Systems of Index Numbers for International Price Comparisons Based on the Stochastic Approach

Gholamreza Hajargasht
D.S. Prasada Rao

Centre for Efficiency and Productivity Analysis
School of Economics
University of Queensland
Brisbane, Australia

Outline

- **Motivation – multilateral comparisons of prices**
- **Standard formulae and the new index formula**
- **Stochastic approach to index numbers**
- **Derivation of index numbers using stochastic approach**
- **Empirical application – OECD data**
- **Concluding remarks**

Multilateral Comparisons

- Simultaneous comparisons of price and quantity levels or changes
  - Spatial comparisons - absence of an ordering
  - Temporal comparisons - use chain comparisons

- Major Multilateral Comparison projects
  - International Comparison Programme (ICP)
    - OECD, Eurostat, World bank
  - Int. Comp. of Output and Productivity (ICOP)
    - Univ. of Groningen
  - FAO - Agricultural Output Comparisons
  - ABS - Inter-city Price comparisons

Role of Multilateral Comparisons

- **Purchasing Power Parities**
  - **Cross-country comparisons of price levels**
  - **Real GDP and Expenditure components**
    - **ICP, OECD and Eurostat**
    - **Penn World Tables (PWT) - Heston and Summers**
    - **Real income comparisons, HDI etc.**
    - **Global inequality**
    - **Growth and Productivity**
      - **Catch-up and Convergence studies**
    - **Global poverty - World Bank**
- **Output and Input index numbers**
  - **Productivity comparisons (TFP, DEA, SFA)**

Index Number Problem

Price and Quantity Data

Countries	Commodities				
	1	2	3	i	N
1	p_{11} q_{11}	p_{21} q_{21}	p_{31} q_{31}	p_{i1} q_{i1}	p_{N1} q_{N1}
2	p_{12} q_{12}	p_{22} q_{22}	p_{32} q_{32}	p_{i2} q_{i2}	p_{N2} q_{N2}
3	p_{13} q_{13}	p_{23} q_{23}	p_{33} q_{33}	p_{i3} q_{i3}	p_{N3} q_{N3}
. j .				p_{ij} q_{ij}	
M	p_{1M} q_{1M}	p_{2M} q_{2M}	p_{3M} q_{3M}	p_{iM} q_{iM}	p_{NM} q_{NM}

$$I_{MxM} = \begin{bmatrix} I_{11} & I_{12} & . & . & I_{1M} \\ I_{21} & I_{22} & . & . & I_{2M} \\ . & . & . & . & . \\ . & . & . & . & . \\ I_{M1} & . & . & . & I_{MM} \end{bmatrix}$$

- **Transitivity:** $I_{jk} = I_{jl} \times I_{lk}$
  - a consistency requirement
- **Base invariance - country symmetry**

Implications of Transitivity

- **There exist numbers PPP_1, PPP_2, …., PPP_M such that any index I_{jk} can be expressed as:**

$$I_{jk} = \frac{PPP_k}{PPP_j}$$

- **These PPP's can be determined only upto a factor of proportionality.**

- **Once the PPPs are given, it is possible to define "international average Prices" for each of the commodities: P_1, P_2, …., P_N**

Notation

- We have M countries and N commodities
- p_{ij} observed price of the ith commodity in the jth country
- q_{ij} quantity of the ith commodity in the jth country
- PPP_j purchasing power parity of jth country
- P_i Average price for ith commodity
- shares

$$w_{ij} = \frac{p_{ij} q_{ij}}{\sum_{i=1}^{N} p_{ij} q_{ij}} \qquad w_{ij}^{*} = \frac{w_{ij}}{\sum_{j=1}^{M} w_{ij}}$$

Index number methods for international comparisons

- **Commonly used methods like Laspeyres, Paasche, Fisher and Tornqvist methods are for bilateral comparisons – not transitive.**
- **Geary-Khamis method – Geary (1958), Khamis (1970)**
- **Elteto-Koves-Szulc (EKS) method – 1968**
  - EKS method constructs transitive indexes from bilateral Fisher indexes
  - Weighted EKS index – Rao (2001)

Index number methods - continued

- **Variants of Geary-Khamis method – Ikle (1972), Rao (1990)**
- **Methods based on stochastic approach:**
  - Country-product-dummy (CPD) method – Summers (1973)
  - Weighted CPD method – Rao (1995)
  - Generating indexes using Weighted CPD method – Rao (2005), Diewert (2005)
  - Using CPD to compute standard errors – Rao (2004), Deaton (2005)

Geary-Khamis Method

- Geary (1958) and Khamis (1970)
- Based on twin concepts:
  - PPPs of currencies - PPP_j's
  - International averages of prices - P_i's
- Computations based on a simultaneous equation system:

$$P_i = \frac{\sum_{j=1}^{M} (p_{ij}\, q_{ij}) / PPP_j}{\sum_{j=1}^{M} q_{ij}} \qquad PPP_j = \frac{\sum_{i=1}^{N} p_{ij}\, q_{ij}}{\sum_{i=1}^{N} P_i\, q_{ij}}$$

Rao and Ikle variants

- Rao Index

$$\begin{cases} PPP_j = \prod_{i=1}^{N} \left(\frac{p_{ij}}{P_i} \right)^{w_{ij}} \\ P_i = \prod_{j=1}^{M} \left(\frac{p_{ij}}{PPP_j} \right)^{w_{ij}^*} \end{cases}$$

Geometric Mean

- Ikle Index

$$\begin{cases} \frac{1}{PPP_j} = \sum_{i=1}^{N} \left(\frac{P_i}{P_{ij}} w_{ij} \right) \\ \frac{1}{P_i} = \sum_{j=1}^{M} \left(\frac{PPP_j}{p_{ij}} w_{ij}^* \right) \end{cases}$$

Harmonic Mean

The New Index

Why not an arithmetic mean

$$\begin{cases} PPP_j = \sum_{i=1}^{N} \left(\frac{p_{ij}}{P_i} w_{ij} \right) \\ P_i = \sum_{j=1}^{M} \frac{p_{ij}}{PPP_j} w_{ij}^{*} \end{cases}$$

Comments:

- **For all these methods it is necessary to establish the existence of solutions for the simultaneous equations.**
- **Existence of Rao and Ikle indexes was established in earlier papers.**
- **Existence of the new index is considered in this paper.**

Relationship between the indices and stochastic approach

- **The stochastic approach to multilateral indexes is based on the CPD model – discussed below.**
- **The indexes described above are all based on the Geary-Khamis framework which has no stochastic framework.**
- **Rao (2005) has shown that the Rao (1990) variant of the Geary-Khamis method can be derived using the CPD model.**
- **Rest of the presentation focuses on the connection between the CPD model and the indexes described above.**

The Law of One Price

- Following Summers (1973), Rao (2005) and Diewert (2005) we consider

$$p_{ij} = P_i \, PPP_j \, u_{ij}$$

-

- price of i-th commodity in j-th country

purchasing power parity

World price of i-th commodity

•random disturbance

The CPD model may be considered as a "hedonic regression model" where the only characteristics considered are the "country" and the "commodity".

CPD Model

- Rao (2005) has shown that applying a weighted least square to the following equation results in Rao's System

$$\ln p_{ij} = \ln P_i + \ln PPP_j + \varepsilon_{ij}$$

where $\ln P_i$ and $\ln PPP_j$ are treated as parameters

- The same result is obtained if we assume a log-normal distribution for u_{ij} and use a weighted maximum likelihood estimation approach which is the same as the weighted least squares estimator.

Stochastic Approach to New Index

- Again

$$p_{ij} = P_i PPP_j u_{ij}$$

- This time assume

$$u_{ij} \sim Gamma(r, r)$$

Maximum Likelihood

- It can be shown

$$f(p_{ij}) = \frac{r^r}{\Gamma(r)} \frac{p_{ij}^{\;r-1}}{P_i^{\,r} PPP_j^{\;r}} e^{-r\frac{p_{ij}}{P_i PPP_j}}$$

- Taking logs we obtain

$$LnL_{ij} = r\ln r - \ln\Gamma(r) + (r-1)\ln p_{ij} - r\ln P_i - r\ln PPP_j - r\frac{p_{ij}}{P_i PPP_j}$$

Weights and M-Estimation

- Define a weighted likelihood as

$$LnWL = \sum_{i=1}^{N}\sum_{j=1}^{M}\frac{w_{ij}}{M}LnL_{ij}$$

- Then we have

$$\ln WL \propto (r-1)\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\ln p_{ij} - r\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\ln P_i - r\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}\ln PPP_j -$$

$$r\sum_{i=1}^{N}\sum_{j=1}^{M}\frac{p_{ij}w_{ij}}{P_i PPP_j} + r\ln r(\sum_{i=1}^{N}\sum_{j=1}^{M} w_{ij}) - \ln\Gamma(r)\sum_{i=1}^{N}\sum_{j=1}^{M} w_{ij}$$

First Order Conditions

$$\left\{\begin{array}{l} \left.\begin{array}{l} P_i - \sum_{j=1}^{M} \dfrac{p_{ij} w_{ij}^*}{PPP_j} = 0 \\ PPP_j - \sum_{i=1}^{N} \dfrac{p_{ij} w_{ij}}{P_i} = 0 \end{array}\right\} \\ \dfrac{\partial}{\partial r} \ln \Gamma(r) - \ln r = \dfrac{1}{M} \left(\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \ln p_{ij} - \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \ln P_i - \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \ln PPP_j - \sum_{i=1}^{N} \sum_{j=1}^{M} \dfrac{p_{ij} w_{ij}}{P_i PPP_j} + M \right) \end{array}\right.$$

The new Index independent of r

- The advantage of MLE is that we can calculate standard errors for PPPs

Stochastic Approach to Ikle

- Now we consider the model

$$\frac{1}{p_{ij}} = \frac{1}{P_i PPP_j} u_{ij}$$

- where

$$u_{ij} \sim Gamma(r, r)$$

First Order Conditions

- Applying a weighted maximum likelihood we obtain the following first order conditions
- $$\begin{cases} \dfrac{1}{PPP_j} = \sum_{i=1}^{N} \left(\dfrac{P_i}{P_{ij}} w_{ij} \right) \\ \dfrac{1}{P_i} = \sum_{j=1}^{M} \left(\dfrac{PPP_j}{p_{ij}} w_{ij}^{*} \right) \end{cases}$$
- i.e. Ikle's Index

Standard Errors

- Computation of standard errors is an important motivation for considering stochastic approach.
- An M-Estimator is defined as an estimator that maximizes

$$Q_N(\boldsymbol{\theta}) = \frac{1}{N}\sum_{i=1}^{N} h_i(y_i, \mathbf{x}_i; \boldsymbol{\theta})$$

- It has the following asymptotic distribution

$$\sqrt{N}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) \xrightarrow{d} N[\mathbf{0}, \mathbf{A}_0^{-1}\mathbf{B}_0\mathbf{A}_0^{-1}]$$

- where

$$\mathbf{A}_0 = \text{plim}\frac{1}{N}\sum_{i=1}^{N}\left.\frac{\partial^2 h_i}{\partial\boldsymbol{\theta}'\partial\boldsymbol{\theta}}\right|_{\boldsymbol{\theta}_0}$$

$$\mathbf{B}_0 = \text{plim}\ \frac{1}{N}\sum_{i=1}^{N}\left.\frac{\partial h_i}{\partial\boldsymbol{\theta}}\right|_{\boldsymbol{\theta}_0}\left.\frac{\partial h_i}{\partial\boldsymbol{\theta}'}\right|_{\boldsymbol{\theta}_0}$$

- In practice, a consistent estimator can be obtained as

$$\mathbf{VAR}(\hat{\boldsymbol{\theta}}) = \frac{1}{N}\hat{\mathbf{A}}^{-1}\hat{\mathbf{B}}\hat{\mathbf{A}}^{-1}$$

- Where

$$\hat{\mathbf{A}} = \frac{1}{N}\sum_{i=1}^{N}\frac{\partial^2 h}{\partial\boldsymbol{\theta}'\partial\boldsymbol{\theta}}\bigg|_{\hat{\boldsymbol{\theta}}}$$

$$\hat{\mathbf{B}} = \frac{1}{N}\sum_{i=1}^{N}\frac{\partial h_i}{\partial\boldsymbol{\theta}}\bigg|_{\hat{\boldsymbol{\theta}}}\frac{\partial h_i}{\partial\boldsymbol{\theta}'}\bigg|_{\hat{\boldsymbol{\theta}}}$$

- In special cases like the standard maximum likelihood we have $\mathbf{A}_0^{-1} = \mathbf{-B_0}$ therefore

$$\mathbf{VAR}(\hat{\boldsymbol{\theta}}) = -\frac{1}{N}\hat{\mathbf{A}}^{-1}$$

- Many software report this as their default variance estimator
- This Variance is not valid in our context and the general formula should be used
- For example if we apply to a weighted linear model (e.g. CPD)

$$\mathbf{VAR}(\hat{\boldsymbol{\theta}}) = \hat{\sigma}^2 (\mathbf{X}'\boldsymbol{\Omega}\mathbf{X})^{-1}$$

- But if we apply the general formula

$$\mathbf{VAR}(\boldsymbol{\theta}) = \hat{\hat{\sigma}}^2 (\mathbf{X}'\boldsymbol{\Omega}\mathbf{X})^{-1} (\mathbf{X}'\boldsymbol{\Omega}'\boldsymbol{\Omega}\mathbf{X}) (\mathbf{X}'\boldsymbol{\Omega}\mathbf{X})^{-1}$$

Application to OECD countries

- OECD data from 1996.
- The price information was in the form of PPPs at the basic heading level for 158 basic headings, with US dollar used as the numeraire currency.
- The estimates of PPPs based on the new index, Ikle's and the weighted CPD for 24 OECD countries along with their standard errors are presented in the following table.

Country	MLE Estimates					
	New Index		CPD		Ikle	
	PPP	S.E	PPP	S.E	PPP	S.E.
GER	1.887	0.136	2.034	0.144	2.187	0.147
FRA	6.092	0.429	6.554	0.455	7.035	0.466
ITA	1425.96	109.727	1504.02	115.509	1584.381	119.196
NLD	1.921	0.150	2.056	0.155	2.205	0.156
BEL	35.491	2.577	37.890	2.698	40.450	2.728
LUX	33.578	2.488	35.816	2.618	38.191	2.700
UK	0.603	0.043	0.642	0.044	0.682	0.045
IRE	0.637	0.051	0.669	0.055	0.696	0.060
DNK	8.525	0.586	9.131	0.615	9.762	0.631
GRC	180.470	13.452	188.482	13.891	196.640	14.005
SPA	112.414	8.304	118.546	8.606	124.799	8.738
PRT	126.043	10.400	129.037	10.994	130.317	12.002
AUT	12.770	0.881	13.730	0.928	14.728	0.948
SUI	2.050	0.168	2.183	0.177	2.320	0.180
SWE	9.424	0.686	10.075	0.720	10.758	0.742
FIN	6.159	0.432	6.598	0.453	7.070	0.462
ICE	86.828	7.000	89.541	6.975	92.329	6.810
NOR	8.807	0.684	9.238	0.736	9.642	0.764
TUR	6304.23	579.128	6321.42	544.907	6357.003	506.991
AUS	1.264	0.099	1.333	0.103	1.407	0.104
NZL	1.464	0.111	1.530	0.113	1.596	0.115
JAP	182.031	13.622	187.429	14.282	192.392	14.780
CAN	1.168	0.090	1.229	0.094	1.295	0.096
USA	1.00		1.00		1.00	

Derivation of Geary-Khamis Method from CPD model

- **Recall that Geary (1958) and Khamis (1970) index is given by:**

$$P_i = \frac{\sum_{j=1}^{M} (p_{ij}\, q_{ij}) / PPP_j}{\sum_{j=1}^{M} q_{ij}} \qquad PPP_j = \frac{\sum_{i=1}^{N} p_{ij}\, q_{ij}}{\sum_{i=1}^{N} P_i\, q_{ij}}$$

- **Rao and Selvanathan (1994) used a conditional regression model to derive standard errors for PPPs.**

- **Diewert (2005) derived G-K PPPs using stochastic approach**

- **Here, we show that the G-K method is related to the CPD model and G-K PPPs are indeed Method of Moments estimators of PPPs from the CPD model!**

Estimation of Non-additive Models

- We consider the CPD model:

$$p_{ij} = P_i \, PPP_j \, u_{ij}$$

- We show that this is a non-additive regression model with p_{ij} as the dependent variable, y, the country and product dummy variables as regressors, X, and P_i *and* PPP_j as the parameter vector, β.
- Then a non-linear model

$$r(y_i, \mathbf{x}_i, \boldsymbol{\beta}) = u_i$$

is said to be non-additive if it cannot be written as:

$$y_i - g(\mathbf{x}_i, \boldsymbol{\beta}) = u_i$$

Estimation of Non-additive Models

- The non-linear least squares estimator of the parameters of non-additive models are not consistent.
- The method of moments estimator may be considered in this case. Consider a set of moment conditions:

$$E(\mathbf{R}(\mathbf{x},\boldsymbol{\beta})'\mathbf{u}) = \mathbf{0}$$

where **R** is a matrix representing K moment conditions.

The method of moments estimator is obtained by solving the sample moment conditions:

$$\frac{1}{N}\mathbf{R}(\mathbf{X},\hat{\boldsymbol{\beta}})'\mathbf{r}(\mathbf{y},\mathbf{X},\hat{\boldsymbol{\beta}}) = \mathbf{0}$$

Estimation of Non-additive Models

- The method of moments estimator is asymptotically normal with variance matrix

$$Var(\hat{\boldsymbol{\beta}}_{MM}) = \hat{\sigma}^2 \left[\hat{\mathbf{D}}'\hat{\mathbf{R}}\right]^{-1} \hat{\mathbf{R}}'\hat{\mathbf{R}} \left[\hat{\mathbf{R}}'\hat{\mathbf{D}}\right]^{-1}$$

where

$$\hat{\mathbf{D}} = \left.\frac{\partial \mathbf{r}(\mathbf{y},\mathbf{X},\boldsymbol{\beta})}{\partial \boldsymbol{\beta}'}\right|_{\hat{\boldsymbol{\beta}}} \qquad \hat{\mathbf{R}} = \mathbf{R}(\mathbf{X},\hat{\boldsymbol{\beta}}) \quad \text{and} \quad \hat{\sigma}^2 = \frac{\hat{\mathbf{u}}'\hat{\mathbf{u}}}{N}$$

- The most efficient choice of moment conditions is

$$\mathbf{R}(\mathbf{X},\boldsymbol{\beta})^* = E\left[\frac{\partial \mathbf{r}(\mathbf{y},\mathbf{X},\boldsymbol{\beta})'}{\partial \boldsymbol{\beta}} \mid \mathbf{X}\right]$$

- In our case, this choice leads to the unweighted index.

GK Index as a method of moments estimator

- The CPD model is

$$p_{ij} = P_i PPP_j u_{ij}^* \quad \text{with} \quad E(u_{ij}^*) = 1$$

- We rewrite the CPD model as a non-additive model

$$\frac{p_{ij}}{P_i PPP_j} - 1 = u_{ij} \quad \text{with} \quad E(u_{ij}) = 0$$

- We consider two sets of moment conditions: (i) optimal set; and (ii) weighted moment conditions

Optimal method of moments estimator

- Considering the CPD model:

$$r_{ij} = \frac{p_{ij}}{P_i PPP_j} - 1 = u_{ij}$$

and deriving the moment conditions leading to optimal estimator, we have the moment conditions defined by the matrix **R** which is defined as:

$$\mathbf{R}' = \mathbf{E}\begin{bmatrix} -\frac{p_{11}}{P_1^2 PPP_1} & & -\frac{p_{12}}{P_1^2 PPP_2} & & \cdots & -\frac{p_{1m}}{P_1^2 PPP_m} & \\ & & & . & & & \\ & -\frac{p_{n1}}{P_n^2 PPP_1} & & -\frac{p_{n2}}{P_n^2 PPP_2} & \cdots & & -\frac{p_{nm}}{P_n^2 PPP_m} \\ & & & \cdots\cdots & & & \\ -\frac{p_{11}}{P_1 PPP_1^3} \;\; .. & -\frac{p_{n1}}{P_n PPP_1^3} & 0 & \cdots\cdots & & \cdots & 0 \\ & \cdots & & \cdots & & & \\ 0 & \cdots & \cdots & & 0 & -\frac{p_{1m}}{P_1 PPP_{1m}^3} \;\; \cdots & -\frac{p_{nm}}{P_n PPP_m^3} \end{bmatrix}$$

Optimal method of moments estimator

- Noting that: $E\left[\frac{p_{ij}}{P_i PPP_j}\right]=1$

and using the moment conditions $E(\mathbf{R}(\mathbf{x},\boldsymbol{\beta})' r)=\mathbf{0}$

we have the following normal equations to solve.

$$\begin{cases} -\frac{1}{P_i}\sum_{j=1}^{M}\left(\frac{p_{ij}}{P_i PPP_j}-1\right)=0 \\ -\frac{1}{PPP_j}\sum_{i=1}^{N}\left(\frac{p_{ij}}{P_i PPP_j}-1\right)=0 \end{cases} \Rightarrow \begin{cases} P_i=\frac{1}{m}\sum_{j=1}^{m}\left(\frac{p_{ij}}{PPP_j}\right) \\ PPP_j=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{p_{ij}}{P_i}\right) \end{cases}$$

These are exactly the same equations that define the arithmetic index introduced earlier.

Estimated standard errors can be computed by substituting the MOM estimator in the formula for the asymptotic variance.

Geary-Khamis method as a method of moments estimator

- We consider weighted moment conditions where quantities are used as weights.

and deriving the moment conditions leading to optimal estimator, we have the moment conditions defined by the matrix **R** which is defined as:

$$\mathbf{R'} = \begin{bmatrix} -\frac{q_{11}}{P_1} & & -\frac{q_{12}}{P_1} & & \cdots & -\frac{q_{1m}}{P_n} & \\ & & & \cdot & & & \\ & -\frac{q_{n1}}{P_n} & & -\frac{q_{n2}}{P_n} & \cdots & & -\frac{q_{nm}}{P_n} \\ -\frac{q_{11}P_1}{PPP_1} & \cdots & -\frac{q_{n1}P_n}{PPP_1} & 0 & \cdots\cdot & & 0 \\ & & \cdots & \cdots & & & \\ 0 & \cdots\cdot & \cdots\cdot & 0 & -\frac{q_{1m}P_1}{PPP_m} & \cdots & -\frac{q_{nm}P_n}{PPP_m} \end{bmatrix} \qquad r = \begin{bmatrix} \frac{p_{11}}{P_1\, PPP_1} - 1 \\ \frac{p_{12}}{P_2\, PPP_1} - 1 \\ \vdots \\ \frac{p_{nm}}{P_n\, PPP_m} - 1 \end{bmatrix}$$

Geary-Khamis as MOM estimator

- Using the moment conditions **R′r = 0;** we can write the implied normal equations as:

$$\begin{cases} PPP_j = \dfrac{\sum_{i=1}^{n} p_{ij} q_{ij}}{\sum_{i=1}^{n} P_i q_{ij}} \\ P_i = \dfrac{\sum_{j=1}^{m} \left(p_{ij} q_{ij} \big/ PPP_j \right)}{\sum_{j=1}^{m} q_{ij}} \end{cases}$$

- These are the equations that define the G-K method.
- The asymptotic standard errors can be derived using the R matrix in the variance-covariance matrix formula.

MOM estimators of PPPs and SE's – OECD Example

	Arithmetic Index	GMM SE Arithmetic	MLE SE Arithematic	G-K Index	GMM SE G-K
GER	1.878	0.109442	0.136	2.08316	0.15474
FRA	6.067	0.606755	0.429	6.679491	0.516194
ITA	1419	79.25337	109.727	1537.168	129.5046
NLD	1.909	0.11156	0.150	2.032161	0.156602
BEL	35.3	1.946125	2.577	38.70436	2.700867
LUX	33.35	2.454269	2.488	36.7877	3.446165
UK	0.5996	0.036311	0.043	0.679564	0.053761
IRE	0.633	0.037709	0.051	0.657754	0.056569
DNK	8.481	0.591807	0.586	9.457703	0.872669
GRC	179.5	9.271153	13.452	187.3352	13.14857
SPA	111.8	7.726502	8.304	122.1712	10.59001
PRT	125.4	6.56711	10.400	124.7745	9.307088
AUT	12.71	0.731266	0.881	14.40264	1.098328
SUI	2.037	0.146331	0.168	2.220059	0.179608
SWE	9.382	0.726701	0.686	10.56069	1.024583
FIN	6.12	0.404593	0.432	6.895726	0.638499
ICE	86.15	6.142211	7.000	90.02853	9.473389
NOR	8.751	0.457666	0.684	9.119335	0.764748
TUR	6251	393.9744	579.128	5967.556	549.1221
AUS	1.259	0.08598	0.099	1.351173	0.106996
NZL	1.455	0.106893	0.111	1.545069	0.140098
JAP	181	12.52263	13.622	179.0048	15.83708
CAN	1.16	0.085695	0.090	1.271441	0.115112

Conclusion

- A new system for international price comparison is proposed. Existence and uniqueness established
- A stochastic framework for generating the Rao, Ikle and the new index has been established.
- Using the framework of M-estimators, standard errors are obtained for the PPPs from each of the methods.
- The Geary-Khamis method is shown to be a Method of moments estimator of PPPs in the CPD model. Standard errors for the GK estimator are obtained.
- Empirical application using the OECD data generates PPPs from different methods along with their standard errors.
- Further work is necessary to address the problem of choosing between different stochastic specifications.