Systems of Index Numbers for International Price Comparisons Based on the Stochastic Approach

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Multilateral Comparisons

- Simultaneous comparisons of price and quantity levels or changes
  - Spatial comparisons - absence of an ordering
  - Temporal comparisons - use chain comparisons

- Major Multilateral Comparison projects
  - International Comparison Programme (ICP)
    - OECD, Eurostat, World bank
  - Int. Comp. of Output and Productivity (ICOP)
    - Univ. of Groningen
  - FAO - Agricultural Output Comparisons
  - ABS - Inter-city Price comparisons
Role of Multilateral Comparisons

- Purchasing Power Parities
  - Cross-country comparisons of price levels
  - Real GDP and Expenditure components
    - ICP, OECD and Eurostat
    - Penn World Tables (PWT) - Heston and Summers
    - Real income comparisons, HDI etc.
    - Global inequality
    - Growth and Productivity
      - Catch-up and Convergence studies
    - Global poverty - World Bank

- Output and Input index numbers
  - Productivity comparisons (TFP, DEA, SFA)
### Index Number Problem

**Price and Quantity Data**

<table>
<thead>
<tr>
<th>Countries</th>
<th>Commodity Data</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>i</th>
<th>N</th>
</tr>
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<tbody>
<tr>
<td>1</td>
<td>p_11, q_11</td>
<td>p_{21}, q_{21}</td>
<td>p_{31}, q_{31}</td>
<td>p_{i1}, q_{i1}</td>
<td>p_{N1}, q_{N1}</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>p_{12}, q_{12}</td>
<td>p_{22}, q_{22}</td>
<td>p_{32}, q_{32}</td>
<td>p_{i2}, q_{i2}</td>
<td>p_{N2}, q_{N2}</td>
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<tr>
<td>3</td>
<td>p_{13}, q_{13}</td>
<td>p_{23}, q_{23}</td>
<td>p_{33}, q_{33}</td>
<td>p_{i3}, q_{i3}</td>
<td>p_{N3}, q_{N3}</td>
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</tr>
<tr>
<td>.</td>
<td>p_{j1}, q_{j1}</td>
<td>p_{j2}, q_{j2}</td>
<td>p_{j3}, q_{j3}</td>
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<td>.</td>
<td></td>
</tr>
<tr>
<td>M</td>
<td>p_{1M}, q_{1M}</td>
<td>p_{2M}, q_{2M}</td>
<td>p_{3M}, q_{3M}</td>
<td>p_{iM}, q_{iM}</td>
<td>p_{NM}, q_{NM}</td>
<td></td>
</tr>
</tbody>
</table>

\[ \text{I}_{\text{MxM}} = \begin{bmatrix} \text{I}_{11} & \text{I}_{12} & \cdots & \text{I}_{1M} \\ \text{I}_{21} & \text{I}_{22} & \cdots & \text{I}_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ \text{I}_{M1} & \cdots & \cdots & \text{I}_{MM} \end{bmatrix} \]

- **Transitivity:** \( I_{jk} = I_{jl} \times I_{lk} \)
  - a consistency requirement
- **Base invariance** - country symmetry
Implications of Transitivity

- There exist numbers $\text{PPP}_1, \text{PPP}_2, \ldots, \text{PPP}_M$ such that any index $I_{jk}$ can be expressed as:

  $$I_{jk} = \frac{\text{PPP}_k}{\text{PPP}_j}$$

- These PPP’s can be determined only upto a factor of proportionality.

- Once the PPPs are given, it is possible to define “international average Prices” for each of the commodities: $P_1, P_2, \ldots, P_N$
Notation

- We have $M$ countries and $N$ commodities
- $P_{ij}$ observed price of the $i$th commodity in the $j$th country
- $q_{ij}$ quantity of the $i$th commodity in the $j$th country
- $PPP_j$ purchasing power parity of $j$th country
- $P_i$ Average price for $i$th commodity
- shares

$$w_{ij} = \frac{p_{ij}q_{ij}}{\sum_{i=1}^{N} p_{ij}q_{ij}}$$

$$w_{ij}^* = \frac{w_{ij}}{\sum_{j=1}^{M} w_{ij}}$$
Index number methods for international comparisons

- Commonly used methods like Laspeyres, Paasche, Fisher and Tornqvist methods are for bilateral comparisons – not transitive.
- Elteto-Koves-Szulc (EKS) method – 1968
  - EKS method constructs transitive indexes from bilateral Fisher indexes
  - Weighted EKS index – Rao (2001)
Index number methods - continued


- Methods based on stochastic approach:
Geary-Khamis Method

Geary (1958) and Khamis (1970)

Based on twin concepts:

- PPPs of currencies - PPP$_j$’s
- International averages of prices - P$_i$’s

Computations based on a simultaneous equation system:

\[
P_i = \frac{\sum_{j=1}^{M} (p_{ij} q_{ij}) / PPP_j}{\sum_{j=1}^{M} q_{ij}}
\]

\[
PPP_j = \frac{\sum_{i=1}^{N} p_{ij} q_{ij}}{\sum_{i=1}^{N} P_i q_{ij}}
\]
### Rao and Ikle variants

#### Rao Index

\[ PPP_j = \prod_{i=1}^{N} \left( \frac{p_{ij}}{P_i} \right)^{w_{ij}} \]

\[ P_i = \prod_{j=1}^{M} \left( \frac{p_{ij}}{PPP_j} \right)^{w_{ij}^*} \]

#### Geometric Mean

\[ \frac{1}{PPP_j} = \sum_{i=1}^{N} \left( \frac{P_i}{p_{ij}} \right)^{w_{ij}} \]

#### Ikle Index

\[ \frac{1}{P_i} = \sum_{j=1}^{M} \left( \frac{PPP_j}{p_{ij}} \right)^{w_{ij}^*} \]

#### Harmonic Mean
The New Index

Why not an arithmetic mean

\[
PPP_j = \sum_{i=1}^{N} \left( \frac{p_{ij}}{P_i} w_{ij} \right)
\]

\[
P_i = \sum_{j=1}^{M} \frac{p_{ij}}{PPP_j} w_{ij}^*
\]

Comments:

• For all these methods it is necessary to establish the existence of solutions for the simultaneous equations.

• Existence of Rao and Ikle indexes was established in earlier papers.

• Existence of the new index is considered in this paper.
The stochastic approach to multilateral indexes is based on the CPD model – discussed below.

The indexes described above are all based on the Geary-Khamis framework which has no stochastic framework.

Rao (2005) has shown that the Rao (1990) variant of the Geary-Khamis method can be derived using the CPD model.

Rest of the presentation focuses on the connection between the CPD model and the indexes described above.
The Law of One Price

- Following Summers (1973), Rao (2005) and Diewert (2005) we consider

\[ p_{ij} = P_i \text{PPP}_j u_{ij} \]

- price of i-th commodity in j-th country
- purchasing power parity
- World price of i-th commodity
- random disturbance

The CPD model may be considered as a “hedonic regression model” where the only characteristics considered are the “country” and the “commodity”.
Rao (2005) has shown that applying a weighted least square to the following equation results in Rao’s System

$$\ln p_{ij} = \ln P_i + \ln PPP_j + \varepsilon_{ij}$$

where \( \ln P_i \) and \( \ln PPP_j \) are treated as parameters.

The same result is obtained if we assume a log-normal distribution for \( u_{ij} \) and use a weighted maximum likelihood estimation approach which is the same as the weighted least squares estimator.
Stochastic Approach to New Index

- Again

\[ p_{ij} = P_i p_{PPP} u_{ij} \]

- This time assume

\[ u_{ij} \sim \text{Gamma}(r, r) \]
Maximum Likelihood

- It can be shown

\[ f(p_{ij}) = \frac{r^r p_{ij}^{r-1}}{\Gamma(r) P_i^r P_{PPP}^j} e^{-r \frac{p_{ij}}{P_i P_{PPP}^j}} \]

- Taking logs we obtain

\[ \ln L_{ij} = r \ln r - \ln \Gamma(r) + (r - 1) \ln p_{ij} - r \ln P_i - r \ln P_{PPP}^j - r \frac{p_{ij}}{P_i P_{PPP}^j} \]
Weights and M-Estimation

- Define a weighted likelihood as

$$\ln WL = \sum_{i=1}^{N} \sum_{j=1}^{M} \frac{W_{ij}}{M} \ln L_{ij}$$

- Then we have

$$\ln WL \propto (r - 1) \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \ln p_{ij} - r \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \ln P_i - r \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \ln PPP_j -$$

$$r \sum_{i=1}^{N} \sum_{j=1}^{M} \frac{p_{ij} w_{ij}}{P_i PPP_j} + r \ln r \left( \sum_{i=1}^{N} \sum_{j=1}^{M} w_{ij} \right) - \ln \Gamma(r) \sum_{i=1}^{N} \sum_{j=1}^{M} w_{ij}$$
First Order Conditions

\[
P_i - \sum_{j=1}^{M} \frac{P_{ij}w_{ij}^*}{PPP_j} = 0
\]
\[
PPP_j - \sum_{i=1}^{N} \frac{P_{ij}w_{ij}}{P_i} = 0
\]

The new Index independent of r

\[
\frac{\partial}{\partial r} \ln \Gamma(r) - \ln r = \frac{1}{M} \left( \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \ln P_{ij} - \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \ln P_i - \sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij} \ln PPP_j - \sum_{i=1}^{N} \sum_{j=1}^{M} \frac{P_{ij}w_{ij}}{P_iPPP_j} + M \right)
\]

The advantage of MLE is that we can calculate standard errors for PPPs
Stochastic Approach to Ikle

- Now we consider the model

\[
\frac{1}{p_{ij}} = \frac{1}{P_i PPP_j} u_{ij}
\]

- where

\[
u_{ij} \sim \text{Gamma}(r,r)
\]
First Order Conditions

- Applying a weighted maximum likelihood we obtain the following first order conditions

\[
\frac{1}{PPP_j} = \sum_{i=1}^{N} \left( \frac{P_i}{P_{ij}} w_{ij} \right)
\]

- \[
\frac{1}{P_i} = \sum_{j=1}^{M} \left( \frac{PPP_j}{p_{ij}} w^*_{ij} \right)
\]

- i.e. Ikle’s Index
Standard Errors

- Computation of standard errors is an important motivation for considering stochastic approach.

- An M-Estimator is defined as an estimator that maximizes

\[ Q_N(\theta) = \frac{1}{N} \sum_{i=1}^{N} h_i(y_i, x_i; \theta) \]

- It has the following asymptotic distribution

\[ \sqrt{N}(\hat{\theta} - \theta_0) \xrightarrow{d} N[0, A_0^{-1}B_0A_0^{-1}] \]

- where

\[ A_0 = \text{plim} \frac{1}{N} \sum_{i=1}^{N} \left. \frac{\partial^2 h_i}{\partial \theta^i \partial \theta} \right|_{\theta_0} \]

\[ B_0 = \text{plim} \frac{1}{N} \sum_{i=1}^{N} \left. \frac{\partial h_i}{\partial \theta} \right|_{\theta_0} \left. \frac{\partial h_i}{\partial \theta'} \right|_{\theta_0} \]
In practice, a consistent estimator can be obtained as

\[
\text{VAR}(\hat{\theta}) = \frac{1}{N} \hat{A}^{-1} \hat{B} \hat{A}^{-1}
\]

Where

\[
\hat{A} = \frac{1}{N} \sum_{i=1}^{N} \frac{\partial^2 h}{\partial \theta^i \partial \theta^i} 
\]

\[
\hat{B} = \frac{1}{N} \sum_{i=1}^{N} \frac{\partial h_i}{\partial \theta^i} \frac{\partial h_i}{\partial \theta^i} 
\]

In special cases like the standard maximum likelihood we have \( A_0^{-1} = -B_0 \) therefore

\[
\text{VAR}(\hat{\theta}) = -\frac{1}{N} \hat{A}^{-1}
\]
Many software report this as their default variance estimator.

This Variance is not valid in our context and the general formula should be used.

For example if we apply to a weighted linear model (e.g. CPD):

\[
\text{VAR}(\hat{\theta}) = \hat{\sigma}^2 (X'\Omega X)^{-1}
\]

But if we apply the general formula:

\[
\text{VAR}(\theta) = \hat{\sigma}^2 (X'\Omega X)^{-1} (X'\Omega'\Omega X)(X'\Omega X)^{-1}
\]
OECD data from 1996.

The price information was in the form of PPPs at the basic heading level for 158 basic headings, with US dollar used as the numeraire currency.

The estimates of PPPs based on the new index, Ikle’s and the weighted CPD for 24 OECD countries along with their standard errors are presented in the following table.
<table>
<thead>
<tr>
<th>Country</th>
<th>MLE Estimates</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>New Index</td>
<td>CPD</td>
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<tr>
<td></td>
<td>PPP</td>
<td>S.E.</td>
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<tr>
<td>GER</td>
<td>1.887</td>
<td>0.136</td>
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<td>FRA</td>
<td>6.092</td>
<td>0.429</td>
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<tr>
<td>ITA</td>
<td>1425.96</td>
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<td>NLD</td>
<td>1.921</td>
<td>0.150</td>
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<tr>
<td>BEL</td>
<td>35.491</td>
<td>2.577</td>
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<tr>
<td>LUX</td>
<td>33.578</td>
<td>2.488</td>
</tr>
<tr>
<td>UK</td>
<td>0.603</td>
<td>0.043</td>
</tr>
<tr>
<td>IRE</td>
<td>0.637</td>
<td>0.051</td>
</tr>
<tr>
<td>DNK</td>
<td>8.525</td>
<td>0.586</td>
</tr>
<tr>
<td>GRC</td>
<td>180.470</td>
<td>13.452</td>
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<tr>
<td>SPA</td>
<td>112.414</td>
<td>8.304</td>
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<tr>
<td>PRT</td>
<td>126.043</td>
<td>10.400</td>
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<td>SWE</td>
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<td>NOR</td>
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<td>NZL</td>
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<td>CAN</td>
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</tr>
<tr>
<td>USA</td>
<td>1.00</td>
<td>1.00</td>
</tr>
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</table>
Derivation of Geary-Khamis Method from CPD model

Recall that Geary (1958) and Khamis (1970) index is given by:

\[ P_j = \frac{\sum_{j=1}^{M} (p_{ij} q_{ij}) / PPP_j}{\sum_{j=1}^{M} q_{ij}} \]

\[ PPP_j = \frac{\sum_{i=1}^{N} p_{ij} q_{ij}}{\sum_{i=1}^{N} P_i q_{ij}} \]

Rao and Selvanathan (1994) used a conditional regression model to derive standard errors for PPPs.


Here, we show that the G-K method is related to the CPD model and G-K PPPs are indeed Method of Moments estimators of PPPs from the CPD model!
We consider the CPD model:

\[ p_{ij} = P_i \cdot PPP_j \cdot u_{ij} \]

We show that this is a non-additive regression model with \( p_{ij} \) as the dependent variable, \( y \), the country and product dummy variables as regressors, \( X \), and \( P_i \) and \( PPP_j \) as the parameter vector, \( \beta \).

Then a non-linear model

\[ r(y_i, x_i, \beta) = u_i \]

is said to be non-additive if it cannot be written as:

\[ y_i - g(x_i, \beta) = u_i \]
The non-linear least squares estimator of the parameters of non-additive models are not consistent.

The method of moments estimator may be considered in this case. Consider a set of moment conditions:

\[ E(R(x, \beta)' u) = 0 \]

where \( R \) is a matrix representing \( K \) moment conditions.

The method of moments estimator is obtained by solving the sample moment conditions:

\[ \frac{1}{N} R(X, \hat{\beta})' r(y, X, \hat{\beta}) = 0 \]
The method of moments estimator is asymptotically normal with variance matrix

\[ \text{Var}(\hat{\beta}_{MM}) = \sigma^2 \left[ \hat{D}' \hat{R} \right]^{-1} \hat{R}' \hat{R} \left[ \hat{R}' \hat{D} \right]^{-1} \]

where

\[ \hat{D} = \left. \frac{\partial r(y, X, \beta)}{\partial \beta} \right|_{\hat{\beta}} \quad \hat{R} = R(X, \hat{\beta}) \quad \text{and} \quad \hat{\sigma}^2 = \frac{\hat{u}' \hat{u}}{N} \]

The most efficient choice of moment conditions is

\[ R(X, \beta)^* = E \left[ \left. \frac{\partial r(y, X, \beta)'}{\partial \beta} \right| X \right] \]

In our case, this choice leads to the unweighted index.
GK Index as a method of moments estimator

- The CPD model is
  \[ p_{ij} = P_iPPP_j u_{ij}^* \quad \text{with} \quad E(u_{ij}^*) = 1 \]

- We rewrite the CPD model as a non-additive model
  \[ \frac{p_{ij}}{P_iPPP_j} - 1 = u_{ij} \quad \text{with} \quad E(u_{ij}) = 0 \]

- We consider two sets of moment conditions: (i) optimal set; and (ii) weighted moment conditions
Optimal method of moments estimator

- Considering the CPD model:

\[ r_{ij} = \frac{p_{ij}}{P_{i}^{PPP}} - 1 = u_{ij} \]

and deriving the moment conditions leading to optimal estimator, we have

the moment conditions defined by the matrix \( R \) which is defined as:

\[
R = E \begin{bmatrix}
\frac{p_{11}}{P_{1}^{PPP}} & -\frac{p_{12}}{P_{2}^{PPP}} & \cdots & -\frac{p_{1m}}{P_{m}^{PPP}} \\
-\frac{p_{21}}{P_{2}^{PPP}} & \frac{p_{22}}{P_{2}^{PPP}} & \cdots & \cdots \\
\vdots & \vdots & \ddots & \vdots \\
-\frac{p_{m1}}{P_{m}^{PPP}} & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots \\
\end{bmatrix}
\]
Optimal method of moments estimator

- Noting that:

\[ E \left( \frac{p_{ij}}{P_{PPP_j}} \right) = 1 \]

and using the moment conditions

\[ E(\mathbf{R}(\mathbf{x}, \beta)' \mathbf{r}) = 0 \]

we have the following normal equations to solve.

\[
\begin{align*}
& -\frac{1}{P_i} \sum_{j=1}^{M} \left( \frac{p_{ij}}{P_i P_{PPP_j}} - 1 \right) = 0 \\
& -\frac{1}{PPP_j} \sum_{i=1}^{N} \left( \frac{p_{ij}}{P_i P_{PPP_j}} - 1 \right) = 0
\end{align*}
\]

\[
\Rightarrow \quad \begin{cases} 
\begin{align*}
P_i &= \frac{1}{m} \sum_{j=1}^{m} \left( \frac{p_{ij}}{PPP_j} \right) \\
PPP_j &= \frac{1}{n} \sum_{i=1}^{n} \left( \frac{p_{ij}}{P_i} \right)
\end{align*}
\end{cases}
\]

These are exactly the same equations that define the arithmetic index introduced earlier.

Estimated standard errors can be computed by substituting the MOM estimator in the formula for the asymptotic variance.
Geary-Khamis method as a method of moments estimator

- We consider weighted moment conditions where quantities are used as weights.

and deriving the moment conditions leading to optimal estimator, we have the moment conditions defined by the matrix $\mathbf{R}$ which is defined as:

$$
\mathbf{R} = \begin{bmatrix}
-\frac{q_{11}}{P_1} & -\frac{q_{12}}{P_1} & \cdots & -\frac{q_{1m}}{P_m} \\
-\frac{q_{n1}}{P_n} & -\frac{q_{n2}}{P_n} & \cdots & -\frac{q_{nm}}{P_m} \\
0 & \frac{q_{n1} P_1}{PPP} & \cdots & 0 \\
0 & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & 0 \\
0 & \frac{q_{nm} P_m}{PPP} & \cdots & 0 \\
(\frac{p_{11}}{P_1 PPP} - 1) & (\frac{p_{12}}{P_2 PPP} - 1) & \cdots & (\frac{p_{nm}}{P_m PPP} - 1)
\end{bmatrix}
$$

$$
r = \begin{bmatrix}
\frac{p_{11}}{P_1 PPP} - 1 \\
\frac{p_{12}}{P_2 PPP} - 1 \\
\vdots \\
\frac{p_{nm}}{P_m PPP} - 1
\end{bmatrix}
$$
Geary-Khamis as MOM estimator

- Using the moment conditions $R'r = 0$; we can write the implied normal equations as:

\[
\begin{align*}
\mathbb{P}_{PP_i} &= \frac{\sum_{i=1}^{n} p_{ij}q_{ij}}{\sum_{i=1}^{n} P_i q_{ij}} \\
\mathbb{P}_{P_i} &= \frac{\sum_{j=1}^{m} \left( p_{ij} q_{ij} / \mathbb{P}_{PP_j} \right)}{\sum_{j=1}^{m} q_{ij}}
\end{align*}
\]

- These are the equations that define the G-K method.

- The asymptotic standard errors can be derived using the $R$ matrix in the variance-covariance matrix formula.
## MOM estimators of PPPs and SE’s – OECD Example

<table>
<thead>
<tr>
<th></th>
<th>Arithmetic Index</th>
<th>GMM SE Arithmetic</th>
<th>MLE SE Arithmetic</th>
<th>G-K Index</th>
<th>GMM SE G-K</th>
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</thead>
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Conclusion

- A new system for international price comparison is proposed. Existence and uniqueness established.
- A stochastic framework for generating the Rao, Ikle and the new index has been established.
- Using the framework of M-estimators, standard errors are obtained for the PPPs from each of the methods.
- The Geary-Khamis method is shown to be a Method of moments estimator of PPPs in the CPD model. Standard errors for the GK estimator are obtained.
- Empirical application using the OECD data generates PPPs from different methods along with their standard errors.
- Further work is necessary to address the problem of choosing between different stochastic specifications.