Construction of World Tables of Purchasing Power Parities and Real Incomes Based on Multiple Benchmarks and Auxiliary Information: A State-Space Approach

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Presentation Outline

- Introduction
- Proposed Econometric Approach
  - Underlying Economic Theory
  - Derived Econometric Model
  - Estimation and Computational issues
- Useful Properties of the Approach
- Empirical Results (141 countries), including ICP 2005

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Introduction

To construct a panel of PPPs for ICP participating and non-participating countries that optimally combines all available information

Current Methodology for Extrapolations:

1. Extrapolate to non-benchmark countries

Based on predictions from a price level regression or alternative regressions from a recent benchmark

2. Use "implicit growth rates" to extrapolate PPPs over time

Using the published National Accounts data on GDP Deflators

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Introduction (cont.)

Our Approach:

- Use all available benchmark information – an unbalanced panel
- Set up an econometric model to predict *PPPs combining ICP benchmark with other available information*
- Express the model in state-space form
- Use a spatio-temporal filter to produce predictions and associated standard errors

See (Doran, Rambaldi, Rao (2006))

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Introduction (cont.)

- Salient features of our approach
  - Produce PPP estimates that combine all the past information in a systematic manner
  - Produce PPP estimates that preserve the movements in the implicit price deflators
  - A flexible method that can be made to closely track either the benchmarks or the growth rates
  - Produce PPP estimates and associated standard errors.
  - Method is invariant to the chosen reference country.

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Proposed Methodology

Sources of information for $p_{it} = ln(PPP_{it})$

1. **ICP Benchmark PPPs**: Observation of the variable of interest contaminated with measurement error.
2. **A Model Derived from the Theory of Price Levels**: Links national price levels to variable of interest.
3. **Growth rates from movements in national GDP Deflators levels**: Links national accounts data to variables of interest
4. **Reference Country Definition**: A restriction that must hold, $p_{reference\ country,t} = 0$ for all t

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Proposed Methodology
Combining Theory and Noisy Data (Source 1)

- Surveys are very resource intensive,
  - Carried out by national statistical agencies of those countries that participate in the ICP.
  - Internationally comparable baskets are priced
- We can then write

$$\tilde{p}_{it} = p_{it} + \xi_{1it} \qquad (1)$$

$\tilde{p}_{it} = \ln(P\tilde{P}P_{it})$ ICP Benchmark observation for participating country *i* at time *t*.

ξ_{1it} is a random error accounting for measurement error in ICP Benchmark data collection

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Proposed Methodology
Combining Theory and Noisy Data (Source 2)

$$r_{it} = \mathbf{x}'_{it}\boldsymbol{\beta}_{it} + u_{it} \qquad (2)$$

where,

$r_{it} = \ln(PPP_{it} / ER_{it})$; $\mathbf{x}'_{it}$ a set of conditioning variables

$\boldsymbol{\beta}_{it}$ a vector of parameters

u_{it} a random disturbance with specific distributional characteristics

We obtain a prediction:

$$\hat{p}_{it} = \mathbf{x}'_{it}\hat{\boldsymbol{\beta}}_{it} + \ln(ER_{it})$$

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Proposed Methodology
Combining Theory and Noisy Data (Source 3)

- We assume some measurement errors are present in national accounts data and thus use

$$PPP_{i,t} = PPP_{i,t-1} \times \frac{GDPDef_{i,[t-1,t]}}{GDPDef_{US,[t-1,t]}}$$

- to define:

$$p_{it} = p_{i,t\text{-}1} + c_{it} + \eta_{it} \qquad (3)$$

where,

$$c_{it} = \ln\left(\frac{GDPDef_{i,[t,t-1]}}{GDPDef_{US,[t,t-1]}}\right)$$

η_{it} is a random error accounting for measurement error in the growth rates

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Proposed Methodology
Combining Theory and Noisy Data (Source 4)

- The definition of PPP requires choice of a reference country.
- The reference country is defined to have a PPP = 1 for all time periods.
- As the US is taken as the reference country,

$$p_{US,t} = 0 \qquad (4)$$

- The procedure and results are invariant to the choice of the numeraire.

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Econometric Model - Assumptions

a) The errors in the regression relationship (2) are assumed to be spatially correlated

$$\mathbf{u}_t = \phi \mathbf{W}_t \mathbf{u}_t + \mathbf{e}_t \quad \phi < 1 \text{ and } \mathbf{W}\ (N \times N) \text{ is a spatial weights matrix}$$

b) measurement errors in the observation of benchmarks, (1), PPP_{it} are heteroskedastic

$$E(\xi_{1it}^2) = \sigma_\xi^2 V_{it} \qquad \sigma_\xi^2 \text{ is a constant of proportionality}$$

c) measurement error in the growth rates, (3), are heteroskedastic

$$\mathrm{E}\left(\eta_{it}^2\right) = \sigma_\eta^2 V_{it} \qquad \sigma_\eta^2 \text{ is a constant of proportionality}$$

$v_{ii,t}$ is an inverse measure of development of country i

Spatial Model

$$r_{it} = \mathbf{x}'_{it}\boldsymbol{\beta}_{it} + u_{it}$$

$$\mathbf{u}_t = \phi \mathbf{W}_t \mathbf{u}_t + \mathbf{e}_t$$

where $\phi < 1$ and $\mathbf{W}_t$ $(N \times N)$ is a spatial weights matrix

$w_{ii,t} = 0$

$w_{ij,t} \neq 0$ if countries i and j are "neighbours"

The rows of $\mathbf{W}_t$ add up to 1

We define "neighbours" as "close" five closest trading partners.

A State-Space Representation

Rewrite models (1)-(4) as a state-space model:

1. **Observation Equations**

   Map observations of the "state variable", $\tilde{p}_{it}$ and $\hat{p}_{it}$, $p_{US,t}$ to the unobservable "state variable", p_{it}

   Equations (1), (2) and (4)

2. **Transition Equations**

   Show the evolution of the state variable over time

   Equations (3)

See paper for detailed specification

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Estimation and Computational Issues

The algorithm we use can be described in 5 steps.

Step 1: Construct an initial prediction, $\hat{p}_{it}^{0}$, using equation (3).

Step 2: Run SS through KF (or KF/GLS) to obtain estimates of the parameters.

Step 3: Use updated estimate of $\boldsymbol{\beta}_{it}$, $\hat{\beta}_{0t} = \hat{\beta}_{t}^{0} - \hat{\theta}_{0t}$ $\hat{\boldsymbol{\beta}}_{t} = \hat{\boldsymbol{\beta}}_{t}^{0} - \hat{\boldsymbol{\theta}}_{t}$, obtain an updated $\hat{p}_{it}$

Step 4: Repeat 2 and 3 until $\hat{\boldsymbol{\theta}}$ are sufficiently close to zero.

Step 5: Run KF and Kalman smoother one more time, obtain p_{it}^{*} and standard errors.

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Estimation and Computational Issues

Prediction of the PPPs

Given the smoothed predictions of p_{it}

$$\hat{PPP}_{it} = e^{p^*_{i,t/T}}$$

$$se(\hat{PPP}_{it}) = \sqrt{e^{2p^*_{i,t/T}} e^{\psi^*_{ii,t}} (e^{\psi^*_{ii,t}} - 1)}$$

$\psi^*_{ii,t}$ is the *ith* diagonal element of the estimated covariance of the smoothed state vector, $\mathbf{p}^*_{t/T}$.

This is under normality of the residuals.

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Main Features of the Approach

1. The approach proposed is fairly flexible and it can handle many different scenarios. For example

- Constrain to make the PPP estimates **track benchmark PPPs** (Section 4)
- Constrain to produce estimates that **track the movements in implicit price deflators** (Appendix 1).
- Use regression information only at benchmark years
  - Under this simplified model, the resulting **estimates are weighted averages of the benchmark-year PPPs of the given country** with weights determined by the error structure of the model (Appendix 2)
- **Produce standard errors for extrapolated PPPs**

2. The **method is invariant to the choice of reference country**. Successfully conducted an empirical test. Complete proof of the result will be included in the next version of the paper.

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Empirical Application

141 COUNTRIES, 1970 – 2005

Benchmark years: 1975,1980, 1985, 1990, 1993, 1996, 1999, 2002, 2005 (Sources: ICP and OECD)

ICP 2005 Participating countries included in the study: 110 countries

Data sources:

WDI= World Development Indicators (WDI, 2007);

CIA= The Central Intelligence Agency Factbook (CIA, 2008);

UN=United Nations National Accounts Main Aggregates Database (UN, 2007);

IMF=International Monetary Fund (IMF, 2007);

PWT 6.2=Penn World Tables Version 6. (Heston, Summers and Aten, 2006);

Rose=Bilateral trade data from Andrew K. Rose (Rose, 2004);

CNTS=Cross-national Time Series data (Banks, 2006);

FAO=Food and Agricultural Organization (FAO, 2006).

BLACKIND was calculated from black market premium data from Easterly (2006).

See paper Tables DA.1, DA.2 and DA.3

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Empirical Application (cont.)

Explanatory Variables in the regression (paper Table 1 estimates of parameters):

•***Dummy Controls (time, trade and monetary agreements)***

dum80_84,dum85_89,dum90_92, dum93_95, dum96_98, dum99_01, dum02_04, dum05, D_anz, D_asean, D_cac, D_euro, D_mercsr, D_nafta, D_scucar, D_spr, D_usd

•***Other Explanatory Variables***

Age dependency ratio (dependents to working-age population); Agriculture, value added (as a percentage of GDP); Index of distortions in exchange rates; Exports of goods and services (% of GDP); Labor force as percentage of total population; Life expectancy at birth; Population aged 15 and over which is literate (per 1,000 population); Manufactures exports (as a percentage of merchandise exports); Manufactures imports (as a percentage of merchandise imports); Non-tradable sector value added (as a percentage of GDP); Telephone mainlines (per 1,000 people); Radios per capita; Rural population (as a percentage of total population); Agricultural machinery, tractors per agricultural worker; Trade (as a percentage of GDP)

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Empirical Application (cont.)

Covariance Variables

Spatial Correlation (W_t): Five closest trading partners to country *i* in year *t* is assigned a weight of 0.2; zero otherwise. Weights were computed every five years from 1970 (ie neighbours change every five years).

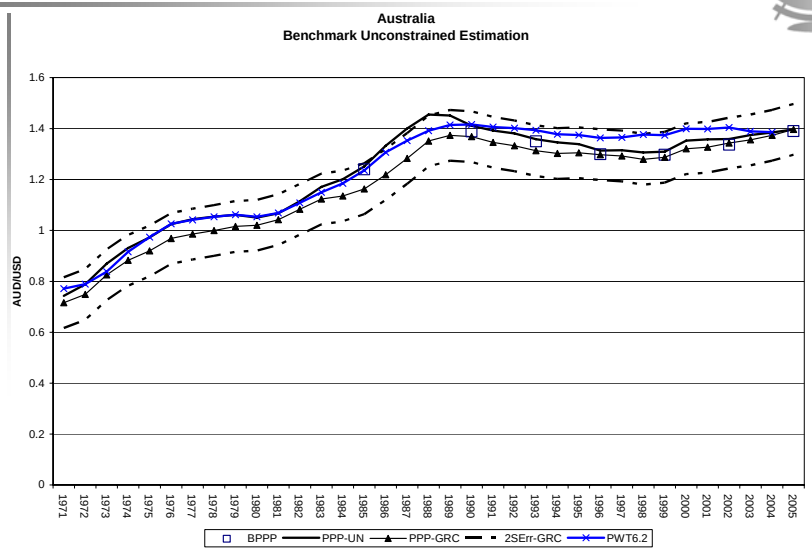
Accuracy of data collection (V_t): (real GDP per capita in thousands, USD exchange rate adjusted)$^{-1}$

Empirical Application (cont.)

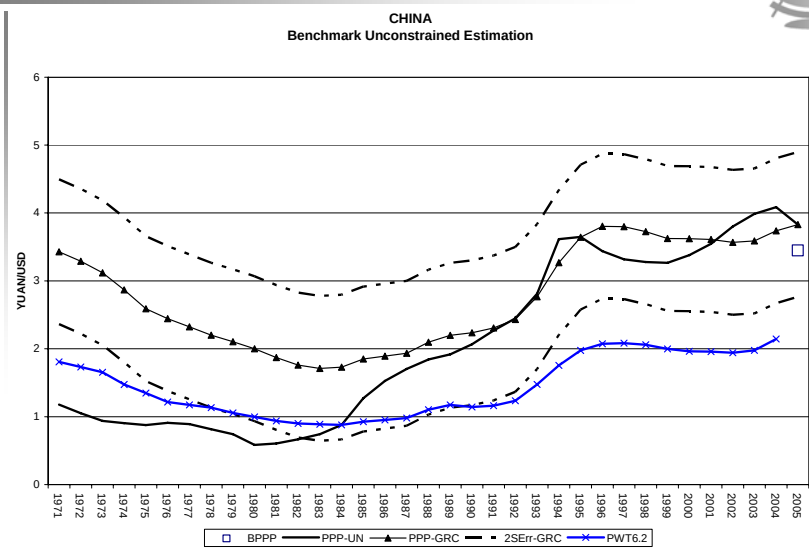
- Results for different specifications:
  - **Model Estimation: Benchmarks and Growth Rates Unconstrained**
    - Smoothing: Growth Rates unconstrained (**PPP-UN**)
    - Smoothing: Growth Rates constrained (Appendix 1) (**PPP-GRC**)
  - **Model Estimation: Constrained to track benchmarks, Growth Rates Unconstrained**

    (H_0: There is no error in benchmarks. H_0 is rejected)
    - Smoothing: Growth Rates unconstrained (**PPP-CON**)
    - Smoothing: Growth Rates constrained (Appendix 1) (**PPP-CON-GRC**)
  - **Model Estimation: Unconstrained – No Regression information for non-benchmark years**
    - Smoothing: Growth Rates unconstrained (**PPP-SIM**)
    - Smoothing: Growth Rates constrained (Appendix 1) (**PPP-SIM-GRC**)
- **Selected countries shown:**
  - OECD Developed: Australia
  - Africa: Nigeria
  - Asia: China, India
  - The Americas: Honduras (did not participate in 2005)

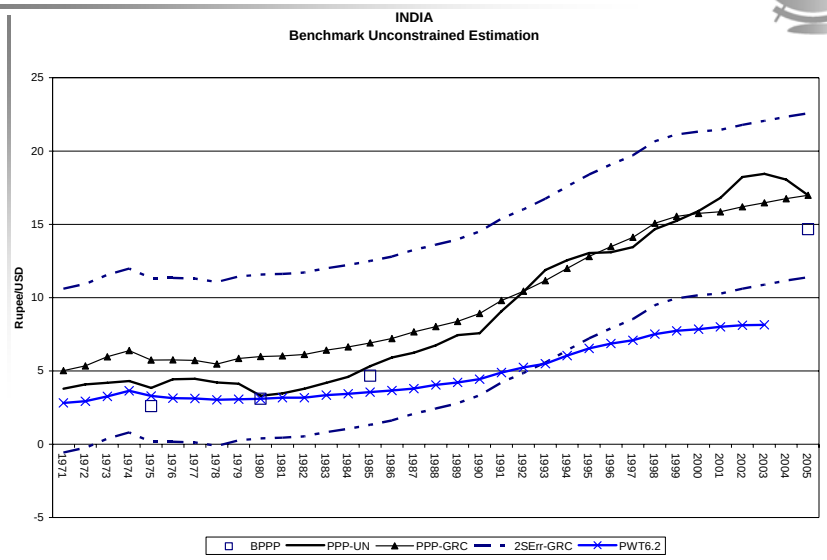
Model Estimation: Unconstrained



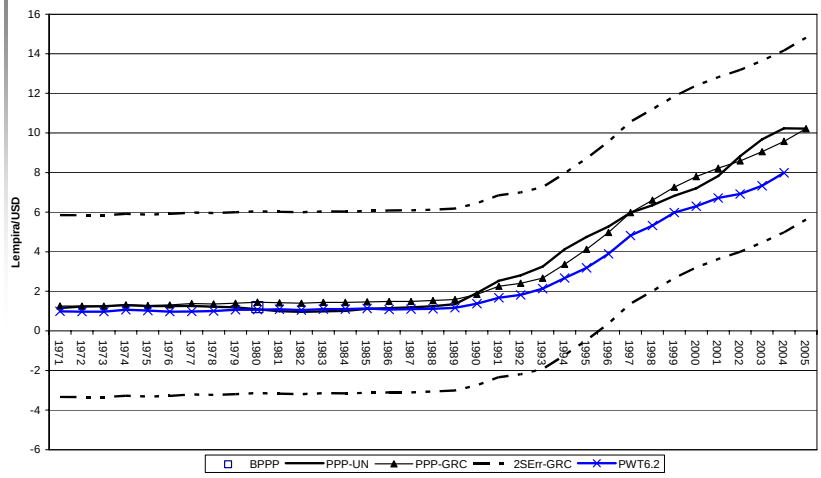
Model Estimation: Unconstrained



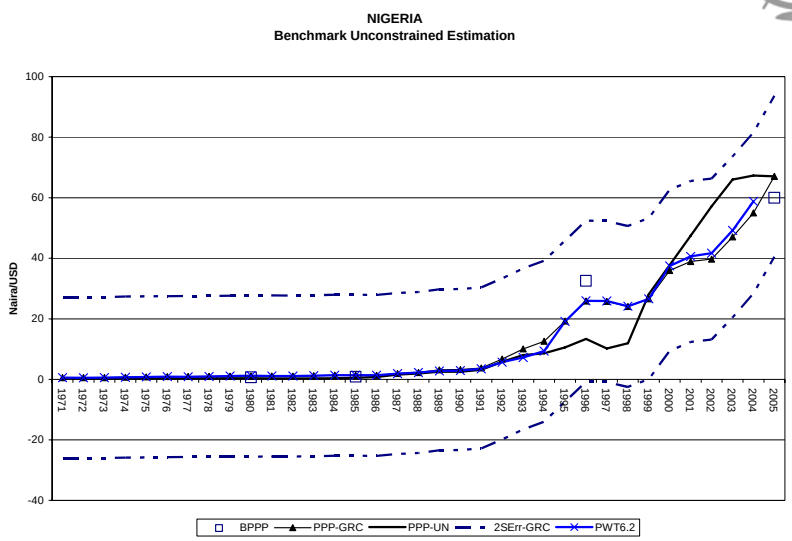
Model Estimation: Unconstrained



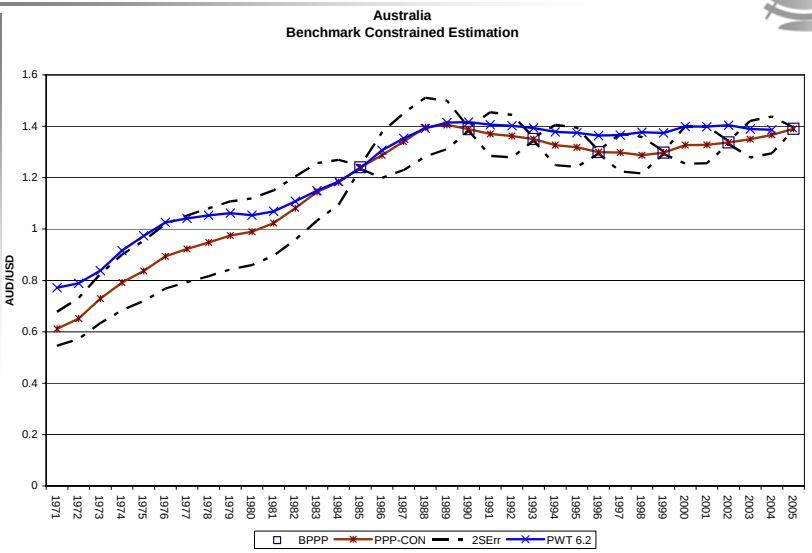
Model Estimation: Unconstrained



Model Estimation: Unconstrained

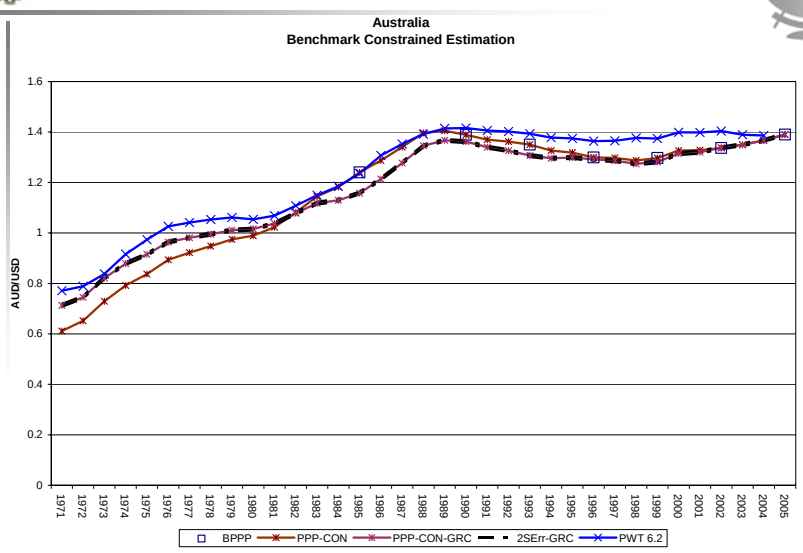


Model Estimation: Benchmark Constrained



This model (imposed zero error in benchmarks) is rejected by the data

Model Estimation: Benchmark Constrained



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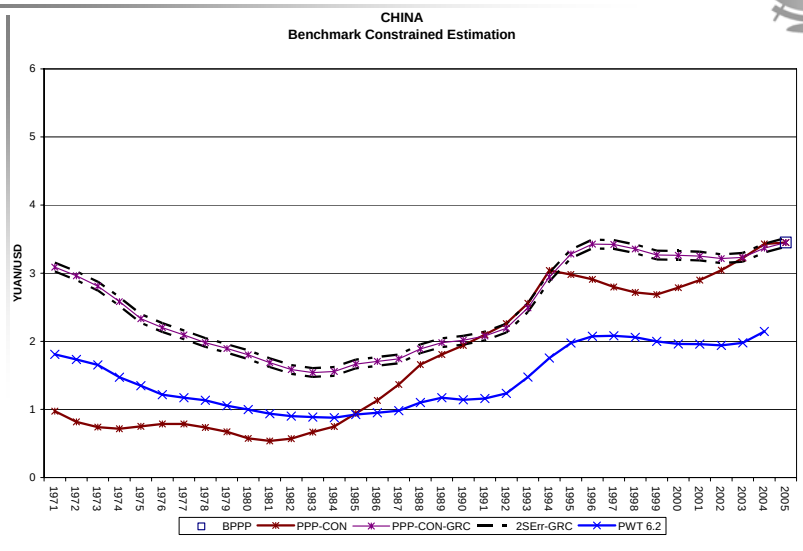
Model Estimation: Benchmark Constrained



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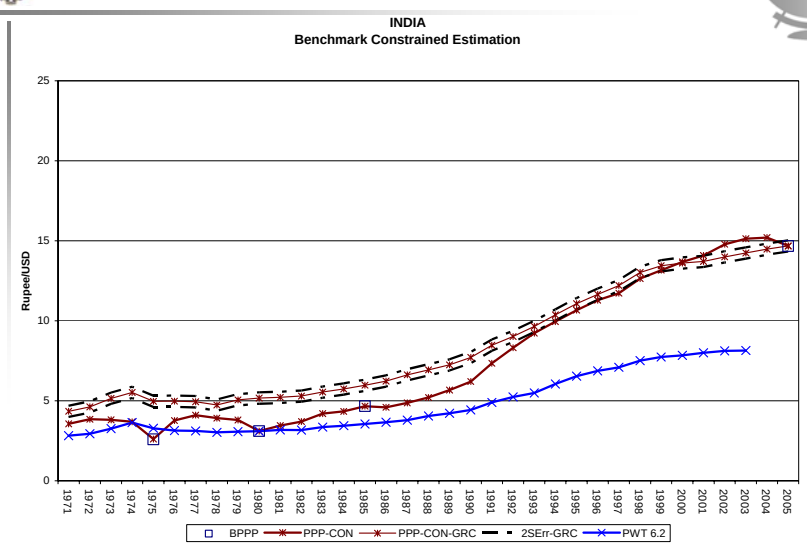
Model Estimation: Benchmark Constrained



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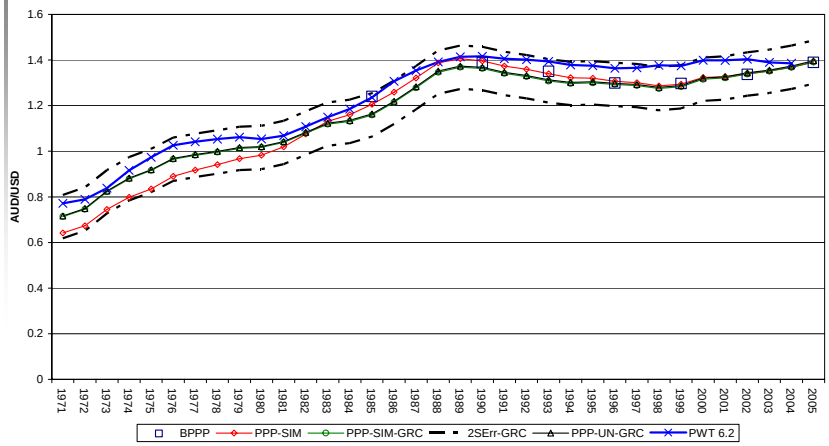
Model Estimation: No Regression in non-benchmark years.



Estimates from full model (unconstrained benchmarks) with growth rates preserved shown for comparison

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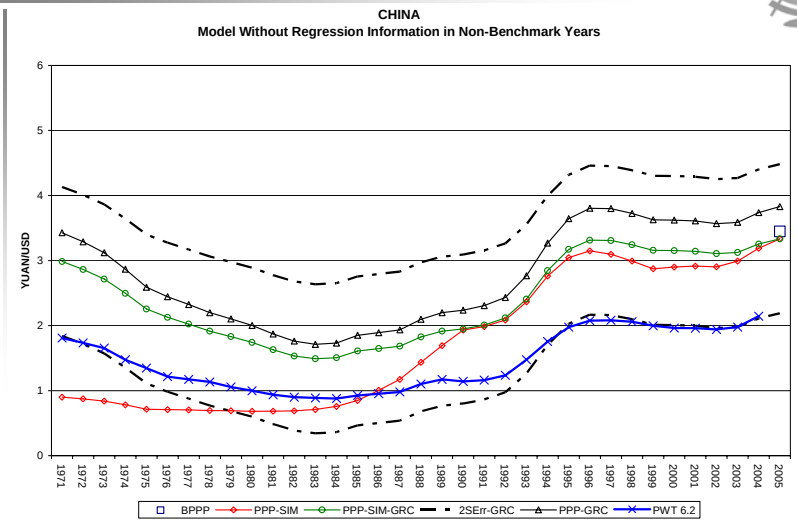
Model Estimation: No Regression in non-benchmark years.



Estimates from full model (unconstrained benchmarks) with growth rates preserved shown for comparison

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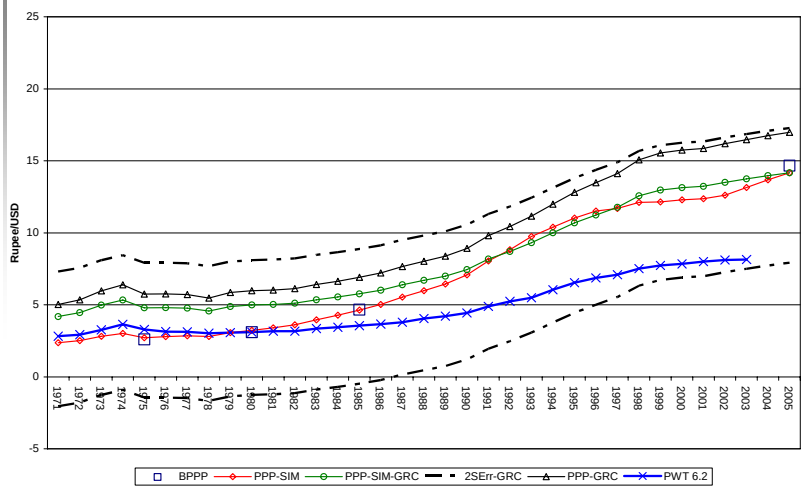
Model Estimation: No Regression in non-benchmark years.



Estimates from full model (unconstrained benchmarks) with growth rates preserved shown for comparison

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SUMMARY

- The proposed method uses a spatio-temporal smoothing filter to combine:
  - All benchmark information
  - Socio-Economic data from all countries
  - GDP Deflators implied price level changes
- Results include the 2005 ICP Benchmarks
- We have demonstrated the flexibility of this procedure through algebraic proofs and the empirical examples.
  - The extrapolations can be made to be consistent with Benchmark PPPs. The constrain of zero error in Benchmark PPPs is rejected by the data.
  - The extrapolations can be made to be consistent with implied price level changes.
- The method produces standard errors.
  - Standard errors are small when all sources of information (Benchmark PPPs, implied price level changes, price level model) provide consistent information (e.g. developed countries).