Identifying Non-Cooperative Behavior Among Spouses: Child Outcomes in Migrant-Sending Households

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Abstract
In the presence of asymmetric information, allocations can only be coordinated to the extent that each can be monitored, and household decision-making may not be fully cooperative. Because this information problem is particularly acute when individuals are not co-resident, I examine households in which the father migrates without his spouse and children. Results from the China Health and Nutrition Survey indicate that, when the father is away, girls' household labor increases while mothers' total work hours decrease. This is inconsistent with a unitary model in which there is no non-cooperative behavior and household members simply reallocate time to compensate for the father's absence. Furthermore, outcomes that are easily observed by the father - child schooling and health - are not affected by migration, controlling for changes in income. This is also inconsistent with a non-unitary model in which mothers' bargaining power increases when fathers migrate, given existing evidence which suggests that mothers have stronger preferences than fathers for these goods. I propose a simple model of contracting under asymmetric information and argue that this is consistent with the data. Additional implications are then tested regarding how and which allocations are likely to be affected.

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I. Introduction.

Economic studies of households have increasingly moved away from a unitary model (Becker, 1991) and toward a collective model in which expenditures are determined through some bargaining process. In this collective framework, decision-makers within a household may have divergent preferences, and thus both the monetary value and the ownership of income streams will be important. Given that household members bargain over decisions and that control over resources affects the allocation of those resources, it is natural to consider whether and how individuals may behave strategically in order to increase their own utility. I examine an information problem that permits an individual to conceal expenditures and/or allocations from his/her spouse. This may lead to non-cooperative behavior, as intra-household allocations can only be coordinated to the extent that they can be monitored. Migration presents a clear opportunity for such behavior. The migrant has limited ability to observe expenditure and allocation decisions made by the spouse remaining at home but may also be able to conceal his own expenditures by determining the amount of money that will be remitted to the household.

The economic literature on the impact of remittances on migrant-sending households (*e.g.* Yang, 2004; Edwards and Ureta, 2003) has largely neglected a key feature of such income, *i.e.* the difficulty inherent in monitoring the disbursement and allocation of remittances (for an exception, see Chami *et. al.*, 2003). With the rising trends in both rural-urban and international migration, it is essential to understand the implications of such an information problem in order to assess the ultimate impact on sending families, child welfare and gender disparities. Identifying this type of non-cooperation among spouses will provide additional information about how men's and women's preferences differ, as well as the extent to which the transparency of income matters for the distribution of resources among household members. The existence of such behavior among household members would suggest that expanding opportunities for migration will have different effects on expenditure patterns than simply increasing the amount of income received by the household. Changes in earned income and the potential to earn income will affect bargaining among spouses, but non-cooperative behavior will have an additional effect on the final distribution of expenditures and allocations. That is, changes in the scope for non-cooperative behavior have different implications for household consumption than changes in the distribution of bargaining power among spouses because, contrary to a shift in bargaining power, information asymmetries can change the choice set for an individual without changing the pool of resources he/she controls. Non-cooperative behavior would also have important implications for policy and program design because it implies that the channel through which income is received can have important spillover effects, even beyond any direct effect on income or bargaining power. For example, direct subsidies are easily observed by other household members, whereas micro-credit loans and the proceeds of micro-credit enterprises could be concealed from one's spouse and used to finance expenditures that otherwise would not be undertaken.

Non-cooperative behavior may occur on either an extensive or intensive margin. The former reduces the pool of resources over which bargaining occurs, whereas the latter circumvents the distribution of resources which was initially agreed upon via the bargaining process. This paper will focus on non-cooperative behavior that occurs on the intensive margin, *i.e.* with respect to the allocation of resources among goods and individuals. I introduce asymmetric information into a model of household decision-making such that a migrant cannot observe all actions taken by his spouse and also may not be able to deduce these actions from observable outcomes. In this case, the non-migrant can benefit by shifting resources towards goods that she prefers and then concealing those allocations from her spouse. If the migrant has complete information about his spouse's preferences, he can provide her with appropriate incentives to behave cooperatively, but intra-household allocation decisions will more closely resemble the non-migrant spouse's preferences. Conversely, if the migrant does not have complete information regarding his spouse's preferences, it is possible to have an equilibrium in which only the non-migrant spouse behaves non-cooperatively. In either case, the optimal allocations will be responsive to the degree of observability, provided that non-cooperative behavior is detected with some positive probability. That is, goods which are more difficult to monitor should exhibit larger changes. This is distinct from a change in bargaining power, which would lead to a shift in consumption towards all goods the individual prefers more than does his/her spouse.

Data are drawn from the China Health and Nutrition Survey. Both the unobservable time- and child-invariant characteristics of the household can be accounted for with the panel aspect of these data, and controls for household full income are included to differentiate any non-cooperative behavior from income effects. Results indicate that wives of migrant workers are, in fact, engaging in non-cooperative behavior, in a somewhat surprising way. Observable outcomes such as children's school enrollment, schooling attainment and anthropometric measures exhibit no significant change with a change in the father's migrant status. The stability of these outcomes is inconsistent with a case in which migration increases mother's bargaining power, given existing evidence that mothers have stronger preferences than fathers for these goods (see Qian, 2005; Duflo, 2003 and Thomas, 1990). However, unobservable allocations such as nutritional intake and time spent in household chores do change. Because migration of the father also reduces the total time available for household production, these results alone cannot confirm the presence of non-cooperative behavior. Extending the analysis to mothers, I find that their time in both household and labor market activities falls when the husband is away. The increase in mothers' leisure is inconsistent with a model in which there is no non-cooperative behavior and all household members simply reallocate time to compensate for the father's absence.

The following section will situate this paper in the existing literature on non-cooperative behavior within households. Section III discusses the implications of migration in standard cooperative models of the household, both unitary and non-unitary. Section IV discusses the empirical implementation and shows that the data are inconsistent with cooperative models of the household. In Section V, I present an alternative model of contracting under asymmetric information and argue that this is consistent with the data. Additional implications of this model are tested in Section VI, and Section VII concludes.

II. Non-Cooperative Decision-Making

Within a collective model of the household, individuals may be either cooperative or non-cooperative. Cooperation implies that household members can negotiate and then commit to binding and costlessly enforceable agreements. The starting point for negotiation is each individual's "threat point", the maximum utility he/she could expect to attain in the absence of a cooperative agreement. Manser and Brown (1980) and McElroy and Horney (1981) take divorce as the threat point. Any circumstances that affect the individual's welfare upon dissolution of the marriage (e.g. value of personal assets, labor market opportunities, divorce law, transfers, etc.) will also affect the threat point and, consequently, his/her bargaining power within the marriage. Alternatively, Lundberg and Pollak (1993) propose a non-cooperative equilibrium as the threat point, in which each spouse maximizes his/her own welfare, given the behavior of his/her spouse. In the non-cooperative equilibrium, the marriage remains intact, but the individuals do not coordinate their actions or pool their resources. This is a more plausible model of household bargaining when divorce is costly, both in monetary and emotional terms. The non-cooperative equilibrium may also be preferred over the cooperative equilibrium when the transaction costs of negotiating, monitoring and enforcing cooperative arrangements are very high or when the gains from cooperation are relatively low.

Household public goods are the distinguishing feature of a household, whether members are cooperative or non-cooperative. When the potential contributors to a public good each make strictly positive contributions, control over resources will not affect the equilibrium level of the public good or the equilibrium utilities of the individuals, even if the individuals do not coordinate (Warr, 1983 and Bergstrom, 1986 as cited in Lundberg and Pollak, 1993). However, if the provision of household public goods is organized along "separate spheres", *i.e.* there is specialization by gender, such that one or both spouses make zero contributions to some public good, control over resources will affect the equilibrium outcome. This is true even when control over resources does not affect the utilities that individuals could obtain outside of the marriage, as in the case of a child allowance which is provided to either married mothers or married fathers but is always provided to mothers upon divorce. Uncertainty about income realizations and hence threat points does not affect the basic result, but the authors do not consider the case where income realizations and/or allocation decisions are not perfectly observable by both household members. Dubois and Ligon (2004) introduce asymmetric information into a unitary model of the household and examine which factors determine the intra-household allocation of calories. They reject the hypothesis that food is efficiently allocated among household members and find suggestive evidence that it is, instead, allocated both to create incentives for individuals and as a form of nutritional investment.

These models are taken as the starting point for this paper. I consider a weaker form of non-cooperative behavior in which household members coordinate allocations only to the extent that they can be contracted on, *i.e.* monitored and verified at reasonable cost. When the asymmetric information is such that certain allocations are not fully contractible, spouses will attempt to deviate from the cooperative outcome, but deviations will be constrained by the possibility of detection. Recent evidence from Ashraf (2004) confirms that individuals do attempt to conceal expenditures from their spouses when presented with the opportunity. The prevalence of such non-cooperative behavior in an experimental setting suggests that this is an important phenomenon to consider in a more general context. de Laat (2005) finds evidence that migrants living in Nairobi invest in costly monitoring technologies to mitigate moral hazard on the part of their spouses in rural villages, but increased monitoring has little effect on their wives' behavior. This paper focuses, instead, on outcomes in the migrant-sending household and shows that allocations are, in fact, responsive to the efficacy of monitoring.

III. Implications of Migration in Cooperative Households

In the absence of non-cooperative behavior, the effect of migration on intra-household allocation consists of three components: a reduction in the amount of time available for household production, an increase in household income and, in a non-unitary model, a change in the distribution of bargaining power between spouses. The appropriate counterfactual for identifying non-cooperative behavior is, then, the set of allocations that would be chosen by the household, conditional on these changes to bargaining power and the time-budget constraint, if both spouses could costlessly commit to cooperation. Thus, before considering the possibility of non-cooperation, the implications of migration in standard cooperative models of the household, both unitary and non-unitary, must first be established.

Unitary Model

A household consists of two adults, a migrant (*m*) and a non-migrant (*n*), and one child (*k*). The household has preferences over adult private consumption (*x*) and a household public good (*z*). Production of the household good depends on the time contributed by each individual, as well as person-specific productivities (τ). Adults may engage in labor market activities that earn a wage of *w* per unit of time, and children cannot participate in income-generating activities but can assist with household production. Rather than specifying a utility of leisure, I allow time spent in productive activities to provide some disutility.

$$\max\ U(x_m, x_n, z, t_m, t_n, t_k) \qquad [1]$$

where $x_m + x_n = w_m t_m^w + w_n t_n^w$, $t_i = t_i^w + \mu t_i^h$ for $i = m, n$ and $z = z(t_m^h, t_n^h, t_k; \tau_m, \tau_n, \tau_k)$

Because household and wage labor may have differing effects on utility, the time spent in household production is scaled by a parameter μ in the utility function which reflects the disutility of household labor relative to wage labor. Total private consumption must be equal to total earnings; there is no savings and no borrowing.

Comparative statics (see Technical Appendix, Section A for derivation) indicate that a reduction in father's household labor, holding wages constant, increases mother's time in household production and has an ambiguous effect on child labor. This is because, for a compensated increase in wages, fathers increase market labor supply and reduce household labor supply. With a utility function concave in *x* and *z*, total household utility can be increased by reallocating mother's labor from the wage sector to the household. The effect on child labor is ambiguous because an increase in child labor has both a direct negative effect on household utility and an indirect positive effect through the increase in z. The income effect of an increase in father's wages, holding father's household labor fixed, increases mother's time in household production and reduces child labor.

Non-Unitary Model

Next, I allow the adults in the household to have different preferences and assume that household members can negotiate binding agreements with zero transaction costs, *i.e.* decision-making is fully cooperative. In this case, the household maximizes a weighted sum of the individual utility functions.

$$\max \ \lambda U_m(x_m, z, t_m, t_k) + (1-\lambda)U_n(x_n, z, t_n, t_k) \qquad [2]$$

where $x_m + x_n = w_m t_m^w + w_n t_n^w$, $t_m = t_m^h + \mu_m t_m^w$, $t_n = t_n^h + \mu_n t_n^w$, $z = z(t_m^h, t_n^h, t_k; \tau_m, \tau_n, \tau_k)$

The bargaining weights (λ,1 – λ) are a function of the individuals' outside options, θ_m and θ_n, respectively. Changes in fathers' wages and the time available for household production have the same effects on household labor supply in both the unitary and non-unitary models, although the magnitude of the effects differs (see Technical Appendix, Section B). However, in the non-unitary model, migration may also affect intra-household allocation via a change in the distribution of bargaining power between spouses. The change in bargaining power is ambiguous; husbands have an increase in wages, but wives must bear a larger burden in household production when their husbands are away. Because non-resident husbands must rely on their wives for provision of the public good, migration may increase θ_n relative to θ_m. Comparative statics (see Technical Appendix, Section B) show that an increase in father's bargaining power, holding his wage and household labor fixed, increases mother's time in household production and reduces child labor. Finally, an increase in father's bargaining power reduces mother's private consumption.

To summarize, the unitary model predicts that mothers' household labor should increase when fathers migrate, and child labor may either increase or decrease, depending on the extent to which children are required substitute for fathers in household production. The cooperative non-unitary model predicts the same, except when migration increases mothers' bargaining power. These implications are sensitive to two simplifying assumptions: (1) goods are separable in utility, and (2) there are no complementarities in production of the household public good. The following section examines whether these predictions are consistent with the data and considers whether relaxing the above assumptions could generate the observed pattern.

IV. Empirical Application

This section first provides descriptions of the data and empirical specification. I then present the main results and argue that the findings are inconsistent with standard cooperative models of the household. A robustness check is also presented to determine if data limitations may be driving the results.

Data and Background

Data are drawn from the China Health and Nutrition Survey (CHNS), which includes roughly 4,000 households (15,000 individuals), drawn from nine diverse provinces. The sample of interest is households with at least one child between the ages of 6 and 16 in which both spouses are typically co-resident. Households were first surveyed in 1989, with follow-ups in 1991, 1993, 1997 and 2000. Attrition at the household level is less than 5% between waves, and replacement households were added in 1997 and 2000. The timing of the survey is well-suited for the study of migration, as the 1990s were a period of rapid growth in intra-national labor migration. Using population surveys, Liang and Ma (2004) find that the number of inter-county migrants in China increased from 20 million in 1990 to 45 million in 1995 and 79 million in 2000. This was, in large part, due to a relaxation of migration restrictions in 1988, which allowed individuals to obtain legal temporary residence in other localities. Increased openness and marketization in the 1990s also spurred economic growth, which increased the demand for construction and service workers in urban areas (de Brauw and Giles, 2005).

The panel nature of the data allows for inclusion of individual and community-year fixed effects to control for unobserved characteristics of the household and/or child as well as unmeasured local economic shocks that may influence the migration decision. Migrants are defined as individuals living away from the household for at least one full month in the previous year. The sample of migrant-sending households is further limited to those in which the father was away from the household for all seven days in the week prior to enumeration, because most outcomes of interest are defined over the previous week. Descriptive statistics are presented in Tables 1 through 3, with observations at the person-year or household-year level. Differences in observable characteristics between migrant and non-migrant households are relatively minor. Children in migrant households are somewhat more likely to be enrolled in school, and also more likely be engaged in household labor. The migrants themselves appear to be positively selected on schooling, as are their spouses. As would be expected, migrant households also hold less value in productive assets and have higher household income, on average.

Empirical Specification

I estimate reduced-form demand equations for children's schooling and health and household labor of both children and mothers. Data on the quantity of time spent in various household activities was collected inconsistently across surveys, so identification must rest on changes in household labor on the extensive rather than intensive margins. For individual *i* in household *j* in community *a* at time *t*, the demand for good *y* can be expressed as

$$y_{ijkt} = \alpha + \beta \cdot h_{jkt} + \phi \cdot c_{ijkt} + \delta \cdot away_{jkt} + \gamma \cdot (d_{ijkt} \cdot away_{jkt}) + \rho \cdot f(months\ away_{jkt}) + \upsilon_{ij} + \eta_{kt} + \pi_t + \varepsilon_{ijt}$$

where *h* is a vector of time-varying household characteristics, *c* is a vector of individual covariates, and *d* is a subset of those covariates which are allowed to vary with father's migration status. The error term consists of four components – an individual effect that is fixed over time (υ), a community-level effect that varies across periods (π), a period effect (η), and a mean-zero i.i.d. disturbance (ε). Controls for the father and mother's current wages are included to account for changes in household income over time. For individuals engaged in occupations that do not pay by time or piece rate, predominantly agricultural work, the wage is imputed as the prevailing daily wage for an unskilled farm laborer. Additional control variables include a quadratic in age, parents' ages (for child-level regressions), assets owned (farm land, farming equipment, value of small business capital and area of owned home), household size, number of children (number of siblings for child-level regressions), the sex composition of children (siblings), as well as month of survey. Parents' schooling attainment changes very little over time and is therefore subsumed into the fixed effect.

A quadratic in the months the father is away is included for two reasons. First, investments in human capital and the allocation of household labor likely require some time to adjust. That is, measures of health (body mass index, skin fold, arm circumference) reflect prior investments and do not necessarily adjust instantaneously to changes in inputs. For household tasks that require learning by doing, there may be fixed costs involved with reallocating labor. Second, the wage variables reflect labor market opportunities available at the time of the survey.

If migrants earn higher wages only while living away from home, including measures of the duration of migration episodes will provide better controls for changes in total household incomes. Age and the number of siblings, by gender, are allowed to vary with father's migrant status because these characteristics affect demand for the child's household labor. The effects differ with migration status because fathers cannot contribute to household production when they are not co-resident. Returns on investments in schooling and health are also likely to vary with age, in which case remittance income may not be allocated identically to children of varying ages. Furthermore, children in larger families receive a smaller share of household resources, implying that the income effect of migration varies with the number of siblings.

Basic Results

Column I of Table 4 presents the child-fixed effects estimates of the effect of migration on household chores for children aged 6 to 16. The direct effect of having the father away is positive for boys and negative for girls, but the length of time the father is away has the opposite effect. That is, when the father initially leaves the household, boys take over some of the father's usual chores whereas girls are relieved of some tasks as the total demand for household production falls. However, as the duration of the father's absence increases, the gain to reorganizing patterns of household production increases, increasing the probability that girls engage in household chores, with the opposite effect for boys. When fathers are away for a sufficiently long period of time (>5 months),

the probability that daughters do any household chores (purchasing food, preparing food, laundry) is increasing in the number of months the father is away, and the opposite is true for sons. This pattern suggests that children, particularly younger children, are required to do more household chores increases when fathers are away for a sufficiently long period of time (>5 months). It is difficult to obtain more precise estimates with such a coarse measure of household labor, and data on actual hours were collected inconsistently across waves. Estimates for specific chores (see Table 8) provide more conclusive evidence that children's household labor increases when fathers are away; these results will be discussed in more detail in Section VI. The changes in chores also do not appear to be offset by changes in other tasks. I find no significant effects of migration on the probability that children engage in non-wage labor activities such as gardening, household farming, livestock care, fishing, or handicrafts. The point estimates are quite small in magnitude and generally smaller than the point estimates for chores. An increase in child household labor would be consistent with a standard unitary model of the household in which individuals must reallocate time in order to compensate for the father's absence.

The unitary model also predicts that mothers should unambiguously increase time in household activities when fathers migrate. Columns I and II of Table 5 present estimates of the effect of migration on mothers' time allocation. The probability that mothers do any of the enumerated household chores (purchasing food, preparing food, laundry) is decreasing in months the father is away, although the point estimate is not statistically significant. Again, estimates for specific chores, presented in Table 9, provide more conclusive evidence that mothers spend less time in household maintenance when fathers are away. Furthermore, the number of months the father is away has a significant negative effect on the total time mothers spend in income-generating activities (wage labor plus "other" non-wage work activities such as gardening, household farming, livestock care, fishing, or handicrafts). When taken together, these results suggest that mothers are consuming more leisure. An increase in mothers' leisure is inconsistent with migration in a unitary model of the household, and it is difficult to imagine a pattern of complementarities in utility or production that could generate both a reduction in mothers' labor and an increase in child labor. If private and public goods are strong complements in utility, an increase in income would increase the demand for household public goods and thus the demand for children's time. However, because child labor provides direct disutility, this complementarity could not produce an increase in child household labor without an increase in mothers' time in income-generating activities, assuming mothers' wages do not increase when their spouses migrate.

Complete time diaries are not available in the CHNS, making it difficult to conclude that the observed reduction in mothers' work hours signifies an increase in mothers' leisure. It is possible that fathers engage in other household activities which are not enumerated and migration forces mothers to substitute into these tasks while children substitute for mothers in the enumerated household tasks. To investigate this possibility, I utilize an alternate sample of households in which the father experiences an illness or injury sometime in the four weeks prior to the survey date. The number of days a health complaint disrupted the individual's normal activities is used to measure the extent to which time available for household production was affected, and both individual and community-year fixed effects are again included.

If the results presented above are driven by increased demand for mothers' time in activities typically carried out by fathers, this alternate sample should yield similar findings. That is, if wives of migrants reduce time in laundry, food preparation and food purchase in order to substitute for husbands' time in other, non-enumerated activities, the same reduction should be evident when husbands' household labor is reduced by illness or injury. Estimates in Table 6 indicate that this is not the case; the probability that mothers do any of these three chores is largely unaffected when fathers experience a debilitating illness or injury, and the point estimates are, in fact, positive. However, the number of days that fathers are debilitated by illness or injury is relatively short, on average. The sample mean is 13 days, and roughly 45% of fathers are debilitated for less than one week. It is possible that households simply do not adjust time allocation in such short periods. This hypothesis is not supported by the estimates in the child-level regression. Younger sons and older daughters are more likely to be engaged in some form of household labor when fathers are debilitated (estimates for specific household chores are similar and not presented here). Thus, the estimated effect of migration on mothers' and children's time allocation cannot be fully explained by the existence of other non-enumerated tasks which are typically carried out by the father.

Under a non-unitary model of the household, an increase in mother's bargaining power could explain an increase in child household labor accompanied by a decrease in mother's household labor. However, an increase in mothers' bargaining power should also be accompanied by changes in other goods favored by the mother, *e.g.* children's human capital (see Duflo, 2003; Thomas, 1990; Qian, 2005), irrespective of the ease with which those goods can be monitored. Columns III and IV of Table 4 present the child-fixed effects estimates of the relative effect of migration on easily observable outcomes for children aged 6 to 16. Migration of the father has no statistically significant effects on school enrollment, and the average marginal effect is quite small. Similarly, migration has no statistically significant effects on children's body mass index, and the absolute effects implied by the point estimates are relatively small. For a 4-foot tall child with BMI in the normal range (weighing 60-80 pounds), a half point change in BMI is equivalent to change in weight of approximately 2 pounds.

Given findings in other studies, the observed stability in schooling and health for both boys and girls appears inconsistent with a model in which mothers' bargaining power increases when fathers migrate. Qian (2005) finds that, among agricultural households in China, an exogenous increase in the share of female income has a significant positive effect on educational attainment for all children, whereas increasing the share of male income reduces educational attainment for girls. Chen (2005) finds that girls' school enrollment increases relative to boys when mothers have increased bargaining power, and Duflo (2003) and Thomas (1990) find that an increase in female income improves health outcome for all children and has a disproportionately positive effect on girls. Finally, the third column of Table 5 indicates that migration of the father does not improve mothers' health, which again appears inconsistent with an increase in mothers' bargaining power.

V. Migration with Imperfect Information

The results presented above cannot be easily explained by standard cooperative models of the household, either unitary or non-unitary. I argue that this is because migration introduces imperfect monitoring – the migrant has limited ability to observe allocation decisions made in his absence, and the spouse remaining at home may not be able to observe the wages or expenditures of the migrant. Imperfect information increases transaction costs associated with enforcing cooperative bargaining agreements and thus reduces the gains from cooperation. It also affects the utility associated with non-cooperation, as each spouse can only react to the allocations that are observed, not necessarily those that actually occurred. Thus, an individual who chooses not to cooperate will not always receive a commensurate response from his/her spouse.

In this section, I model intra-household allocation as the result of a contracting problem, allowing for asymmetric information. I will first describe the contracts that will be offered in equilibrium and the conditions under which the contracted allocations will or will not be chosen. I then derive testable implications by examining how the optimal non-cooperative strategy varies with the parameters of the model and, consequently, how the optimal contract must differ from the allocations that would be obtained in the absence of imperfect monitoring. Because the CHNS provides data only on sending households, I focus on the case in which there is imperfect monitoring of allocations under the mother's control and assume that there is perfect information regarding the earnings and expenditures of the migrant. A more complete dynamic model in which wives update beliefs about husbands' wage realizations in each period is left to future research. Furthermore, imperfect monitoring of the migrant's actions would not obviate non-cooperative behavior on the part of his spouse, provided that asymmetric information prevents the couple from attaining a fully cooperative equilibrium.

Description of Game

There are two players, a migrant (*m*) and a non-migrant (*n*). Utility and production functions are the same as in the basic non-unitary model described above; both players have preferences over own private consumption (x), a household public good (z) that is produced with household members' time, own time spent in productive activities (t^w and t^h), and their child's time spent in household production (t_k). Each household member has a time endowment T. Adults may engage in labor market activities that earn a wage of w per unit of time, and children cannot participate in the labor market but can assist with household production.

$U_i = U_i(x_i, z, t_i, t_k)$ for $i = m,n$

where $T = t_i^w + t_i^h + l$, $t_i = t_i^w + \mu t_i^h$, $x_m + x_n = w_m t_m^w + w_n t_n^w$, $z = z(t_m^h, t_n^h, t_k; \tau_m, \tau_n, \tau_k)$,

l denotes leisure and μ reflects the disutility of household labor relative to wage labor. When the father migrates, allocations move into "separate spheres" such that each spouse has direct control over only a subset of goods. This division is determined by residence patterns, *i.e.* the absence of the father from the sending household. While away, the migrant can only imperfectly monitor his spouse's actions. Before leaving the household, the migrant can contract with his spouse for a set of allocations to be implemented in his absence. The contract also stipulates a transfer (*s*) to be made to the wife upon the migrant's return, and the value of this transfer may be contingent upon the outcome of a monitoring process.

> **Definition.** The non-migrant's action space includes own private consumption $x_n \in [0, w_n t_n^w]$ (equivalently, own market labor $t_n^w \in [0,T]$), own household labor, $t_n^h \in [0,T]$ and the child's household labor, $t_k \in [0,T]$. The migrant cannot contribute to household production ($t_m^h = 0$), and thus the non-migrant's choices of t_n^h and t_k fully determine z. The migrant's action space is limited to the choice of $t_m^w \in [0,T]$ and a contingent contract $\{t_n^{h^c}, t_k^{\ c}, s^c, s^{nc}\}$ that includes a transfer to his wife, $s \in [0, w_m t_m^w]$ expressed in units of x, where s^c is the transfer if the contracted allocations are observed, and s^{nc} is the transfer otherwise.

Note that, because the father cannot contribute to household production, a contract specifying t_n^h and t_k also implicitly specifies z. Transfers from the migrant to his spouse are bounded from below by zero; this condition is analogous to a participation constraint such that the non-migrant would decline any contracts that do not provide an expected payoff greater than or equal to her reservation utility. The migrant's strategy thus consists of a contingent contract, and the non-migrant's strategies are to either play *cooperate* and choose the contracted allocations or disregard the contract and play *don't cooperate*.

The game then proceeds as follows. First, player *m*, the migrant, offers a contingent contract to his spouse, player *n*, that specifies all intra-household allocations and the transfer the non-migrant spouse will receive contingent on the outcome of the monitoring process. Both players then choose the allocations associated with their respective spheres. Player *n*'s choices are monitored, and a transfer is then made from player *m* to player *n* contingent on the outcome and consistent with the contract offered prior to migration. I assume that player *m*'s actions are perfectly monitored by player *n* and that player *m* cannot renege on the contract.[1] If player *n* plays *don't cooperate*, the contracted allocations are revealed with probability one; otherwise, monitoring reveals player *n*'s actions with probability $q \leq 1$ and the contracted allocations with probability $(1 - q)$. The probability of detection (q) depends on the actions of both players as well as a set of exogenous parameters $\boldsymbol{\omega}$, and both players have complete information regarding the structure of this q-function.

Because the husband receives zero utility from his wife's consumption, he would always agree to an increase in t_n^h holding t_k constant, or a reduction in t_k holding z constant.[2] Thus, the *don't cooperate* strategy can only yield a higher payoff for player *n* when $t_n^h \leq t_n^{h\,c}$ and $t_k \geq t_k^{\,c}$. That is, when it is optimal for the non-migrant spouse to play *don't cooperate*, it must be the case that she shifts household labor away from herself and to the child, relative to the contracted allocations. Whether the optimal actions associated with the *don't cooperate* strategy yield a higher or lower level of household production (z) depends on player *n*'s utility of z relative to her disutility of t_n^h and t_k. Although the contract between spouses does not explicitly specify z and $x_n\,(t_n^w)$, observations of these allocations provide useful signals to player *m* about player *n*'s actions. Given that $t_n^h \leq t_n^{h\,c}$ and $t_k \geq t_k^{\,c}$ when player *n* chooses *don't cooperate*, a level of

[1] More formally, enforcement of the contract could occur through repeated interaction between spouses; this extension is discussed below.

[2] If the migrant's utility depended directly on his spouse's utility, he would still accept such a change as long as the arrangement provides more utility to his spouse as well. The only cases in which this claim would not hold are (1) if the migrant's disutility from his spouse's time in household production exceeds his disutility from children's time in household production, or (2) children's time in household production provides fathers with positive utility over some range.

household production that is higher than the contracted value signals that player n has chosen $t_k > t_k^c$, and a lower level of household production signals $t_n^h < t_n^{h\,c}$. Similarly, because the migrant would always agree to an increase in x_n via an increase in t_n^w while holding t_n^h constant, $x_n > w_n(T - t_n^{h^c} - l)$ also signals that player n has chosen $t_n^h < t_n^{h\,c}$. Therefore, for each possible contract, there exist unique values of z, x_n and t_n^w that are consistent with the *cooperate* strategy, even though these allocations are not explicitly contracted upon. Based on the above discussion, define q as follows.

Definition. $q = q(x_n, z, t_n^h, t_k; t_n^{h^c}, t_k{}^c, \omega_q, \omega_x, \omega_z, \omega_h, \omega_k)$ is the probability that non-cooperative behavior is detected, where

$$\frac{\partial q}{\partial x_n} > 0, \frac{\partial^2 q}{\partial x_n{}^2} > 0 \text{ for } x_n > w_n(T - t_n^{h^c} - l); \ \frac{\partial q}{\partial x_n} = 0 \text{ for } x_n \le w_n(T - t_n^{h^c} - l)$$

$$\frac{\partial q}{\partial z} < 0, \frac{\partial^2 q}{\partial z^2} < 0 \text{ for } z < z(t_n^{h^c}, t_k{}^c; \tau_n, \tau_k); \ \frac{\partial q}{\partial z} > 0, \frac{\partial^2 q}{\partial z^2} > 0 \text{ for } z > z(t_n^{h^c}, t_k{}^c; \tau_n, \tau_k);$$

$$\text{and } \frac{\partial q}{\partial z} = 0 \text{ for } z = z(t_n^{h^c}, t_k{}^c; \tau_n, \tau_k)$$

$$\frac{\partial q}{\partial t_n^h} < 0, \frac{\partial^2 q}{\partial t_n^{h2}} < 0 \text{ for } t_n^h < t_n^{h^c}; \ \frac{\partial q}{\partial t_n^h} = 0 \text{ for } t_m^h \ge t_m^{h^c}$$

$$\frac{\partial q}{\partial t_k} > 0, \frac{\partial^2 q}{\partial t_k{}^2} > 0 \text{ for } t_k > t_k^c; \ \frac{\partial q}{\partial t_k} = 0 \text{ for } t_k \le t_k^c$$

$$\frac{\partial q}{\partial \omega_q} > 0, \ \frac{\partial q}{\partial \omega_x} > 0, \frac{\partial q}{\partial \omega_h} < 0, \frac{\partial q}{\partial \omega_k} > 0$$

$$\frac{\partial q}{\partial \omega_z} < 0 \text{ for } z < z(t_n^{h^c}, t_k{}^c; \tau_n, \tau_k) \text{ and } \frac{\partial q}{\partial \omega_z} > 0 \text{ for } z > z(t_n^{h^c}, t_k{}^c; \tau_n, \tau_k)$$

The probability of detection is convex for all goods. An increase in ω_q increases the marginal probability of detection for all goods symmetrically, and good-specific ω parameters serve to increase the observability of any given allocation and thus have the same sign as the marginal probability of detection for each good.

I assume no costly monitoring technologies are available, *i.e.* the $\boldsymbol{\omega}$ factors are characteristic of the specific goods in question, the marriage match and/or the specific migration opportunity and cannot be affected by either player. This assumption should not alter the general theoretical results, provided any available monitoring technologies are either prohibitively costly or cannot fully reveal all hidden actions.

Because changes in t_n^w are exactly proportional to changes in x_n, I assume that the choice of t_n^w does not have an independent effect on the probability of detection, *i.e.* changes in t_n^w do not affect q, holding x_n constant. An increase in x_n is indicative of a decrease in t_n^h and, consequently, the probability of detection must be increasing in x_n. The probability of detection is also increasing in t_k because, for player n, the individual utility-maximizing value is greater than the cooperative value. Conversely, the optimal values of t_n^h is less than the cooperative values, and thus any increase in t_n^h will decrease the probability of detection. The probability of detection is increasing and convex in the absolute difference between z and z^c because the optimal level of household production associated with *don't cooperate* may be higher or lower than the contracted value. Whenever a contracted allocation is chosen, the marginal probability of detection for that good is zero. For simplicity, I have also assumed that the probability of detection is zero for any value of t_n^h greater than $t_n^{h^c}$ and any value of x_n or t_k less than $x_n{}^c$ or $t_k{}^c$, respectively, because any allocations satisfying these conditions would increase the utility of player m. In practice, this assumption simply assures that the wife would not be punished for any non-cooperative behavior that benefits her spouse.

I assume that, for each value of ω_q, there is a unique best response associated with *don't cooperate* and thus a unique value of q. The other $\boldsymbol{\omega}$ parameters (ω_x, ω_z, ω_h, ω_k) will be taken as fixed. Payoffs are

$$V_m = U_m(t_m^w, z^c, t_k{}^c, x_m - s^c),\; V_n = U_n(t_n^{w^c}, z^c, t_n^{h^c}, t_k{}^c, x_n{}^c + s^c)$$

if player n chooses the contracted allocations and

$$V_m = (1-q^*)U_m(t_m^w, z, t_k, x_m - s^c) + q^* U_m(t_m^w, z, t_k, x_m - s^{nc}),$$

$$V_n = (1-q^*)U_n(t_n^w, z, t_n^h, t_k, x_n + s^c) + q^* U_n(t_n^w, z, t_n^h, t_k, x_n + s^{nc})$$

otherwise, where q^* is the probability of detection associated with player n's best response within the space of non-cooperative actions.

Possible Equilibria

<u>Case 1. Homogeneous Wives</u>

When the migrant has complete information about his spouse's preferences, he can always induce her to choose the contracted allocations. Imperfect monitoring, however, will affect the equilibrium payoffs and can still lead to non-cooperative behavior under certain parameter values.

Proposition 1. For $\omega_q \geq \overline{\omega_q}$, the allocations that would be obtained with perfect monitoring are feasible and will be obtained in equilibrium. For $\omega_q < \underline{\omega_q}$, allocations will be fully non-cooperative. That is, the migrant and his spouse will not make joint consumption decisions, but the resultant allocations are equivalent to the contracted allocations. For $\underline{\omega_q} \leq \omega_q < \overline{\omega_q}$, player n will also choose the contracted allocations, but the equilibrium payoff for player n exceeds the payoff she would obtain under perfect monitoring and conversely for player m.

Proof. First note that, for player n, the payoff to *don't cooperate* is monotonically decreasing in ω_q up to some $\overline{\omega_q}$ (non-increasing in ω_q if $s^c = s^{nc}$, see Technical Appendix for derivation). Now consider the contract $\{t_n^{h^*}, t_k^*, s^*, 0\}$, where * denotes the allocations that would be obtained in the absence of imperfect monitoring. For $\omega_q \geq \overline{\omega_q}$, the optimal actions associated with player n's *don't cooperate* strategy are equivalent to the actions associated with *cooperate*. That is, when the probability of detection is sufficiently high, player n cannot increase her own utility by deviating from the contracted allocations. Thus, for $\omega_q \geq \overline{\omega_q}$, the fully cooperative allocations, *i.e.* the allocations that would be obtained under perfect monitoring can be enforced even when monitoring is imperfect. Furthermore, because $\{t_n^{w^*}, z^*, t_n^{h^*}, t_k^*, t_m^{w^*}, s^*\}$ are the allocations that would be obtained when $q = 1$, the value of q^* associated with $\overline{\omega_q}$ must be strictly less than one. In contrast, for $\omega_q < \overline{\omega_q}$, the above contract cannot be enforced because the payoff to *don't cooperate* exceeds the payoff to *cooperate* for player n. For these parameter values, the migrant can only induce his spouse to play *cooperate* by offering an alternative contract $\{t_n^{h^c}, t_k^c, s^c, 0\}$ that provides her higher utility than playing *don't cooperate* and thus also higher utility than she would obtain under perfect monitoring. This contract will be determined as follows:

$$\max_{t_m^w, t_n^{h^c}, t_k^c, s^c, s^{nc}} V_m = U_m(t_m^w, z^c, t_k^c, x_m - s^c) \text{ subject to } x_m = w_m t_m^w - s^c \text{ and}$$

$$U_n(t_n^{w^c}, z^c, t_n^{h^c}, t_k^c, x_n^c + s^c) \geq (1-q^*)U_n(t_n^w, z, t_n^h, t_k, x_n + s^c) + q^* U_n(t_n^w, z, t_n^h, t_k, x_n)$$

The migrant is willing to do this as long as he can extract some of the gains from cooperation,

$$U_m(t_m^w, z^c, t_k^c, x_m - s^c) \geq U_m(t_m^{w}{}', z', t_k{}', x_m{}' - s')$$

where ' denotes the fully non-cooperative allocations, *i.e.* the allocations that would be chosen if there were no joint consumption decisions and each individual maximized his/her own utility, taking the other player's actions and $q = 1$ as given. The left-hand side of the above inequality is inversely related to ω_q; the payoff to *don't cooperate* increases for player n as ω_q decreases and thus the migrant must offer increasingly more favorable contracts to induce cooperation. For $\omega_q < \underline{\omega_q}$, the only contracts under which player n will choose *cooperate* are such that $U_m(t_m^w, z^c, t_k{}^c, x_m - s^c) < U_m(t_m^w{}', z', t_k{}', x_m{}' - s')$, and player m's optimal strategy will be to offer the contract $\{t_n^h{}', t_k{}', s', s'\}$. Allocations will be fully non-cooperative, but the contracted allocations will be chosen by player n in equilibrium. Since the migrant knows with certainty the allocations that his spouse will choose in equilibrium, he offers s' irrespective of the outcome of monitoring. However, it is not necessarily the case that $s' \leq 0$, as $s' > 0$ may increase the payoff for both players (see Lundberg and Pollak, 1993). ■

The contracted allocations will always be chosen in equilibrium and behavior is not non-cooperative *per se*, except in case of $\omega_q < \underline{\omega_q}$. However, for $\underline{\omega_q} \leq \omega_q < \overline{\omega_q}$, the difference between the equilibrium allocations and the allocations that would be obtained under perfect monitoring provide a measure of the extent of the incentive problem.

Proposition 2. For $\underline{\omega_q} \leq \omega_q < \overline{\omega_q}$, the optimal contract offered by player m is such that $t_n^{hc} \leq t_n^h*, t_k{}^c \geq t_k*, s^c \geq s*$. That is, the optimal contract provides player n with more leisure and a larger transfer than the values that would be obtained in the absence of imperfect monitoring. In this range of ω_q, the optimal contract may also implicitly specify values of $t_n^w(x_n)$ and z that provide higher utility to player n. However, the contracted values of more easily monitored goods will be closer to the values that would be obtained in the absence of imperfect monitoring.

Proof. Increasing the transfer player n receives when the contracted allocations are observed increases the payoff to *cooperate* more than it increases the payoff to *don't cooperate*, and thus incentivizes player n to choose the contracted allocations (see Technical Appendix, Part C for derivations). Decreasing the contracted value of t_n^h and increasing the contracted values of $t_k, t_n^w(x_n)$ also make *cooperate* a more appealing strategy for player n, provided that changing the contracted values has a sufficiently small effect on the probability of detection q^*. Similarly, decreasing the contracted value of z will increase the payoff to *cooperate* relative to *don't cooperate*, if player n prefers a lower level of z than her spouse. However, note that, for goods that are easier to monitor, changing the contracted value will have a larger effect on the probability of detection and thus be less effective as an incentive for player n to choose *cooperate*. ■

Thus, for $\underline{\omega_q} \leq \omega_q < \overline{\omega_q}$, the difference between the equilibrium allocations and the allocations that would be obtained under perfect monitoring will be larger for goods that are more difficult to monitor. In contrast, when decision-making is fully non-cooperative ($\omega_q < \underline{\omega_q}$), the equilibrium allocations will reflect proportional changes in *all* goods favored by the higher-weighted spouse, irrespective of the degree of transparency.

Case 2. Heterogeneous Wives

Now suppose player n may be one of two types, A and B, drawn exogenously with probability p and $(1 - p)$, respectively, with p taken as fixed. Type A has payoffs as defined above, but Type B incurs a fixed cost (c) when she chooses to play *don't cooperate*.[3] Payoffs for player n are thus

$$V_n^A = V_n^B = U_n(t_n^{w^c}, z^c, t_n^{h^c}, t_k^{\ c}, x_n^{\ c} + s^c)$$

when playing *cooperate*, and

$$V_n^A = (1-q^*)U_n(t_n^w, z, t_n^h, t_k, x_n + s^c) + q^* U_n(t_n^w, z, t_n^h, t_k, x_n + s^{nc})$$

$$V_n^B = (1-q^*)U_n(t_n^w, z, t_n^h, t_k, x_n + s^c) + q^* U_n(t_n^w, z, t_n^h, t_k, x_n + s^{nc}) - c$$

when playing *don't cooperate*. When both types play the same strategy, player m's payoffs are as defined above, otherwise player m's expected payoff is

$$V_m = (1-p)U_m(t_m^w, z, t_k, x_m - s^c) + p[(1-q^*)U_m(t_m^w, z, t_k, x_m - s^c) + q^* U_m(t_m^w, z, t_k, x_m - s^{nc})]$$

when type A plays *don't cooperate* and type B plays *cooperate* and

$$V_m = pU_m(t_m^w, z, t_k, x_m - s^c) + (1-p)[(1-q^*)U_m(t_m^w, z, t_k, x_m - s^c) + q^* U_m(t_m^w, z, t_k, x_m - s^{nc})]$$

when type B plays *don't cooperate* and type A plays *cooperate*. In this case, the migrant has incomplete information about his spouse's payoffs, and the contracted allocations may not always be chosen in equilibrium.

Proposition 3. For $\omega_q \geq \overline{\omega_q}$, the allocations that would be obtained with perfect monitoring are feasible and will be obtained in equilibrium. For $\omega_q < \underline{\underline{\omega_q}}$, allocations will be fully non-cooperative. That is, the migrant and his spouse will not make joint consumption decisions, but the resultant allocations are equivalent to the contracted allocations. For $\underline{\omega_q} \leq \omega_q < \overline{\omega_q}$, the equilibrium payoff for player n is weakly greater than the payoff she would obtain under perfect monitoring and conversely for player m;

[3] This assumption ensures that, for any given contract, the payoff functions for types A and B cross only at the value of ω_q at which the *don't cooperate* strategy yields a higher payoff than the *cooperate* strategy for type A. Alternative formulations for player heterogeneity are discussed below.

however, whether or not the contracted allocations are obtained in equilibrium depends on the probability that player *n* is type A. For $\underline{\underline{\omega}}_q{}' \le \omega_q < \underline{\omega}_q$, the contracted allocations will be chosen in equilibrium by type B but not by type A and, for $\underline{\underline{\omega}}_q \le \omega_q < \underline{\underline{\omega}}_q{}'$, the contracted and equilibrium allocations will be fully non-cooperative, where the cutoff point $\underline{\underline{\omega}}_q{}'$ depends on the value of *p*.

Proof. Because type A receives a weakly higher payoff from playing *don't cooperate* than type B, the first statement follows from Proposition 2. That is, with a fixed cost for type B, whenever type A finds it optimal to play *cooperate*, type B will also find it optimal to play *cooperate*. For $\omega_q < \overline{\omega_q}$, player *m* can incentivize both types to behave cooperatively, *i.e.* choose the contracted allocations, by offering a contract that provides type A higher utility than playing *don't cooperate*. As in the previous case, this contract also provides player *n* higher utility than she would obtain under perfect monitoring. Alternatively, the migrant can offer a contract that induces cooperation only from type B. The latter contract will be optimal if

$$\begin{aligned}(1-p)U_m(t_m^w, z^{c,B}, t_k{}^{c,B}, x_m - s^{c,B}) + p[(1-q^*)U_m(t_m^w, z, t_k, x_m - s^{c,B}) + q^* U_m(t_m^w, z, t_k, x_m - s^{nc,B})] \\ > U_m(t_m^w, z^{c,A}, t_k{}^{c,A}, x_m - s^{c,A})\end{aligned}$$

or

$$p < \frac{U_m(t_m^w, z^{c,B}, t_k{}^{c,B}, x_m - s^{c,B}) - U_m(t_m^w, z^{c,A}, t_k{}^{c,A}, x_m - s^{c,A})}{[U_m(t_m^w, z^{c,B}, t_k{}^{c,B}, x_m - s^{c,B}) - (1-q^*)U_m(t_m^w, z, t_k, x_m - s^{c,B}) - q^* U_m(t_m^w, z, t_k, x_m - s^{nc,B})]}.$$

That is, if the probability that player *n* is type A is sufficiently low, player *m* finds it optimal to offer a contract that cannot always be enforced. This contract provides type B with weakly higher utility than she would obtain under perfect monitoring.[4] Type A necessarily receives higher utility because *don't cooperate* yields a higher payoff than *cooperate*, and *cooperate* yields weakly higher utility than she would obtain under perfect monitoring (recall that payoffs for type A and type B are identical when *cooperate* is played). For $\omega_q < \underline{\underline{\omega}}_q$, the only contracts that induce either type of player *n* to choose the contracted allocations provide player *m* with less utility than the fully non-cooperative allocations, and thus the optimal contract is $\{t_n^h{}', t_k', s', s'\}$, as in the previous case. Again because type A receives a weakly higher payoff from playing *don't cooperate* than type B, it must be the case that $\underline{\underline{\omega}}_q < \underline{\omega}_q$. That is, the contract that induces type A to cooperate when $\omega_q = \underline{\omega}_q$ will induce type B to cooperate for $\omega_q > \underline{\omega}_q$. Finally, for $\underline{\underline{\omega}}_q \leq \omega_q < \underline{\omega}_q$, there exists a contract that induces type B to cooperate and, conditional on player *n* being type B, yields higher utility for player *m* than the fully non-cooperative allocations. The migrant will prefer this contract to a contract specifying the fully non-cooperative allocations if

$$p \leq \frac{U_m(t_m^w, z^{c,B}, t_k^{c,B}, x_m - s^{c,B}) - U_m(t_m^w, z', t_k', x_m - s')}{U_m(t_m^w, z^{c,B}, t_k^{c,B}, x_m - s^{c,B}) - (1 - q^*(\omega_q))U_m(t_m^w, z, t_k, x_m - s^{c,B}) - q^*(\omega_q)U_m(t_m^w, z, t_k, x_m - s^{nc,B})}.$$

Then, the cutoff point $\underline{\underline{\omega}}_q'$ is the value of ω_q, for a fixed value of *p*, at which the above inequality is reversed, *i.e.* the point at which it becomes optimal for the migrant to offer a contract specifying the fully non-cooperative allocations rather than a contract that incentive-compatible only for type B. Again, when the probability that player *n* is type A is sufficiently small, the contracted allocations will not always be chosen in equilibrium. Note that, if the

[4] For $\overline{\omega}_q' \leq \omega_q < \overline{\omega}_q$ where $\overline{\omega}_q'$ is the value of ω_q at which type B is just indifferent between the *cooperate* and *don't cooperate* strategies, the probability of detection is sufficiently high that type B cannot increase her own utility by deviating from the cooperative allocations. That is, she will find it optimal to cooperate when offered a contract that specifies the fully cooperative allocations and a zero transfer conditional upon discovery of non-cooperative behavior.

above inequality holds for $\omega_q = \underline{\underline{\omega}}_q$, then $\underline{\underline{\omega}}_q' = \underline{\underline{\omega}}_q$ and contracts and allocations will be fully non-cooperative only for $\omega_q < \underline{\underline{\omega}}_q$. Conversely, if the above inequality does not hold for $\omega_q = \underline{\omega}_q$, then contracts and allocations will be fully non-cooperative for all $\omega_q < \underline{\omega}_q$. ■

Thus, when the probability of having a type A spouse is sufficiently low, the migrant offers contracts in equilibrium that are not necessarily incentive compatible. That is, non-cooperative behavior may occur in equilibrium. Even if the optimal contract targets both types, the difference between the contracted allocations and those that would be obtained in the absence of imperfect monitoring provide a measure of the incentive problem. Furthermore, the contracted allocations will closely resemble the allocations that would have been chosen by player *n* if she had chosen to behave non-cooperatively. This is for two reasons: (1) an incentive-compatible contract must provide player *n* with a higher payoff than playing *don't cooperate* and thus must provide her with larger amounts of the goods that she prefers, and (2) changing the contracted value of goods that are more easily monitored provides better incentives for player *n* to behave cooperatively.

Testable Implications

The equilibrium strategies discussed above are consistent with the empirical results presented in the previous section. Non-migrant spouses reduce own household labor and increase child household labor. Allocations that are easily observed – child schooling and health – exhibit no change with migration, conditional on income, but allocations that are difficult to verify – participation in household chores – exhibit relatively large changes, even on the extensive margin.

I now examine how player *n*'s optimal non-cooperative strategy varies with the parameters of the model. Note that these comparative statics also provide insight into how the optimal contract offered by player *m* varies with the parameters of the model. When playing *don't cooperate*, player *n*'s actions are determined as follows:

$$\max\ V_n = (1-q)U_n(t_n^w, z, t_n^h, t_k, x_n + s^c) + qU_n(t_n^w, z, t_n^h, t_k, x_n + s^{nc}),$$

where $q = q(x_n, z, t_n^h, t_k; t_n^{hc}, t_k{}^c, \omega_q, \omega_x, \omega_z, \omega_h, \omega_k)$, $t_n = t_n^h + \mu_n t_n^w$ and $z = z(t_n^h, t_k; \tau_n, \tau_k)$.

Transfers from the husband, s^c and s^{nc}, are taken as given. The first order conditions are

$$\left(\frac{\partial U_n}{\partial t_n} + \frac{\partial U_n}{\partial x_n} w_n\right) - q\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right) - \frac{\partial q}{\partial x_n} w_n (U_n - U_n^{'}) = 0,$$

$$\left(\frac{\partial U_n}{\partial t_n}\mu_n + \frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_n^h}\right) - \left(\frac{\partial q}{\partial t_n^h} + \frac{\partial q}{\partial z}\frac{\partial z}{\partial t_n^h}\right)(U_n - U_n^{'}) = 0 \text{ and}$$

$$\left(\frac{\partial U_n}{\partial t_k} + \frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_k}\right) - \left(\frac{\partial q}{\partial t_k} + \frac{\partial q}{\partial z}\frac{\partial z}{\partial t_k}\right)(U_n - U_n^{'}) = 0.$$

where $U_n = U_n(t_n^w, z, t_n^h, t_k, x_n + s^c)$ and $U_n' = U_n(t_n^w, z, t_n^h, t_k, x_n + s^{nc})$. Clearly, the optimal amount of household labor provided by the non-migrant spouse exceeds the amount she would provide when $q = 0$ because the gains from decreasing t_n^h are offset by an increase in the probability of detection. Whether the optimal amount of child household labor is higher or lower than when $q = 0$ depends on both the productivity of the child and the probability of detection for t_k relative to *z*. The optimal amount of wage labor will be less than that chosen when $q = 0$ only if the expected punishment for increasing t_n^w exceeds the gain to smoothing consumption between states. I assume that the parameters are such that the values of t_k and t_n^w are less than the values that would be chosen when $q = 0$.

Comparative statics (see Technical Appendix for derivation and full set of assumptions) are utilized to derive the following testable implications:

$$\frac{\partial t_n^w}{\partial \omega_q} < 0,\ \frac{\partial t_n^h}{\partial \omega_q} > 0 \text{ and } \frac{\partial t_k}{\partial \omega_q} < 0$$

$$\frac{\partial t_n^w}{\partial \omega_x} < 0,\ \frac{\partial t_n^h}{\partial \omega_x} > 0 \text{ and } \frac{\partial t_k}{\partial \omega_x} < 0$$

$$\frac{\partial t_n^w}{\partial \omega_\mu} > 0,\ \frac{\partial t_n^h}{\partial \omega_\mu} < 0 \text{ and } \frac{\partial t_k}{\partial \omega_\mu} > 0$$

$$\frac{\partial t_n^w}{\partial \tau_n} < 0,\ \frac{\partial t_n^h}{\partial \tau_n} > 0 \text{ and } \frac{\partial t_k}{\partial \tau_n} < 0$$

$$\frac{\partial t_n^w}{\partial \tau_k} > 0\ ,\ \frac{\partial t_n^h}{\partial \tau_k} < 0 \text{ and } \frac{\partial t_k}{\partial \tau_k} > 0$$

Variation in the general factor ω_q can be thought of as differences across households in the distance and duration of migration episodes. More frequent return visits allow more frequent observations of intrahousehold allocations and increase both the overall and marginal probability of detecting any given deviation from the contract. An increase in ω_q brings all allocations closer to the cooperative values.

Private consumption is often in the form of durable items (*e.g.* clothing) which can be easily observed. To examine the case where deviations in x_n are easily detected relative to other allocations, we can consider the effect of an increase in ω_x. Under the assumption that the expected punishment for increasing t_n^w exceeds the gain to smoothing consumption between states, an increase in the probability of detection specifically for private consumption reduces the optimal value of t_n^w and thus x_n , bringing both closer to their cooperative values. The reduction in t_n^w reduces the level of disutility associated with labor hours and thus makes an increase in t_n^h in less costly which, in turn, makes a decrease in t_k less costly as well. An increase in ω_x thus brings all allocations closer to the cooperative values. The magnitude of these effects, however, is smaller than the effect of an increase in ω_q; this is because a general increase in the probability of detection induces more feedback effects between allocations.

In contrast, an increase in ω_z increases z but has an ambiguous effect on t_n^h and t_k. Holding constant the marginal probability of detection for time inputs, increasing the observability of z may actually induce additional reallocation of household labor between mothers and children. The marginal productivity of mothers and children in the household will also affect non-cooperative behavior via changes in the expected utility gain for any given deviation. When mothers are more productive, a decrease in t_n^h results in a larger reduction in z and thus a larger increase in the probability of detection; however, any given level of z can be provided with less labor and less disutility. The converse is true for children. Accordingly, the formal comparative statics indicate that an increase in mothers' productivity brings labor allocations closer to the cooperative values, whereas an increase in children's productivity does the opposite. Finally, an increase in mothers' relative disutility of own household labor pulls labor allocations further away from the cooperative values. Generalizing to multiple household goods and multiple children, this suggests that deviations in mothers' and children's labor inputs should be larger for goods in which the child has relatively higher productivity and for goods which involve higher disutility for own time spent in production.

Extensions

A variety of extensions to the above model will be discussed briefly here, with more rigorous treatment left to future research. First, allowing the migrant to offer a menu of contracts to his spouse will not affect the main results. The assumption that payoffs for types A and B differ only by a fixed cost associated with non-cooperative behavior ensures that any contract that induces type A to cooperate will also induce type B to cooperate and, because the payoffs are identical for type A and B conditional on playing *cooperate*, both types will have identical preferences for any such contracts. Therefore, with this specific form of player heterogeneity, there does not exist a separating (*i.e.*, the two types choose different contracts) equilibrium in which both types cooperate. If, however, one or both players are risk averse, the migrant can induce type A and B to separate by offering one contract that induces cooperation only from type B and a second contract that is identical but has a smaller spread between s^c and s^{nc}. The second contract still does not offer appropriate incentives for type A to cooperate, but she and the migrant will both prefer this contract if they are risk-averse. If only one player is risk averse, a separating equilibrium can still occur with the risk-averse player paying a premium to reduce the spread between s^c and s^{nc}. The ability to offer a menu of contracts can increase the payoff for the migrant provided that at least one player is risk-averse, but it does not eliminate the range of parameter values for which non-cooperative behavior can occur in equilibrium.

Introducing heterogeneity in a different form will not affect the main results provided that the payoff to playing *don't cooperate* is always weakly greater for type A. As long as this assumption holds, the migrant cannot utilize separate contracts to simultaneously induce cooperation from both types, and thus non-cooperative behavior will occur in equilibrium for certain parameter values. For example, heterogeneity could be characterized as differences in the efficacy of the monitoring technology – the wife may be "good" or "bad" at hiding allocations from her spouse, or the husband may enlist members of his social network to monitor his spouse's actions without knowing ex ante whether they are "good" or "bad" monitors – without eliminating the range of parameter values for which non-cooperative behavior will occur in equilibrium. Alternatively, if types A and B have different preferences for time allocation, household public goods and/or child labor such that *don't cooperate* is, under some contracts, a dominant strategy for type B but not for type A (*e.g.* type B has stronger preferences for the household public good an is willing to trade a smaller amount of child labor for a large reduction in private consumption), there can exist a separating equilibrium in which both types cooperate but under different contracts. In order for this type of heterogeneity to be feasible in the current model, however, it must be the case that the differences in preferences between A and B cannot be observed prior to migration, *i.e.* the differences in preferences do not generate differences in the fully cooperative allocations that would be obtained if monitoring were perfect.

Extending the game to multiple periods will provide the migrant with more latitude in designing incentive-compatible contracts to elicit cooperative behavior. An infinitely repeated version of the above stage game would be a better framework for describing intra-household allocation, as spouses typically interact over long periods of time and external enforcement of contracts is often infeasible. With multiple periods, the migrant would be able to impose more stringent punishments when non-cooperative behavior is detected, and non-cooperative behavior could only occur in equilibrium if such punishments have some lower bound, *e.g.* as imposed by social norms. A dynamic model would also raise additional issues related to limited commitment, as in Ligon (2002), such that the wife cannot commit to cooperate in subsequent periods when facing multiple periods of punishment for prior non-cooperative behavior. Finally, if migration occurs over multiple periods, the migrant may be willing to accept a lower payoff in the first period in order to implement contracts that will enable him to separate types A and B.

VI. Tests of the Non-Cooperative Model

Non-cooperative behavior on the part of the mother implies a reduction in her own household labor and an increase in children's household labor. This is consistent with the findings in Tables 4 and 5. However, the focus on a composite measure of household chores masks substantial variation in time allocation, as indicated in Table 8. One implication of the non-cooperative model is that children who are more productive will be more likely to increase their time in household production, and this effect will be larger for those goods for which children are relatively more productive. As a proxy for productivity, we can examine the frequency with which children engage in various tasks. Laundry is the most common household chore, followed by food preparation. This ordering does not vary by gender of the child, but girls are more likely to be engaged in all three household chores.

A second implication of the non-cooperative model is that changes in household labor will be largest for those tasks which provide the highest disutility. One measure of the degree of disutility associated with a task is the extent to which it depletes body mass. To determine the effort expended for each of the three household chores, I estimate a health production function. Pitt *et. al.* (1990), however, find that work activities and calories are allocated among household members according to unmeasured health-related endowments. Therefore, following Foster and Rosenzweig (1994), variables reflecting household budget constraints – productive assets, household composition, food prices and month and year of survey – are utilized as instruments for activities, calorie intake, and lagged health. Table 7 presents two-stage least squares estimates of the health (BMI) production function. Because data on actual hours in household chores were collected inconsistently, these activities are included as binary regressors, and thus the effect of household chores on BMI can only be estimated imprecisely. Nonetheless, laundry appears to be the most energy intensive household chore.

The child fixed-effects estimates in Table 8 are consistent with these predictions. Longer migration episodes significantly increase the probability that daughters do laundry and have the opposite and also statistically significant effect for sons. The point estimates are quite large in magnitude – the average marginal effects indicate that the probability that sons do laundry is 6.1

percentage points lower and the probability that daughters do laundry is 19.1 percentage points higher, compared to the baseline in which approximately 7.3% of boys and 18.7% of girls aged 6-16 do laundry. The scope for non-cooperative behavior also increases with the number of children in the household because the probability of detection depends on the magnitude of each deviation. However, while the number of children will increase the probability and frequency of non-cooperative behavior, the magnitude of any single deviation is likely to be smaller when there are more children in the household. Thus, for changes on extensive margins, *i.e.* the probability of engaging in some household task, the number of siblings should reinforce non-cooperative behavior, whereas the opposite would be true for changes on intensive margins such as nutritional intake. Consistent with this, results in Table 8 suggest that siblings reinforce the effect of months away, with own gender siblings having a larger effect, although the point estimates are not statistically significant.

The findings for laundry are mirrored in the estimates for food preparation; girls are more likely to be engaged in this task when fathers are away, and the opposite is true for boys. The point estimates are generally smaller than those for laundry, which is consistent with descriptive evidence that children have lower productivity in food preparation than in laundry. However, it also appears that simultaneous changes in all three household chores may be somewhat offsetting. The average marginal effect of paternal migration is much larger for each of the specific chores than for the composite measure of chores presented in Table 4. Furthermore, the probability that boys purchase food for the household increases when fathers are away, and the opposite is true for girls. This follows the gendered division of household labor among adults; the most common chore reported by fathers is purchasing food, and laundry is the least common, with the opposite being true for mothers. Turning to mothers' detailed time allocation, Table 9 shows that mothers are less likely to prepare food or do laundry when fathers migrate, and this effect is increasing in the number of months that the father is away. As with the findings for daughters, the average marginal effects are larger for laundry than for food preparation. The probability that mothers purchase food is also decreasing in months away, but this effect does not dominate the direct positive effect for any value of months away in the relevant range (1-12).

When a specific allocation can be more easily monitored, the non-cooperative value chosen by the mother will be closer to the cooperative value negotiated prior to migration. This is also true for expenditures/allocations that can be mapped directly into observable outcomes – in this case, school attendance. Health (body mass index) can also be easily observed but, with a stochastic production function, it may be more difficult to detect whether changes are due to non-cooperative behavior or unobservable shocks and/or endowments. Indeed, the average marginal effects of migration on health are slightly larger, in percentage terms, than those for school enrollment (see Table 4), although both are small in magnitude and not statistically significant. However, time spent in productive activities also affects an individual's health. To the extent that health outcomes can be monitored, inputs to the health production function, *i.e.* labor hours and nutrition, must be adjusted simultaneously in order to keep observable health measures within a certain range. Estimates presented in Table 10 depict exactly this. Migration of the father has large and statistically significant effects on children's nutritional intake. The direct effect is positive for boys and negative for girls, but age effects work in the opposite direction such that the marginal effects are positive for girls and negative for boys at all ages. Estimated coefficients are larger in magnitude for girls than for boys, consistent with the finding that, in absolute terms, changes in household labor are larger for girls than for boys. Own-gender sibling effects are opposite in sign to age effects, suggesting that the magnitude of deviations declines with the number of children in the household, as predicted. Months away does not have a statistically significant effect on nutrition, which suggests that the intensity of household activities varies predominantly with age, although children of all ages are equally likely to shift on the extensive margin.

Robustness Checks

Next, I examine the possibility that the results are driven by unobserved changes in income or bargaining power rather than by non-cooperative behavior. Changes in the distribution of household bargaining power would not be captured by person-fixed effects, and the variables utilized to control for changes in wages may have significant measurement error. By utilizing a sample of migrants who were home for the entire week preceding the survey, I can largely eliminate the scope for non-cooperative behavior and replace fathers' time in household production. If the remaining factors, change in income and change in bargaining power, are the main causes of the changes in time allocation estimated above, this sample should yield similar results. Results presented in Table 11 indicate no significant effects of migration on either mothers' or children's household labor when migrant fathers are present. The point estimates are much smaller in magnitude and, in fact, tend be opposite in sign (estimates for children's participation in specific household chores are again similar and not presented here). These findings support the conclusion of non-cooperative behavior.

Finally, to determine the generality of the main results, I further restrict the sample of migrant households to those in which the father migrates in multiple survey periods. If migration is less likely to occur in households with strong tendencies towards non-cooperation, households in the restricted sample should exhibit a lesser degree of non-cooperative behavior and therefore smaller changes in time allocation. In fact, estimates in Table 12 suggest the opposite. The negative effect of months away on mothers' household labor is more pronounced, and the average marginal effect is much larger and statistically significant. Estimates for children are less precise than for the main sample in Table 4 but display the same sign pattern, and the average marginal effects are much larger. Taken together, these findings suggest that repeat migration in fact increases the scope for non-cooperative behavior.

Non-Cooperation versus Incentive Compatible Contracts

The results presented above show that migration of one spouse increases the scope for non-cooperation by making it more difficult to monitor certain intrahousehold allocation decisions. However, it is unclear whether non-cooperative behavior is realized in equilibrium, *i.e.* whether contracts are fully incentive compatible such that the non-migrant spouse chooses the allocations stipulated in the contract. These alternatives can be distinguished by examining changes in private consumption for the non-migrant spouse. That is, when contracts are not fully incentive compatible, the non-migrant receives a smaller transfer, even when her non-cooperative behavior is not detected. In the extreme, a zero effect of migration on the non-migrant's private consumption would indicate that the migrant is providing his spouses with very little incentive to behave cooperatively, perhaps because he has strong prior beliefs about his spouse's type. The measures of private consumption available in these data are current nutritional intake and health which, of course, can provide only suggestive evidence on the question of whether non-cooperative behavior is realized in equilibrium. If limited monitoring occurs while the migrant is away, current consumption would not reflect the total transfer ultimately received by the non-migrant. Alternatively, the non-migrant might devote her additional discretionary income to other goods if the marginal utility of nutritional intake and BMI is very low.

Estimates of the effect of migration on nutritional intake and health for non-migrant spouses are presented in Table 13. There are no statistically significant effects of migration on calorie or protein intake. While the point estimates are rather large in magnitude, the overall effect is quite small. Daily intake falls by 20 to 190 calories, and changes in protein intake range from and increase of 0.4 to a reduction of 8 grams. Of course, these changes are also consistent with the finding that mothers take more leisure when fathers are away and therefore require less nutrition. Body mass index provides a better measure, in terms of utility, of changes in private consumption. But, again, migration has no significant effect, and the point estimates are very small in magnitude. The total effect on mothers' BMI ranges from -0.25 to +0.25 points, which implies a change in weight of approximately one pound for an average-height woman with BMI in the normal range. The slight negative effect on BMI might suggest that allocations are, in fact, fully non-cooperative. However, in a fully non-cooperative equilibrium, mothers adjust all allocations to better meet their preferences, irrespective of the degree to which an allocation can be easily monitored. Evidence from Qian (2005), Chen (2006), Duflo (2003) and Thomas (1990) suggests that mothers have stronger preferences for children's schooling and health. This pattern is not evident in the current context (see Table 4), which suggests that mothers are still taking measures to conceal allocations from their spouses and decision-making is not fully non-cooperative.

VII. Conclusion

Non-cooperative behavior among spouses is common in anecdotes but difficult to identify in typical survey data. In this paper, I use the incidence of migration to examine such behavior. Migration by one spouse presents a clear opportunity for non-cooperation by introducing imperfect monitoring and increasing the transaction costs associated with enforcing a cooperative equilibrium. I find evidence that wives of migrants do attempt to conceal allocations from their husbands. In particular, mothers shift household chores to children and consume more leisure themselves when fathers are away. To limit the probability that such behavior is detected, mothers also adjust the distribution of nutrition among children in order to keep observable health outcomes stable. This change in household labor is not consistent with a simple reallocation of time in order to compensate for the father's absence, nor is it consistent with a pure income effect. Furthermore, given existing evidence on rural Chinese households (Qian, 2005), the observed stability in children's health and schooling outcomes is not consistent with an increase in mother's bargaining power due to the absence of the father. These conclusions are also robust to several alternative interpretations: (1) an increase in the demand for mothers' time in non-enumerated household tasks, (2) unobserved changes in bargaining power and/or inadequate controls for changes in income, and (3) self-selection of migrants on the propensity for non-cooperative behavior.

The type of non-cooperative behavior observed in this setting appears relatively innocuous; children's school enrollment is unaffected by the changes in time allocation, and changes in household labor are compensated by changes in nutritional intake in order to maintain child health. However, increasing opportunities for international migration, *i.e.* migration over longer distances and for longer periods of time, will exacerbate informational asymmetries. The ultimate effect on intrahousehold allocation will depend on the capacity for monitoring and the preferences of decision-makers remaining in the sending household. To the extent that this information problem constrains the allocation of remittance income to easily observable goods, non-cooperative behavior may generate inefficiencies in investment and hinder growth (see

Chami *et. al.*, 2003). Development agencies may also wish to consider how the efficacy of targeted transfers and subsidies is affected by the transparency of those income sources.

Further research should consider the effect of non-cooperative behavior on a broader range of allocations which have larger implications for economic growth, *e.g.* schooling-related expenditures and investments in income-generating activities. To do so, it will be crucial to understand how remittance flows are affected by non-cooperative behavior on the part of both recipients and senders. If migrants' earnings are difficult for sending households to monitor, migrants face a trade-off when determining the value of remittance flows. An increase in remittances will increase the migrant's bargaining power in the household but, because the migrant must then bargain with other household members over the allocation of this income, remittance flows will effectively be taxed, even when there is no non-cooperative behavior on the part of recipients. Better data on the distance of migration and frequency of visits would shed light on the sensitivity of non-cooperative behavior to the capacity for monitoring and permit clearer extrapolation to households in which all decision-makers are co-resident.

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Table 1. Characteristics of Children Age 6-16 by Gender and Migrant Status

	Father Never Migrates		Father Migrates at Least Once		Father Currently Away	
	Sons	Daughters	Sons	Daughters	Sons	Daughters
Age	11.28	11.40	11.46 *	11.34	11.63	11.50
	(3.093)	(3.074)	(3.073)	(3.062)	(3.131)	(3.121)
School Enrollment	0.861	0.827	0.882 *	0.868 ***	0.893	0.884 **
	(0.346)	(0.378)	(0.323)	(0.339)	(0.310)	(0.321)
Body Mass Index	17.14	17.11	17.02	17.21	16.88	17.33
	(3.609)	(2.937)	(2.527)	(3.124)	(2.588)	(2.964)
Upper Arm Circumference	19.04	19.03	18.87	19.11	18.97	19.06
	(4.423)	(3.628)	(3.409)	(3.517)	(3.521)	(3.926)
Skin Fold	8.320	10.29	8.573	10.58	8.952	11.26 *
	(4.906)	(5.622)	(4.951)	(5.276)	(5.104)	(5.366)
Buy Food for the Hh	0.024	0.032	0.021	0.040	0.021	0.041
	(0.153)	(0.176)	(0.142)	(0.197)	(0.143)	(0.198)
Prepare Food for the Hh	0.057	0.117	0.059	0.106	0.087	0.130
	(0.231)	(0.321)	(0.236)	(0.308)	(0.283)	(0.338)
Do Laundry for the Hh	0.073	0.187	0.061	0.159 *	0.084	0.173
	(0.259)	(0.390)	(0.239)	(0.366)	(0.278)	(0.379)
Do Any Chores	0.115	0.228	0.102	0.201 *	0.125	0.209
(buy/prep food or laundry)	(0.319)	(0.420)	(0.303)	(0.401)	(0.332)	(0.408)
Engage in Other Work	0.063	0.077	0.050	0.062 *	0.043	0.068
	(0.243)	(0.267)	(0.219)	(0.241)	(0.203)	(0.252)
Daily Calorie Intake	1851	1711	1880	1724	1838	1663
	(636.3)	(551.2)	(595.1)	(532.2)	(551.3)	(706.9)
Daily Protein Intake	63.01	58.14	63.84	57.95	64.49	54.68 *
	(25.40)	(21.79)	(22.82)	(19.59)	(23.11)	(21.63)
Daily Fat Intake	28.37	25.99	29.20	25.95	35.01 ***	22.81 *
	(26.71)	(22.24)	(24.34)	(20.63)	(27.89)	(17.81)
Months Away in the Year					6.681	6.340
					(3.908)	(3.923)
Number of Observations	4474	4116	1073	969	210	162

Notes: Standard deviations reported in parentheses. (*) indicates significantly different from column [1] or column [2] at the 10%, (**) 5% or (***) 1% level. Observations at the person-year level.

Table 2. Characteristics of Households by Migrant Status

	Husband Never Migrates	Husband Migrates at Least Once	Husband Currently Away
Number of Children	2.116	2.099	2.000 **
	(0.941)	(0.927)	(0.923)
Sex Ratio of Children	0.544	0.537	0.577
	(0.357)	(0.354)	(0.361)
% with Only One Child	0.278	0.278	0.318
	(0.448)	(0.448)	(0.467)
Mother's Age	38.34	38.07	38.20
	(6.374)	(5.743)	(6.075)
Father's Age	40.26	39.80 **	39.95
	(7.018)	(6.078)	(6.595)
Mother's Schooling	5.636	6.285 ***	6.195 **
	(4.129)	(4.061)	(3.744)
Father's Schooling	7.539	7.989 ***	7.899 *
	(3.497)	(3.232)	(3.034)
Household Size	4.313	4.237 **	4.106 ***
	(1.118)	(1.027)	(0.969)
Mother's Wage	9.072	9.060	9.154
	(12.79)	(13.30)	(9.439)
Father's Wage	11.79	12.68 *	15.24 ***
	(21.42)	(16.56)	(15.55)
Area of Owned Home	66.31	64.85	65.12
	(54.86)	(57.73)	(50.82)
Farm Land	3.636	2.810 ***	3.121
	(8.807)	(4.697)	(6.662)
Value of Business Equip.	213.1	209.3	58.77 ***
	(2234)	(3087)	(388.4)
Adj. Per Capita Hh Income	1344	1430 ***	1590 ***
	(1053)	(1045)	(1052)
Months Away in the Year			6.606
			(3.878)
Number of Observations	5666	1344	264

Notes: Standard deviations reported in parentheses. (*) indicates significantly different from column [1] at the 10%, (**) 5% or (***) 1% level. Observations at the household-year level.

Table 3. Mothers' Outcomes of Interest by Migrant Status

	Husband Does Not Migrate	Husband Migrates at Least Once	Husband Currently Away
Total Work Hours	43.50	44.78	44.43
(excl. household chores)	(29.25)	(30.38)	(29.48)
Body Mass Index	22.41	22.27	22.31
	(2.963)	(2.819)	(2.841)
Upper Arm Circumference	25.08	25.01	25.14 **
	(3.090)	(2.767)	(2.729)
Skin Fold	14.67	14.78	15.78
	(7.086)	(6.989)	(7.094)
Daily Calorie Intake	2119	2121	2027 **
	(660.8)	(598.7)	(633.8)
Daily Protein Intake	71.62	71.40	69.40
	(26.13)	(21.89)	(24.10)
Daily Fat Intake	29.99	31.36 *	31.72
	(26.38)	(24.06)	(26.67)
Buy Food for the Hh	0.614	0.673 ***	0.760 ***
	(0.487)	(0.469)	(0.428)
Prepare Food for the Hh	0.912	0.916	0.939 *
	(0.284)	(0.277)	(0.240)
Do Laundry for the Hh	0.915	0.929 *	0.935
	(0.279)	(0.257)	(0.247)
Do Any Chores	0.974	0.969	0.973
(buy/prep food or laundry)	(0.158)	(0.172)	(0.162)
Number of Observations	5677	1344	264

Notes: Standard deviations reported in parentheses. (*) indicates significantly different from column [1] at the 10%, (**) 5% or (***) 1% level. Observations at the person-year level.

Table 4. Outcomes for Children Age 6-16, Child-Fixed Effects Estimates

	I Do Any Chores	II Engage in Other Work	III School Enrollment	IV Body Mass Index
Father Away	0.255	0.032	0.056	-0.671
	(0.209)	(0.119)	(0.172)	(0.844)
Months Father Away	-0.043	0.000	-0.053	0.203
	(0.053)	(0.026)	(0.045)	(0.253)
Months Away Squared	0.003	-0.001	0.005 *	-0.025
	(0.004)	(0.002)	(0.003)	(0.022)
(Age-6)*Away	-0.006	0.029	0.019	0.155
	(0.049)	(0.029)	(0.045)	(0.257)
(Age-6) Squared*Away	0.000	-0.003	0.000	-0.015
	(0.004)	(0.003)	(0.004)	(0.026)
Male Siblings*Away	-0.056	-0.034	0.016	0.525
	(0.079)	(0.031)	(0.068)	(0.362)
Female Siblings*Away	-0.052	-0.060	-0.048	0.388
	(0.047)	(0.052)	(0.078)	(0.326)
Marginal Effect of Away	**0.008**	**0.004**	**0.006**	**0.351**
	(0.080)	**(0.041)**	**(0.071)**	**(0.366)**
Relative Effect for Girls				
Father Away	-0.415	0.060	-0.041	0.657
	(0.299)	(0.161)	(0.275)	(1.750)
Months Father Away	0.100	-0.025	0.008	-0.180
	(0.077)	(0.047)	(0.065)	(0.447)
Months Away Squared	-0.007	0.002	-0.001	0.020
	(0.006)	(0.003)	(0.005)	(0.038)
(Age-6)*Away	-0.019	0.023	-0.030	-0.539
	(0.077)	(0.037)	(0.073)	(0.522)
(Age-6) Squared*Away	0.001	-0.003	0.004	0.063
	(0.007)	(0.004)	(0.006)	(0.052)
Male Siblings*Away	0.137	0.066	-0.074	-0.490
	(0.111)	(0.052)	(0.091)	(0.950)
Female Siblings*Away	0.101	-0.050	0.051	-0.147
	(0.109)	(0.068)	(0.105)	(0.754)
Marginal Effect of Away	**-0.005**	**0.017**	**-0.068**	**-0.898**
	(0.135)	**(0.062)**	**(0.116)**	**(0.852)**
Number of Observations	8739	9794	9056	6121

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for age of parents, assets owned, household size, parents' wages, month and year of survey, and community-year fixed effects.

Table 5. Mothers' Time Allocation, Mother-Fixed Effects Estimates

	I	II	III
	Do Any Chores	Work Hours (excl. chores)	Body Mass Index
Father Away	0.077 *	12.19	0.317
	(0.046)	(8.002)	(0.442)
Months Father Away	-0.031	-5.579 *	-0.071
	(0.019)	(3.085)	(0.172)
Months Away Squared	0.002 **	0.459 *	0.002
	(0.001)	(0.241)	(0.014)
Marginal Effect of Away	**-0.041**	**-4.395**	**-0.106**
	(0.030)	**(4.274)**	**(0.241)**
Number of Observations	6450	5996	5777

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for own and husband's age, own and husband's wages, assets owned, household size, month and year of survey, and community-year fixed effects.

Table 6. Household Time Allocation, Father Debilitated

	Do Any Chores		
	I	II	
	Mothers	Sons	Girls (Relative)
Father Debilitated	0.016	0.246 *	-0.461 *
	(0.017)	(0.137)	(0.258)
Days Father Debilitated	0.001	0.002	-0.005
	(0.002)	(0.006)	(0.010)
Days Debilitated Squared	0.000	0.000	0.000
	(0.000)	(0.000)	(0.000)
(Age-6)*Sick		-0.064	0.188 *
		(0.052)	(0.096)
(Age-6) Squared*Sick		0.004	-0.015 *
		(0.005)	(0.009)
Male Siblings*Sick		-0.026	0.116
		(0.064)	(0.113)
Female Siblings*Sick		0.002	-0.129
		(0.049)	(0.087)
Marginal Effect of Sick	**0.025**	**-0.011**	**0.104**
	(0.016)	**(0.050)**	**(0.089)**
Number of Observations	5396	7393	

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for own and husband's age, own and husband's wages, assets owned, household size, month and year of survey, and community-year fixed effects. Includes children age 6-16.

Table 7. 2SLS Estimates
of BMI Production Function

Lagged BMI	0.8330	***
	(0.0327)	
Hours in Wage Labor	0.0012	
Professional and Administrative	(0.0052)	
Hours in Wage Labor	-0.0041	
Skilled and Semi-Skilled	(0.0067)	
Hours in Wage Labor	-0.0270	
Farmers, Fishermen, etc.	(0.0344)	
Hours in Wage Labor	0.0061	
Unskilled	(0.0057)	
Hours in Wage Labor	-0.0046	
Service and Other Misc.	(0.0066)	
Hours in Gardening	-0.0139	***
	(0.0039)	
Hours in Farming	-0.0051	*
	(0.0030)	
Hours in Livestock Care	-0.0370	***
	(0.0105)	
Hours in Fishing	0.0557	
	(0.0932)	
Hours in Handicrafts	0.0029	
	(0.0050)	
Buy Food for the Hh	-0.1588	
	(0.2238)	
Prepare Food for the Hh	0.8183	***
	(0.2540)	
Do Laundry for the Hh	-0.7689	***
	(0.2579)	
Daily Calorie Intake * 10^{-3}	0.1527	
	(0.1266)	
Age	0.0077	
	(0.0224)	
Age Squared	-0.0001	
	(0.0003)	
Female	0.2149	
	(0.1715)	
Constant	3.5513	***
	(0.6720)	
Number of Observations	3464	

Notes: Standard errors clustered at the individual level and reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Activities, calorie intake, and lagged health instrumented with assets, household composition, food prices, community of residence, and year and month of survey. Includes individuals age 16 to 60.

Table 8. Children's Detailed Time Allocation, Child-Fixed Effects Estimates

	I Buy Food	II Prepare Food	III Do Laundry
Father Away	0.030	0.253	0.228
	(0.084)	(0.168)	(0.195)
Months Father Away	0.024	-0.047	-0.084 *
	(0.017)	(0.048)	(0.051)
Months Away Squared	-0.002 **	0.003	0.007 *
	(0.001)	(0.004)	(0.004)
(Age-6)*Away	-0.018	-0.044	0.013
	(0.021)	(0.038)	(0.042)
(Age-6) Squared*Away	0.001	0.004	-0.001
	(0.002)	(0.004)	(0.004)
Male Siblings*Away	-0.008	0.001	-0.057
	(0.018)	(0.052)	(0.084)
Female Siblings*Away	-0.027	-0.006	-0.055
	(0.023)	(0.035)	(0.046)
Marginal Effect of Away	**0.046**	**-0.064**	**-0.061**
	(0.031)	**(0.087)**	**(0.072)**
Relative Effect for Girls			
Father Away	-0.247	-0.116	-0.365
	(0.154)	(0.208)	(0.287)
Months Father Away	-0.008	0.065	0.164 **
	(0.041)	(0.064)	(0.078)
Months Away Squared	0.000	-0.003	-0.013 **
	(0.003)	(0.005)	(0.006)
(Age-6)*Away	0.010	0.006	-0.015
	(0.039)	(0.054)	(0.070)
(Age-6) Squared*Away	0.000	0.000	0.002
	(0.003)	(0.005)	(0.006)
Male Siblings*Away	0.106 *	-0.091	0.034
	(0.056)	(0.078)	(0.117)
Female Siblings*Away	0.082	-0.077	0.142
	(0.063)	(0.079)	(0.112)
Marginal Effect of Away	**-0.136** *	**0.120**	**0.191**
	(0.073)	**(0.112)**	**(0.123)**
Number of Observations	8723	8476	8329

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for age of parents, assets owned, household size, parents' wages, month and year of survey, and community-year fixed effects. Includes children age 6-16.

Table 9. Mothers' Detailed Time Allocation, Mother-Fixed Effects Estimates

	I	II	III
	Buy Food	Prepare Food	Do Laundry
Father Away	0.125	0.105 *	0.067
	(0.147)	(0.062)	(0.056)
Months Father Away	-0.033	-0.045 *	-0.041
	(0.052)	(0.024)	(0.025)
Months Away Squared	0.003	0.003	0.003
	(0.004)	(0.002)	(0.002)
Marginal Effect of Away	**0.031**	**-0.059**	**-0.078 ***
	(0.070)	**(0.040)**	**(0.040)**
Number of Observations	6440	6430	6436

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for own and husband's age, own and husband's wages, assets owned, household size, month and year of survey, and community-year fixed effects.

Table 10. Children's Nutrition, Child-Fixed Effects Estimates

	I Daily Calorie Intake	II Daily Protein Intake	III Daily Fat Intake
Father Away	288.0	18.51	7.558
	(276.6)	(13.94)	(11.34)
Months Father Away	10.96	-1.025	0.070
	(93.99)	(4.205)	(3.855)
Months Away Squared	-3.574	0.010	-0.117
	(8.319)	(0.355)	(0.327)
(Age-6)*Away	-181.0 **	-8.521 **	-3.800
	(88.37)	(3.841)	(3.372)
(Age-6) Squared*Away	18.11 **	0.867 **	0.387
	(9.166)	(0.418)	(0.360)
Male Siblings*Away	23.31	-6.843	-1.367
	(140.0)	(5.762)	(4.980)
Female Siblings*Away	187.3	5.489	6.205
	(132.6)	(5.389)	(4.782)
Marginal Effect of Away	**-139.7**	**-8.776**	**-4.167**
	(142.6)	**(5.501)**	**(4.937)**
Relative Effect for Girls			
Father Away	-409.8	-4.813	-12.96
	(444.8)	(20.32)	(15.33)
Months Father Away	53.31	0.160	3.424
	(138.5)	(5.914)	(4.959)
Months Away Squared	-2.433	-0.008	-0.113
	(10.98)	(0.457)	(0.396)
(Age-6)*Away	211.4 *	6.591	1.380
	(119.8)	(5.547)	(4.400)
(Age-6) Squared*Away	-24.68 **	-0.871	-0.104
	(12.38)	(0.561)	(0.462)
Male Siblings*Away	130.5	9.081	0.817
	(188.3)	(7.596)	(6.594)
Female Siblings*Away	-257.7	-5.289	-8.785
	(209.9)	(8.966)	(6.543)
Marginal Effect of Away	**160.5**	**5.985**	**6.033**
	(229.6)	**(9.167)**	**(6.901)**
Number of Observations	7303	7283	7173

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for age of parents, assets owned, household size, parents' wages, month and year of survey, and community-year fixed effects. Includes children age 6-16.

Table 11. Household Time Allocation, Migrant Home at Survey

	Do Any Chores		
	I	II	
	Mothers	Sons	Girls (Relative)
Father Away	-0.008	0.147	0.035
	(0.047)	(0.132)	(0.207)
Months Father Away	0.008	0.017	-0.109
	(0.024)	(0.063)	(0.086)
Months Away Squared	-0.001	-0.003	0.011
	(0.002)	(0.005)	(0.007)
(Age-6)*Away		-0.050	0.015
		(0.044)	(0.072)
(Age-6) Squared*Away		0.004	-0.001
		(0.005)	(0.008)
Male Siblings*Away		-0.047	0.072
		(0.092)	(0.113)
Female Siblings*Away		-0.018	0.078
		(0.063)	(0.097)
Marginal Effect of Away	**0.020**	**-0.063**	**-0.049**
	(0.037)	**(0.111)**	**(0.148)**
Number of Observations	6405	8670	

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for own and husband's age, own and husband's wages, assets owned, household size, month and year of survey, and community-year fixed effects. Includes children age 6-16.

Table 12. Household Time Allocation, Father Migrates Multiple Times

	Do Any Chores		
	I	II	
	Mothers	Sons	Girls (Relative)
Father Away	0.113	-0.069	-0.314
	(0.075)	(0.290)	(0.435)
Months Father Away	-0.060 *	-0.005	0.110
	(0.032)	(0.079)	(0.117)
Months Away Squared	0.004 **	0.001	-0.008
	(0.002)	(0.006)	(0.009)
(Age-6)*Away		0.004	-0.009
		(0.070)	(0.099)
(Age-6) Squared*Away		-0.001	-0.002
		(0.006)	(0.009)
Male Siblings*Away		0.005	0.255
		(0.144)	(0.287)
Female Siblings*Away		0.024	-0.049
		(0.080)	(0.176)
Marginal Effect of Away	**-0.108 ****	**-0.040**	**0.056**
	(0.053)	**(0.132)**	**(0.220)**
Number of Observations	5604	7482	

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for own and husband's age, own and husband's wages, assets owned, household size, month and year of survey, and community-year fixed effects. Includes children age 6-16.

Table 13. Mothers' Nutrition, Mother-Fixed Effects Estimates

	I Daily Calorie Intake	II Daily Protein Intake	III Body Mass Index
Father Away	-156.9	-6.087	0.317
	(141.9)	(5.439)	(0.442)
Months Father Away	48.59	2.285	-0.071
	(62.08)	(2.249)	(0.172)
Months Away Squared	-4.232	-0.202	0.002
	(4.958)	(0.176)	(0.014)
Marginal Effect of Away	-24.19	-0.012	-0.106
	(97.90)	(3.541)	(0.241)
Number of Observations	6065	6051	5777

Notes: Robust standard errors reported in parentheses. (*) indicates significant at the 10%, 5% (**) or 1% (***) level. Marginal effects calculated at values approximate to the sample average. Includes controls for own and husband's age, own and husband's wages, assets owned, household size, month and year of survey, and community-year fixed effects.

Technical Appendix

A. Cooperative Case – Unitary Household

First Order Conditions

$$\frac{\partial U}{\partial t_m} + \frac{\partial U}{\partial x_m} w_m = 0$$
$$\frac{\partial U}{\partial t_n} + \frac{\partial U}{\partial x_n} w_n = 0$$
$$\frac{\partial U}{\partial x_m} - \frac{\partial U}{\partial x_n} = 0$$
$$\frac{\partial U}{\partial t_m} \mu + \frac{\partial U}{\partial z} \frac{\partial z}{\partial t_m^h} = 0$$
$$\frac{\partial U}{\partial t_n} \mu + \frac{\partial U}{\partial z} \frac{\partial z}{\partial t_n^h} = 0$$
$$\frac{\partial U}{\partial t_k} + \frac{\partial U}{\partial z} \frac{\partial z}{\partial t_k} = 0$$

Assumptions

(1) All goods separable in utility.
(2) No complementarities in household production.

Let H denote the determinant of the Hessian, and define its elements as

$$h_{11} = \frac{\partial^2 U}{\partial t_m{}^2} + \frac{\partial^2 U}{\partial x_m{}^2} w_m{}^2 < 0$$
$$h_{12} = \frac{\partial^2 U}{\partial x_n{}^2} w_m w_n < 0$$
$$h_{13} = -\frac{\partial^2 U}{\partial x_n{}^2} w_m > 0$$
$$h_{14} = \frac{\partial^2 U}{\partial t_m{}^2} \mu < 0$$
$$h_{22} = \frac{\partial^2 U}{\partial t_n{}^2} + \frac{\partial^2 U}{\partial x_n{}^2} w_n{}^2 < 0$$
$$h_{23} = -\frac{\partial^2 U_n}{\partial x_n{}^2} w_n > 0$$

$$h_{25} = \frac{\partial^2 U}{\partial t_n^{\ 2}} \mu < 0$$

$$h_{33} = \frac{\partial^2 U}{\partial x_m^{\ 2}} + \frac{\partial^2 U}{\partial x_n^{\ 2}} < 0$$

$$h_{44} = \frac{\partial^2 U}{\partial t_m^{\ 2}} \mu^2 + \frac{\partial^2 U}{\partial z^2} \left(\frac{\partial z}{\partial t_m^h} \right)^2 + \frac{\partial U}{\partial z} \frac{\partial^2 z}{\partial t_m^{h\,2}} < 0$$

$$h_{45} = \frac{\partial^2 U}{\partial z^2} \frac{\partial z}{\partial t_m^h} \frac{\partial z}{\partial t_n^h} < 0$$

$$h_{46} = \frac{\partial^2 U}{\partial z^2} \frac{\partial z}{\partial t_m^h} \frac{\partial z}{\partial t_k} < 0$$

$$h_{55} = \frac{\partial^2 U}{\partial t_n^{\ 2}} \mu^2 + \frac{\partial^2 U}{\partial z^2} \left(\frac{\partial z}{\partial t_n^h} \right)^2 + \frac{\partial U}{\partial z} \frac{\partial^2 z}{\partial t_n^{h\,2}} < 0$$

$$h_{56} = \frac{\partial^2 U}{\partial z^2} \frac{\partial z}{\partial t_n^h} \frac{\partial z}{\partial t_k} < 0$$

$$h_{66} = \frac{\partial^2 U}{\partial t_k^{\ 2}} + \frac{\partial^2 U}{\partial z^2} \left(\frac{\partial z}{\partial t_k} \right)^2 + \frac{\partial U}{\partial z} \frac{\partial^2 z}{\partial t_k^{\ 2}} < 0$$

$$s_{15} = s_{16} = s_{24} = s_{26} = s_{34} = s_{35} = s_{36} = 0$$

$$\left.\frac{dt_m^h}{dw_m}\right|^{compensated} = \frac{1}{H}\ [((h25h33h56h46 - h25h33h66h45)h12 + (-h25h56h23h46 + h25h66h23h45)h13 + (h66h33h25^2 + h33h56^2h22 - h33h66h55h22 - h56^2h23^2 + h66h23^2h55)h14)a]$$

$$\left.\frac{dt_n^h}{dw_m}\right|^{compensated} = \frac{1}{H}\ [((-h25h33h46^2 + h25h33h66h44)h12 + (h25h23h46^2 - h25h23h66h44)h13 + (-h23^2h66h45 + h33h66h45h22 + h46h23^2h56 - h46h33h56h22)h14)a]$$

$$\left.\frac{dt_k}{dw_m}\right|^{compensated} = \frac{1}{H}\ [((h25h33h45h46 - h25h33h56h44)h12 + (-h25h23h45h46 + h25h23h56h44)h13 + (-h46h23^2h55 + h23^2h56h45 - h33h25^2h46 + h46h33h55h22 - h33h56h45h22)h14)a]$$

$\frac{dt_n^h}{dt_m^h} < 0$, $\frac{dt_k}{dt_m^h}$ *ambiguous*

where $a = -\frac{\partial U}{\partial x_n}$

$$\left.\frac{dt_n^h}{dw_m}\right|_{t_m^h}^{income} = \frac{1}{H}\ [((h12h25h33 - h13h25h23)h66b + (-h11h25h33 + h13^2h25)h66c + (h11h25h23 - h12h25h13)h66d]$$

$$\left.\frac{dt_k}{dw_m}\right|_{t_m^h}^{income} = \frac{1}{H}\ [((-h12h25h33 + h13h25h23)h56b + (h11h25h33 - h13^2h25)h56c + (-h11h25h23 + h12h25h13)h56d]$$

$\left.\frac{dt_n^h}{dw_m}\right|_{t_m^h}^{income} > 0$, $\left.\frac{dt_k}{dw_m}\right|_{t_m^h}^{income} < 0$

where $b = -\frac{\partial^2 U}{\partial {x_n}^2} t_m^w w_n$, $c = -\frac{\partial^2 U}{\partial {x_n}^2} t_m^w w_n$, $d = \frac{\partial^2 U}{\partial {x_n}^2} t_m^w w_n$

B. Cooperative Case – Non-Unitary Household

<u>First Order Conditions</u>

$$\lambda \frac{\partial U_m}{\partial t_m} + (1-\lambda)\frac{\partial U_n}{\partial x_n} w_m = 0$$

$$\frac{\partial U_n}{\partial t_n} + \frac{\partial U_n}{\partial x_n} w_n = 0$$

$$\lambda \frac{\partial U_m}{\partial x_m} - (1-\lambda)\frac{\partial U_n}{\partial x_n} = 0$$

$$\lambda\left(\frac{\partial U_m}{\partial t_m}\mu_m + \frac{\partial U_m}{\partial z}\frac{\partial z}{\partial t_m^h}\right) + (1-\lambda)\left(\frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_m^h}\right) = 0$$

$$\lambda\left(\frac{\partial U_m}{\partial z}\frac{\partial z}{\partial t_n^h}\right) + (1-\lambda)\left(\frac{\partial U_n}{\partial t_n}\mu_n + \frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_n^h}\right) = 0$$

$$\lambda\left(\frac{\partial U_m}{\partial t_k} + \frac{\partial U_m}{\partial z}\frac{\partial z}{\partial t_k}\right) + (1-\lambda)\left(\frac{\partial U_n}{\partial t_k} + \frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_k}\right) = 0$$

<u>Assumptions</u>

(1) All goods separable in utility.
(2) No complementarities in household production.

Let D denote the determinant of the Hessian, and define its elements as

$$s_{11} = \lambda \frac{\partial^2 U_m}{\partial t_m^{\;2}} + (1-\lambda)\frac{\partial^2 U_n}{\partial x_n^{\;2}} w_m^{\;2} < 0$$

$$s_{12} = (1-\lambda)\frac{\partial^2 U_n}{\partial x_n^{\;2}} w_m w_n < 0$$

$$s_{13} = -(1-\lambda)\frac{\partial^2 U_n}{\partial x_n^{\;2}} w_m > 0$$

$$s_{14} = \frac{\partial^2 U_m}{\partial t_m^{\;2}} \mu_m < 0$$

$$s_{22} = \frac{\partial^2 U_n}{\partial t_n^{\;2}} + \frac{\partial^2 U_n}{\partial x_n^{\;2}} w_n^{\;2} < 0$$

$$s_{23} = -\frac{\partial^2 U_n}{\partial x_n^{\;2}} w_n > 0$$

$$s_{25} = \frac{\partial^2 U_n}{\partial t_n{}^2}\mu_n < 0$$

$$s_{33} = \lambda \frac{\partial^2 U_m}{\partial x_m{}^2} + (1-\lambda)\frac{\partial^2 U_n}{\partial x_n{}^2} < 0$$

$$s_{44} = \lambda\left[\frac{\partial^2 U_m}{\partial t_m{}^2}\mu_m{}^2 + \frac{\partial^2 U_m}{\partial z^2}\left(\frac{\partial z}{\partial t_m^h}\right)^2 + \frac{\partial U_m}{\partial z}\frac{\partial^2 z}{\partial t_m^{h\,2}}\right] + (1-\lambda)\left[\frac{\partial^2 U_n}{\partial z^2}\left(\frac{\partial z}{\partial t_m^h}\right)^2 + \frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_m^{h\,2}}\right] < 0$$

$$s_{45} = \lambda\left(\frac{\partial^2 U_m}{\partial z^2}\frac{\partial z}{\partial t_m^h}\frac{\partial z}{\partial t_n^h}\right) + (1-\lambda)\left(\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial t_m^h}\frac{\partial z}{\partial t_n^h}\right) < 0$$

$$s_{46} = \lambda\left(\frac{\partial^2 U_m}{\partial z^2}\frac{\partial z}{\partial t_m^h}\frac{\partial z}{\partial t_k}\right) + (1-\lambda)\left(\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial t_m^h}\frac{\partial z}{\partial t_k}\right) < 0$$

$$s_{55} = \lambda\left[\frac{\partial^2 U_m}{\partial z^2}\left(\frac{\partial z}{\partial t_n^h}\right)^2 + \frac{\partial U_m}{\partial z}\frac{\partial^2 z}{\partial t_n^{h\,2}}\right] + (1-\lambda)\left[\frac{\partial^2 U_n}{\partial t_n{}^2}\mu_n{}^2 + \frac{\partial^2 U_n}{\partial z^2}\left(\frac{\partial z}{\partial t_n^h}\right)^2 + \frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_n^{h\,2}}\right] < 0$$

$$s_{56} = \lambda\left(\frac{\partial^2 U_m}{\partial z^2}\frac{\partial z}{\partial t_n^h}\frac{\partial z}{\partial t_k}\right) + (1-\lambda)\left(\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial t_n^h}\frac{\partial z}{\partial t_k}\right) < 0$$

$$s_{66} = \lambda\left[\frac{\partial^2 U_m}{\partial t_k{}^2} + \frac{\partial^2 U_m}{\partial z^2}\left(\frac{\partial z}{\partial t_k}\right)^2 + \frac{\partial U_m}{\partial z}\frac{\partial^2 z}{\partial t_k{}^2}\right] + (1-\lambda)\left[\frac{\partial^2 U_n}{\partial t_k{}^2} + \frac{\partial^2 U_n}{\partial z^2}\left(\frac{\partial z}{\partial t_k}\right)^2 + \frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_k{}^2}\right] < 0$$

$$s_{15} = s_{16} = s_{24} = s_{26} = s_{34} = s_{35} = s_{36} = 0$$

$$\left.\frac{dt_m^h}{dw_m}\right|^{compensated} = \frac{1}{D}\ [((s25s33s56s46 - s25s33s66s45)s12 + (-s25s56s23s46 + s25s66s23s45)s13 + (s66s33s25^2 + s33s56^2s22 - s33s66s55s22 - s56^2s23^2 + s66s23^2s55)s14)e]$$

$$\left.\frac{dt_n^h}{dw_m}\right|^{compensated} = \frac{1}{D}\ [((-s25s33s46^2 + s25s33s66s44)s12 + (s25s23s46^2 - s25s23s66s44)s13 + (-s23^2s66s45 + s33s66s45s22 + s46s23^2s56 - s46s33s56s22)s14)e]$$

$$\left.\frac{dt_k}{dw_m}\right|^{compensated} = \frac{1}{D}\ [((s25s33s45s46 - s25s33s56s44)s12 + (-s25s23s45s46 + s25s23s56s44)s13 + (-s46s23^2s55 + s23^2s56s45 - s33s25^2s46 + s46s33s55s22 - s33s56s45s22)s14)e]$$

$$\frac{dt_n^h}{dt_m^h} < 0,\ \frac{dt_k}{dt_m^h}\ ambiguous$$

where $e = -(1-\lambda)\frac{\partial U_n}{\partial x_n}$

$$\left.\frac{dt_n^h}{dw_m}\right|_{t_m^h}^{income} = \frac{1}{D}\ [((s12s25s33 - s13s25s23)s66f + (-s11s25s33 + s13^2s25)s66g + (s11s25s23 - s12s25s13)s66i]$$

$$\left.\frac{dt_k}{dw_m}\right|_{t_m^h}^{income} = \frac{1}{D}\ [((-s12s25s33 + s13s25s23)s56f + (s11s25s33 - s13^2s25)s56g + (-s11s25s23 + s12s25s13)s56i]$$

$$\left.\frac{dt_n^h}{dw_m}\right|_{t_m^h}^{income} > 0,\ \left.\frac{dt_k}{dw_m}\right|_{t_m^h}^{income} < 0$$

where $f = -(1-\lambda)\frac{\partial^2 U_n}{\partial x_n{}^2}t_m^w w_n$, $g = -\frac{\partial^2 U_n}{\partial x_n{}^2}t_m^w w_n$, $i = (1-\lambda)\frac{\partial^2 U_n}{\partial x_n{}^2}t_m^w w_n$

$$\left.\frac{dt_n^h}{d\lambda}\right|_{t_m^h} = \frac{1}{D}\ [(s12s25s33 - s13s25s23)s66j + (-s12s25s13 + s11s25s23)s66k + (s11s22s33$$

$$- s11s23^2 - s22s13^2 - s12^2s33 + 2s12s23s13)s66m + (-2s12s23s13 - s11s22s33$$

$$+ s12^2s33 + s11s23^2 + s22s13^2)s56n]$$

$$\left.\frac{dt_k}{d\lambda}\right|_{t_m^h} = \frac{1}{D}\ [(-s12s33 + s13s23)s25s56j + (s12s25s13 - s11s25s23)s56k + (-s11s22s33$$

$$+ s11s23^2 + s22s13^2 + s12^2s33 - 2s12s23s13)s56m + ((s11s22s33 - s11s23^2$$

$$+ 2s12s23s13 - s12^2s33 - s13^2s22)s55 + (-s11s33 + s13^2)s25^2)n]$$

$$\left.\frac{dx_n}{d\lambda}\right|_{t_m^h} = \frac{1}{D}\ [((s12s23s55 - s13s22s55 + s13s25^2)s66 + (-s12s23 + s13s22)s56^2)j + ((s11s22s55$$

$$- s12^2s55 - s11s25^2)s66 + (- s11s22 + s12^2)s56^2)k + (s11s25s23 - s12s25s13)s66m$$

$$+ (s12s25s13 - s11s25s23)s56n]$$

$$\left.\frac{dt_n^h}{d\lambda}\right|_{t_m^h} > 0,\ \left.\frac{dt_k}{d\lambda}\right|_{t_m^h} < 0,\ \left.\frac{dx_n}{d\lambda}\right|_{t_m^h} > 0 \ \text{ if } \left(\frac{\partial U_n}{\partial t_k} + \frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_k}\right) \geq 0$$

where $j = \frac{1}{\lambda^2}\frac{\partial U_n}{\partial x_n} w_m, k = -\frac{1}{\lambda^2}\frac{\partial U_n}{\partial x_n},\ m = -\frac{1}{\lambda^2}\left(\frac{\partial U_n}{\partial t_n}\mu_n + \frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_n^h}\right)$ and

$$n = -\frac{1}{\lambda^2}\left(\frac{\partial U_n}{\partial t_k} + \frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_k}\right)$$

C. Non-Cooperative Case

<u>Derivations for proof of **Proposition 2**</u>.

$$\frac{dV_n^{\ c}}{ds^c}-\frac{dV_n^{\ nc}}{ds^c}=\left[\frac{\partial U_n(t_n^{w^c},z^c,t_n^{h^c},t_k^{\ c},x_n^{\ c}+s^c)}{\partial s^c}-(1-q^*)\frac{\partial U_n(t_n^w,z,t_n^h,t_k,x_n+s^c)}{\partial s^c}\right]ds^c$$

$$\frac{dV_n^{\ c}}{dt_n^{h^c}}-\frac{dV_n^{\ nc}}{dt_n^{h^c}}=\left[\frac{\partial U_n(t_n^{w^c},z^c,t_n^{h^c},t_k^{\ c},x_n^{\ c}+s^c)}{\partial t_n^{h^c}}+\frac{\partial U_n(t_n^{w^c},z^c,t_n^{h^c},t_k^{\ c},x_n^{\ c}+s^c)}{\partial z}\frac{\partial z}{\partial t_n^{h^c}}\right]dt_n^{h^c}$$
$$-\frac{\partial q^*}{\partial t_n^{h^c}}[U_n(t_n^w,z,t_n^h,t_k,x_n+s^{nc})-U_n(t_n^w,z,t_n^h,t_k,x_n+s^c)]$$

$$\frac{dV_n^{\ c}}{dt_k^{\ c}}-\frac{dV_n^{\ nc}}{dt_k^{\ c}}=\left[\frac{\partial U_n(t_n^{w^c},z^c,t_n^{h^c},t_k^{\ c},x_n^{\ c}+s^c)}{\partial t_k^{\ c}}+\frac{\partial U_n(t_n^{w^c},z^c,t_n^{h^c},t_k^{\ c},x_n^{\ c}+s^c)}{\partial z}\frac{\partial z}{\partial t_k^{\ c}}\right]dt_k^{\ c}$$
$$-\frac{\partial q^*}{\partial t_k^{\ c}}[U_n(t_n^w,z,t_n^h,t_k,x_n+s^{nc})-U_n(t_n^w,z,t_n^h,t_k,x_n+s^c)]$$

$$\frac{dV_n^{\ c}}{dt_n^{w^c}}-\frac{dV_n^{\ nc}}{dt_n^{w^c}}=\left[\frac{\partial U_n(t_n^{w^c},z^c,t_n^{h^c},t_k^{\ c},x_n^{\ c}+s^c)}{\partial t_n^{w^c}}+\frac{\partial U_n(t_n^{w^c},z^c,t_n^{h^c},t_k^{\ c},x_n^{\ c}+s^c)}{\partial x_n}w_n\right]dt_n^{w^c}$$
$$-\frac{\partial q^*}{\partial x_n^{\ c}}w_n[U_n(t_n^w,z,t_n^h,t_k,x_n+s^{nc})-U_n(t_n^w,z,t_n^h,t_k,x_n+s^c)]$$

$$\frac{dV_n^{\ c}}{dz^c}-\frac{dV_n^{\ nc}}{dz^c}=\frac{\partial U_n(t_n^{w^c},z^c,t_n^{h^c},t_k^{\ c},x_n^{\ c}+s^c)}{\partial z^c}dz^c+\frac{\partial U_n(t_n^{w^c},z^c,t_n^{h^c},t_k^{\ c},x_n^{\ c}+s^c)}{\partial t_n^h}\left[\left(1-\frac{\partial z}{\partial t_k}\frac{dt_k}{dz}\right)/\frac{\partial z}{\partial t_n^h}\right]$$
$$+\frac{\partial U_n(t_n^{w^c},z^c,t_n^{h^c},t_k^{\ c},x_n^{\ c}+s^c)}{\partial t_k}\left[\left(1-\frac{\partial z}{\partial t_n^h}\frac{dt_n^h}{dz}\right)/\frac{\partial z}{\partial t_k}\right]-\frac{\partial q^*}{\partial z^c}[U_n(t_n^w,z,t_n^h,t_k,x_n+s^{nc})-U_n(t_n^w,z,t_n^h,t_k,x_n+s^c)]$$

where V_n^{nc} denotes the payoff to *don't cooperate* and V_n^{c} denotes the payoff to *cooperate* for player *n*.

First Order Conditions

$$\left(\frac{\partial U_n}{\partial t_n}+\frac{\partial U_n}{\partial x_n}w_n\right)-q\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)-\frac{\partial q}{\partial x_n}w_n(U_n-U_n^{'})=0\,,$$

$$\left(\frac{\partial U_n}{\partial t_n}\mu_n+\frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_n^h}\right)-\left(\frac{\partial q}{\partial t_n^h}+\frac{\partial q}{\partial z}\frac{\partial z}{\partial t_n^h}\right)(U_n-U_n^{'})=0 \text{ and}$$

$$\left(\frac{\partial U_n}{\partial t_k}+\frac{\partial U_n}{\partial z}\frac{\partial z}{\partial t_k}\right)-\left(\frac{\partial q}{\partial t_k}+\frac{\partial q}{\partial z}\frac{\partial z}{\partial t_k}\right)(U_n-U_n^{'})=0$$

where $U_n=U_n(t_n^w,z,t_n^h,t_k,x_n+s^c)$ and $U_n'=U_n(t_n^w,z,t_n^h,t_k,x_n+s^{nc})$.

Assumptions

(1) All goods separable in utility.
(2) No complementarities in household production.
(3) No cross-good effects in $q(\cdot)$.
(4) $q\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)+\frac{\partial q}{\partial x_n}w_n(U_n-U_n^{'})>0$
(5) $\left(\frac{\partial q}{\partial t_k}+\frac{\partial q}{\partial z}\frac{\partial z}{\partial t_k}\right)>0$
(6) $\sigma_{23}<0$

Let Δ denote the determinant of the Hessian, and define its elements as

$$\sigma_{11} = \frac{\partial^2 U_n}{\partial t_n^{\,2}} + \frac{\partial^2 U_n}{\partial x_n^{\,2}} w_n^{\,2} - q\left(\frac{\partial^2 U_n}{\partial x_n^{\,2}} w_n^{\,2} - \frac{\partial^2 U_n^{'}}{\partial x_n^{\,2}} w_n^{\,2}\right) - \frac{\partial^2 q}{\partial x_n^{\,2}} w_n^2 (U_n - U_n^{'}) - 2\frac{\partial q}{\partial x_n} w_n \left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right) < 0$$

$$\sigma_{12} = \frac{\partial^2 U_n}{\partial t_n^{\,2}} \mu - \left(\frac{\partial q}{\partial t_n^h} + \frac{\partial q}{\partial z}\frac{\partial z}{\partial t_n^h}\right)\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right) < 0$$

$$\sigma_{13} = -\left(\frac{\partial q}{\partial t_k} + \frac{\partial q}{\partial z}\frac{\partial z}{\partial t_k}\right)\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right) > 0$$

$$\sigma_{22} = \left(\frac{\partial^2 U_n}{\partial t_n^{\,2}} \mu_n^{\,2} + \frac{\partial^2 U_n}{\partial z^2}\left(\frac{\partial z}{\partial t_n^h}\right)^2 + \frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_n^{h\,2}}\right) - \left[\frac{\partial^2 q}{\partial t_n^{h\,2}} + \frac{\partial^2 q}{\partial z^2}\left(\frac{\partial z}{\partial t_n^h}\right)^2 + \frac{\partial q}{\partial z}\left(\frac{\partial^2 z}{\partial t_n^{h\,2}}\right)\right](U_n - U_n^{'}) < 0$$

$$\sigma_{23} = \left(\frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial t_n^h}\frac{\partial z}{\partial t_k}\right) - \left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial t_n^h}\frac{\partial z}{\partial t_k}\right)(U_n - U_n^{'}) < 0$$

$$\sigma_{33} = \left[\frac{\partial^2 U_n}{\partial t_c^{\,2}} + \frac{\partial^2 U_n}{\partial z^2}\left(\frac{\partial z}{\partial t_k}\right)^2 + \frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_k^{\,2}}\right] - \left[\frac{\partial^2 q}{\partial t_k^{\,2}} + \frac{\partial^2 q}{\partial z^2}\left(\frac{\partial z}{\partial t_k}\right)^2 + \frac{\partial q}{\partial z}\frac{\partial^2 z}{\partial t_k^{\,2}}\right](U_n - U_n^{'}) < 0$$

$$\frac{dt_n^w}{d\omega_q} = \frac{1}{\Delta}\left\{\left[\frac{\partial q}{\partial \omega_q}\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right) + \frac{\partial^2 q}{\partial x_n \partial \omega_q} w_n (U_n - U_n^{'})\right](\sigma_{22}\sigma_{33} - \sigma_{23}{}^2) - \left[\left(\frac{\partial^2 q}{\partial t_n^h \partial \omega_q} + \frac{\partial^2 q}{\partial z \partial \omega_q}\frac{\partial z}{\partial t_m^h}\right)(U_n - U_n^{'})\right](\sigma_{12}\sigma_{33} - \sigma_{13}\sigma_{23}) \right.$$

$$\left. + \left[\left(\frac{\partial^2 q}{\partial t_k \partial \omega_q} + \frac{\partial^2 q}{\partial z \partial \omega_q}\frac{\partial z}{\partial t_k}\right)(U_n - U_n^{'})\right](\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22})\right\} < 0$$

$$\frac{dt_n^h}{d\omega_q} = \frac{1}{\Delta}\left\{-\left[\frac{\partial q}{\partial \omega_q}\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right) + \frac{\partial^2 q}{\partial x_n \partial \omega_q} w_n (U_n - U_n^{'})\right](\sigma_{12}\sigma_{33} - \sigma_{13}\sigma_{23}) + \left[\left(\frac{\partial^2 q}{\partial t_n^h \partial \omega_q} + \frac{\partial^2 q}{\partial z \partial \omega_q}\frac{\partial z}{\partial t_n^h}\right)(U_n - U_n^{'})\right](\sigma_{11}\sigma_{33} - \sigma_{13}{}^2) \right.$$

$$\left. - \left[\left(\frac{\partial^2 q}{\partial t_k \partial \omega_q} + \frac{\partial^2 q}{\partial z \partial \omega_q}\frac{\partial z}{\partial t_k}\right)(U_n - U_n^{'})\right](\sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13})\right\} > 0$$

$$\frac{dt_k}{d\omega_q} = \frac{1}{\Delta}\left\{\left[\frac{\partial q}{\partial \omega_q}\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right) + \frac{\partial^2 q}{\partial x_n \partial \omega_q} w_n (U_n - U_n^{'})\right](\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22}) - \left[\left(\frac{\partial^2 q}{\partial t_n^h \partial \omega_q} + \frac{\partial^2 q}{\partial z \partial \omega_q}\frac{\partial z}{\partial t_n^h}\right)(U_n - U_n^{'})\right](\sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13}) \right.$$

$$\left. + \left[\left(\frac{\partial^2 q}{\partial t_k \partial \omega_q} + \frac{\partial^2 q}{\partial z \partial \omega_q}\frac{\partial z}{\partial t_k}\right)(U_n - U_n^{'})\right](\sigma_{11}\sigma_{22} - \sigma_{12}{}^2)\right\} < 0$$

$$\frac{dt_n^w}{d\omega_x} = \frac{1}{\Delta}\left\{\frac{\partial q}{\partial \omega_x}\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right) + \frac{\partial^2 q}{\partial x_n \partial \omega_x} w_n (U_n - U_n^{'})\right\}(\sigma_{22}\sigma_{33} - \sigma_{23}{}^2) < 0$$

$$\frac{dt_n^h}{d\omega_x} = \frac{1}{\Delta}\left\{\frac{\partial q}{\partial \omega_n}\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right) + \frac{\partial^2 q}{\partial x_n \partial \omega_x} w_n (U_n - U_n^{'})\right\}[-(\sigma_{12}\sigma_{33} - \sigma_{23}\sigma_{13})] > 0$$

$$\frac{dt_k}{d\omega_x} = \frac{1}{\Delta}\left\{\frac{\partial q}{\partial \omega_x}\left(\frac{\partial U_n}{\partial x_n} w_n - \frac{\partial U_n^{'}}{\partial x_n} w_n\right) + \frac{\partial^2 q}{\partial x_n \partial \omega_x} w_n (U_n - U_n^{'})\right\}(\sigma_{12}\sigma_{23} - \sigma_{22}\sigma_{13}) < 0$$

$$\frac{dt_n^w}{d\omega_z}=\frac{1}{\Delta}\left\{\frac{\partial q}{\partial \omega_z}\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)(\sigma_{22}\sigma_{33}-\sigma_{23}{}^2)-\frac{\partial^2 q}{\partial z\partial \omega_z}(U_n-U_n^{'})\left[\frac{\partial z}{\partial t_n^h}(\sigma_{12}\sigma_{33}-\sigma_{13}\sigma_{23})-\frac{\partial z}{\partial t_k}(\sigma_{12}\sigma_{23}-\sigma_{13}\sigma_{22})\right]\right\} \text{ amb.}$$

$$\frac{dt_n^h}{d\omega_z}=\frac{1}{\Delta}\left\{\frac{\partial q}{\partial \omega_z}\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)[-(\sigma_{12}\sigma_{33}-\sigma_{23}\sigma_{13})]+\frac{\partial^2 q}{\partial z\partial \omega_z}(U_n-U_n^{'})\left[\frac{\partial z}{\partial t_n^h}(\sigma_{11}\sigma_{33}-\sigma_{13}{}^2)-\frac{\partial z}{\partial t_k}(\sigma_{11}\sigma_{23}-\sigma_{12}\sigma_{13})\right]\right\} \text{ amb.}$$

$$\frac{dt_k}{d\omega_z}=\frac{1}{\Delta}\left\{\frac{\partial q}{\partial \omega_z}\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)(\sigma_{12}\sigma_{23}-\sigma_{22}\sigma_{13})-\frac{\partial^2 q}{\partial z\partial \omega_z}(U_n-U_n^{'})\left[\frac{\partial z}{\partial t_n^h}(\sigma_{11}\sigma_{23}-\sigma_{12}\sigma_{13})-\frac{\partial z}{\partial t_k}(\sigma_{11}\sigma_{22}-\sigma_{12}{}^2)\right]\right\} \text{ amb.}$$

$$\frac{dt_n^w}{d\omega_h}=\frac{1}{\Delta}\left\{\frac{\partial q}{\partial \omega_h}\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)(\sigma_{22}\sigma_{33}-\sigma_{23}{}^2)-\frac{\partial^2 q}{\partial t_n^h\partial \omega_h}(U_n-U_n^{'})(\sigma_{12}\sigma_{33}-\sigma_{13}\sigma_{23})\right\}<0$$

$$\frac{dt_n^h}{d\omega_h}=\frac{1}{\Delta}\left\{\frac{\partial q}{\partial \omega_h}\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)[-(\sigma_{12}\sigma_{33}-\sigma_{23}\sigma_{13})]+\frac{\partial^2 q}{\partial t_n^h\partial \omega_h}(U_n-U_n^{'})(\sigma_{11}\sigma_{33}-\sigma_{13}{}^2)\right\}>0$$

$$\frac{dt_k}{d\omega_h}=\frac{1}{\Delta}\left\{\frac{\partial q}{\partial \omega_h}\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)(\sigma_{12}\sigma_{23}-\sigma_{22}\sigma_{13})-\frac{\partial^2 q}{\partial t_n^h\partial \omega_h}(U_n-U_n^{'})(\sigma_{11}\sigma_{23}-\sigma_{12}\sigma_{13})\right\}<0$$

$$\frac{dt_n^w}{d\omega_k}=\frac{1}{\Delta}\left\{\frac{\partial q}{\partial \omega_k}\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)(\sigma_{22}\sigma_{33}-\sigma_{23}{}^2)+\frac{\partial^2 q}{\partial t_k\partial \omega_k}(U_n-U_n^{'})(\sigma_{12}\sigma_{23}-\sigma_{13}\sigma_{22})\right\} \text{ amb.}$$

$$\frac{dt_n^h}{d\omega_k}=\frac{1}{\Delta}\left\{\frac{\partial q}{\partial \omega_k}\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)[-(\sigma_{12}\sigma_{33}-\sigma_{23}\sigma_{13})]-\frac{\partial^2 q}{\partial t_k\partial \omega_k}(U_n-U_n^{'})(\sigma_{11}\sigma_{23}-\sigma_{12}\sigma_{13})\right\} \text{ amb.}$$

$$\frac{dt_k}{d\omega_k}=\frac{1}{\Delta}\left\{\frac{\partial q}{\partial \omega_k}\left(\frac{\partial U_n}{\partial x_n}w_n-\frac{\partial U_n^{'}}{\partial x_n}w_n\right)(\sigma_{12}\sigma_{23}-\sigma_{22}\sigma_{13})+\frac{\partial^2 q}{\partial t_k\partial \omega_k}(U_n-U_n^{'})(\sigma_{11}\sigma_{22}-\sigma_{12}{}^2)\right\}<0$$

$$\frac{dt_n^w}{d\mu} = \frac{1}{\Delta}\left(-\frac{\partial U_n}{\partial t_n} - \frac{\partial^2 U_n}{\partial t_n^{\ 2}} t_n^h \mu\right)[-(\sigma_{12}\sigma_{33} - \sigma_{13}\sigma_{23})] > 0$$

$$\frac{dt_m^h}{d\mu} = \frac{1}{\Delta}\left(-\frac{\partial U_n}{\partial t_n} - \frac{\partial^2 U_n}{\partial t_n^{\ 2}} t_n^h \mu\right)(\sigma_{11}\sigma_{33} - \sigma_{13}^{\ 2}) < 0$$

$$\frac{dt_k}{d\mu} = \frac{1}{\Delta}\left(-\frac{\partial U_n}{\partial t_n} - \frac{\partial^2 U_n}{\partial t_n^{\ 2}} t_n^h \mu\right)[-(\sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13})] > 0$$

$$\begin{aligned}\frac{dt_n^w}{d\tau_n} = \frac{1}{\Delta}\Bigg\{&\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_n}\frac{\partial z}{\partial t_n^h} + \frac{\partial q}{\partial z}\frac{\partial^2 z}{\partial t_n^h \partial \tau_n}\right)(U_n - U_n^{'}) - \frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_n^h \partial \tau_n} - \frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_n}\frac{\partial z}{\partial t_n^h}\right][-(\sigma_{12}\sigma_{33} - \sigma_{13}\sigma_{23})] \\ &+ \left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_n}\frac{\partial z}{\partial t_k}\right)(U_n - U_n^{'}) - \frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_n}\frac{\partial z}{\partial t_k}\right](\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22})\Bigg\} < 0\end{aligned}$$

$$\begin{aligned}\frac{dt_n^h}{d\tau_n} = \frac{1}{\Delta}\Bigg\{&\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_n}\frac{\partial z}{\partial t_n^h} + \frac{\partial q}{\partial z}\frac{\partial^2 z}{\partial t_n^h \partial \tau_n}\right)(U_n - U_n^{'}) - \frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_n^h \partial \tau_n} - \frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_n}\frac{\partial z}{\partial t_n^h}\right](\sigma_{11}\sigma_{33} - \sigma_{13}^{\ 2}) \\ &- \left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_n}\frac{\partial z}{\partial t_k}\right)(U_n - U_n^{'}) - \frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_n}\frac{\partial z}{\partial t_k}\right](\sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13})\Bigg\} > 0\end{aligned}$$

$$\begin{aligned}\frac{dt_k}{d\tau_n} = \frac{1}{\Delta}\Bigg\{&\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_n}\frac{\partial z}{\partial t_n^h} + \frac{\partial q}{\partial z}\frac{\partial^2 z}{\partial t_n^h \partial \tau_n}\right)(U_n - U_n^{'}) - \frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_n^h \partial \tau_n} - \frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_n}\frac{\partial z}{\partial t_n^h}\right][-(\sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13})] \\ &+ \left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_n}\frac{\partial z}{\partial t_k}\right)(U_n - U_n^{'}) - \frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_n}\frac{\partial z}{\partial t_k}\right](\sigma_{11}\sigma_{22} - \sigma_{12}^{\ 2})\Bigg\} < 0\end{aligned}$$

$$\frac{dt_n^w}{d\tau_k} = \frac{1}{\Delta}\left\{\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_k} + \frac{\partial q}{\partial z}\frac{\partial^2 z}{\partial t_k \partial \tau_k}\right)(U_n - U_n^{'}) - \frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_k \partial \tau_k} - \frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_k}\right](\sigma_{12}\sigma_{23} - \sigma_{13}\sigma_{22})\right.$$
$$\left. - \left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_n^h}\right)(U_n - U_n^{'}) - \frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_n^h}\right](\sigma_{12}\sigma_{33} - \sigma_{13}\sigma_{23})\right\} > 0$$

$$\frac{dt_n^h}{d\tau_k} = \frac{1}{\Delta}\left\{\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_k} + \frac{\partial q}{\partial z}\frac{\partial^2 z}{\partial t_k \partial \tau_k}\right)(U_n - U_n^{'}) - \frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_k \partial \tau_k} - \frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_k}\right][-(\sigma_{11}\sigma_{23} - \sigma_{13}\sigma_{12})]\right.$$
$$\left. + \left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_n^h}\right)(U_n - U_n^{'}) - \frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_n^h}\right](\sigma_{11}\sigma_{33} - \sigma_{13}{}^2)\right\} < 0$$

$$\frac{dt_k}{d\tau_k} = \frac{1}{\Delta}\left\{\left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_k} + \frac{\partial q}{\partial z}\frac{\partial^2 z}{\partial t_k \partial \tau_k}\right)(U_n - U_n^{'}) - \frac{\partial U_n}{\partial z}\frac{\partial^2 z}{\partial t_k \partial \tau_k} + \frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_k}\right](\sigma_{11}\sigma_{22} - \sigma_{12}{}^2)\right.$$
$$\left. - \left[\left(\frac{\partial^2 q}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_n^h}\right)(U_n - U_n^{'}) - \frac{\partial^2 U_n}{\partial z^2}\frac{\partial z}{\partial \tau_k}\frac{\partial z}{\partial t_n^h}\right](\sigma_{11}\sigma_{23} - \sigma_{12}\sigma_{13})\right\} > 0$$