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Welfare Effect of a Price Change: The Case of Lotto 6/49

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Summary

- Canadians spend a substantial amount on government lottery.

- Ticket price of Lotto 6/49 increased from \$1 to \$2 in 2004.

- Additional rule change was implemented.

- Welfare effect of the price change can be measured using a model in choice under risk.

- Effects of the ticket price increase:

  1. nature of the game has changed,

  2. welfare has increased,

  3. higher revenue for the lottery corporations.

- Other potential applications:

  1. Direct measurement of government output in SNA

  2. Price index in CPI

Government Lotteries in Canada

From the Household Expenditure Surveys:

	Average expenditure (dollars)	Percent of households reporting
1997	165	68.3
1998	170	67.3
1999	165	67.0
2000	156	63.8
2001	155	61.7
2002	162	63.3
2003	157	65.3
2004	163	61.3
2005	154	60.5

Note: Households often under-report spending on gambling.

Rules of Lotto 6/49

Prize	Rule	Probability of Winning, π_i
Jackpot	6 numbers	0.0000000715
Second	5 numbers + bonus	0.000000429
Third	5 numbers	0.00001802
Fourth	4 numbers	0.0009686
Fifth	3 numbers	0.01765
Sixth	2 numbers + bonus	0.0123

Rules of Lotto 6/49 (cont.)

	Before June 2, 2004	After
Ticket price	\$1	\$2
Take-out rate	55%	53%
Prize categories	5	6
Overall odds per ticket	1/54	1/32
Share of the prize fund:		
Jackpot	50%	80.5%
2nd prize	15%	5.75%
3rd prize	12%	4.75%
4th prize	23%	9%
5th prize	\$10	\$10
6th prize	N/A	\$5

Note: Prize fund $=(1-$ takeout rate$)\times$ total wager.

Modelling Choice under Risk

Expected Utility Hypothesis:

$$u = f(g) = \sum_{i=1}^{N} p_i f(y_i)$$

or

$$\sum_{i=1}^{N} p_i f(y_i) - f(g) = 0.$$

Implies that a risk-averse expected utility maximizer will not buy lottery tickets

Diewert's (1993) Implicit Expected-Utility Hypothesis:

$$\sum_{i=1}^{N} p_i \phi_u(x_i) - \phi_u(u) = 0, \quad x_i = f(y_i).$$

Assuming homotheticity for aggregation:

$$\sum_{i=1}^{N} p_i \gamma(x_i/u) - \gamma(1) = 0,$$

where

$$\gamma(z) = \begin{cases} \alpha + (1-\alpha) z^{\beta}, & z \geq 1 \\ 1 - \alpha + \alpha z^{\beta}, & z < 1 \end{cases}$$

with parameter restrictions

$$0 < \alpha < 1/2, \quad \beta < 1, \quad \beta \neq 0.$$

Modelling Lotto 6/49

Notation:

$$
\begin{array}{lcl}
w & = & \text{wager,} \\
n & = & \text{number of tickets purchased per draw,} \\
v & = & \text{price per ticket,} \\
\pi_i & = & \text{probability of winning the } i\text{-th prize for one single ticket,} \\
p_i & = & \text{total probability of winning the } i\text{-th prize,} \\
p_7 & = & \text{probability of not winning any prize,} \\
x_i & = & \text{state contingent consumption,} \\
y & = & \text{real disposable income,} \\
R_i & = & \text{payout for the } i\text{-th prize, } (R_7 = 0)
\end{array}
$$

Then $n = w/v$, $p_i = n\pi_i$,

$$p_7 = 1 - \sum_{i=1}^{6} p_i = 1 - n \sum_{i=1}^{6} \pi_i,$$

and $x_i = y + R_i - vn, i = 1, \ldots, 7$.

Consumers' Expected Utility:

$$u(n) = \left[\frac{(1-\alpha)n \sum_{i=i}^{6} \pi_i (y + R_i - vn)^{\beta} + \alpha \left(1 - n \sum_{i=i}^{6} \pi_i\right) (y - vn)^{\beta}}{\alpha + (1 - 2\alpha) n \sum_{i=i}^{6} \pi_i} \right]^{1/\beta}. \tag{1}$$

Utility maximization:

$$\max_n \{u : 0 \leq 2n \leq y\}$$

Regression equation can be derived from the first-order condition, so that α and β can be estimated.

Output of Lotto 6/49:

- Use estimated values of α and β to calculate utility u^{*t} in period t using equation (1)

- Utility u^{0t} as if there is no lottery is equal to y^t

- Real output of Lotto 6/49, $Q^t = u^{*t} - u^{0t}$

- Implicit price, $P^t = w^t/Q^t$

Data:

- Data source: LotteryCanada.com
- Sales volume and payout prizes from 3/1/98 to 29/5/04 (one dollar game) and 2/6/04 to 30/8/06 (two dollar game)
- Highest Jackpot won in the one-dollar games: 30/9/00, with \$15 million
- Highest Jackpot won in the two-dollar games: 26/10/05, with \$54 million

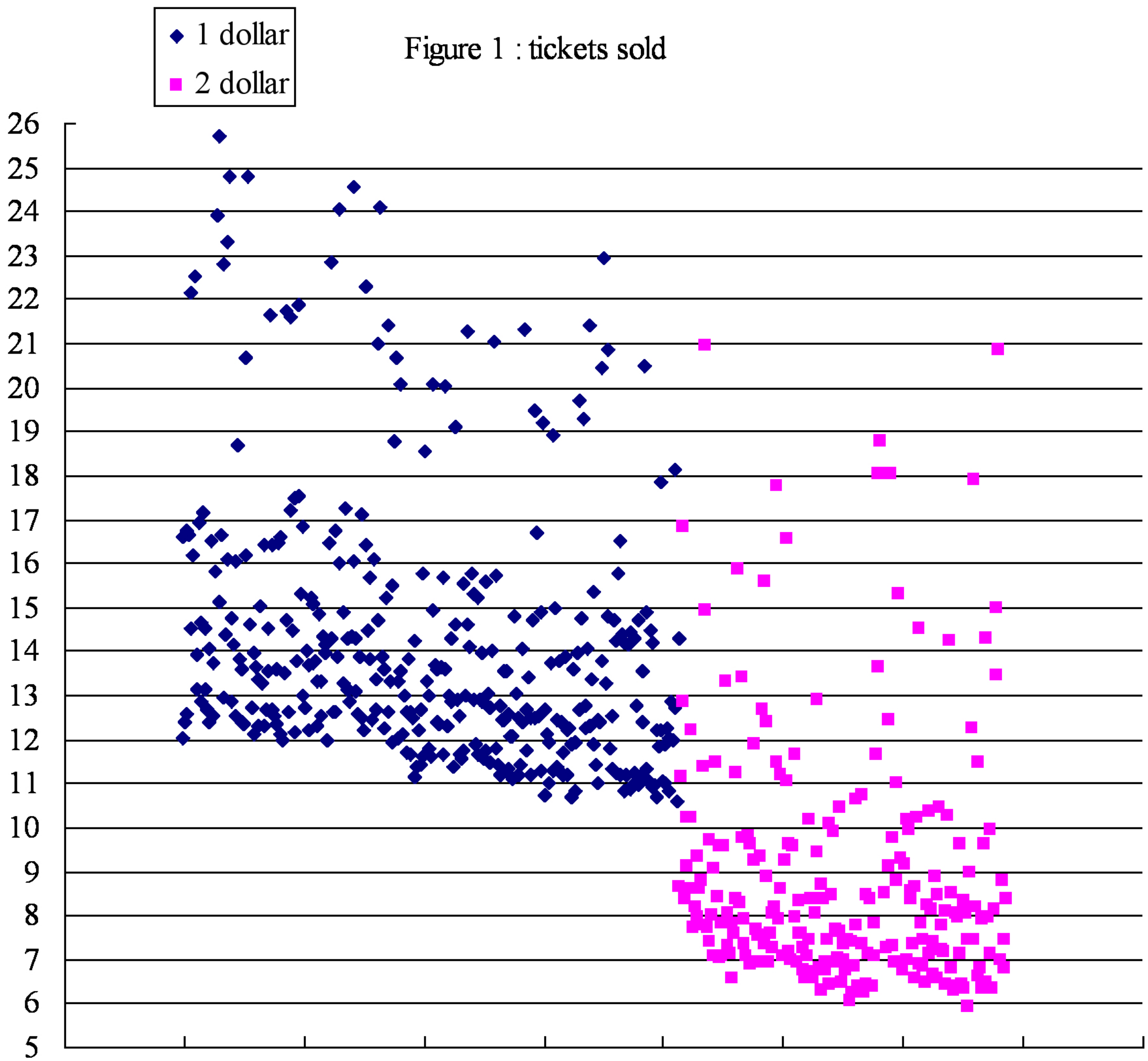
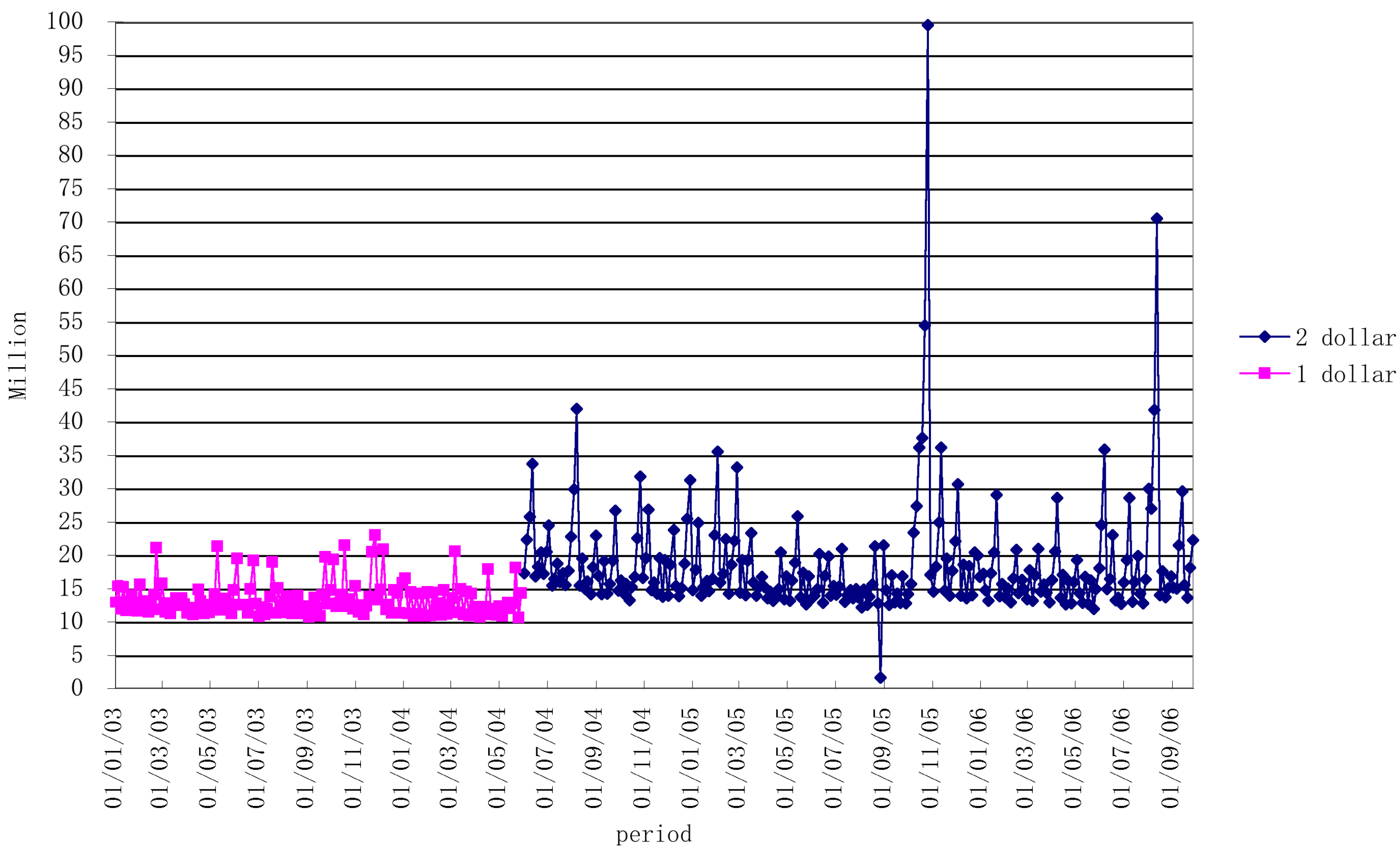

1 dollar
2 dollar
Figure 1 : tickets sold
26
25
24
23
22
21
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5

Figure 2: Sales Revenue



Regression results

	Coefficient	St. Error	t-Ratio
One-Dollar Game			
α	0.1201	0.834×10^{-5}	14,408
β	-31.95	-0.269×10^{-3}	$-118,670$
Two-Dollar Game			
α	0.0319	0.3801×10^{-6}	8,389
β	-0.000338	0.4312×10^{-3}	-0.79

With these estimated values of α and β the output and price can be computed.

Elasticities

	One-Dollar	Two-Dollar
Price elasticity, $\vert\eta\vert$	0.72	0.21
Income elasticity, ϵ	2.1	0.76

Figure 4: Output

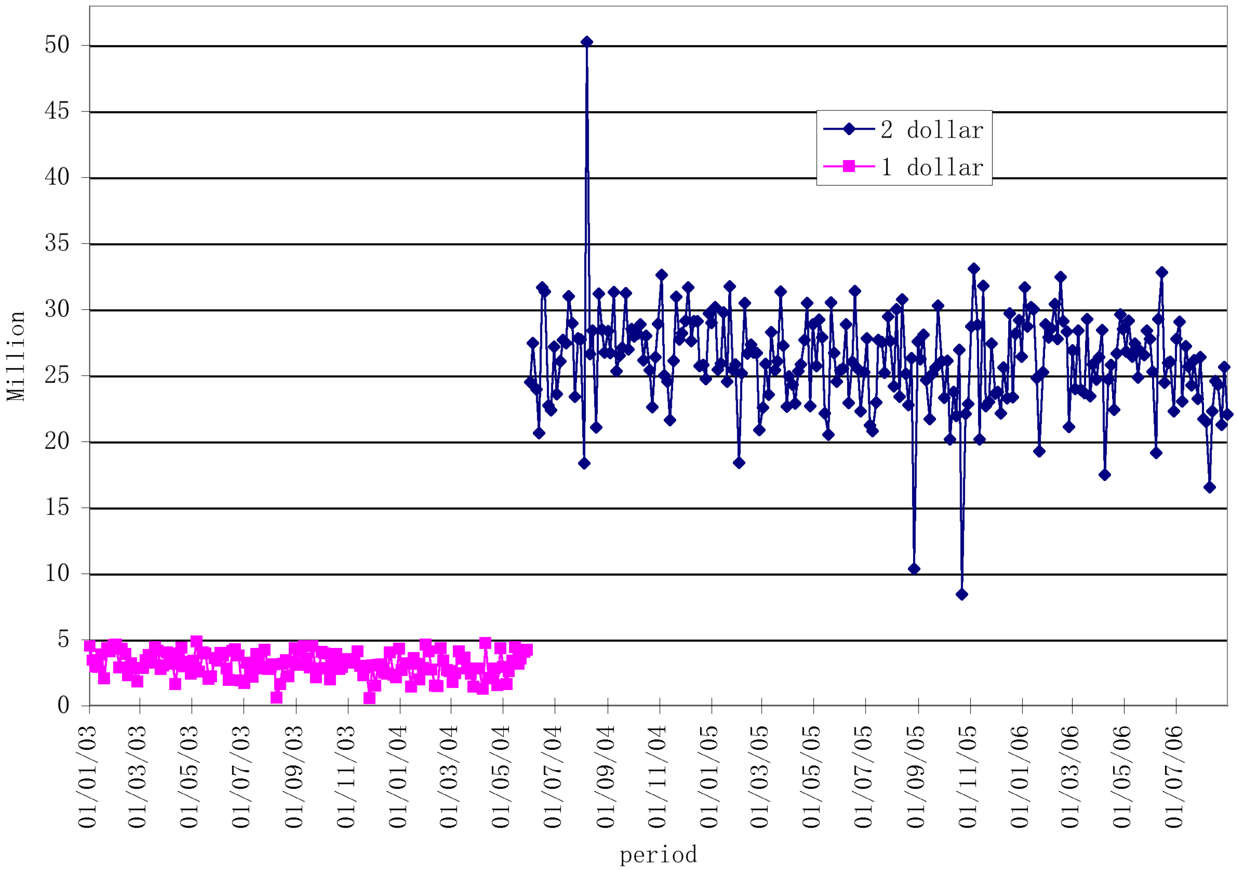


Figure 6: Price Index

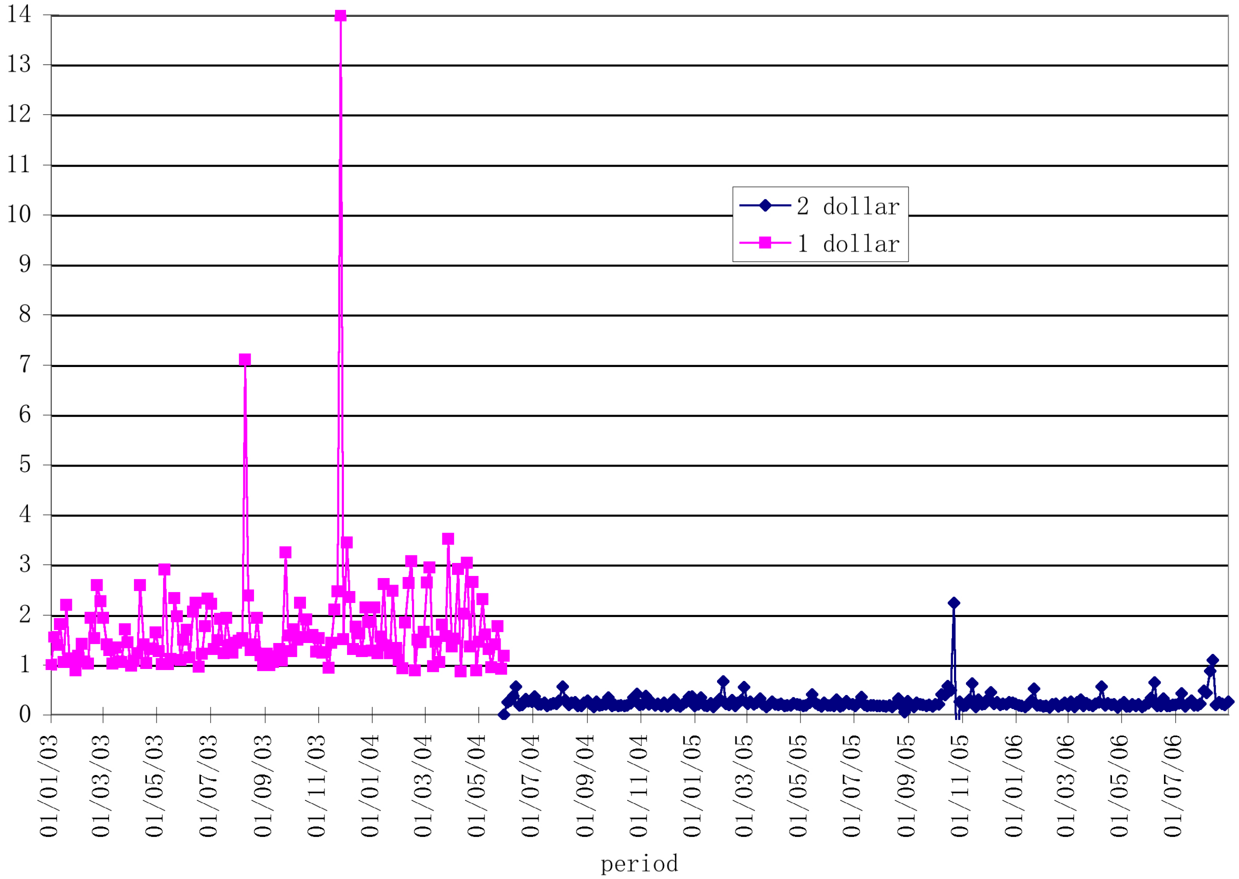
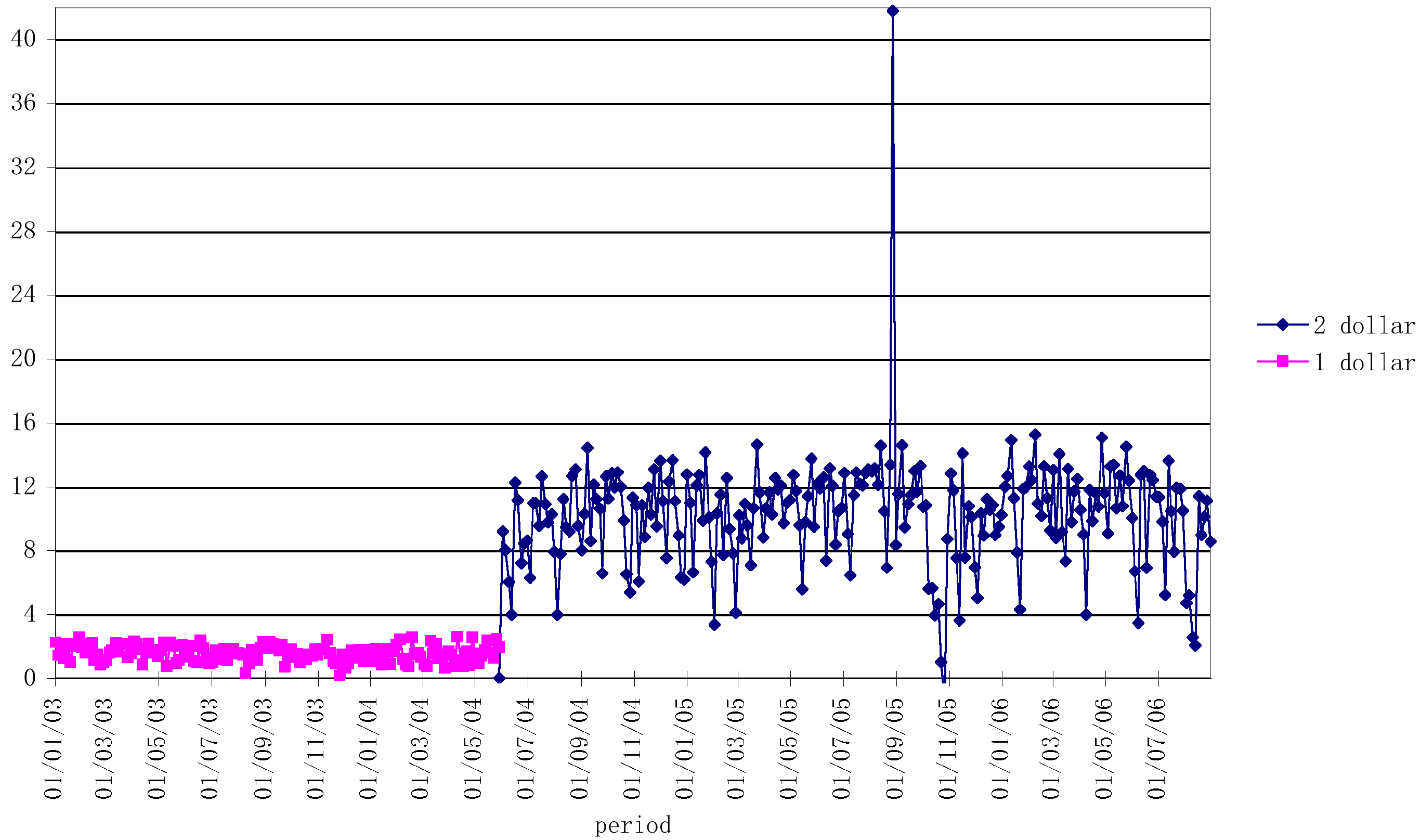


Figure 5: Productivity



Conclusion

- Ticket price increase reduces ticket sales but increase revenue
- Redistribution of the Jackpot and other prizes changes the nature of the game
- Consumer welfare has gone up, effective price of the game has gone down
- Can be interpreted as a productivity improvement