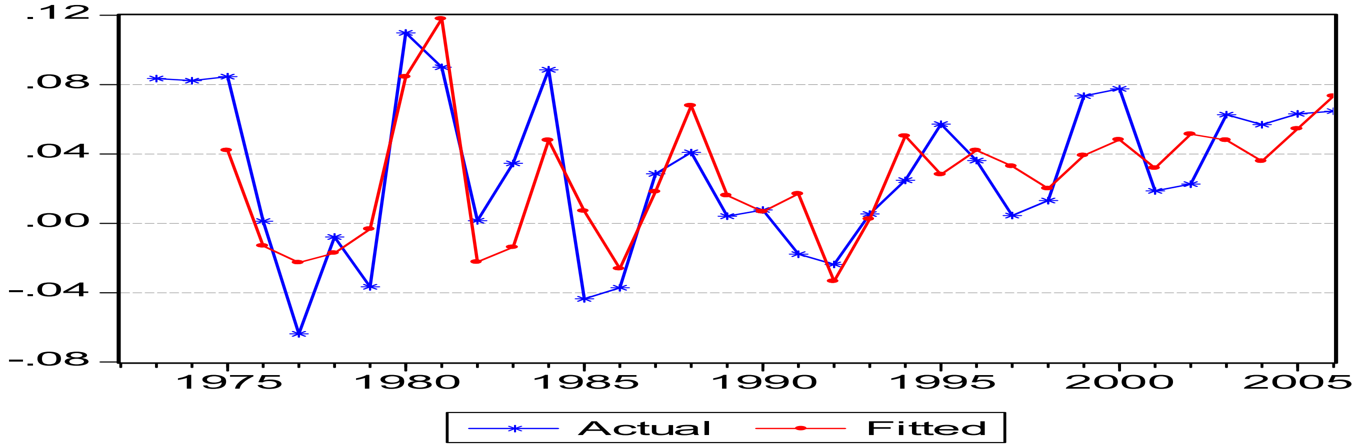
Government



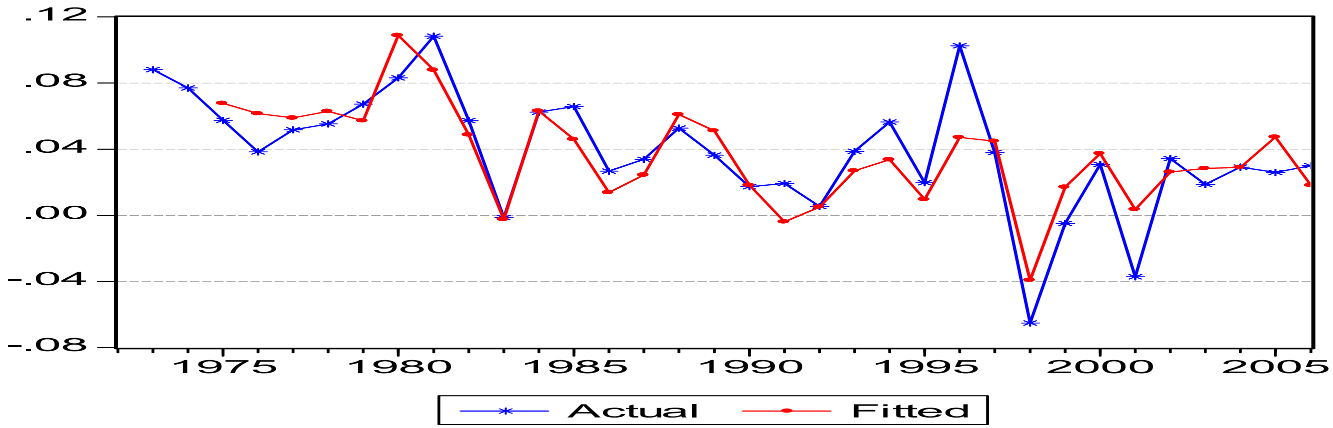
Community services



Financial sector

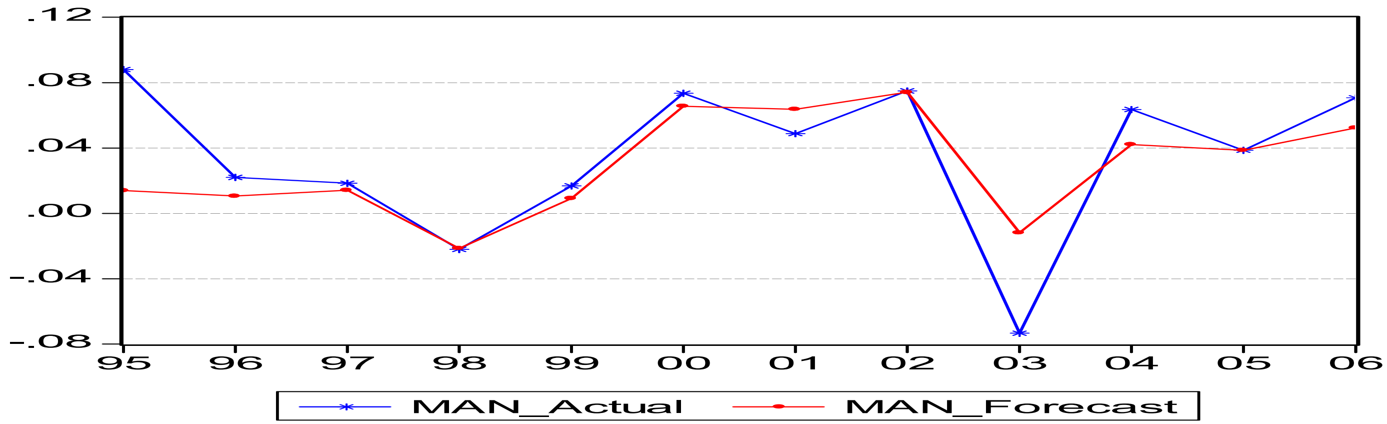


Wholesales

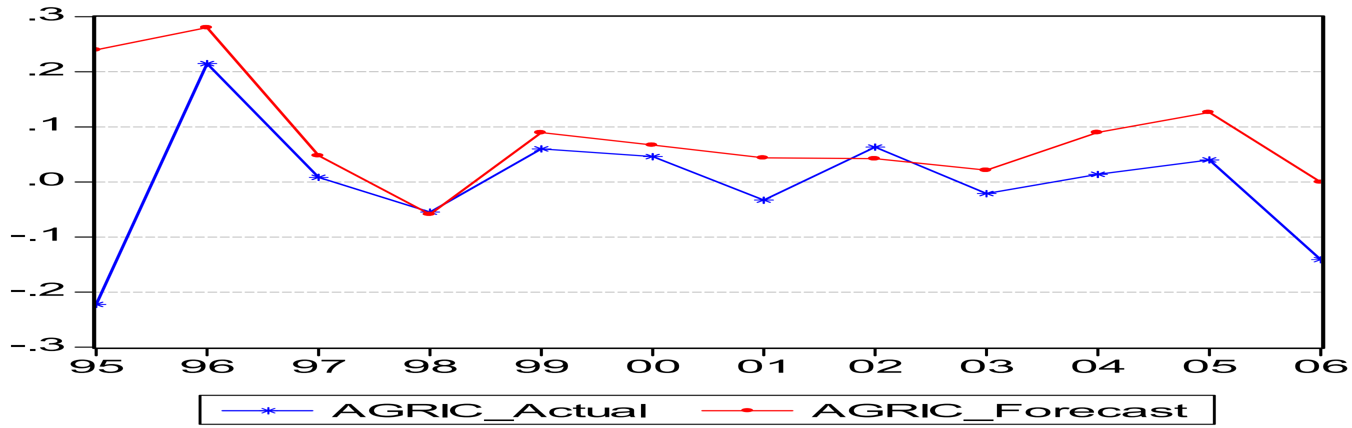


Electricity

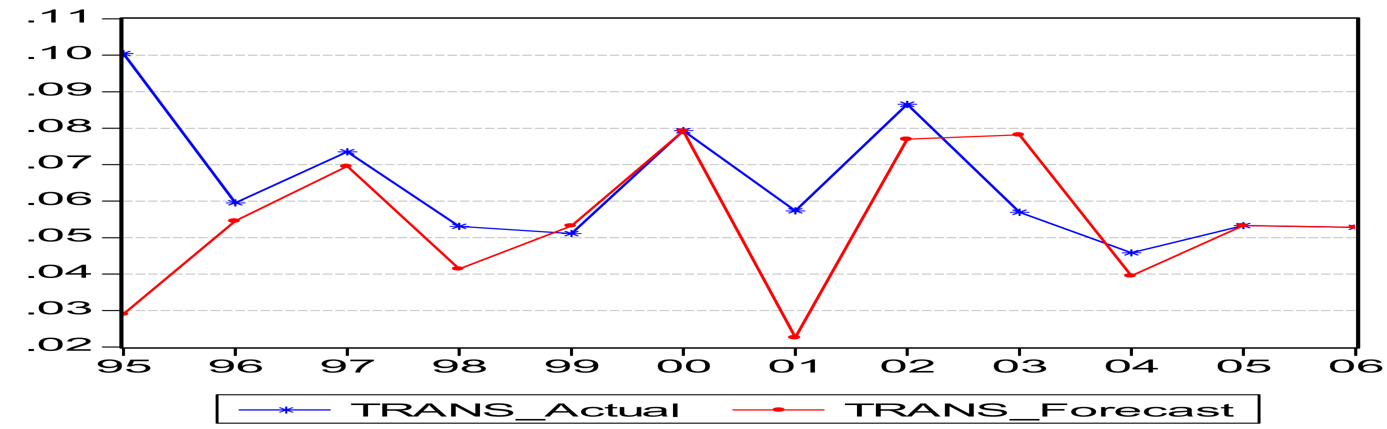
Model's Prediction Ability: Actual versus Predictions (Forecasts)



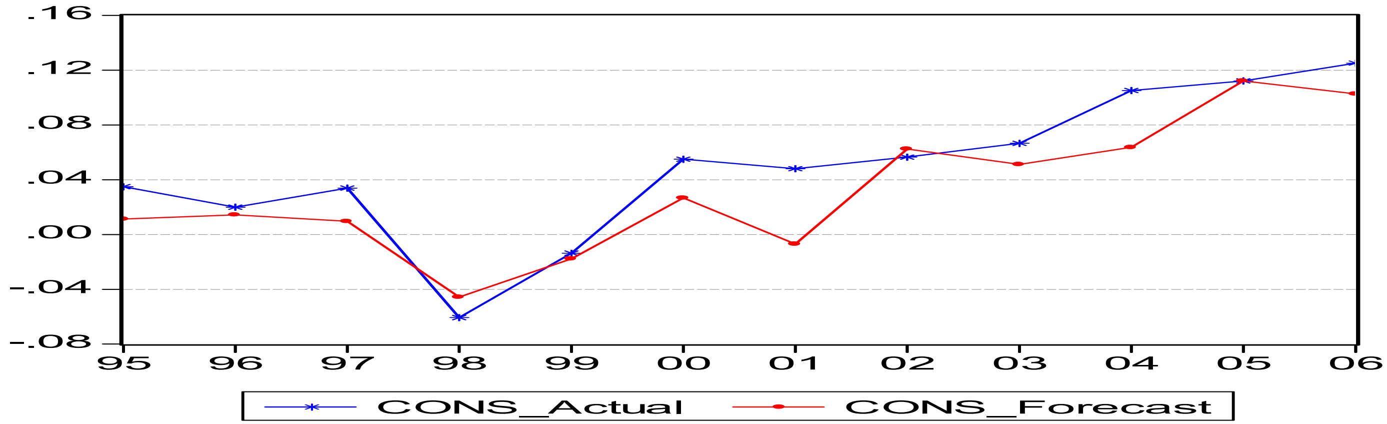
Manufacturing



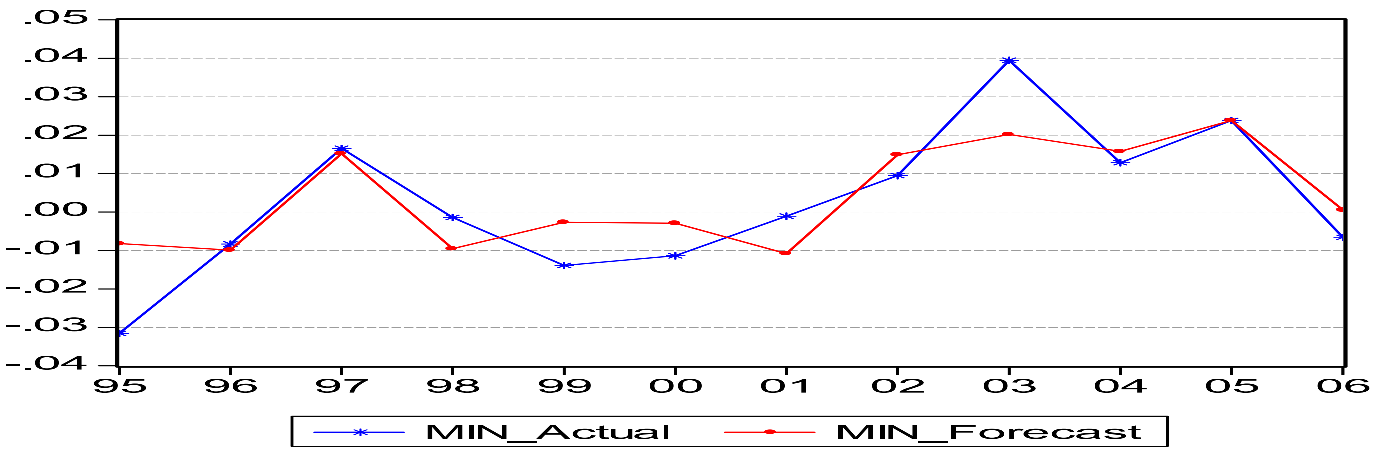
Agriculture



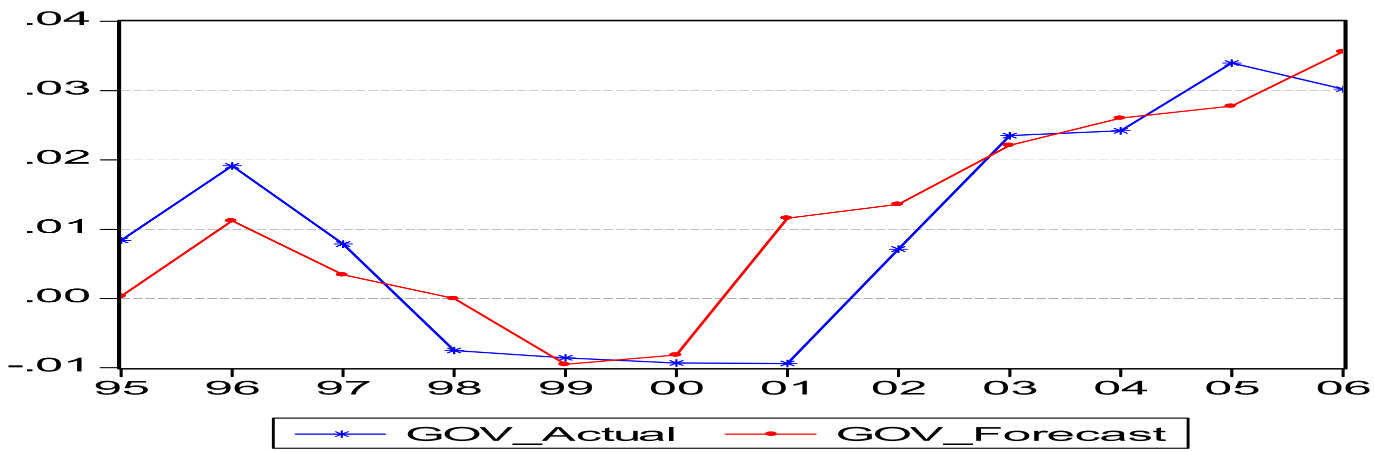
Transport



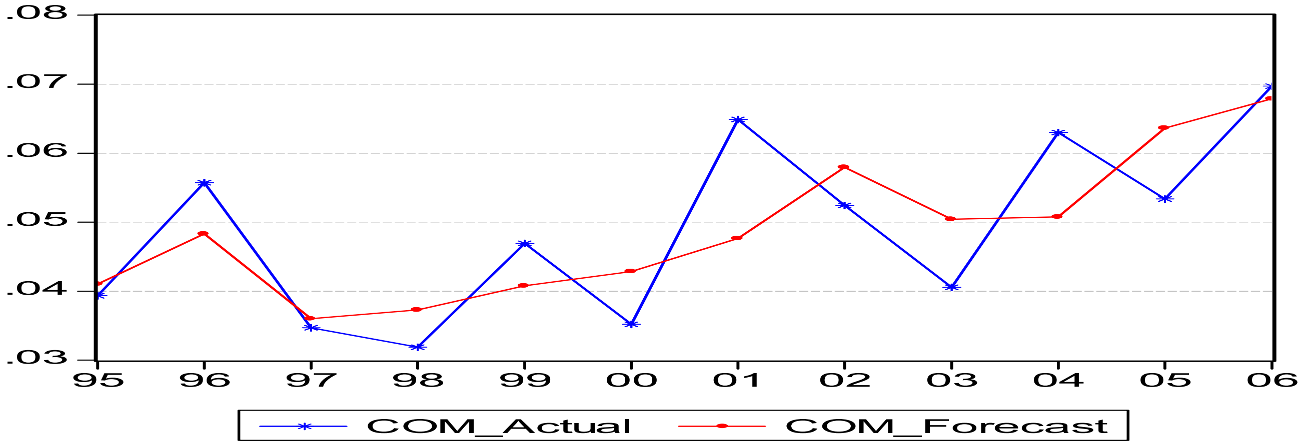
Construction



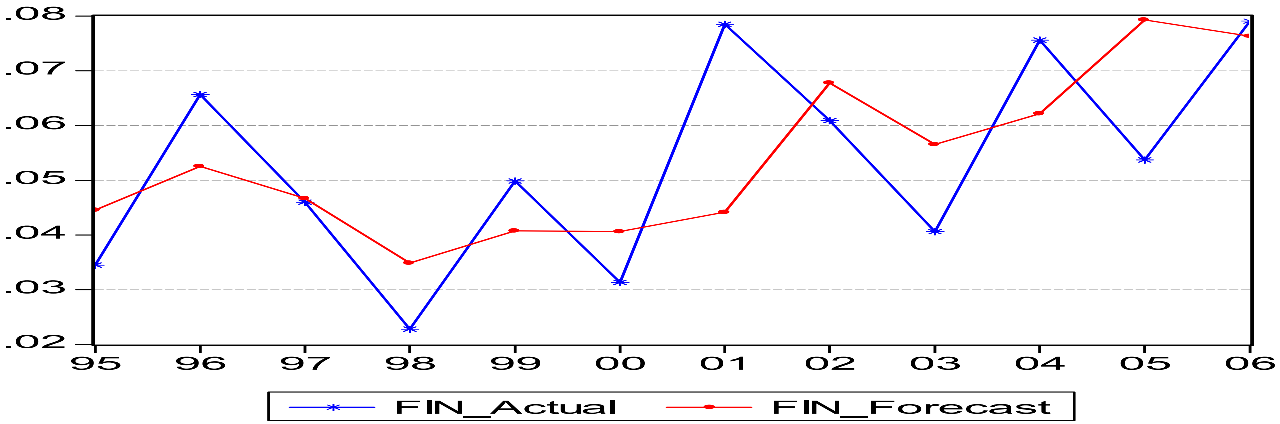
Mining



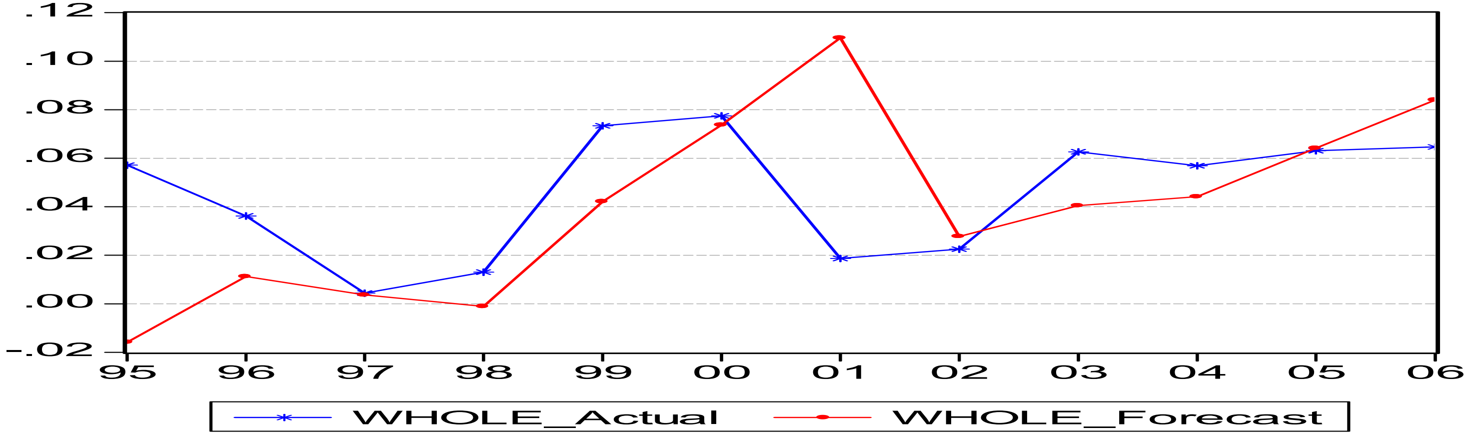
Government



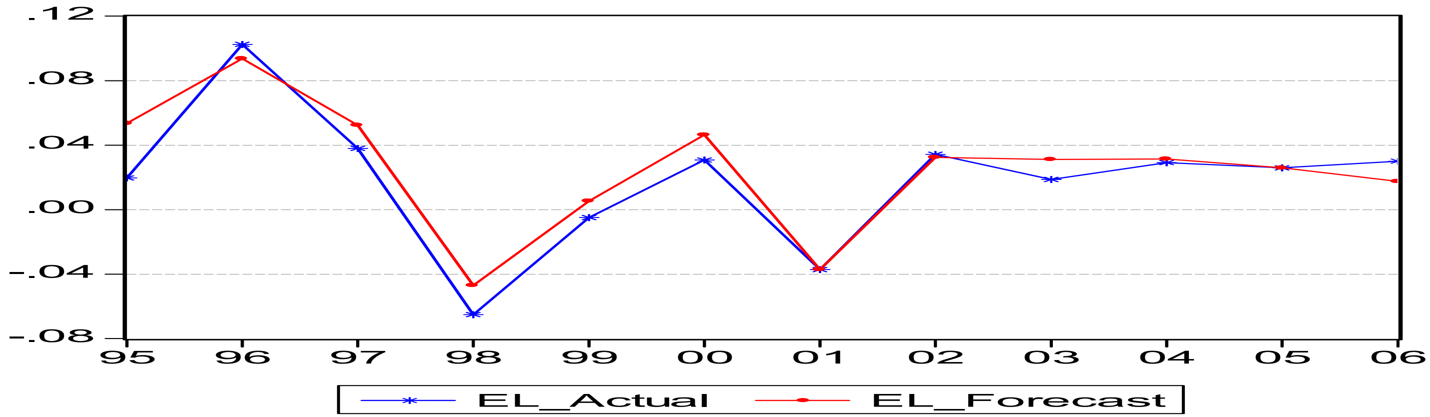
Community services



Financial services



Wholesales

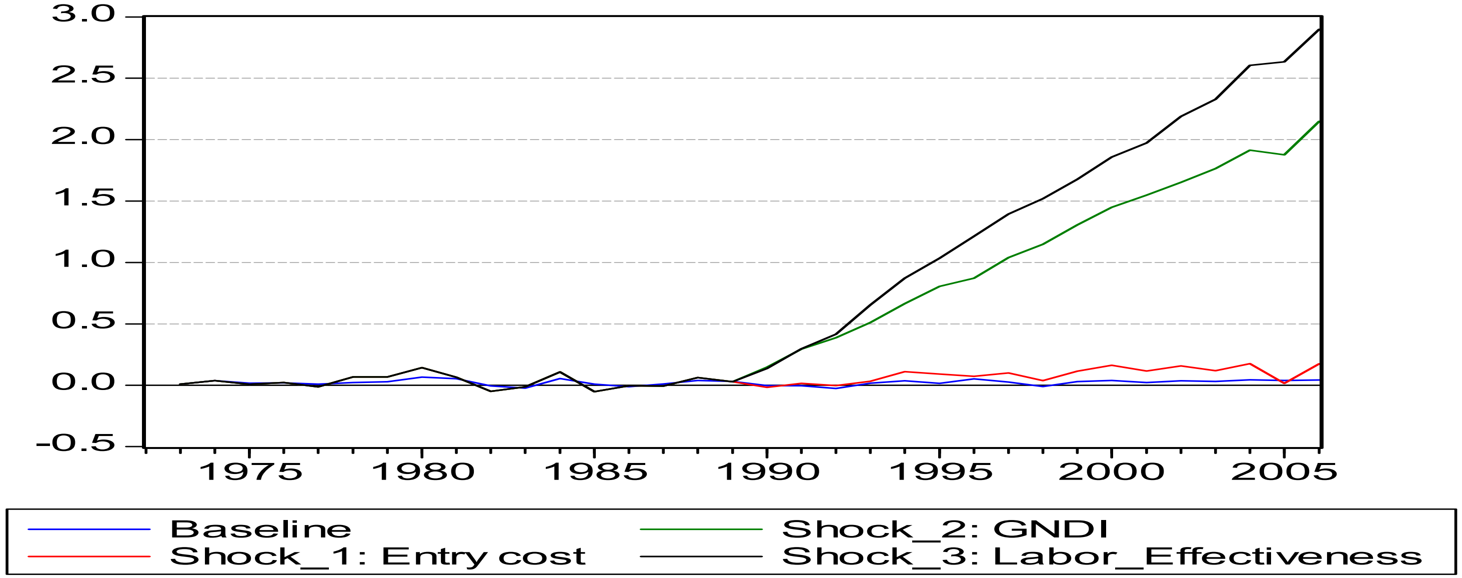


Electricity

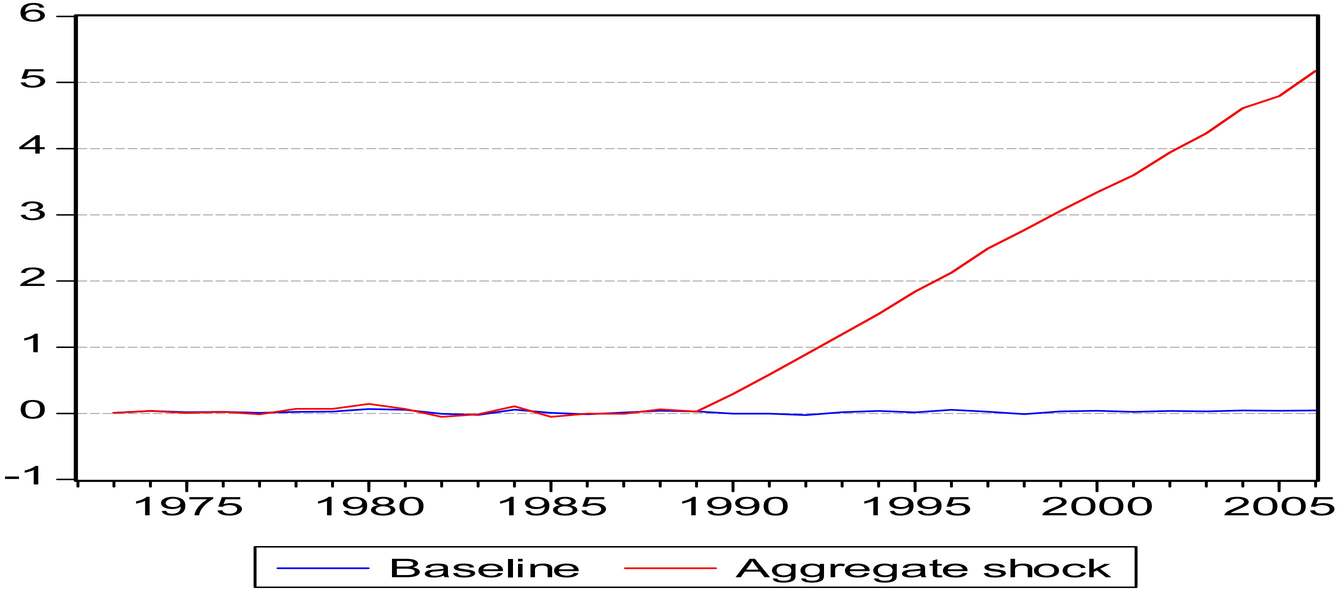
POLICY SHOCKS (permanent shock from 1990)

Manufacturing

a) Individual shocks: 10 percent

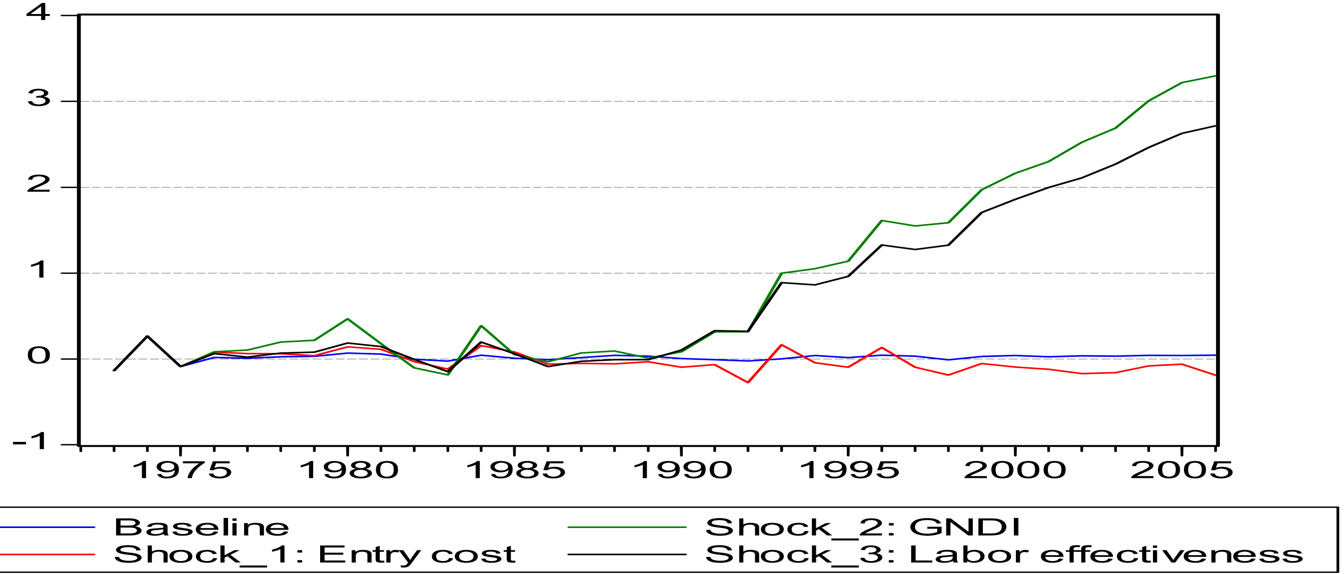


b) Aggregate shock: all three shocks applied simultaneously

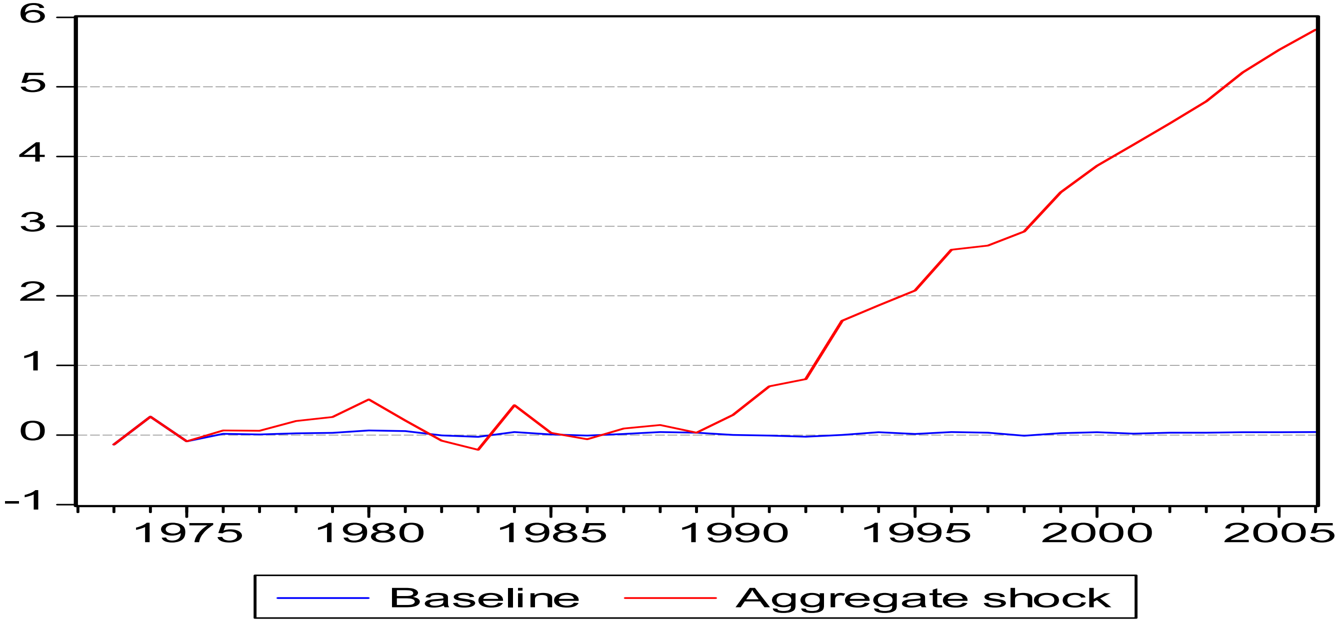


Agriculture

a) Individual shocks: 10 percent

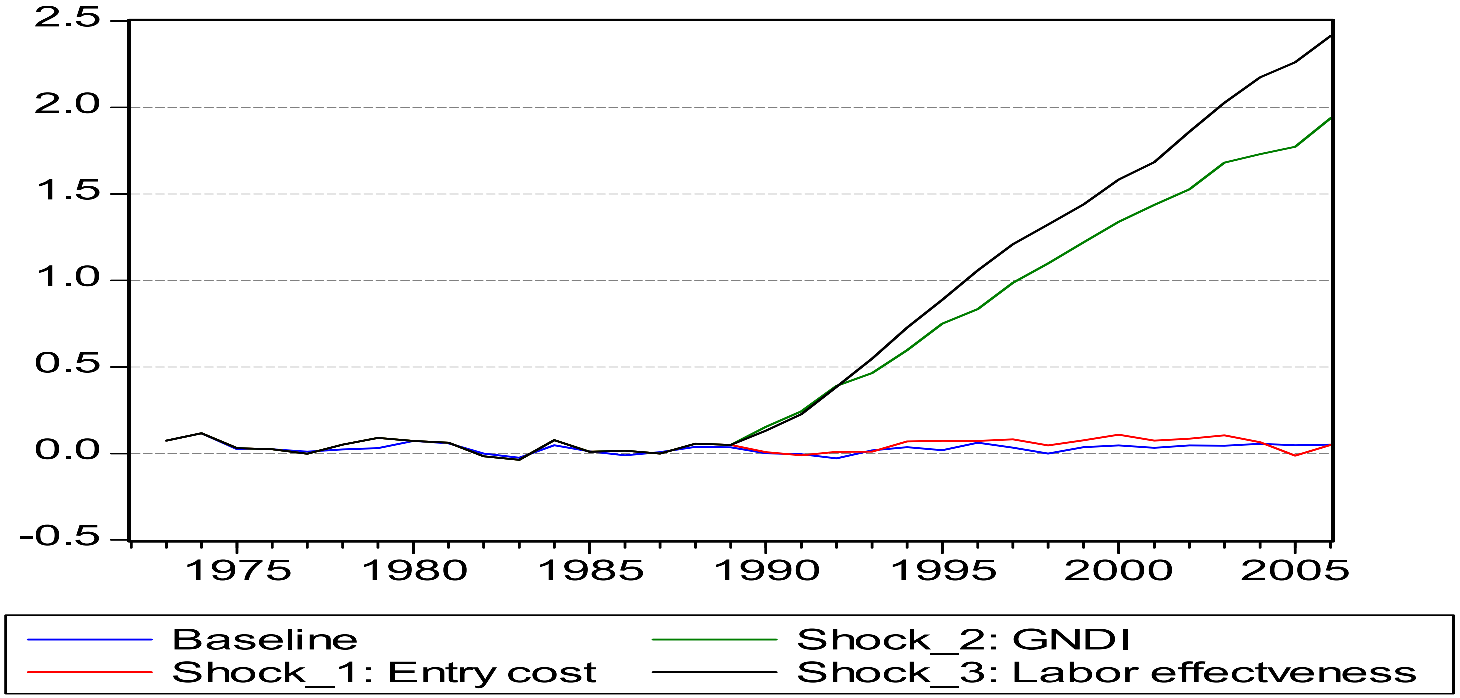


b) Aggregate shock: all three shocks applied simultaneously

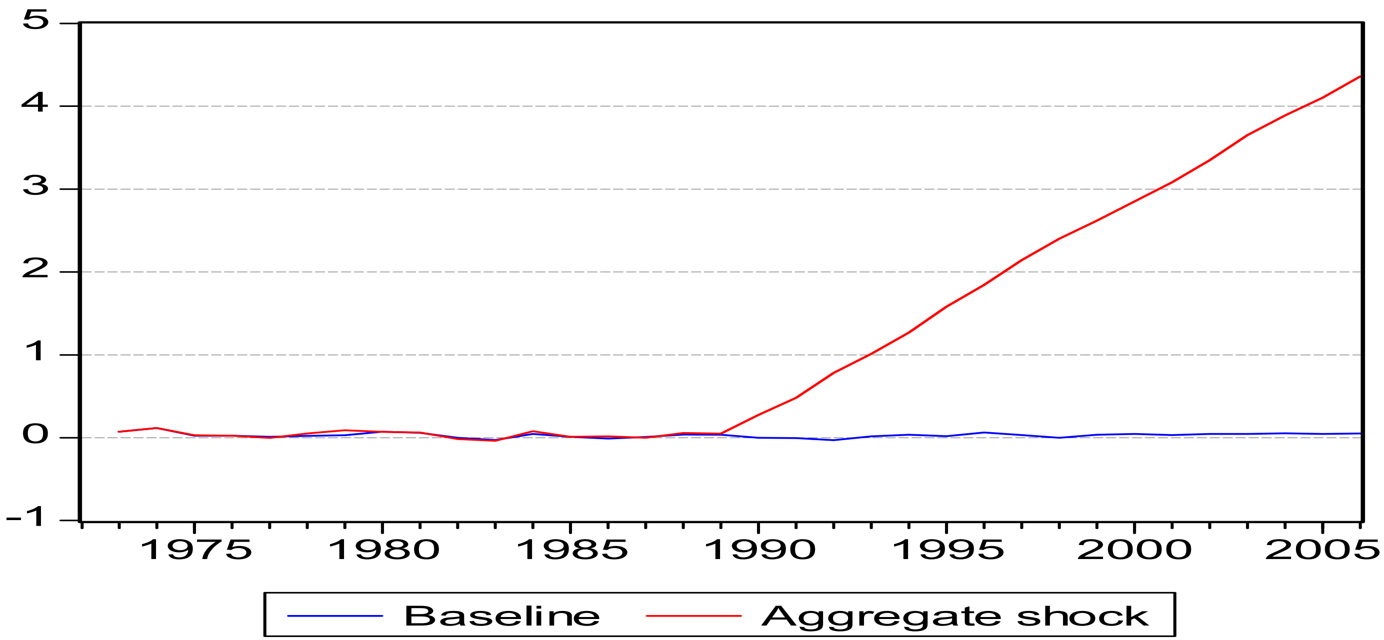


Transport

a) Individual shocks: 10 percent

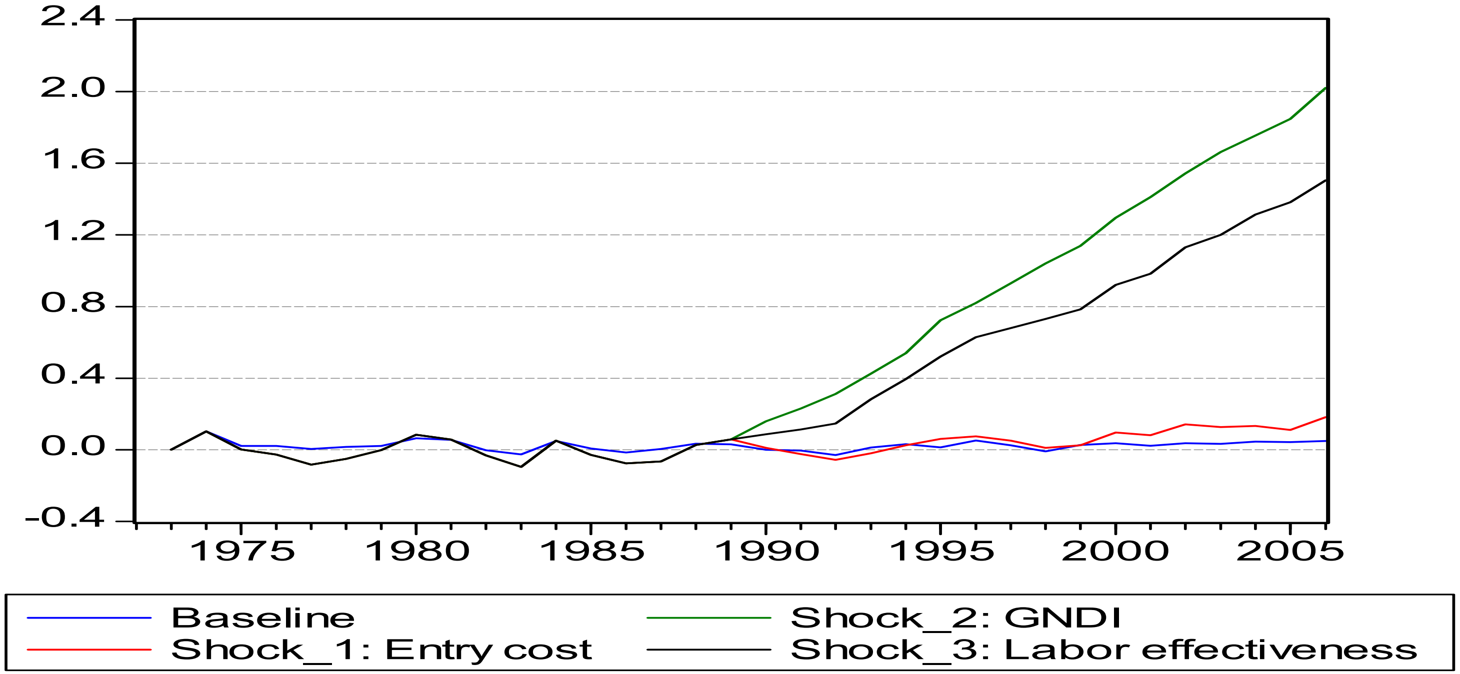


b) Aggregate shock: all three shocks applied simultaneously

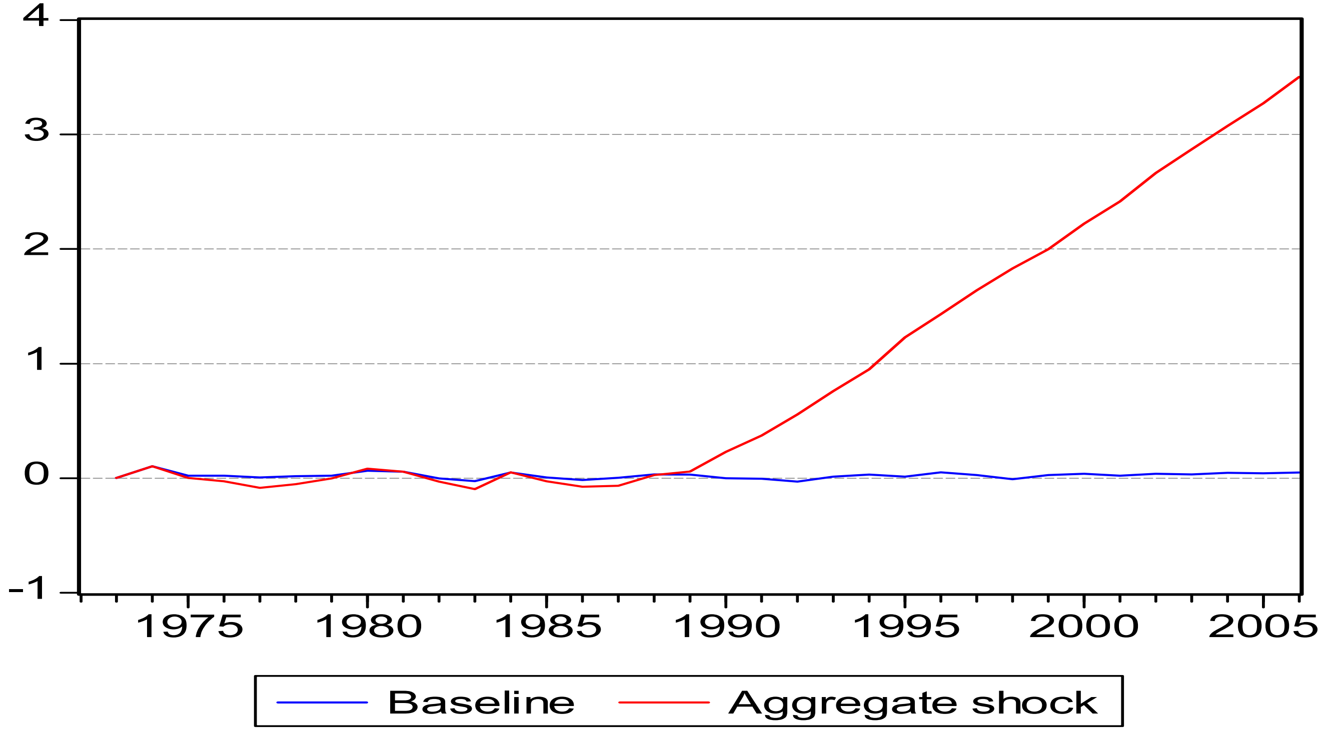


Construction

a) Individual shocks: 10 percent

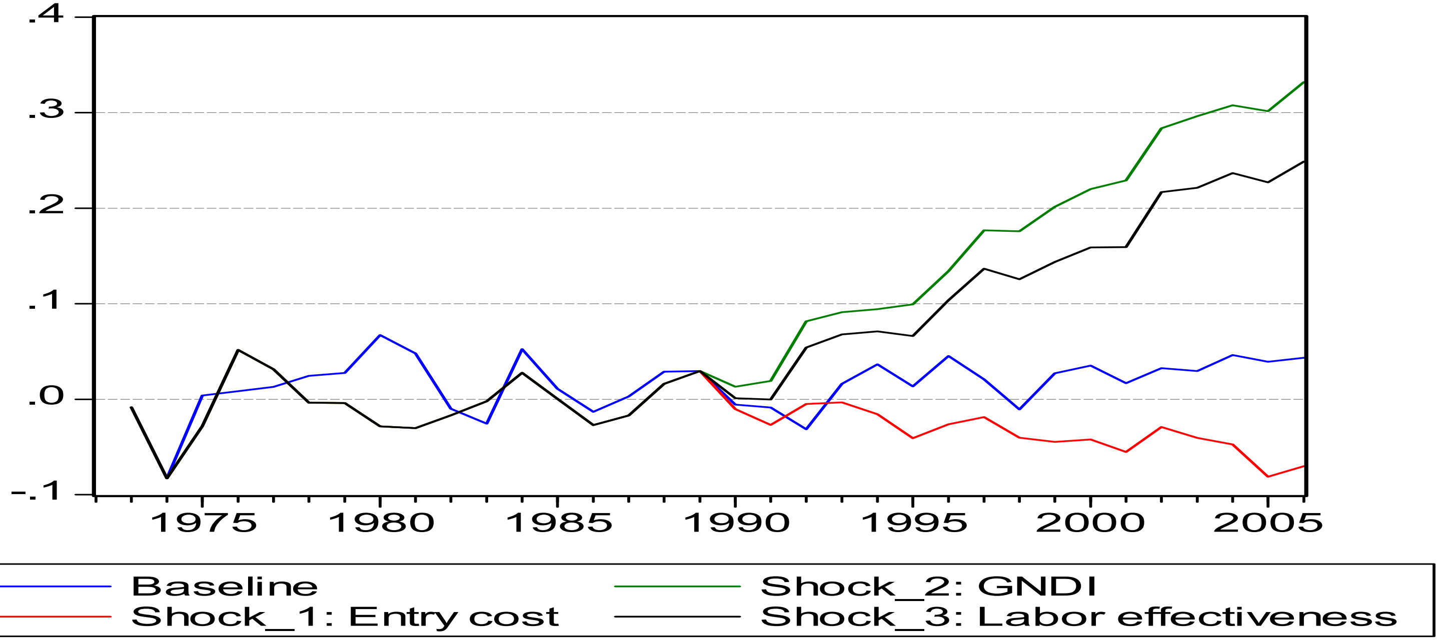


b) Aggregate shock: all three shocks applied simultaneously

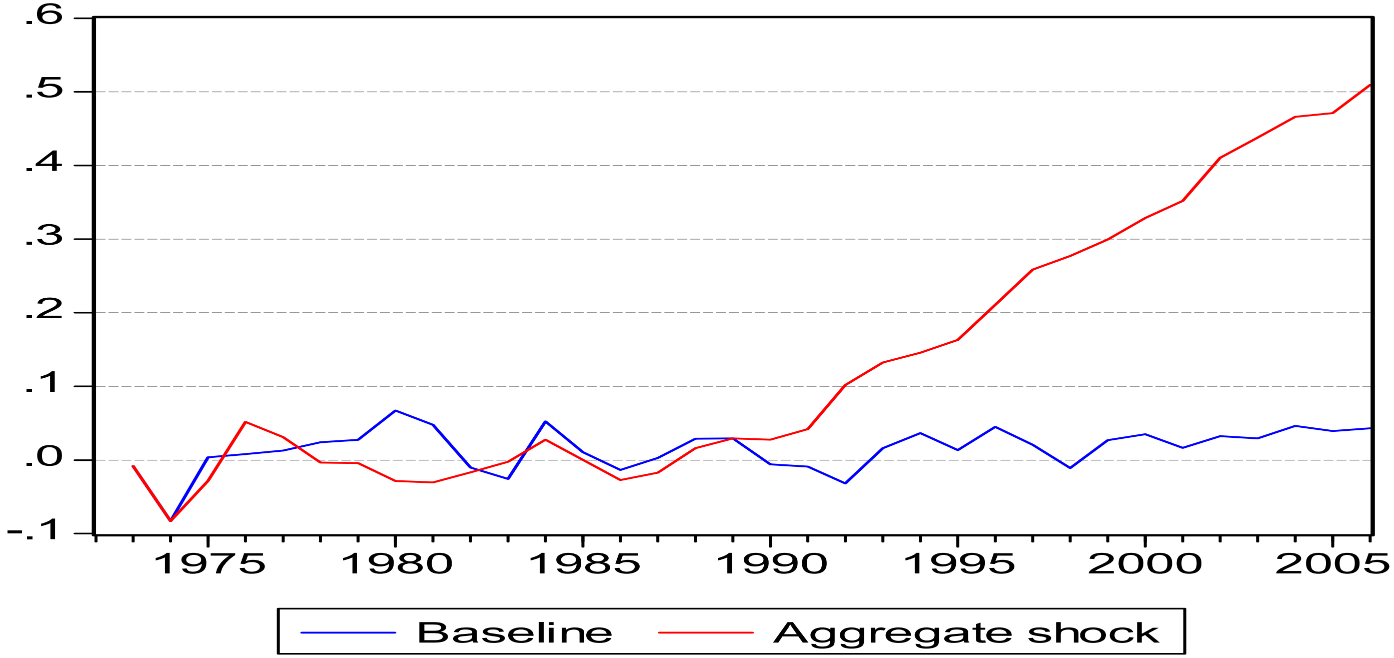


Mining

a) Individual shocks: 10 percent

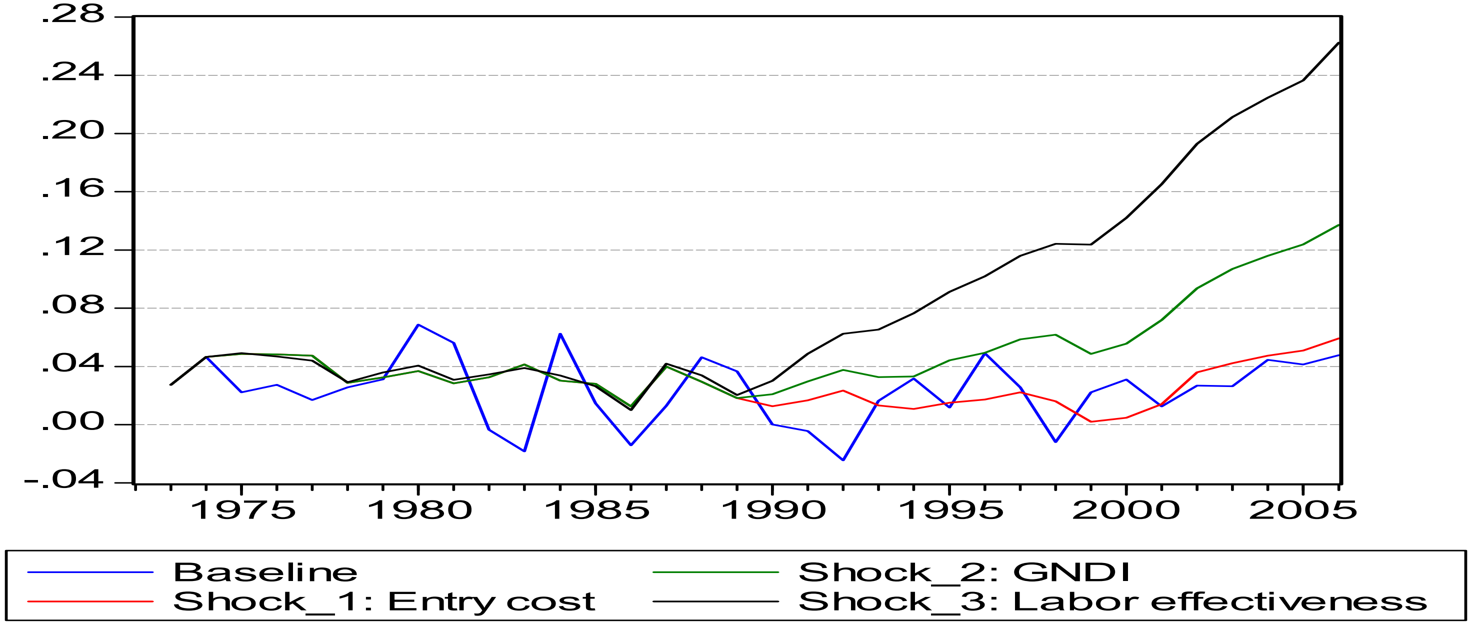


b) Aggregate shock: all three shocks applied simultaneously

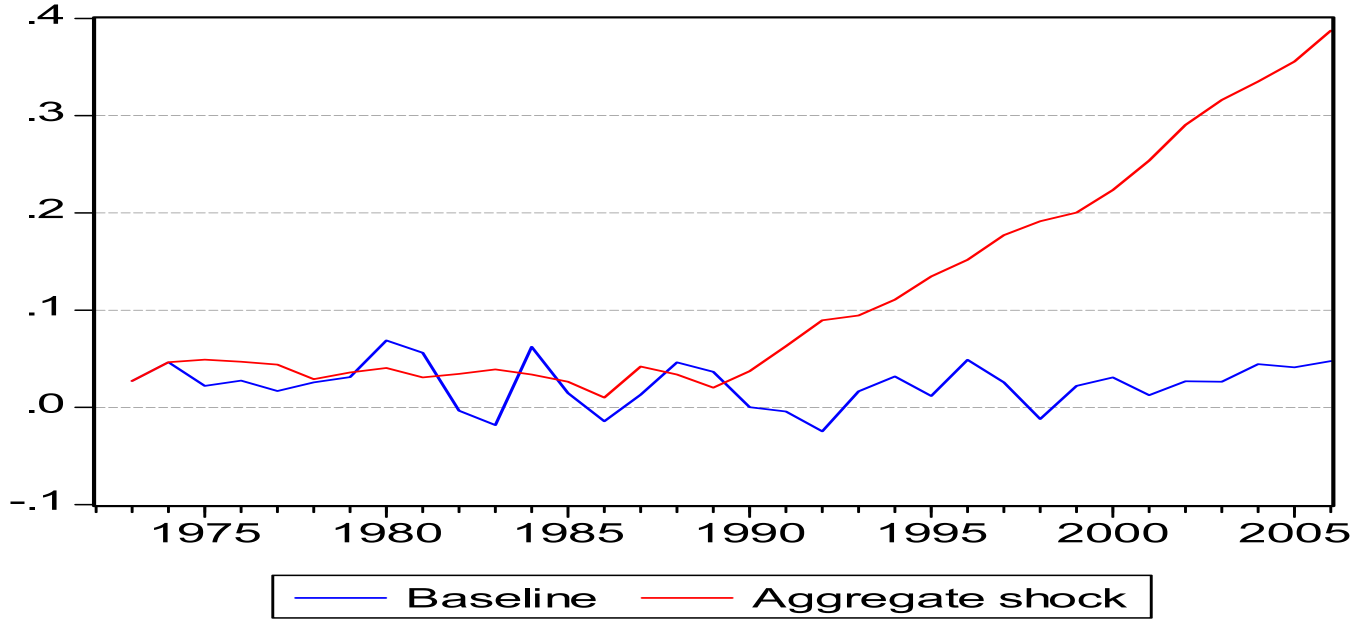


Government

a) Individual shocks: 10 percent

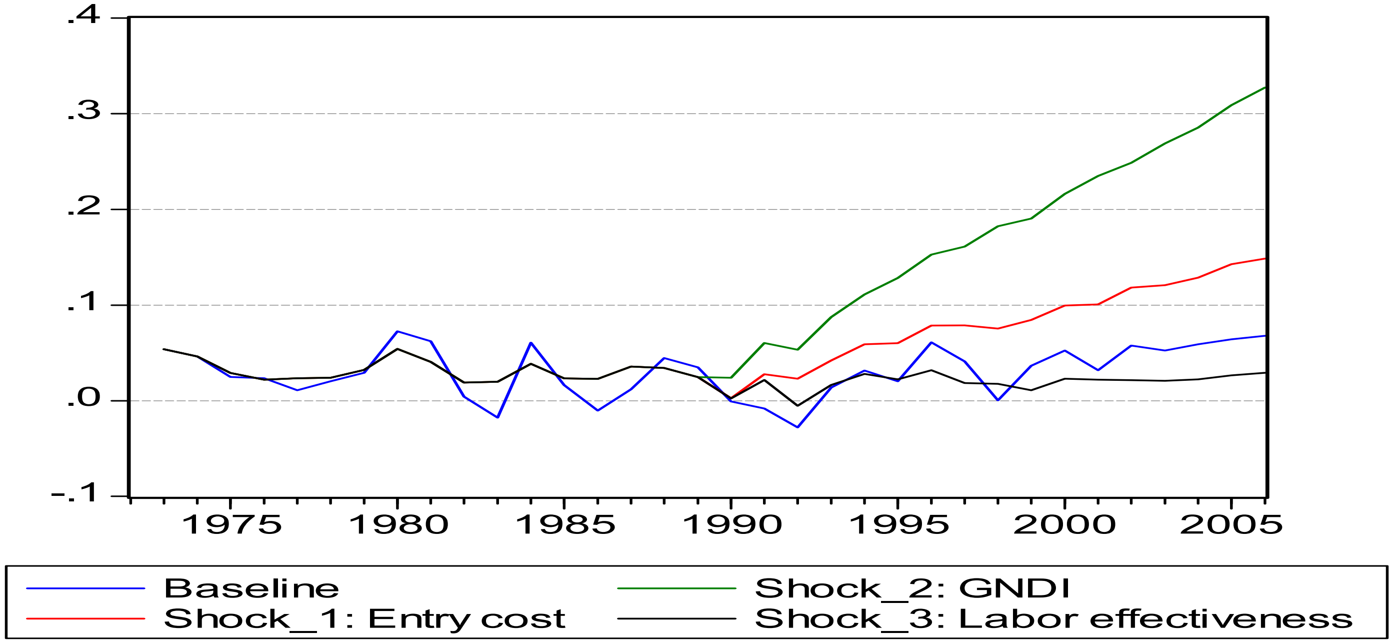


b) Aggregate shock: all three shocks applied simultaneously

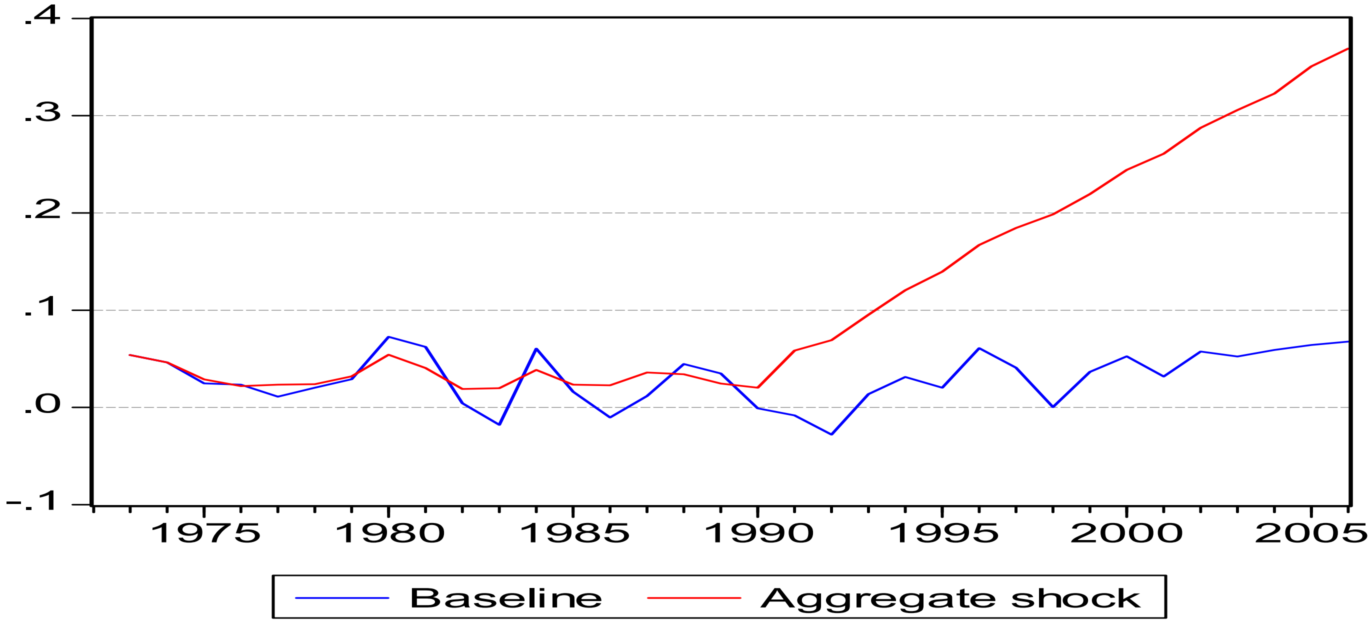


Community

a) Individual shocks: 10 percent

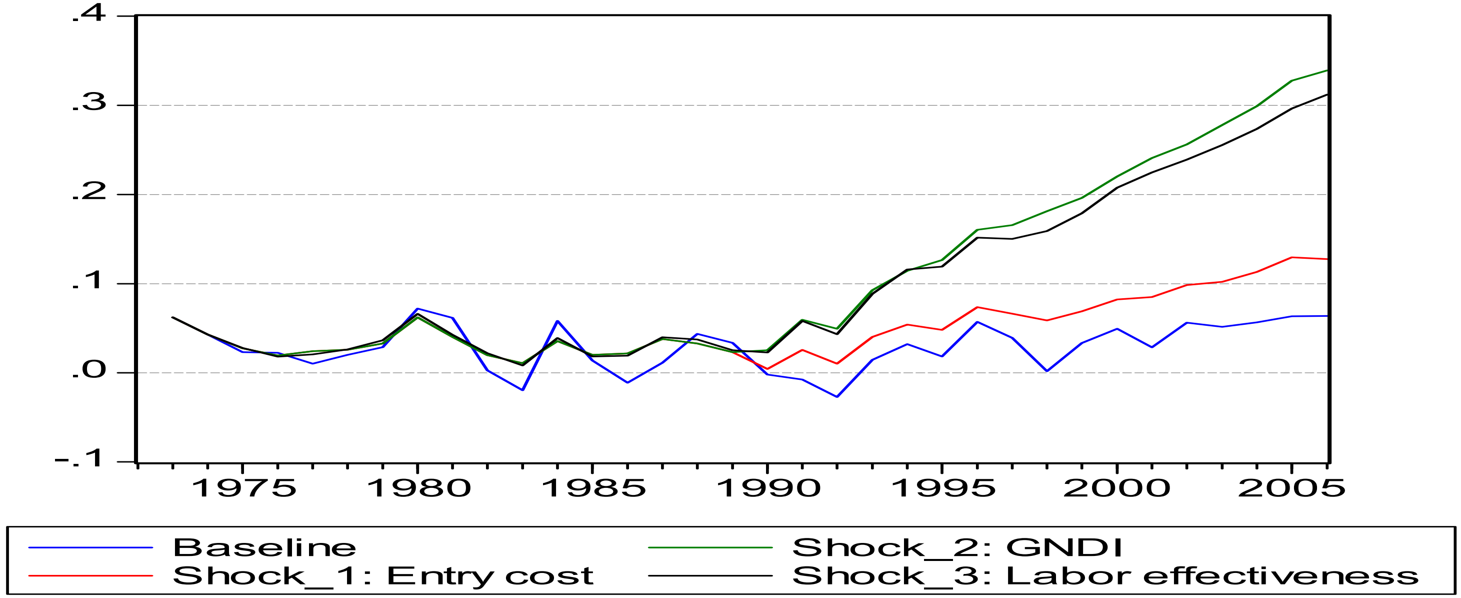


b) Aggregate shock: all three shocks applied simultaneously

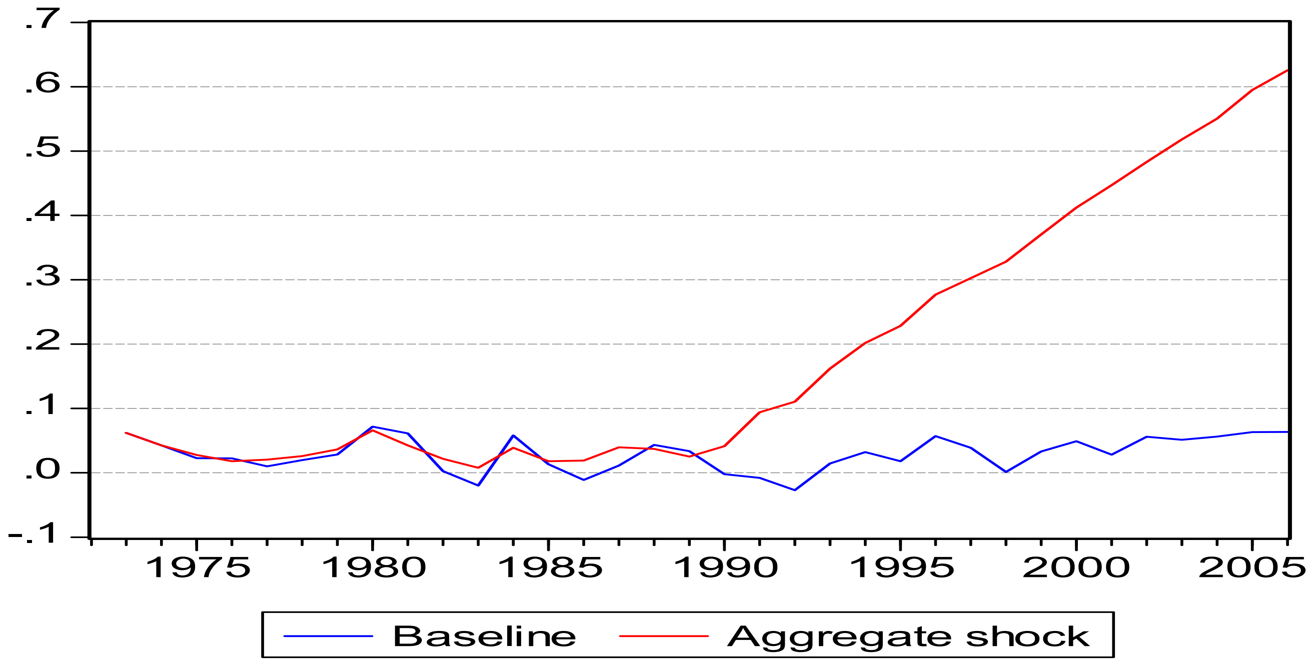


Finance

a) Individual shocks: 10 percent

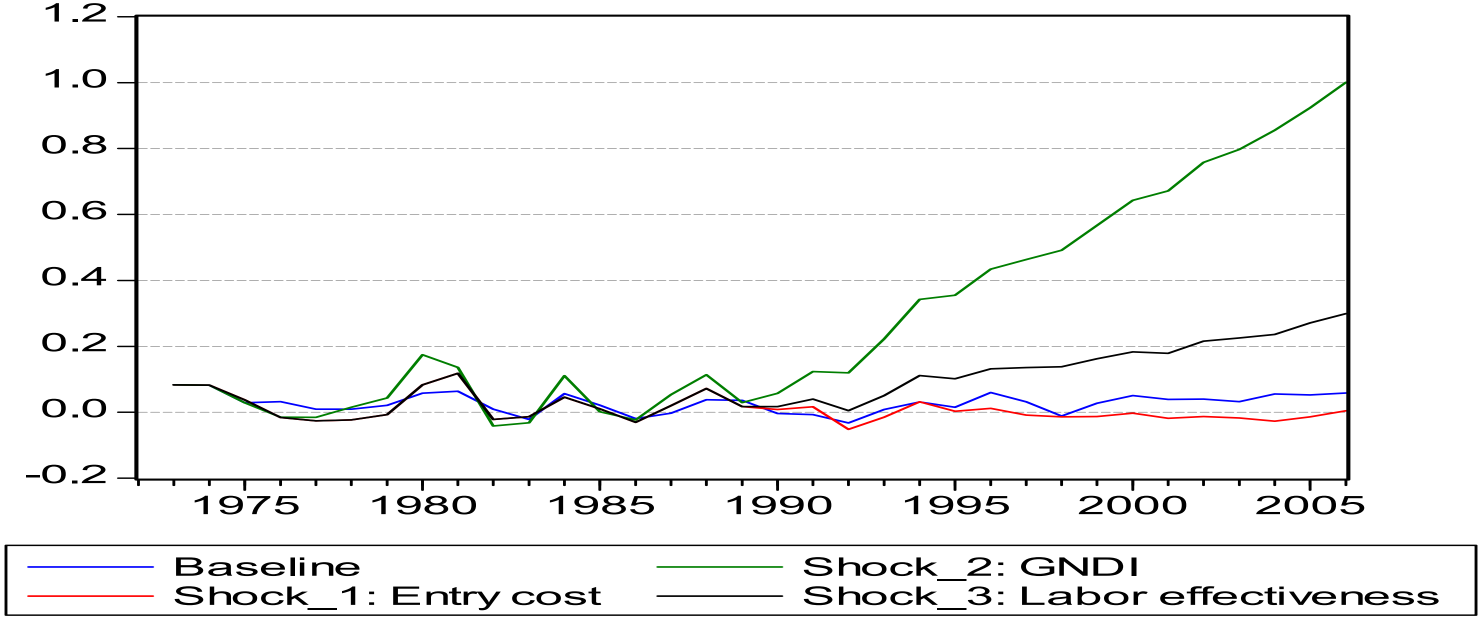


b) Aggregate shock: all three shocks applied simultaneously

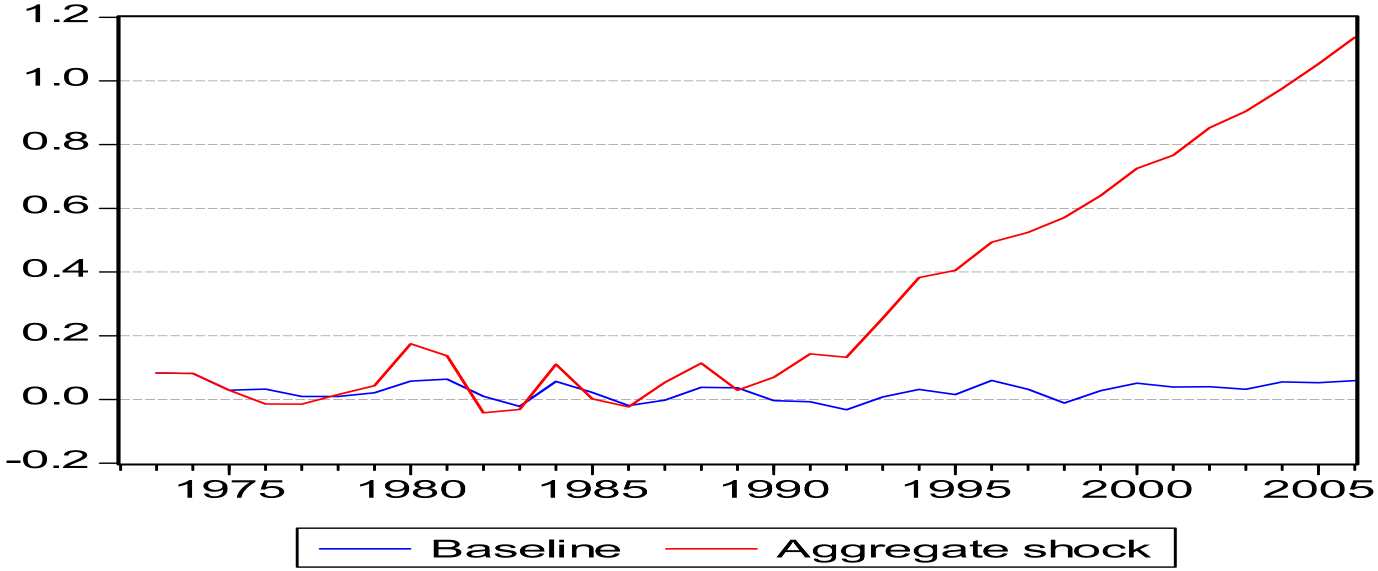


Wholesales

a) Individual shocks: 10 percent



b) Aggregate shock: all three shocks applied simultaneously



CONCLUSIONS AND POLICY RECOMMENDATIONS

- **Extending Marshallian modeling with use of human capital and entry cost provides higher prediction ability;**
- **Thatcher-like reforms are recommendable for SA: GDP can increase as high as 5.1 % (1 percent shock) or 8.1 % (10 percent shock);**
- **Further disaggregation will improve prediction ability;**
- **Improving the quality of public service delivery is an efficient way of promoting growth.**

Contact emails:

- **jacques.kibambe@up.ac.za or jacquesk@dimacs.rutgers.edu**
- **fazellne@uchicago.edu**