Retrospective Approximations of Superlative Price Indexes for Years where Expenditure Data is Unavailable

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April 15, 2008

Abstract: CPIs are typically fixed-weight or Lowe indexes. Several statistical agencies have recently been experimenting with retrospective computations of superlative price index numbers, to provide information on (upper level) substitution bias of their CPIs. This obviously requires expenditure data of all the periods compared. However, detailed expenditure data are often available only for fairly distant benchmark years, particularly for the 'weight-reference periods' of consecutive CPI series. This paper addresses the question of how to approximate a consistent time series of annual superlative price index numbers such that use is made of all the available data. We consider various approximation methods, all of which are based on linear combinations of expenditure shares from benchmark years. The methods are illustrated on a data set consisting of the elementary aggregate price index numbers and expenditure weights that have been used for the computation of the official Danish CPI from 1996 to 2006. We also compare the resulting index numbers with Lloyd-Moulton index numbers.

Keywords: Consumer Price Index; Price Updating and Backdating; Superlative Index.

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[d] The views expressed in this paper are those of the authors and do not necessarily reflect the views of their organizations. We thank Martin B. Larsen (Statistics Denmark) for preparing the data set and David Vellekoop for research assistance.

1. Introduction

The computation of superlative price index numbers is hampered by the fact that this requires quantity or expenditure data for current periods, whereas such data usually become available with considerable time lags. Statistical agencies may want to inform the public about the substitution bias of their Consumer Price Index (CPI) by calculating superlative price index numbers retrospectively. In countries that revise the CPI weights every year, calculating annual superlative index numbers retrospectively is a simple exercise. However, the majority of countries revise the weights less frequently, and their CPIs are typically Lowe indexes where the expenditure weights are fixed for several years. Though detailed expenditure data is lacking for the years in between consecutive weight-reference years, the question can be asked: would it be possible to interpolate superlative price indexes for such intermediate years? This is indeed the case: using a Constant Elasticity of Substitution (CES) framework, a superlative price index can be approximated once we have estimated the elasticity of substitution. The Lloyd-Moulton price index does not make use of current-period expenditure data, so it is even possible to approximate a superlative index in real time and extrapolate the time series.

In this paper we present several alternative methods which make use of all the available data and approximate a consistent time series of annual superlative price index numbers. The idea – which may be appealing to statistical agencies that are reluctant to rely on the assumptions underlying the CES theory – is to approximate the expenditure shares relating to the intermediate years by using linear combinations of expenditure shares from the weight-reference years. Our main aim is to clarify some issues that arise when approximating retrospectively a superlative price index. Furthermore, we argue that the methods applied by a number of researchers could be improved.

By way of introducing the subject, in Section 2 we recall that the Fisher and Törnqvist indexes are instances of a general class of superlative indexes and show how the Lloyd-Moulton index fits in. In Section 3 we describe our approximation methods as well as the Lloyd-Moulton method. The approximations are generalized in Section 4 to the case when the price reference period differs from the weight reference period (as with Lowe CPIs). Section 5 extends the analysis to three or more weight-reference years and suggests chain linking during these years. Section 6 provides an illustrative example based on data that have been used for compiling the Danish CPI. Section 7 concludes.

2. Superlative and Lloyd-Moulton Price Indexes

The Quadratic Mean (QM) of order *r* price index was defined by Diewert (1976) as

$$P_{QM}^{0t}(r) \equiv \left[\frac{\sum_i s_i^0 (p_i^t / p_i^0)^{r/2}}{\sum_i s_i^t (p_i^t / p_i^0)^{-r/2}} \right]^{1/r} = \left\{ \left[\sum_i s_i^0 (p_i^t / p_i^0)^{r/2} \right]^{2/r} \left[\sum_i s_i^t (p_i^t / p_i^0)^{-r/2} \right]^{-2/r} \right\}^{1/2}$$

$$(r \neq 0), \qquad (1)$$

where p_i^0, p_i^t and s_i^0, s_i denote the price and expenditure share of commodity *i* in base period 0 and current or comparison period *t* ($t > 0$), respectively. It is a superlative index. By setting $r = 2(1-\sigma)$ expression (1) becomes

$$P_{QM}^{0t}(2(1-\sigma)) = \left\{ \left[\sum_i s_i^0 (p_i^t / p_i^0)^{1-\sigma} \right]^{1/(1-\sigma)} \left[\sum_i s_i^t (p_i^t / p_i^0)^{-(1-\sigma)} \right]^{-1/(1-\sigma)} \right\}^{1/2}, \qquad (2)$$

which is the geometric mean of the Lloyd (1975)-Moulton (1996) price index

$$P_{LM}^{0t}(\sigma) = \left[\sum_i s_i^0 (p_i^t / p_i^0)^{1-\sigma} \right]^{1/(1-\sigma)} \qquad (3)$$

and its ‘current weight (CW) counterpart’

$$P_{CW}^{0t}(\sigma) = \left[\sum_i s_i^t (p_i^t / p_i^0)^{-(1-\sigma)} \right]^{-1/(1-\sigma)}. \qquad (4)$$

The price index $P_{LM}^{0t}(\sigma)$ monotonically decreases and $P_{CW}^{0t}(\sigma)$ monotonically increases as σ increases, which implies that there exists a unique value σ^{0t} such that[1]

$$P_{LM}^{0t}(\sigma^{0t}) = P_{CW}^{0t}(\sigma^{0t}) = P_{QM}^{0t}(2(1-\sigma^{0t})). \qquad (5)$$

Thus for $\sigma = \sigma^{0t}$ the Lloyd-Moulton index becomes superlative. The drawback of (5) is of course that, unless σ^{0t} happens to be constant over time, we would be using different superlative index number formulas for different periods.

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[1] The Lloyd-Moulton index is a generalized mean of order $1-\sigma$, which is strictly increasing in $1-\sigma$ and thus strictly decreasing in σ. Its ‘current-weight counterpart’ can be written as the inverse of a generalized mean of order $1-\sigma$ and is thus strictly increasing in σ. The solution σ^{0t} must be obtained by some numerical method.

For $\sigma = 0$ the Lloyd-Moulton price index and its CW counterpart reduce to the Laspeyres and Paasche price indexes, respectively, and the QM index reduces to the Fisher price index

$$P_F^{0t} = \left\{ \left[\sum_i s_i^0 (p_i^t / p_i^0) \right] \left[\sum_i s_i^t (p_i^0 / p_i^t) \right]^{-1} \right\}^{1/2} = \left\{ \left[\frac{\sum_i p_i^t q_i^0}{\sum_i p_i^0 q_i^0} \right] \left[\frac{\sum_i p_i^t q_i^t}{\sum_i p_i^0 q_i^t} \right] \right\}^{1/2}, \tag{6}$$

where q_i^0 and q_i^t denote the quantities consumed or purchased in periods 0 and *t*, respectively. If we replace the arithmetic averages of the price relatives in equation (6) by corresponding geometric averages then we obtain the Törnqvist index

$$P_T^{0t} = \left\{ \left[\prod_i (p_i^t / p_i^0)^{s_i^0} \right] \left[\prod_i (p_i^0 / p_i^t)^{s_i^t} \right]^{-1} \right\}^{1/2} = \prod_i (p_i^t / p_i^0)^{(s_i^0 + s_i^t)/2}. \tag{7}$$

As a matter of fact this index would also be obtained if in expression (2) the arithmetic averages were replaced by geometric averages, so this 'trick' is independent of σ. Notice that the QM index is not defined for $\sigma = 1$. It can be shown that for $\sigma \to 1$ $P_{LM}^{0t}(\sigma)$ and $P_{CW}^{0t}(\sigma)$ tend to the Geometric Laspeyres and Paasche price indexes, so that $P_{QM}^{0t}(2(1-\sigma))$ tends to the Törnqvist price index.

In empirical studies, particularly when the price and quantity data exhibit smooth trends, the differences between Fisher or Törnqvist index numbers are often negligible. This seems to corroborate Diewert's (1978, p. 884) finding that "all superlative indexes closely approximate each other".[2]

3. Approximating Superlative Price Indexes

We now consider two distant years 0 and *T* and one or more intermediate years (years in between 0 and *T*). Suppose that price data for all $t = 0,...,T$ are known, and expenditure shares for years 0 and *T*, but that the expenditures shares for the intermediate years are

[2] Yet, not all the superlative price indexes are necessarily numerically similar. The problem is that "as the parameter *r* increases in absolute value, the superlative price (quantity) index number formula becomes increasingly sensitive to outliers in the price-relatives (quantity-relatives) distribution" (Hill, 2006, p. 38). Anyway, for small absolute values of *r*, which is the usual case, we do expect small numerical differences between different superlative indexes.

unavailable. This will be the case for countries that do not annually revise their weights in the CPI but instead revise them, say, every three to five years. Suppose 0 and T are those weight-reference or benchmark years. Obviously, superlative price indexes cannot be computed for intermediate years $t = 1,...,T-1$. The problem addressed here is how to approximate, or interpolate, superlative price indexes, for example P_F^{0t} or P_T^{0t}, given the lack of expenditure data.

3.1 Using Lloyd-Moulton Price Indexes

A first possibility would be estimating Lloyd-Moulton price index numbers. As shown by equation (5), for $\sigma = \sigma^{0t}$ the Lloyd-Moulton formula produces a superlative index going from 0 to t. Due to the unavailability of data, we cannot compute σ^{0t}, but we can compute σ^{0T} and then assume that $\sigma^{0t} \cong \sigma^{0T}$ for $t = 1,...,T-1$. That is, we assume σ^{0t} (which makes the Lloyd-Moulton index equal to the CW index) to be constant over time. Since the Lloyd-Moulton index $P_{LM}^{0T}(\sigma^{0T})$ will be numerically close to Fisher or Törnqvist indexes, we could also compute the value of σ for which $P_{LM}^{0T}(\sigma)$ is equal to P_F^{0T} or P_T^{0T}. The last method was used by Shapiro and Wilcox (1997) and is suggested in the international CPI Manual (ILO *et al*., 2004).

Assuming constancy of the parameter σ is consistent with a Constant Elasticity of Substitution (CES) framework in which σ figures as the elasticity of substitution, which is assumed to be the same for all pairs of commodities. Balk (2000) proposed a two-level, nested CES approach: at the upper aggregation level there is a fixed set of product groups (strata, or elementary aggregates), whereas at the lower level (that is, within the strata or elementary aggregates) the set of commodities is allowed to change over time.[3] An interesting result is that the value of the elasticity of substitution should be less than 1 at the upper level but greater than 1 at the lower level. In this paper we are dealing with the upper level, hence expect a value of σ less than 1. The estimated value will depend on the actual aggregation level. Shapiro and Wilcox (1997), who employed a U.S. data set consisting of 9,108 item-area strata, found that a value of 0.7 generated price index numbers very similar to those computed with the Törnqvist formula.

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[3] Balk (2000) addressed substitution effects as well as the treatment of new and disappearing goods in a nested CES price index. See De Haan (2005), Melser (2006) and Ivancic (2007) for empirical evidence on these topics at the lower aggregation level.

3.2 Using Estimated Expenditure Shares

A more statistically-oriented approach to approximating a superlative price index is the following.[4] Suppose that expenditure shares exhibit reasonably smooth trends. If year t is close to year 0 we would expect s_i^t to be close to s_i^0; moving from benchmark year 0 to benchmark year T the expenditure share s_i will move toward s_i^T. This suggests that we approximate s_i by a moving linear combination of s_i^0 and s_i^T:

$$\hat{s}_i = [ts_i^T + (T-t)s_i^0]/T = (t/T)s_i^T + (1-t/T)s_i^0 \qquad (t = 0,1,....,T), \tag{8}$$

which is a weighted mean of s_i^0 and s_i^T with t/T and $1-t/T$ as weights. The limiting cases of (8) are $\hat{s}_i^t = s_i^0$ and $\hat{s}_i^t = s_i^T$. Thus it is rather natural to approximate the Fisher price index for year t by

$$\hat{P}_F^{0t} = \left\{ \left[\sum_i s_i^0 (p_i^t / p_i^0) \right] \left[\sum_i \hat{s}_i^t (p_i^0 / p_i^t) \right]^{-1} \right\}^{1/2} \tag{9}$$

and the Törnqvist price index by

$$\hat{P}_T^{0t} = \left\{ \left[\prod_i (p_i^t / p_i^0)^{s_i^0} \right] \left[\prod_i (p_i^0 / p_i^t)^{\hat{s}_i^t} \right]^{-1} \right\}^{1/2} = \prod_i (p_i^t / p_i^0)^{(s_i^0 + \hat{s}_i^t)/2} . \tag{10}$$

Of course for $t = T$ we have $\hat{P}_F^{0T} = P_F^{0T}$ and $\hat{P}_T^{0T} = P_T^{0T}$.

A related alternative approach goes as follows. The average shares $(s_i^0 + \hat{s}_i^t)/2$ in (10) can be written as

$$(s_i^0 + \hat{s}_i^t)/2 = (1 - t/2T)s_i^0 + (t/2T)s_i^T , \tag{11}$$

so that expression (10) becomes

$$\hat{P}_T^{0t} = \left[\prod_i (p_i^t / p_i^0)^{s_i^0} \right]^{1-t/2T} \left[\prod_i (p_i^t / p_i^0)^{s_i^T} \right]^{t/2T} . \tag{12}$$

Substituting the identity

$$\prod_i (p_i^t / p_i^0)^{s_i^T} = \prod_i (p_i^t / p_i^T)^{s_i^T} \left[\prod_i (p_i^0 / p_i^T)^{s_i^T} \right]^{-1} \tag{13}$$

into expression (12), we obtain

[4] This subsection draws heavily from an unpublished paper by Balk (1990a).

$$\hat{P}_T^{0t} = \left[\prod_i (p_i^t / p_i^0)^{s_i^0} \right]^{1-t/2T} \left[\frac{\prod_i (p_i^t / p_i^T)^{s_i^T}}{\prod_i (p_i^0 / p_i^T)^{s_i^T}} \right]^{t/2T} . \quad (14)$$

Now, replacing the geometric averages in expression (14) by arithmetic averages, using $s_i^\tau = p_i^\tau q_i^\tau / \sum_i p_i^\tau q_i^\tau$ ($\tau = 0, T$) and doing some rearranging, we define the Quasi Fisher (QF) index:

$$\hat{P}_{QF}^{0t} \equiv \left[\frac{\sum_i p_i^t q_i^0}{\sum_i p_i^0 q_i^0} \right]^{1-t/2T} \left[\frac{\sum_i p_i^t q_i^T}{\sum_i p_i^0 q_i^T} \right]^{t/2T} = \left[\sum_i s_i^0 (p_i^t / p_i^0) \right]^{1-t/2T} \left[\sum_i s_i^{T*} (p_i^t / p_i^0) \right]^{t/2T}, \quad (15)$$

where $s_i^{T*} = (p_i^0 / p_i^T) p_i^T q_i^T / \sum_i (p_i^0 / p_i^T) p_i^T q_i^T = (p_i^0 / p_i^T) s_i^T / \sum_i (p_i^0 / p_i^T) s_i^T$ are price backdated expenditure shares. Expression (15) is a weighted geometric average of a Laspeyres price index and a Lowe price index. Since its limiting values are $\hat{P}_{QF}^{00} = 1$ and $\hat{P}_{QF}^{0T} = P_F^{0T}$, $\hat{P}_{QF}^{0t}$ should be seen as an approximation of the Fisher index.

Triplett (1989a) proposed the following approximation formula, called the Time-series Generalized Fisher Ideal (TGFI) index:

$$\hat{P}_{TGFI}^{0t} \equiv \left[\frac{\sum_i p_i^t q_i^0}{\sum_i p_i^0 q_i^0} \right]^{1/2} \left[\frac{\sum_i p_i^t q_i^T}{\sum_i p_i^0 q_i^T} \right]^{1/2} = \left[\sum_i s_i^0 (p_i^t / p_i^0) \right]^{1/2} \left[\sum_i s_i^{T*} (p_i^t / p_i^0) \right]^{1/2} . \quad (16)$$

Whereas in (15) the exponents depend on *t*, they are fixed at the value 1/2 in (16).[5] Note that $1 - t/2T > 1/2$ and $t/2T < 1/2$ for $0 < t < T$. This suggests that the TGFI index is biased in the sense that it places too less weight on the first component, the Laspeyres index, and too much weight on the second component, the Lowe index. Note also that the Lowe index in (16) should be an approximation of the Paasche index if $\hat{P}_{TGFI}^{0t}$ is meant to approximate the Fisher index. Under normal circumstances the Paasche index will be less than the Laspeyres. Thus, the TGFI index most likely understates the Fisher index if the Lowe index in (16), which has been given too much weight, will be less than the Paasche. Such a situation is likely to happen if long-run trends in relative price

[5] All indexes are 'forward looking'; they are going from benchmark year 0 to year *t*. Retrospectively we could also calculate 'backward looking' indexes going from benchmark year *T* to year *t*; see also Triplett (1989b). Given some approximation method, the product of the forward looking index going from 0 to *t* and the inverse of the backward looking index going from *T* to *t* will in general not be equal to the index going from 0 to *T*.

changes exist. To illustrate this point, suppose the price change between the benchmark years, p_i^T / p_i^0, of commodity i is relatively large. When relative price changes are persistent, the price change between year 0 and year t, p_i^t / p_i^0, is also relatively large as is the price change between year t and year T, p_i^t / p_i^T. Due to substitution effects we then expect $q_i^T < q_i^t$ for such commodities, and hence the Lowe index to be less than the Paasche index.

4. Lowe CPIs and Approximate Superlative Price Indexes

In practice it takes some time to compile the weighting scheme of a CPI. Consequently, the typical CPI is a Lowe index instead of a Laspeyres index, based on quantities or expenditure shares pertaining to one or more years preceding the index reference period. Let 0 be the quantity reference year and b ($b > 0$) the price reference year. The Lowe CPI going from year b to year t ($t > b$) with quantity reference year 0 is then

$$P_L^{bt} = \frac{\sum_i p_i^t q_i^0}{\sum_i p_i^b q_i^0} = \sum_i s_i^{0*} (p_i^t / p_i^b), \tag{17}$$

where $s_i^{0*} = (p_i^b / p_i^0) p_i^0 q_i^0 / \sum_i (p_i^b / p_i^0) p_i^0 q_i^0 = (p_i^b / p_i^0) s_i^0 / \sum_i (p_i^b / p_i^0) s_i^0$ are price updated expenditure shares.[6] Now we show how the approximations of a superlative index described in Sub-section 3.2 can be generalized to the situation in which $0 < b < t$ ($t = 0,1,....,T$).

First, similar to expression (8) the expenditure shares pertaining to period b are approximated by

$$\hat{s}_i^b = (b/T) s_i^T + (1 - b/T) s_i^0 . \tag{18}$$

Then, natural approximations of the Fisher and Törnqvist indexes, P_F^{bt} and P_T^{bt}, are

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[6] In some countries the CPI is a Young index, which results from replacing the price-updated expenditure shares in (17) by the actual shares of period 0. A Young index can be interpreted in different ways. For example, statistical agencies computing a Young index may target at a Laspeyres index with price and weight reference period b and use the period 0 shares as estimates of the period b shares. Essentially they assume that the elasticity of substitution between any two commodities is equal to 1. In Denmark the Young CPI is interpreted as an approximation of the (superlative) Walsh index (Boldsen Hansen, 2006; 2007). In Section 6 we present evidence for Denmark on the value of the elasticity of substitution.

$$\hat{P}_F^{bt} = \left\{ \left[\sum_i \hat{s}_i^b (p_i^t / p_i^b) \right] \left[\sum_i \hat{s}_i^t (p_i^b / p_i^t) \right]^{-1} \right\}^{1/2}, \tag{19}$$

$$\hat{P}_T^{bt} = \prod_i (p_i^t / p_i^b)^{(\hat{s}_i^b + \hat{s}_i^t)/2}, \tag{20}$$

respectively.

Second, the average shares in (20) are equal to

$$(\hat{s}_i^b + \hat{s}_i^t)/2 = [1-(b+t)/2T]s_i^0 + [(b+t)/2T]s_i^T, \tag{21}$$

so that (20) becomes

$$\hat{P}_T^{bt} = \left[\prod_i (p_i^t / p_i^b)^{s_i^0} \right]^{1-(b+t)/2T} \left[\prod_i (p_i^t / p_i^b)^{s_i^T} \right]^{(b+t)/2T}. \tag{22}$$

Substituting

$$\prod_i (p_i^t / p_i^b)^{s_i^0} = \prod_i (p_i^t / p_i^0)^{s_i^0} \left[\prod_i (p_i^b / p_i^0)^{s_i^0} \right]^{-1} \tag{23}$$

and

$$\prod_i (p_i^t / p_i^b)^{s_i^T} = \prod_i (p_i^t / p_i^T)^{s_i^T} \left[\prod_i (p_i^b / p_i^T)^{s_i^T} \right]^{-1} \tag{24}$$

into (22), we obtain

$$\hat{P}_T^{bt} = \left[\frac{\prod_i (p_i^t / p_i^0)^{s_i^0}}{\prod_i (p_i^b / p_i^0)^{s_i^0}} \right]^{1-(b+t)/2T} \left[\frac{\prod_i (p_i^t / p_i^T)^{s_i^T}}{\prod_i (p_i^b / p_i^T)^{s_i^T}} \right]^{(b+t)/2T}. \tag{25}$$

Then, replacing all four geometric averages in (25) by arithmetic averages gives rise to the alternative (Quasi Fisher) approximation:

$$\hat{P}_{QF}^{bt} \equiv \left[\frac{\sum_i p_i^t q_i^0}{\sum_i p_i^b q_i^0} \right]^{1-(b+t)/2T} \left[\frac{\sum_i p_i^t q_i^T}{\sum_i p_i^b q_i^T} \right]^{(b+t)/2T}$$

$$= \left[\sum_i s_i^{0*} (p_i^t / p_i^b) \right]^{1-(b+t)/2T} \left[\sum_i \tilde{s}_i^{T*} (p_i^t / p_i^b) \right]^{(b+t)/2T}. \tag{26}$$

$\hat{P}^{bt}_{QF}$ is a weighted geometric mean of the Lowe index (17) and a Lowe index based on price backdated shares $\tilde{s}_i^{T*} = (p_i^b / p_i^T)s_i^T / \sum_i (p_i^b / p_i^T)s_i^T$. Note that these shares differ from those in expression (15). However, (26) reduces to (15) for $b = 0$.

To obtain a price index going from *b* to *t* one could also use (15) two times and divide $\hat{P}^0$ by $\hat{P}^{0b}$. However, the resulting index cannot be called an approximation of a superlative index and will most likely differ from $\hat{P}^b$.

Finally, the unweighted counterpart to (26) is

$$\hat{P}^{bt}_{TGFI} \equiv \left[\frac{\sum_i p_i^t q_i^0}{\sum_i p_i^b q_i^0}\right]^{1/2} \left[\frac{\sum_i p_i^t q_i^T}{\sum_i p_i^b q_i^T}\right]^{1/2} = \left[\sum_i s_i^{0*}(p_i^t / p_i^b)\right]^{1/2} \left[\sum_i \tilde{s}_i^{T*}(p_i^t / p_i^b)\right]^{1/2}. \qquad (27)$$

Nimmo *et al.* (2007) retrospectively approximated quarterly Fisher price index numbers for New Zealand from 2002 to 2006 but seem to have used a slightly adjusted version of expression (27).[7] Their first weight-reference period is not a calender year but a broken year 2000/01, and these weights were price updated to the June 2002 quarter. Most interestingly, Nimmo *et al.* (2007) extended their time series beyond the second weight-reference period (2003/04) to the June 2006 quarter.[8] In an annual framework, this implies extrapolating the time series by applying (27) to years $t > T$, which yields

$$\hat{P}^{bt}_{TGFI} = \left[\frac{\sum_i p_i^T q_i^0}{\sum_i p_i^b q_i^0}\right]^{1/2} \left[\frac{\sum_i p_i^T q_i^T}{\sum_i p_i^b q_i^T}\right]^{1/2} \left[\frac{\sum_i p_i^t q_i^0}{\sum_i p_i^T q_i^0}\right]^{1/2} \left[\frac{\sum_i p_i^t q_i^T}{\sum_i p_i^T q_i^T}\right]^{1/2}$$

$$= \hat{P}^{bT}_{TGFI} \left[\frac{\sum_i p_i^t q_i^0}{\sum_i p_i^T q_i^0}\right]^{1/2} \left[\frac{\sum_i p_i^t q_i^T}{\sum_i p_i^T q_i^T}\right]^{1/2} = \hat{P}^{bT}_{TGFI} \left\{\left[\sum_i \tilde{s}_i^{0*}(p_i^t / p_i^T)\right]\left[\sum_i s_i^T (p_i^t / p_i^T)\right]\right\}^{1/2}, \qquad (28)$$

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[7] Unfortunately their description is not entirely clear on this point. Quarterly index numbers can indeed be calculated using (27). However, the computation of quarterly or monthly (approximate) superlative indexes may not be very useful as seasonality disturbs their interpretation. Diewert (2000) discusses the problems faced when constructing annual superlative index numbers using monthly price data. The U.S. Bureau of Labor Statistics publishes an experimental monthly superlative index (see Cage *et al*., 2003).

[8] It should be mentioned that "in some cases adjustments were made to reflect quantity changes since 2003/04" (Nimmo *et al*., 2007, p. 4).

where $\tilde{s}_i^{0*} = (p_i^T / p_i^0)s_i^0 / \sum_i (p_i^T / p_i^0)s_i^0$ are price updated shares of benchmark year 0. The right-hand side of (28) is the product of two factors: $\hat{P}_{TGFI}^{bT}$, given by expression (27) for benchmark year *T*, and the unweighted geometric mean of two price indexes going from year *T* to year *t*, a Lowe index (based on year 0 quantities) and a Laspeyres index.[9] The first factor is an *unweighted* mean of two Lowe indexes instead of the preferred weighted mean. The second factor will generally be a biased measure of price change between year *T* and year *t*. It is obvious that $\hat{P}_{TGFI}^{bt}$, given by (28), cannot be considered as an approximation of a superlative index.

5. Three or More Benchmark Years

Except for the Lloyd-Moulton approach, a time series of approximate superlative index numbers between two benchmark years (weight-reference years) can only be extended when expenditure data from a third benchmark year become available. Suppose we have expenditure data of three benchmark years: 0, T^1 and T^2. Extending expression (8), expenditure shares for intermediate years *t* ($t = 0,...,T^2$) are approximated as

$$\hat{s}_i^t = (t/T^1)s_i^{T^1} + (1 - t/T^1)s_i^0 \qquad \text{for } 0 \le t \le T^1; \qquad (29a)$$

$$\hat{s}_i^t = [(t - T^1)/(T^2 - T^1)]s_i^{T^2} + [1 - (t - T^1)/(T^2 - T^1)]s_i^{T^1} \qquad \text{for } T^1 < t \le T^2. \qquad (29b)$$

There are two main scenarios for extending the time series to year T^2: 1) a direct index going from 0 to T^2, or 2) a chained index as the product of a direct index going from 0 to T^1 and a direct index going from T^1 to T .

Let us start with the first scenario. Using expressions (29a) and (29b) the natural approximations, given by (9) and (10), are still valid. The alternative approximation becomes a little bit more complicated. For $T^1 < t \le T^2$ the average of year 0 and year *t* expenditure shares can be written as

$$(s_i^0 + \hat{s}_i^t)/2 = s_i^0/2 + \{[1 - (t - T^1)/(T^2 - T^1)]s_i^{T^1} + [(t - T^1)/(T^2 - T^1)]s_i^{T^2}\}/2. \qquad (30)$$

A similar line of reasoning as in Subsection 3.2 then yields the following approximation of the Fisher index for $T^1 < t \le T^2$:

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[9] For a discussion of the substitution bias of a Lowe price index, see Balk and Diewert (2004).

$$\hat{P}_{QF}^{0t} = \left[\frac{\sum_i p_i^t q_i^0}{\sum_i p_i^0 q_i^0} \right]^{1/2} \left\{ \left[\frac{\sum_i p_i^t q_i^{T^1}}{\sum_i p_i^0 q_i^{T^1}} \right]^{1-\frac{t-T^1}{T^2-T^1}} \left[\frac{\sum_i p_i^t q_i^{T^2}}{\sum_i p_i^0 q_i^{T^2}} \right]^{\frac{t-T^1}{T^2-T^1}} \right\}^{1/2}$$

$$= \left[\sum_i s_i^0 (p_i^t / p_i^0) \right]^{1/2} \left\{ \left[\sum_i s_i^{T^{1*}} (p_i^t / p_i^0) \right]^{1-\frac{t-T^1}{T^2-T^1}} \left[\sum_i s_i^{T^{2*}} (p_i^t / p_i^0) \right]^{\frac{t-T^1}{T^2-T^1}} \right\}^{1/2}, \qquad (31)$$

where $s_i^{T^{1*}} = (p_i^0 / p_i^{T^1}) s_i^{T^1} / \sum_i (p_i^0 / p_i^{T^1}) s_i^{T^1}$ and $s_i^{T^{2*}} = (p_i^0 / p_i^{T^2}) s_i^{T^2} / \sum_i (p_i^0 / p_i^{T^2}) s_i^{T^2}$ are price backdated expenditure shares. The first component, between square brackets, at the right-hand side of (31) is a Laspeyres price index, and the factor between braces approximates a Paasche price index. Expression (31) can easily be extended to four or more benchmark years.

Another possibility to approximate a direct superlative index would be to act as if (benchmark) year T^1 was just another intermediate year and ignore the observed expenditure shares. Replacing actually observed expenditure shares by estimated values is clearly not advisable. We will nevertheless try it out in Section 6 to get an impression of how well the various approximation methods perform in case of two very distant benchmark years.

The second scenario is to calculate a chained version of $\hat{P}_{QF}^{0t}$ for $T^1 < t \leq T^2$, defined by $\hat{P}_{QF,chain}^{0t} \equiv \hat{P}_{QF}^{0T^1} \hat{P}_{QF}^{T^1 t}$. Using (15) and the fact that $\hat{P}_{QF}^{0T^1} = P_F^{0T^1}$, we obtain

$$\hat{P}_{QF,chain}^{0t} = P_F^{0T^1} \left[\frac{\sum_i p_i^t q_i^{T^1}}{\sum_i p_i^{T^1} q_i^{T^1}} \right]^{1-\frac{t-T^1}{2(T^2-T^1)}} \left[\frac{\sum_i p_i^t q_i^{T^2}}{\sum_i p_i^{T^1} q_i^{T^2}} \right]^{\frac{t-T^1}{2(T^2-T^1)}} . \qquad (32)$$

$\hat{P}_{QF}^{0t}$ and $\hat{P}_{QF,chain}^{0t}$ will usually differ, just like the unknown Fisher index and its chained counterpart will differ, though in practice the differences might be limited. Chaining has practical advantages. For example, there is no need to price backdate the expenditure shares relating to T^1 and T^2 to year 0. Further, statistical agencies must regularly revise commodity classification schemes. Changes in the number of commodity groups at the upper level of aggregation, or in their definitions, would make the computation of direct index numbers problematic.[10]

[10] As will be explained in Section 6, our data have been adjusted to account for such changes.

Chaining is also useful when estimating retrospectively Lloyd-Moulton price index numbers. The use of a direct index combined with a fixed value for the elasticity of substitution σ for $0 \le t \le T^1$ *and* $T^1 < t \le T^2$ means that the index numbers in year T^1 or in year T^2 (or both) will differ from the true Fisher or Törnqvist index numbers. While not entirely consistent with the CES theory, it seems better to estimate separate values for σ for $0 \le t \le T^1$ and $T^1 < t \le T^2$ so that $P_{LM}^{0T^1}(\sigma) = P_F^{0T^1}$ (or alternatively $P_{LM}^{0T^1}(\sigma) = P_T^{0T^1}$) and $P_{LM}^{T^1T^2}(\sigma) = P_F^{T^1T^2}$ (or $P_{LM}^{T^1T^2}(\sigma) = P_T^{T^1T^2}$).

6. Data and Empirical Evidence

6.1 Some Facts and Figures

The various methods will be illustrated on building blocks for the official Danish CPI. Our data set concerns 444 elementary aggregates. Monthly price index numbers are available from January 1996 to December 2006 (1996=100), and expenditure shares (CPI weights) for the weight-reference years 1994, 1996, 1999, and 2003. During this ten-year period Statistics Denmark made a number of changes in the set of elementary aggregates. To establish a coherent data set that allows us to calculate direct as well as chained index numbers, some elementary aggregates have been left out and some have been merged. In a few cases price changes have been imputed from those of similar elementary aggregates. These modifications, however, have a limited effect because the elementary aggregates concerned have low weights in the CPI.[11] Annual price index numbers are computed as arithmetic means of the twelve monthly index numbers. Price index numbers for 1994 and 1995 (1994=100) are not available. Using the expenditure shares for 1996, 1999, and 2003, we calculate 'true' direct and chained superlative price index numbers for 1999 and 2003 and approximate superlative index numbers for the intermediate years 1997, 1998, 2000, 2001, and 2002.

Table 1 contains Laspeyres, Paasche, Geometric Laspeyres, Geometric Paasche, Fisher and Törnqvist price index numbers for 1999 and 2003. Let us focus first on the direct indexes for 1999 (1996=100) and 2003 (1999=100) shown in the first and third

[11] The resulting 444 elementary aggregates account for over 98% of the total CPI weight. Boldsen Hansen (2007) has shown that re-calculating the Danish (Young) CPI with this data set produces index numbers that differ only marginally from the officially published figures.

column. As expected, the Laspeyres index numbers are greater than the Paasche numbers. The Geometric Laspeyres is less than the ordinary Laspeyres – as it should be according to Jensen's Inequality – while the Geometric Paasche is greater than the ordinary Paasche. Notice that the Geometric Laspeyres and Paasche indexes in 1999 (1996=100) coincide while in 2003 (1999=100) the difference between them is very small. This suggests that the elasticity of substitution has a value of (almost) 1, which is extraordinarily high. In turn this suggests that, for this particular data set, the Geometric Laspeyres might be an acceptable approximation of a superlative price index for the intermediate years. We will come back to this issue in Sub-section 6.2.

In accordance with our expectations the Fisher and Törnqvist index numbers are quite similar for 1999 (1996=100) and 2003 (1999=100). There is a surprisingly large difference, however, between the direct Fisher and Törnqvist price indexes going from 1996 to 2003 (116.58 and 116.85, respectively). The upper level substitution bias of the Laspeyres index, as measured by the difference with the Fisher, amounts to 0.11 %-points on average per year during 1996-1999 and 0.17 %-points during 1999-2003.[12]

Table 1. Direct and chained price index numbers, 1999 and 2003

	Direct indexes			Chained indexes (1996=100)	
	1996=100		1999=100		
	1999	2003	2003	1999	2003
Laspeyres	106.69	117.90	110.74	106.69	118.15
Paasche	106.00	115.27	109.40	106.00	115.96
Fisher	106.34	116.58	110.07	106.34	117.05
Geometric Laspeyres	106.38	116.54	109.96	106.38	116.97
Geometric Paasche	106.38	117.15	110.16	106.38	117.20
Törnqvist	106.38	116.85	110.06	106.38	117.08

[12] These figures are in between the estimates of 0.2 %-points per year on average by Shapiro and Wilcox (1997) for the U.S. and 0.1 %-points per year by Balk (1990b) and De Haan (1999) for the Netherlands. The differences will be partly due to differences in the aggregation level. The Dutch figures are based on approximately 100 product categories, the U.S. figures on more than 9,000 item-area strata.

The difference between the chained Laspeyres and Fisher price index numbers in 2003 (1996=100), shown in the last column of Table 1, is 0.15 %-points on average per year. The difference between their direct counterparts in the second column is as large as 0.19 %-points per year. What is surprising as well is that chain linking in 1999 raises the Laspeyres index in 2003 from 117.90 to 118.15, and also raises the Paasche, thereby raising the Fisher index from 116.58 to 117.05. The upward effect of chaining, though less strong, goes for the Törnqvist as well.

A number of 'unusual' data seem to have contributed to some of these unexpected findings. During 1999-2003 several services showed extreme price increases: the prices of financial services, car insurance and gardening rose by 33%, 47% and 152%, respectively. At the same time their expenditure shares increased sharply, which is counterintuitive – one would expect consumers to substitute away from services that have become relatively much more expensive. We decided not to exclude these data as unusual things do happen now and then and reflect reality (assuming the expenditures were correctly measured).

6.2 Empirical Results

As mentioned in Section 5 we prefer chain linking in 1999 to calculating direct indexes. Table 2 lists approximate chained price index numbers for the intermediate years 1997-1998 and 2000-2002; the true numbers for the benchmark years 1999 and 2003, copied from Table 1, are also presented. The Fisher and Törnqvist index numbers in the third and sixth row are based on the 'natural' approach of expressions (9) and (10) and differ only marginally from each other. Particularly during 2000-2002 the alternative 'Quasi Fisher' index numbers based on expression (15) are slightly higher. In line with what was suggested in Sub-section 3.2, the price index numbers estimated by the Time-series Generalized Fisher Ideal (TGFI) method (16) are lower.

Table 2 also contains two different versions of chained Lloyd-Moulton price index numbers. The value for the elasticity of substitution σ is estimated separately for each subperiod (1996-1999 and 1999-2003) such that the Lloyd-Moulton index equals either the Fisher or the Törnqvist in 1999 and 2003. The estimated elasticities differ appreciably; the values used for computing the Lloyd-Moulton index numbers in row nine are 1.11 (1996-1999) and 0.85 (1999-2003), whereas those used for computing the Lloyd-Moulton index numbers in row ten are 0.99 (1996-1999) and 0.87 (1999-2003).

The estimates for the first sub-period are rather high and might be related to anomalies in the data set used. Notwithstanding these estimates, the resulting Lloyd-Moulton index numbers for 1997 and 1998, as well as those for 2000, 2001 and 2002, are quite similar to the earlier approximations. This suggests that the 'theoretically-oriented' CES-type method and our 'statistically-oriented' methods based on taking linear combinations of the expenditures shares of the benchmark years 1996, 1999 and 2003, might both be considered by statistical agencies.

Table 2. Chained price index numbers (1996=100)*

	1997	1998	1999	2000	2001	2002	2003
Laspeyres	102.11	104.03	106.69	109.88	112.59	115.62	118.15
Paasche	*102.03*	*103.74*	106.00	*108.86*	*111.20*	*113.81*	115.96
Fisher	*102.07*	*103.88*	106.34	*109.37*	*111.90*	*114.71*	117.05
Geometric Laspeyres	102.06	103.88	106.38	109.41	111.94	114.71	116.97
Geometric Paasche	*102.08*	*103.90*	106.38	*109.40*	*111.97*	*114.83*	117.20
Törnqvist	*102.07*	*103.89*	106.38	*109.41*	*111.96*	*114.77*	117.08
Quasi Fisher	*102.09*	*103.90*	106.34	*109.47*	*112.03*	*114.82*	117.05
TGFI	*102.04*	*103.83*	106.34	*109.29*	*111.83*	*114.68*	117.05
Lloyd-Moulton a)	*102.06*	*103.86*	106.34	*109.40*	*111.95*	*114.75*	117.05
Lloyd-Moulton b)	*102.06*	*103.88*	106.38	*109.43*	*111.99*	*114.79*	117.08

* Approximations are shown in italics; a) Fisher index as benchmark; b) Törnqvist index as benchmark.

Table 3. Direct price index numbers (1996=100)*

	1997	1998	1999	2000	2001	2002	2003
Laspeyres	102.11	104.03	106.69	109.88	112.55	115.39	117.90
Paasche	*102.03*	*103.74*	106.00	*108.68*	*110.89*	*113.25*	115.27
Fisher	*102.07*	*103.88*	106.34	*109.28*	*111.72*	*114.31*	116.58
Geometric Laspeyres	102.06	103.88	106.38	109.29	111.72	114.29	116.54
Geometric Paasche	*102.08*	*103.90*	106.38	*109.39*	*111.95*	*114.80*	117.15
Törnqvist	*102.07*	*103.89*	106.38	*109.34*	*111.83*	*114.54*	116.85
Quasi Fisher	*102.09*	*103.90*	106.34	*109.39*	*111.84*	*114.45*	116.58

* Approximations are shown in italics.

Table 3 shows the direct counterparts to the approximations presented in Table 2. The Quasi Fisher (alternative) approximations, shown in the last row, overestimate the corresponding natural approximations, shown in the third row, especially for 2000-2002. This may, at least partially, be caused by the fact that the Quasi Fisher approach essentially approximates a Törnqvist index – which in our case is greater than the Fisher – and then converts the result into a 'Fisher-type' formula. Notice that the Geometric Laspeyres index numbers nearly coincide with the natural Fisher approximations.

Table 4 contains approximate direct index numbers (1996=100), computed as if expenditure data for 1999 were unavailable. Thus, the true 1999 index numbers in Table 3 are replaced by estimated values. Yet the index numbers in Table 4 calculated with the natural method are very similar to the corresponding numbers in Table 3, including the true 1999 numbers. This seems to indicate that this method works well even for rather distant benchmark years. As before, the Quasi Fisher index numbers are greater than those computed with the natural approach, the difference being 0.10 %-points in 2000.

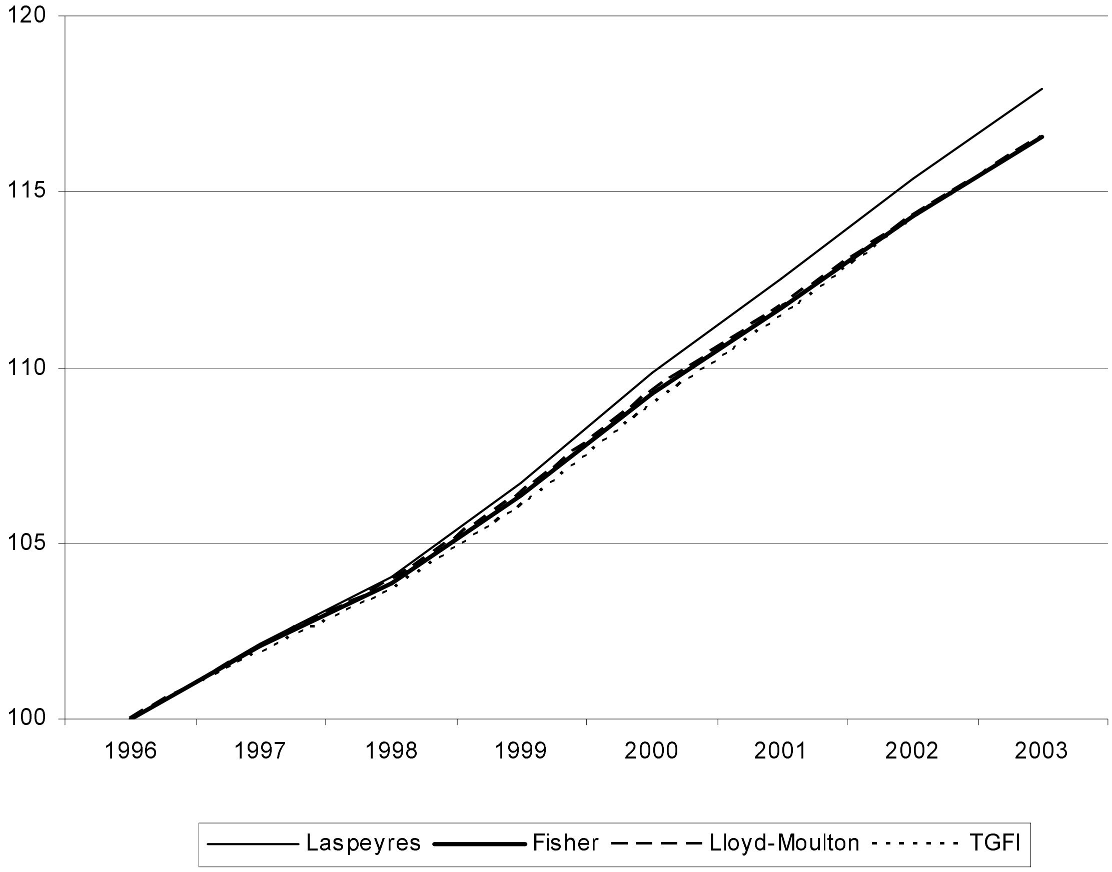
Table 4. Direct price index numbers (1996=100), excluding observed expenditure shares for 1999*

	1997	1998	1999	2000	2001	2002	2003
Laspeyres	102.11	104.03	106.69	109.88	112.55	115.39	117.90
Paasche	*102.02*	*103.74*	*105.99*	*108.62*	*110.84*	*113.22*	115.27
Fisher	*102.06*	*103.88*	*106.34*	*109.25*	*111.69*	*114.30*	116.58
Geometric Laspeyres	102.06	103.88	106.38	109.29	111.72	114.29	116.54
Geometric Paasche	*102.07*	*103.90*	*106.37*	*109.32*	*111.88*	*114.75*	117.15
Törnqvist	*102.06*	*103.89*	*106.37*	*109.30*	*111.80*	*114.52*	116.85
Quasi Fisher	*102.08*	*103.92*	*106.40*	*109.35*	*111.78*	*114.39*	116.58
TGFI	*101.92*	*103.66*	*106.03*	*108.96*	*111.48*	*114.22*	116.58
Lloyd-Moulton a)	*102.07*	*103.89*	*106.39*	*109.31*	*111.74*	*114.32*	116.58
Lloyd-Moulton b)	*102.07*	*103.91*	*106.45*	*109.42*	*111.91*	*114.54*	116.85

* Approximations are shown in italics; a) Fisher index as benchmark; b) Törnqvist index as benchmark.

Table 4 confirms that the TGFI method understates our alternative, Quasi Fisher approximations. Moreover, in 1999 the difference with the true Fisher index (106.34) is as large as -0.31. The Lloyd-Moulton price index numbers have been calculated using $\sigma = 0.98$ (row nine) and $\sigma = 0.79$ (row ten) which make the numbers in 2003 equal to the true Fisher and Törnqvist index numbers. The Lloyd-Moulton estimates are slightly greater than our natural approximations, up to 0.12 %-points for the Törnqvist in 2000. Also, the true Törnqvist index number in 1999 is 106.38 whereas the (Törnqvist-based) Lloyd-Moulton estimate is 106.45. Of course we should not draw the conclusion that the Lloyd-Moulton method in general overstates the true numbers.

Figure 1. Direct price index numbers (1996=100), excluding observed expenditure shares for 1999



The (direct) Laspeyres price index numbers, the natural Fisher approximations, the TGFI approximations and the Lloyd-Moulton (Fisher-based) estimates from Table 4 are depicted in Figure 1. The figure nicely illustrates that, although there are differences between the natural and Lloyd-Moulton approximations, these differences are negligible compared to the differences with the Laspeyres index numbers. Figure 1 again makes clear that the TGFI method most likely produces (slightly) downward biased estimates of the Fisher price index, particularly for years in the middle of the period between two benchmark years.[13]

7. Conclusion

The results of the approximation methods discussed in this paper are all numerically similar to those obtained with the Lloyd-Moulton approach. Ideally each method should be assessed on a data set that enables to calculate superlative price index numbers for intermediate years also. If both sorts of approaches work well then statistical agencies that wish to approximate retrospectively some superlative index can choose either. The Lloyd-Moulton price index has the advantage of being grounded in economic theory. Statistical agencies that are reluctant to rely on CES-assumptions or the like may find our pragmatic stance more attractive.

Suppose the Fisher index would be the preferred target. There is now a choice between the natural approach, for two benchmark years given by expression (9), and the Quasi Fisher alternative, given by expression (15). An advantage of the natural method is its greater flexibility. Data permitting, important expenditure shares can be estimated directly from available price and quantity data.[14] Linear combinations of benchmark year shares can then be used for the remaining shares.

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[13] Balk (1990a) employed Triplett's (1989a) U.S. data on office, computing and accounting machinery for 1972-1986. This data set had four benchmark (weight-reference) years, 1972, 1977, 1982, and 1986, so that there were three sub-periods: 1972-1977, 1977-1982, and 1982-1986. For the first sub-period the TGFI method actually yielded greater index numbers than the natural and Quasi Fisher methods, whereas for the second and third sub-periods the TGFI method produced smaller index numbers. Thus, though there are reasons to expect that the TGFI method in 'normal circumstances' generates downward biased index numbers, this is not necessarily the case.

[14] This is what Nimmo *et al*. (2007) apparently did (see footnote 8).

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