Spurious Investment Prices

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Abstract

We introduce a vintage capital model in which workers are matched with machines of increasing quality. Quality improvements of the machines are the sole source of technological change in this economy. However, the matching of workers with machines implies that there is no well defined capital aggregate in this economy. Hence, investment price indices are a spurious measure of price changes in capital goods. We show that the use of such spurious measures of investment price changes can lead to misleading conclusions about (changes in) the trend properties of the economy.

keywords: imperfect competition, price indices, vintage capital.

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1 Introduction

Many recent empirical studies of technological change have used changes in the relative price of investment goods with respect to consumption goods as a measure of the degree of investment specific or embodied technological change. These studies include, among others, Greenwood, Hercowitz, and Krusell (1997,2000), Violante, Ohanian, Ríos-Rull, and Krusell (2000), Cummins and Violante (2002), Fisher (2002), Altig, Christiano, Eichenbaum, and Linde (2005), Ireland and Shuh (2006).

Other studies, like Jorgenson and Stiroh (2000) and Oliner and Sichel (2000), emphasize the importance of information technology (IT) production and capital for aggregate U.S. productivity growth.

What these studies have in common is that their main conclusions in large part hinge on the way investment prices are measured. In particular, they hinge on the assumption of the existence of an aggregate or IT capital stock, the price of which is properly reflected by the price index used.

A large literature has evolved around the question whether price indices properly reflect the quality improvements embodied in capital. An example of the recent contributions along this strand of the literature is Pakes (2003).

In this paper we examine this issue from another angle. Instead of considering whether price indices properly reflect capital price changes, we consider a case in which the assumed capital aggregate, the price of which is supposed to be approximated by the price index, does not even exist in the first place.

To illustrate our case, we introduce a vintage capital model, in the spirit of Johansen (1959), Arrow (1962), and Jovanovic (1999,2004). In it, workers of different skill levels are matched with machines of different and increasing quality. The quality improvements of machines are the sole source of economic growth in our model. Each worker can only use one machine, such that the capital labor ratio is fixed. The assignment of workers across machines means that capital vintages and labor are intertwined to such a degree that there is no aggregate production function representation in terms of labor and an aggregate capital stock.

The non-existence of an aggregate capital stock is nothing new. Fisher (1969) showed that in the case of embodied technological change such a capital stock only exists if the vintage-specific production functions are Cobb-Douglas. Because of the fixed capital labor ratio, in our model the vintage specific production functions are Leontief instead, just like in Solow, Tobin, von Weizsacker, and Yaari (1966). The problem is that the application of a capital price index in the absence of an aggregate capital stock can lead to deceiving conclusions about (the sources of) economic growth in our model.

The trend in our model is fully determined by the exogenous rate of embodied technological change. However, we show that, when one applies conventional investment price indices, the measured trend properties of the economy depend on the competitive and cost structure of the capital goods producing sector as well as on the skill distribution of workers that use the capital goods.

For example, we show that in our model an increase in competition in the capital goods producing sector that does not affect the rate of embodied technological change, leads to an increase in the rate of decline in the relative price of investment goods and increases in the growth rates of real investment, real GDP, and investment specific technological change. It would thus result in a shift in the measured trend properties of the economy, while the actual theoretical underlying trend does not change.

The discussion about the existence or non-existence of appropriate capital aggregates has a long history in macroeconomics. It was at the heart of what Harcourt (1969) termed the Cambridge Capital Controversy. This controversy was initiated by Robinson's (1959) criticism of the neoclassical assumption of the existence of an aggregate capital stock.

This paper points out a specific new issue where it might be relevant: The measurement of the growth rate of aggregate economic activity.

The assumption of the existence of proper capital aggregates underlies almost all of the existing measures of economic activity. However, we show how applying price index methods when such aggregates do not exist might lead to misleading results about economic growth and productivity.

We do not provide any direct measurement solutions for what to do when one suspects the identifying assumption of the existence of a proper capital aggregate is invalid. Therefore, we would like to emphasize that this paper merely provides a counterexample as a note of caution. That is, we do not claim that all studies that rely on investment price indices have yielded spurious results. Instead, we would just like to point out that there might be an alternative explanation for the 'stylized facts' that these studies claim to document.

The structure of this paper is as follows. In the next section we introduce our model economy. Because our argument does not hinge on transitional dynamics, we consider an economy that is always on its balanced growth path. In Section 3 we derive the equilibrium balanced growth path of the economy and prove its relevant properties. In Section 4 we consider what we would actually measure in terms of economic aggregates in our model economy. We do so in two stages. First, we derive an aggregate production function representation and show that there is no aggregate capital stock. Secondly, we show how the spurious application of a capital price index in this case can yield misleading statistics on the decline of the relative price of capital, and the growth rates of real investment, real GDP, total factor productivity, and investment specific technological change. Section 5 contains a numerical example that we use to illustrate the issues presented in Section 4. We discuss the empirical relevance of our theoretical results in Section 6. Finally, we conclude in Section 7. Mathematical details are left for Appendix A.

2 Model

The model that we introduce is a model of embodied technological change. In our model, a continuum of workers with heterogenous levels of human capital in each period choose a type of machine that they use to produce a homogenous final good. The machines are supplied by a set of firms that compete monopolistically. The final good is used as a consumption good as well as the input in the production of machines.

The main results of this paper are easiest explained along a balanced growth path. For this reason, we develop a model economy that is always on its balanced growth path. This allows us to make the simplifying assumptions of linear preferences and innovations of equal size at a constant frequency.

The following four subsections introduce the household sector, final goods sector, capital goods sector, as well as the type of exogenous embodied technological progress in our model economy respectively.

2.1 Households

A household in our economy consists of a single infinitely-lived worker. All households have linear preferences in the sense that a household, which, for reasons explained below, we index by h, that consumes $c_{t+s}(h)$ for $s = 0, 1, 2, \ldots$ gets the following level of utility

$$\sum_{s=0}^{\infty} \beta^s c_{t+s}(h) \quad \text{where } 0 < \beta < 1 \tag{1}$$

The household maximizes this objective subject to the intertemporal budget constraint that

$$a_{t+s+1}(h) = (1 + r_{t+s}) a_{t+s}(h) + w_{t+s}(h) + \pi_{t+s} - c_{t+s}(h) \tag{2}$$

Here $a_{t+s}(h)$ denotes the real assetholdings of the household at the beginning of period $t+s$, r_{t+s} is the real interest rate at time $t+s$, $w_{t+s}(h)$ is the real wage rate the household earns, π_{t+s} are the dividend payments that the household receives over the shares it owns in capital goods producing firms[1].

The intertemporal optimality condition for the households in this economy implies that for consumption to be positive in each period, the real interest rate has to satisfy

$$r_t = \frac{1-\beta}{\beta} \equiv r \text{ for all } t \tag{3}$$

which is what we assume throughout the rest of this paper.

[1] We assume that the shares in these firms are equally distributed among the households, because of which they all get equal dividend payments. However, as Caselli and Ventura (2000) show, the aggregate behavior of our economy will not depend on the distribution of shares.

2.2 Final goods producers

Firms produce a homogenous final (consumption) good by matching workers of different skill-levels with machines of different quality. The capital labor ratio is fixed such that each worker is only matched up with a single machine.

We take a certain degree of heterogeneity among workers as given. The relevant dimension of heterogeneity across workers is their human capital levels. We denote the human capital level of a particular worker by h. There is a continuum of workers of measure one whose human capital levels are uniformly distributed on the interval $(\underline{h}, \overline{h}]$, such that $h \backsim unif\left(\underline{h}, \overline{h}\right)$.

Just like workers, machines are also heterogenous in this economy. There is a countably finite number of types of machines supplied in each period. We denote a particular type, or vintage, of machine by τ[2]. Each vintage of machine embodies a different quality, where $A_{t-\tau} > 0$ denotes the number of efficiency units embodied in a machine of vintage age τ. Throughout, we will assume that there is no technological regress such that $A_t - A_{t-1} > 0$ for all t.

The combination of a worker of type h and a machine of vintage age τ yields $hA_{t-\tau}$ units of output[3].

In order to avoid having to consider intractable intertemporal optimization problems and having to make assumptions about possible second hand markets, we will assume that machines fully depreciate in one period. This assumption basically implies that the machines considered here are equivalent to intermediate goods in the sense of Aghion and Howitt (1992) and Romer (1990).

Firms cannot use these machines for nothing. The price of a machine of quality $A_{t-\tau}$ at time t is $P_{t,\tau}$. This price is measured in units of the final good, which we use as the numeraire

[2]The notational convention that we use in this paper follows Chari and Hopenhayn (1991) in the sense that τ represents 'vintage age'. That is, A_t represents the frontier technology level at time t and $A_{t-\tau}$ is the frontier technology level of τ periods ago.

[3]This setup of the production function is similar to the preference setup used by Bresnahan (1981) to estimate marginal cost profiles and markups in the American automobile industry.

good throughout.

Given this production technology, vintage profile of prices, and the menu of available vintages of machines, in each period a firm that employs a worker with skill level h chooses, from this menu, the type of machine that maximizes labor service flows. These labor service flows are the difference between the revenue generated by the sale of the final goods produced and the cost of the machine used to produce them.

That is, if $\mathbf{T}_t$ denotes the set of available technology vintage ages and $\mathbf{A}_t$ the set of associated productivity levels of the machines, then firms will assign a worker with human capital level h to a technology from the technology choice set $\Upsilon_t(h)$, which is defined as

$$\Upsilon_t(h) = \left\{ \tau \in \mathbf{T}_t \,\middle|\, \tau \in \arg\max_{s \in \mathbf{T}_t} \{hA_{t-s} - P_{t,s}\} \right\} \tag{4}$$

Let $w_t(h)$ be the wage rate of a worker with human capital level h, then competition and free entry on the demand side of the labor market implies zero profits such that the wage rate of a worker with skill level h equals revenue minus capital expenditures. Mathematically, this implies

$$w_t(h) = hA_{t-\tau} - P_{t,\tau}, \text{ for all } \tau \in \Upsilon_t(h) \tag{5}$$

When we aggregate over workers of all human capital levels, we obtain the relevant capital demand sets. Let $\mathbf{P}_t$ be the vector of prices charged for the available machines, then the set of buyers of machines of vintage age τ, which we denote by $D_t(\tau, \mathbf{P}_t, \mathbf{A}_t)$, is given by

$$D_t(\tau, \mathbf{P}_t, \mathbf{A}_t) = \left\{ h \in (\underline{h}, \overline{h}] \,\middle|\, \tau \in \arg\max_{s \in \mathbf{T}_t} (hA_{t-s} - P_{t,s}) \right\} \tag{6}$$

These sets determine the demand for each of the available vintages of machines.

2.3 Capital goods producers

Machine designs are assumed to be patented for M periods and each period there is one new machine design patented.

During the first M periods of a machine design's life, the particular machine is supplied by a monopolist firm. After the patent expires the machine design is public domain and there is perfect competition in the supply of these machines.

In order to show the generality of our results, we allow for one monopolist selling all M patented machines, M monopolistic competitors that each sell one particular vintage of machine, or any case in between.

Hence, each supplier may supply more than one patented machine design. We denote the number of suppliers of patented machines by $N \leq M$ and index them by n. The function $\iota_t(\tau)$ identifies the supplier of machines of vintage τ.

The technology used to produce machines is as follows. Units of the final (consumption) good are the only input needed in machine production. We make this assumption to avoid having to deal with the selection of workers across the final goods and capital good producing sectors. Production of a continuum of mass $K_{t,\tau}$ of machines of type $A_{t-\tau}$ requires the use of

$$\underline{h} A_{t-\tau} K_{t,\tau} + \frac{c}{2} A_{t-\tau} K_{t,\tau}^2 \tag{7}$$

units of the final good. The cost parameter $c > 0$ determines the degree to which the capital goods producers are subject to decreasing returns to scale.

The question that is left is how these machine producers end up choosing the prices of their machines. Suppose supplier n supplies a total of v_n vintages. Let $\tau_{n_1}, \tau_{n_2}, \tau_{n_3}, ...\tau_{n_{v_n}}$ be the vintages supplied by supplier n. Then the vector of prices chosen by supplier n can be denoted

$$\mathbf{P}_{t,n} = \left\{ P_{t,\tau_{n_1}}, \ldots, P_{t,\tau_{n_{v_n}}} \right\}. \tag{8}$$

Throughout this paper, we focus on Pure Strategy Nash (PSN) equilibria. For the particular problem at hand here this implies that supplier n takes the prices set by the other supplies, which we denote by the vector $\mathbf{P}'_{t,n}$, and the productivity levels of the machines, i.e. the $A_{t-\tau}$ for $\tau \in \mathbf{T}_t$, as given.

Given these variables, producer n chooses the prices of his machines to maximize profits. This implies that $\mathbf{P}_{t,n}$ is an element of the best response set

$$BR_t\left(\tau; \mathbf{P}'_{t,n}, \mathbf{A}_t\right) = \left\{ \mathbf{P}_{t,n} \in \mathbb{R}_+^{v_n} \right. \tag{9}$$

$$\left| \mathbf{P}_{t,n} \in \underset{\mathbf{P} \in \mathbb{R}_+^{v_n}}{\arg\max} \left\{ \sum_{i=1}^{v_n} \left(\mathbf{P}_i K_{t,\tau_{n_i}} - \underline{h} A_{t-\tau_{n_i}} K_{t,\tau_{n_i}} - \frac{c}{2} A_{t-\tau_{n_i}} K_{t,\tau_{n_i}}^2 \right) \right\} \right\} \tag{10}$$

Where $K_{t,\tau_{n_i}}$ equals the mass of workers that demand machines of vintage age τ_{n_i} at the prices set[4].

Because patents expire after M periods, these best response sets only apply to $\tau = 0, \ldots, M-1$. For machines that were designed M or more periods ago, perfect competition implies that price must equal average cost and that free entry drives both to $\underline{h}A_{t-\tau}$. Hence, $P_{t,\tau} = \underline{h}A_{t-\tau}$ for $\tau \geq M$.

The corresponding profits are

$$\pi_{t,n} = \sum_{i=1}^{v_n} \left(P_i K_{t,\tau_{n_i}} - \underline{h}A_{t-\tau_{n_i}} K_{t,\tau_{n_i}} - \frac{c}{2} A_{t-\tau_{n_i}} K^2_{t,\tau_{n_i}} \right) \text{ for all } \mathbf{P}_{t,n} \in BR_t \left(\tau; \mathbf{P}'_{t,n}, \mathbf{A}_t \right) \tag{11}$$

for $\tau = 0, \ldots, M-1$ and are zero for $\tau \geq M$.

2.4 Technological progress

What generates the improvements in the quality of machines does not matter for the argument in this paper. Therefore, we simply assume that the quality of machines grows exogenously. In particular, we assume that the quality of machines grows at a constant rate g, such that $A_{t+1} = (1+g) A_t$, where $g > 0$. Thus, g is the exogenous rate of embodied productivity growth in our economy. We also assume that the patent for this innovation is randomly awarded to one of the households in the economy[5].

Throughout, we consider two cases. The first is the one in which the household hangs on to this patent and becomes the owner of one of the monopolistically competing machine suppliers. Because there is a continuum of households, the probability that one household holds the patent for two of the M patented machines is a zero probability event. Hence, this is the case in which there is monopolistic competition between M different suppliers. We refer to this as the 'monopolistic competition' case.

The second is the one in which the market for machines is dominated by a monopoly firm that buys out the patentholders of new machine designs in each period. In this market the

[4] Formally, $K_{t,\tau_{n_i}}$ is the Lebesque measure of the demand set $D\left(\tau_{n_i}, \left(\mathbf{P}'_{t,n}, \mathbf{P}_{t,n}\right), \mathbf{A}_t\right)$.

[5] In principle, one could include an R&D sector in the model that is financed using the profits made by the monopolistic competitors. This would significantly complicate the equilibrium conditions in this economy and would not change the main results that are the focus of this paper.

same firm will hold a monopoly in the supply of all machines along the equilibrium path. We refer to this as the 'monopoly' case.

3 Equilibrium

In this section we derive the equilibrium outcome and prove the relevant properties of this economy along its balanced growth path. These are the properties that drive our main results on measurement issues that are presented in the next section.

We derive the equilibrium in three steps. First of all, we solve for the machine demand decisions made by the workers in the final goods sector. Secondly, we obtain the optimal price setting strategies by the suppliers of the different vintages of machines in response to the demand functions derived in the first step. Finally, we combine the results of the first two steps to derive the balanced growth path of our model economy. We only describe the main results and their intuition here. The details of the derivations are left for Appendix A.

3.1 Demand for machines

Because our setup in the final goods sector is similar to that of the car market in Bresnahan (1981), so are the resulting demand functions. They satisfy the following two main properties, independent of the set of technologies sold, i.e. $\mathbf{A}_t$, and the prices set for the patented designs, i.e. $\mathbf{P}_t$.

First of all, better workers end up using better machines. That is, there is endogenous assortative matching between workers and machines. Mathematically, this can be written as

$$\text{For } h' > h, \text{ if } h \in D_t(\tau, \mathbf{P}_t, \mathbf{A}_t) \text{ then } h' \notin D_t(\tau', \mathbf{P}_t, \mathbf{A}_t) \text{ for all } \tau' > \tau. \tag{12}$$

Assortative matching between machines and workers is a natural outcome when a technology exhibits capital-skill complementaries, like in the final goods sector in our model. Jovanovic (1999,2004) are examples where this is the case as well.

This assortative matching result also implies that the demand sets are connected. That is, for vintages of machines for which there is positive demand, they are of the form

$$D_t(\tau, \mathbf{P}_t, \mathbf{A}_t) = \left(\underline{h}_{t,\tau}, \overline{h}_{t,\tau}\right] \text{ where } \underline{h} < \underline{h}_{t,\tau} < \overline{h}_{t,\tau} \leq \overline{h} \tag{13}$$

where the upper and lowerbounds of the set are determined by the prices and the productivity levels of the vintages sold. It also follows from this assortative matching result that the set of workers that is indifferent between the choice of two machines is negligible. That is, the size of these demand sets, and thus the demand for each of the different vintages, is uniquely determined by the prices that are set and the productivity levels of the machines.

Secondly, perfect competition for the production of machines of vintage age M and older implies that machines of a design older than M, i.e. a design for which the patent has expired for more than one period, are not demanded anymore. Their demand set is the empty set in equilibrium. That is,

$$D_t\left(\tau, \mathbf{P}_t, \mathbf{A}_t\right) = \emptyset \text{ for } \tau > M \tag{14}$$

The derivation of this result is straightforward. The quadratic production technology for machines implies that perfect competition on the machines for which the patent has expired will drive their prices to $\underline{h}A_{t-\tau}$. Considering only the machines in the public domain, the final goods producing firm's problem becomes

$$\max_{\tau \geq M} \left\{hA_{t-\tau} - \underline{h}A_{t-\tau}\right\} \tag{15}$$

for some $h > \underline{h}$, which is solved by choosing the largest value of $A_{t-\tau}$.

3.2 Price schedule of machines

The properties of the demand sets proven above imply that the amount of machines of vintage age τ equals

$$K_{t,\tau} = \left(\overline{h}_{t,\tau} - \underline{h}_{t,\tau}\right) / \left(\overline{h} - \underline{h}\right) \tag{16}$$

This result can be used to derive the equilibrium price schedule of machines. Before doing so, we first formally define what we mean by the PSN price setting equilibrium in this market.

For a given set of available technologies, $\mathbf{A}_t$, a PSN equilibrium price schedule

$$\mathbf{P}_t^* = \left\{\mathbf{P}_{t,1}^*, \ldots, \mathbf{P}_{t,N}^*\right\} \tag{17}$$

in this market satisfies two properties. First of all, for those vintages for which the patent has expired the price is driven to the minimum average cost level. That is, $P_{t,\tau} = \underline{h}A_{t-\tau}$ for all $\tau \geq M$. Secondly, each of the suppliers of one or more patented vintages of machines chooses its prices as part of its best response set with respect to the prices set by the other producers. That is, let $\mathbf{P}^{*}_{t,n}$ be the prices set by supplier n for the machines it supplies and let $\mathbf{P}^{*'}_{t,n}$ be the prices set by the other producers in the PSN equilibrium, then

$$\mathbf{P}^{*}_{t,n} \in BR_t\left(\tau; \mathbf{P}^{*'}_{t,n}, \mathbf{A}_t\right) \text{ for all } n = 1, \ldots, N \tag{18}$$

It turns out that, for all possible technology menus $\mathbf{A}_t$ and all possible permutations of suppliers over the M patented vintages, there exists a unique equilibrium price schedule. The equilibrium price schedule has several relevant properties that are independent of $\mathbf{A}_t$, of the way suppliers are distributed over the M newest machine designs, and of the cost parameter c. The existence and uniqueness of the price schedule as well as the details underlying the properties are derived in Appendix A. Here we limit ourselves to the description of the properties that are relevant for the rest of our analysis.

The first property is that, in equilibrium, prices are set such that there is strictly positive demand for all M patented vintages. Mathematically, this boils down to

$$D_t\left(\tau, \mathbf{P}^{*}_{t}, \mathbf{A}_t\right) \neq \emptyset \text{ for } \tau = 0, \ldots, M \tag{19}$$

in the PSN price setting equilibrium.

The second property is that, in this equilibrium, suppliers make strictly positive profits of the supply of each of the individual patented designs. That is,

$$P_{t,\tau} > \underline{h}A_{t-\tau} + cA_{t-\tau}K_{t,\tau} > 0 \text{ for all } \tau = 0, \ldots, M-1 \tag{20}$$

such that for each patented vintage, all of which are produced with a decreasing returns to scale technology, price exceeds average cost and thus profits are strictly positive.

The final two properties are most easily written in terms of prices per efficiency units. For this purpose, we define the price per efficiency unit of a machine of vintage age τ as $\widehat{P}_{t,\tau} \equiv P_{t,\tau}/A_{t-\tau}$.

In terms of the price schedule per efficiency unit, the third relevant property for what is to come is that prices per efficiency unit are increasing in the quality of the machines. Formally,

$$\widehat{P}_{t,\tau} \text{ is strictly decreasing in } \tau \tag{21}$$

That is, the older the vintage age of the machine, the lower the quality, and the lower the price per efficiency unit.

The final property of the price per efficiency unit schedule is that it only depends on the cost parameter, c, the patent length, M, and the productivity profile of the vintages, $\mathbf{A}_t = \{A_t, \ldots, A_{t-M}\}$. Moreover the price per efficiency unit schedule is homogenous of degree zero in the productivity levels of the vintages.

Formally, let $\widehat{\mathbf{P}}_t^*$ be the equilibrium schedule of prices per efficiency unit, then this last property implies

$$\widehat{\mathbf{P}}_t^* = \widehat{\mathbf{P}}\left(\mathbf{A}_t, c\right) \tag{22}$$

such that $\widehat{\mathbf{P}}_t^*$ is solely a function of the cost parameter, i.e. c, and the productivity profile, i.e. $\mathbf{A}_t$. Furthermore, the function $\widehat{\mathbf{P}}$ is homogenous of degree zero in $\mathbf{A}_t$. So are the demand sets and the equilibrium demand levels, $K_{t,\tau}$.

3.3 Balanced growth path

What is important for the behavior of this economy on its balanced growth path is the last property. That is, that the equilibrium price per efficiency unit profile, $\widehat{\mathbf{P}}_t^*$, as well as the demand levels, $K_{t,\tau}$, are homogenous of degree zero in $\mathbf{A}_t$. Combined with our assumption that the productivity profile grows at a constant rate over time, this implies that the balanced growth path has several important properties. We will describe them here. They are proven in Appendix A.

First of all, the vintage age distributions of machines and investment and the prices per efficiency units for particular vintage ages are constant over time. That is

$$K_{t,\tau} = \overline{K}_\tau \text{ and } \widehat{P}_{t,\tau} = \overline{\widehat{P}}_\tau \tag{23}$$

Here, $-$ denotes equilibrium values along the balanced growth path and $\hat{}$ denotes detrended equilibrium values in terms of efficiency units.

Secondly, equilibrium consumption is defined as final goods output minus intermediate goods demand. That is,

$$C_t = Y_t - X_t \tag{24}$$

Equilibrium investment, expressed in terms of the (numeraire) final good, is defined as

$$I_t = \sum_{\tau=0}^{M} P_{t,\tau} K_{t,\tau} \tag{25}$$

Gross output of the final goods sector equals

$$Y_t = \int_{\underline{h}}^{\overline{h}} A_{t-\Upsilon_t(h)} h dh \tag{26}$$

While the intermediate inputs demand for final goods is

$$X_t = \sum_{\tau=0}^{M-1} A_{t-\tau} K_{t,\tau} \left(\underline{h} + \frac{c}{2} K_{t,\tau} \right) \tag{27}$$

and reflects the amount of final goods output needed to produce the capital goods.

Along the balanced growth path, consumption, investment expressed in terms of finals goods, gross output of the final goods sector, and intermediate input demand of the capital goods sector all grow at the constant rate g. Mathematically, this means that

$$C_t = A_t \overline{\widehat{C}}\ ,\ I_t = A_t \overline{\widehat{I}},\ Y_t = A_t \overline{\widehat{Y}},\ X_t = A_t \overline{\widehat{X}} \tag{28}$$

In terms of the theoretical equivalents of things that are actually measured in the national accounts, the balanced growth path implies the following. Gross Domestic Product (GDP), expressed in units of the consumption (final) good grows at the constant rate, $g > 0$, over time. GDP here equals the sum of the value added of the final goods sector, which is Y_t, and that of the capital goods sector, which equals $I_t - X_t$. Hence, GDP follows

$$GDP_t = Y_t + (I_t - X_t) = A_t \left(\overline{\widehat{Y}} + \overline{\widehat{I}} - \overline{\widehat{X}} \right) \tag{29}$$

Note that GDP does not take into account capital depreciation. GDP corrected for capital depreciation is known as Net Domestic Product (NDP). Since capital fully depreciates in every period here, NDP in this economy equals net value added for the final goods sector plus net value added for the capital goods sector. That is,

$$NDP_t = (Y_t - I_t) + (I_t - X_t) = GDP_t - I_t = A_t\left(\overline{\widehat{Y}} - \overline{\widehat{X}}\right) = A_t\overline{\widehat{C}} \tag{30}$$

This is again denoted in terms of the consumption good, which we use as the numeraire good.

Finally, for growth accounting purposes, it is useful to consider factor shares, most notably labor shares. The labor shares in both the overall economy as well as in the final goods producing sectors are constant along the balanced growth path. In particular, the aggregate labor share equals

$$s_L = \frac{\text{wages}}{GDP_t} = \frac{Y_t - I_t}{GDP_t} = \frac{\left(\overline{\widehat{Y}} - \overline{\widehat{I}}\right)}{\left(\overline{\widehat{Y}} + \overline{\widehat{I}} - \overline{\widehat{X}}\right)} = 1 - \frac{2\overline{\widehat{I}} - \overline{\widehat{X}}}{\left(\overline{\widehat{Y}} + \overline{\widehat{I}} - \overline{\widehat{X}}\right)} \tag{31}$$

while the share of labor in the final goods sector equals

$$s_L^f = \frac{\text{wages}}{\text{gross value added in final goods sector}} = \frac{\left(\overline{\widehat{Y}} - \overline{\widehat{I}}\right)}{\overline{\widehat{Y}}} = 1 - \frac{\overline{\widehat{I}}}{\overline{\widehat{Y}}} \tag{32}$$

Together, the equilibrium properties along the balanced growth path described above imply that the trend properties of this economy are fully defined by the exogenous growth rate $g > 0$. All other parameters only influence the detrended equilibrium levels, $\overline{\widehat{Y}}$, $\overline{\widehat{C}}$, $\overline{\widehat{I}}$, and $\overline{\widehat{X}}$, as well as the equilibrium factor shares.

4 Measurement

The main point of this paper is that standard measures of the trend properties of our model economy will paint a misleading picture of the actual economic developments. In this section we will show that this is the case because the measured trend properties of this economy turn out to depend on much more than only g. This reveals a potential source for persistent measurement error in the growth rates of several important economic aggregates.

Before we consider the measurement of these aggregate variables, we first consider whether they acually exist. This turns out not to be the case. The crux for our results is that an aggegregate capital stock does not exist in our model economy. In the first part of this section we derive this non-existence result and discuss how it is closely related to the Cambridge Capital Controversy in macroeconomics.

In the subsequent part of this section we consider the measured growth rates of several commonly studied economic aggregates. In particular, we focus on the growth rates of the relative price of capital goods relative to consumption goods, of real investment and the capital stock, of real GDP, and those of both TFP and investment-specific technological change. We deal with them in the order mentioned.

4.1 The absence of an aggregate capital stock

At the heart of the potential measurement errors in the growth rates of economic aggregates in this economy is the fact that, in this economy, there is no theoretically well-defined aggregate capital stock. Because the final goods sector is the only sector that uses capital goods in our economy, we will focus on the non-existence of an appropriate capital aggregate for that sector.

The argument that a proper capital aggregate only exists under very restrictive assumptions goes back to Robinson (1959). Her article was at the heart of the Cambridge Capital Controversy that was a prominent topic in macroeconomics in the 1960's and 1970's[6]. After Robinson's criticism of the neoclassical production function, a large literature evolved in which the conditions under which an aggregate capital stock exists are derived.

Fisher (1969) showed that this was only the case when the vintage specific production functions are Cobb-Douglas with equal capital elasticicities. The final goods sector in our model does not satisfy this assumption, i.e. the vintage specific production functions are Leontief, as in Solow et. al. (1966). Just like in Solow et. al. (1966), this means that no aggregate capital stock exists for this sector. In order to show this, we derive an aggregate

[6] Harcourt (1969, 1976), and Cohen and Harcourt (2003) are three surveys of the capital controversy.

production function for the final goods sector in the way Fisher (1969) proposed. The basic issue is the following.

The common assumption in most statistical measurements of aggregate economic activity as well as empirical applications of neoclassical macroeconomic models is that value added is generated using capital and labor and that the production function can be represented as

$$Y_t = Z_t F\left(K_t, L_t\right) \tag{33}$$

Here Z_t is factor neutral technological progress, also known as Total Factor Productivity, L_t is an appropriately defined aggregate of labor inputs, and K_t is an appropriately defined capital aggregate that is a composite of all the underlying different capital inputs[7]. The composite K_t is assumed to be homogenous of degree one in the underlying capital stocks. These capital stocks would, in our case, be the stocks of different machines used in production. Hence, in our model this capital stock would be a composite of all machine vintages, such that

$$K_t = J\left(\{K_{t,\tau}\}_{\tau=-\infty}^{\infty}\right) \tag{34}$$

What we will show in the following is that such a representation of the production function in the final goods sector does not exist in our model. Instead, the best we can do is write the production function in that sector as

$$Y_t = Z_t F\left(\{K_{t,\tau}\}_{\tau=-\infty}^{\infty}, L_t\right) \tag{35}$$

Because all increases in output in this sector are due to a shift in the distribution of machines used in production towards better vintages, factor neutral technological progress is zero. That is TFP, Z_t, is constant over time.

For the derivation of the aggregate production function, (35), for the final goods sector, we follow Fisher (1969). We consider the decision of a planner that is endowed with a continuum of workers of measure L_t that is uniformly distributed over the interval $(\underline{h}, \overline{h}]$ as well as with a sequence of capital stocks of different vintages $\{K_{t,\tau}\}_{\tau=0}^{M}$. Given these endowments of

[7] We make this argument here for the maximum level of aggregation. Our argument equally applies to the problem of the existence of capital stocks of subaggregates, like computers for example.

production factors, the planner chooses an allocation of labor over the capital stocks to maximize output.

Let $K_\tau(h) \geq 0$ be the amount of capital of vintage age τ that is assigned to workers of type h and,equivalently, let $L_h(\tau) \geq 0$ be the amount of workers of human capital level h that is assigned to machines of vintage age τ.

The planner chooses these allocations to maximize output, which is given by the production function

$$Y_t = \sum_{\tau=0}^{M} A_{t-\tau} \int_{\underline{h}}^{\overline{h}} h \min\{K_\tau(h), L_h(h)\} dh \tag{36}$$

The allocations are chosen subject to the resource constraints that the capital assigned does not exceed the capital available

$$\int_{\underline{h}}^{\overline{h}} K_\tau(h) dh \leq K_{t,\tau} \tag{37}$$

and that the amount of labor assigned does not exceed the amount of labor available

$$\sum_{\tau=0}^{M} L_h(\tau) \leq \frac{L_t}{[\overline{h} - \underline{h}]} \text{ for all } h \in (\underline{h}, \overline{h}) \tag{38}$$

The solution to this optimization problem coincides with the decentralized equilibrium outcome in our model economy. It entails the assortative matching between workers and machines.

Denote the human capital level of the least skilled worker that is still assigned a machine as

$$\underline{h}^* = \overline{h} - (\overline{h} - \underline{h}) \min\left\{\frac{1}{L_t} \sum_{\tau=0}^{M} K_{t,\tau}, 1\right\} \tag{39}$$

and let the oldest vintage of machines assigned to workers be

$$\tau^* = \max_{\tau=0,\ldots,M} \left\{\sum_{s=0}^{\tau-1} K_{t,s} < L_t\right\} \tag{40}$$

These definitions allow us to write the optimal assignment as follows.

$$K_\tau(h) = L_h(\tau) = \begin{cases} L_t & \text{for } \tau \leq \tau^* \text{ and } h \in (h^*_{\tau-1}, h^*_\tau] \\ 0 & \text{otherwise} \end{cases} \tag{41}$$

Where the boundaries of the matching sets are given by

$$h_\tau^* = \begin{cases} \overline{h} & \text{for } \tau = 0 \\ \max\left\{\underline{h}, \overline{h} - \frac{(\overline{h}-\underline{h})}{L} \sum_{s=0}^{\tau-1} K_{t,s}\right\} & \text{otherwise} \end{cases} \tag{42}$$

The level of output that results from this assignment equals

$$\begin{aligned} Y\left(L_t, \{K_{t,\tau}\}_{\tau=0}^M\right) &= \frac{L_t}{\overline{h}-\underline{h}} \sum_{\tau=1}^{\tau^*} \int_{h_{\tau-1}^*}^{h_\tau^*} A_{t-\tau} h \\ &= \frac{1}{2} \frac{L_t}{\overline{h}-\underline{h}} \left[A_t \overline{h}^2 - \sum_{\tau=1}^{\tau^*} \left(A_{t-\tau-1} - A_{t-\tau}\right) h_\tau^{*2} - A_{t-\tau^*} \underline{h}^{*2}\right] \end{aligned} \tag{43}$$

This production function exhibits constant returns to scale. In fact, it is a perfectly Neoclassical production function in labor, L_t, and the heterogenous capital inputs $\{K_{t,\tau}\}_{\tau=0}^M$.[8]

However, because of the assignment of capital over workers, capital and labor are not separable in this production function. On the contrary, the amounts of capital and labor interact in a complex manner through the assignment of machines to workers, which determines the h_τ^*'s.

Hence, there is no aggregate production function representation for the final goods sector in terms of a single capital aggregate $J\left(\{K_{t,\tau}\}_{\tau=-\infty}^\infty\right)$ that is homogenous of degree one in the capital inputs $\{K_{t,\tau}\}_{\tau=-\infty}^\infty$ and the aggregate labor input L_t. Therefore, the concept of a capital price index is ill-defined in this model; a capital price does not exist, because there is no properly defined theoretical aggregate capital stock.

4.2 Measured relative price of investment

The theoretical non-existence of an aggregate capital stock does not mean that one cannot apply price index methods to obtain a spurious estimate of it.

Such an estimate would be spurious in the same sense as the regressions in Granger and Newbold (1974). That is, the spurious regressions in Granger and Newbold (1974) involve

[8] This is in line with the main point of Solow et. al. (1966) who use a similar theoretical setup to argue that the non-existence of an aggregate capital stock does not invalidate the application of neoclassical macroeconomic principles.

the estimation of a non-existent stationary linear relationship between two random walks. Here, the spurious capital measure involves the estimation of a non-existent aggregate capital stock.

There are, in principle, many different ways to construct such a price index $P_{K,t}$, each of which essentially employs a different price index formula. Furthermore, since in every period some machines exit the market while others enter, one also has to decide on how to deal with the inclusion of new goods.

The aim of this paper is not to be an exposition on price index methods. Instead, it is meant to illustrate a conceptual problem with the application of them in the model economy introduced. Therefore, we limit our analysis to one of the most common price index formulas. Furthermore, we consider only two ways of dealing with the inclusion of new goods. The qualitative results derived from the resulting price indices also hold for the application of other common price index methods. That is, we emphasize the conceptual issues with constructing a capital price index in this model and these issues are robust to what type of capital price index is constructed.

The price index formula we use is the Laspeyres formula. It is a useful benchmark, because as Frisch (1936) and Konüs (1939) showed it yields an upperbound on inflation in the standard case in which there are no new capital goods and there exists a proper capital aggregate.

The first way we deal with new goods is to ignore them and simply apply the price index formulas to models of machines that are sold in the two periods between which we calculate capital price inflation. This yields the matched model indices used in, for example, Aizcorbe et al. (2000) and that are commonly applied to capital price indices by the Bureau of Labor Statistics.

The Laspeyres matched model index that aims to measure capital price inflation between $t-1$ and t in our model would yield

$$\pi_t^{(M)} = \frac{\sum_{\tau=1}^{M} P_{t,\tau} K_{t-1,\tau-1}}{\sum_{\tau=0}^{M-1} P_{t-1,\tau} K_{t-1,\tau}} - 1 \tag{44}$$

It measures the percentage change in the cost from $t-1$ to t of buying the period $t-1$

machines that are available in period t.

For this matched model Laspeyres index we find that, on the balanced growth path of our economy, it yields a constant percentage decline in the relative price of capital goods relative to consumption goods. That is,

$$\pi_t^{(M)} = \pi^{(M)} < 0 \quad \text{for all } t \tag{45}$$

The magnitude of the measured price declines depends on cross-vintage profile of the price declines[9]

$$\frac{P_{t,\tau} - P_{t-1,\tau-1}}{P_{t-1,\tau-1}} = \frac{\widehat{P}_{t,\tau} - \widehat{P}_{t-1,\tau-1}}{\widehat{P}_{t-1,\tau-1}} \tag{46}$$

which in its turn depends on the length of the patent M, the cost parameter c and the growth rate g.

The second way we deal with new goods is to include them by using a hedonic regression model to impute the price of the models that enter and exit for the periods that their prices are not observed. This would result in a hedonic price index.

The Laspeyres hedonic price index that aims to measure capital price inflation between $t-1$ and t in our model would yield

$$\pi_t^{(H)} = \frac{\sum_{\tau=1}^{M} P_{t,\tau} K_{t-1,\tau-1} + P'_{t,M+1} K_{t-1,M}}{\sum_{\tau=0}^{M} P_{t-1,\tau} K_{t-1,\tau}} - 1 \tag{47}$$

where $P'_{t,M+1}$ is the imputed price of the machines of vintage age $M+1$ at time t that is imputed using a hedonic regression. In general $P'_{t,M+1}$depends on the specific hedonic regression applied. However, for simplicity we will assume that the price of the obsolete vintage is imputed as $P'_{t,M+1} = P_{t-1,M} = A_{t-M-1}\underline{h}$, then

$$\pi_t^{(H)} = (1 - s_{t-1,M})\, \pi_t^{(M)} \tag{48}$$

where $s_{t-1,M}$ is the share of the vintage of age M at time $t-1$ and $\pi_t^{(M)}$ is the inflation rate measured using the Laspeyres matched model index defined above. Because $s_{t-1,M} > 0$ and $\pi_t^{(M)} < 0$ are both constant over time on the balanced growth path, we obtain that

$$\pi_t^{(H)} = \pi^{(H)} < 0 \quad \text{for all } t \tag{49}$$

[9] Aizcorbe and Kortum (2004) call the price path of a specific vintage over its lifecycle a ‘price contour’.

Thus, just like the matched model index, the hedonic Laspeyres capital price index would yield a constant rate of decline in the relative price of capital compared to the consumption good along the balanced growth path. Note, however, that in this case the hedonic price index measures smaller price declines than the matched model one.

Hence, both price indices that we consider here would find a constant rate of decline in the relative price of investment goods, consistent with the observation that drives the results in Greenwood, Hercowitz, and Krusell (1997) for example.

This is because older vintages of machines are assigned to workers with lower skill levels, their prices decline over their product life-cycle. These price declines are aggregated into a decline in the aggregate capital price index. This decline in the aggregate price index consequently reflects four things in this model.

First of all, it reflects the matching of machines with workers and the price declines thus depend on shape of the skill distribution, which we assumed to be uniform here. Secondly, it captures the decreasing returns to scale, c, to which the monopolistic competitors are subject. Thirdly, it depends on the competitive structure on the supply side of machines. That is, which suppliers supply which vintages and how many vintages are sold in the market. Finally, it depends on the growth rate of embodied technological change g.

4.3 Real investment, capital stock, and output

Since there is full depreciation of machines in every period, the capital stock is equal to gross investment in each period. That is, both the capital aggregate K_t and real investment I_t in our model economy would be measured as the capital expenditures in period t deflated by a capital price index. Since firms in the final goods sector make zero profits in equilibrium, capital expenditures equal total revenue minus the wage bill. That is, capital expenditures equal $(1 - s_L) Y_t$. Consequently, the capital aggregate and real investment are constructed as

$$K_t = I_t = (1 - s_L) \frac{Y_t}{P_{K,t}} \tag{50}$$

where $P_{K,t}$ is the capital price index.

This implies that the growth rates of real investment and the capital stock equal

$$g - \pi \tag{51}$$

Where π is the percentage change in the capital price index derived in the subsection above.

Real GDP in this economy would equal the value added of the final goods sector deflated by the price of the final good, which we normalized to 1 here because it is the numeraire good, plus the value added of the investment goods sector deflated by the investment price index, $P_{K,t}$.

That is real GDP equals

$$\text{real}\, GDP_t = Y_t + \frac{(I_t - X_t)}{P_{K,t}} \tag{52}$$

On the balanced growth path, this implies that the measured growth rate of real GDP equals

$$\frac{\text{real}\, GDP_t - \text{real}\, GDP_{t-1}}{\text{real}\, GDP_{t-1}} = g - \frac{\left(\overline{\widehat{I}} - \overline{\widehat{X}}\right)}{\overline{\widehat{Y}} + \left(\overline{\widehat{I}} - \overline{\widehat{X}}\right)}\pi \tag{53}$$

Hence, beyond the trend in technological progress, g, measured economic growth in this economy depends on the other three factors that affect the spurious investment price inflation rate π. That is, if any of these factors change, measured economic growth will change, even though there is no shift in the rate of technological change in the economy.

4.4 Productivity

We already showed that there is no growth in total factor productivity in the final goods sector of our model economy. This follows from the construction of the aggregate production function above. That is, if the final goods sector uses the same amounts of labor and the same number of machines for each particular vintage at two different points in time, then it would produce the same amount of output at both points in time. There is no technological progress in this model that shifts the productivity of all factors of production in the same way, where each vintage of machine is considered a separate production factor because there is no aggregate capital stock, and thus TFP growth is zero.

All productivity growth in this model is embodied in the new machines that become available over time. Without the adoption of the new machines productivity levels in the final goods sector would not be increasing over time.

Hence, what we would like to get out of an accounting exercise that distinguishes between total factor productivity and embodied technological change is that TFP growth is zero in the final goods sector and that all growth is due to the quality improvements of machines.

Would our current methods of measuring investment specific technological change (and of growth accounting) yield this result in the model economy here? What would happen if we would apply growth accounting techniques in our model economy to assess the contributions of total factor productivity growth and of capital deepening?

Using growth accounting for the final goods sector involves dividing the growth of output in this sector into its three contributing factors. The first is the growth of the labor input. The second is capital deepening, i.e. the growth of capital inputs as measured by the "quality adjusted" real capital stock we discussed in the previous subsection. The final part is TFP growth, i.e. the Solow residual, it is simply the part of output growth that is not attributed to growth of the capital and labor inputs.

In practice, this boils down to applying a log-linear approximation of the neoclassical production function and assuming that marginal products equal factor prices, (33), to obtain the decomposition

$$\left(\frac{Y_t - Y_{t-1}}{Y_{t-1}}\right) \approx \left(\frac{Z_t - Z_{t-1}}{Z_{t-1}}\right) + s^f_{L,t}\left(\frac{L_t - L_{t-1}}{L_{t-1}}\right) + \left(1 - s^f_{L,t}\right)\left(\frac{K_t - K_{t-1}}{K_{t-1}}\right) \tag{54}$$

where Z_t represents the measured level of TFP, $s^f_{L,t}$ is the share of labor in the final goods sector, and K_t is the measured capital aggregate.

As derived above, on the balanced growth path, output of the final goods sector grows at a constant rate g, the labor share in the final goods sector is constant, i.e. $s_{L,t} = s^f_L$, and the labor inputs are constant and equal one, i.e. $L_t = 1$ for all t. This implies that, along the balanced growth path in our model economy, this decomposition simplifies to

$$g = \left(\frac{Z_t - Z_{t-1}}{Z_{t-1}}\right) + \left(1 - s^f_L\right)\left(\frac{K_t - K_{t-1}}{K_{t-1}}\right) \tag{55}$$

Thus, on the balanced growth path our growth accounting exercise will attribute output growth either to TFP growth, i.e. to the growth of Z_t, or to capital deepening, i.e. the growth of K_t. The growth rate of TFP is the residual, after the subtraction of the capital deepening contribution from g.

Substitution of (50) into the above equation yields that TFP will be measured as a weighted average of output growth and the capital price declines. That is,

$$\left(\frac{Z_t - Z_{t-1}}{Z_{t-1}}\right) = s_L^f g + \left(1 - s_L^f\right)\left(\frac{P_{K,t} - P_{K,t-1}}{P_{K,t-1}}\right) \tag{56}$$

Hence, what is crucial for the growth accounting results in our model is the capital price index $P_{K,t}$ used for it.

Since we already argued that all growth in the final goods sector of this economy is due to quality increases in capital and that there is no TFP growth, i.e.

$$\left(\frac{Z_t - Z_{t-1}}{Z_{t-1}}\right) = 0 \tag{57}$$

in the sector, we would like our capital price index to satisfy

$$\pi = \left(\frac{P_{K,t} - P_{K,t-1}}{P_{K,t-1}}\right) = -\frac{s_L^f g}{\left(1 - s_L^f\right)} \tag{58}$$

However, there is nothing in our model that assures us that this is the actual percentage change in the relative price of capital, $P_{K,t}$, measured using common price index methods.

5 Numerical example

In order to illustrate the implications of our theoretical results, we provide a numerical example. Our approach is to calibrate a benchmark set of parameter values. We then proceed by changing the competitive structure in the capital goods market, by changing the number of models sold, M, the cost parameter c, the distribution of suppliers over machine vintages, and show how these changes affect the measured growth rates of economic aggregates. We also show how a shift in the skill distribution of workers affects the measured growth rates of aggregates in our model economy. Finally, we show that an increase in the growth rate of embodied technological change g, leads to an even bigger measured increase in the growth rate of real GDP.

For our benchmark case we use a year as a period and calibrate our parameters based on the machines in our model representing equipment in the U.S. economy. This means that the theoretical labor input in the model should be interpreted as a composite of labor and structures. Because of this interpretation, we would like to emphasize the illustrative nature of the numerical results here. They are by no means meant to quantify any biases in existing empirical analyses. Instead, they are meant to illustrate conceptual measurement issues.

We choose the growth rate, g, to equal the average growth rate of output, which in our case equals the sum of personal consumption expenditures and fixed private non-residential investment, divided by the PCE deflator for 1960-2005. It turns out to equal 3.7% annually. The number of models sold is calibrated to equal the length of a U.S. patent in years, i.e. 20. For illustrative purposes, we chose the monopolist case as our benchmark. We choose c, and the skill distribution parameters, $\underline{h}$ and $\overline{h}$, such that our model approximately satisfies the following three conditions.

The aggregate labor share, i.e. the share of labor and structures, equals 83%, such that the share of equipment in value added in this economy is 17%. This is consistent with the output elasticity of 0.17 that Greenwood, Hercowitz, and Krusell (1997) for their Cobb-Douglas production function. The second condition is that equipment investment as a share of output equals the average of 9% observed over 1960-2005 in the data. The final condition is that measured investment price deflation is about 6% annually, which is in line with the estimates reported in Cummins and Violante (2002).

The column labeled 'benchmark' of Table 1 lists the results for our benchmark case. In this case, the spuriously measured growth rate of real investment equals 10.6%, while that of real GDP is 4.2%. Contrary to the theoretical results derived above, growth accounting does find positive TFP growth, both for the overall economy, by applying the methodology of Greenwood, Hercowitz, and Krusell (1997), as well as in the final goods sector. Even though production in the latter does not exhibit any factor neutral technological change, a standard growth accounting exercise in this case would suggest TFP growth of 2.6% annually. The corresponding price per efficiency unit contour, $\widehat{P}_\tau$, that drives these results is depicted as the solid line in Figure 1.

We consider the effect of five particular changes in the benchmark parameters. The first four are changes to the competitive and cost structure underlying the supply of capital goods. The fifth, and final, change is one in which the growth rate of embodied technological change doubles. In theory, the first four changes should not affect the measured growth rates of economic aggregates, since they do not affect theoretical trend growth in the economy. We will show, however, that it does affect measured growth.

Case (I) is one in which machines are supplied by monopolistic competitors rather than by one monopolist. As one can see from Figure 1, in this case the increased degree of competition lowers all prices. Furthermore, the competition between suppliers of adjacent vintages also leads to more rapid price declines for the most advanced vintages with the highest market shares in this case. As a result, measured investment prices decline 4.4% to 4.7% faster in this case than under the monopoly. This also means an increase in the rate of growth of real investment. The effect of this change in the competitive structure on other measured growth rates is subdued because the increased competition implies that the suppliers of capital goods extract less of the value added produced in the final goods sector. This results in a decrease in the nominal investment ratio as well as increases in the labor shares in both the overall economy and the final goods sector. These changes in the composition of value added almost counterbalance the change in the growth rate of real investment. Consequently, the measured growth rates of real GDP as well as aggregate TFP and TFP in the final goods sector are almost the same in this case as in our benchmark.

In case (II) we illustrate what happens when the product lifecycle is shortened. In this case we halved it from 20 models to 10. This shortening reduces the opportunity for the monopolist to price discriminate between the different workers and thus the capital goods supplier extracts less of the producer surplus in the final goods sector. This leads to a lower investment ratio as well as a steeper decline in prices over the lifecycle. The outcome is that measured investment price declines are almost double that of the benchmark case. Furthermore, the growth rate of real investment increases from 10.6% to 18.4%. Just like in the previous case, this change in the growth rate of real investment is in large part offset by the change in the nominal investment share, leading to little change in the measured growth rates of the other aggregates.

Case (III) is one in which there is an increasing disparity in skills. In particular, we will assume that the best workers become twice as good. In that case the monopolists price differentation is more effective, because of the increased heterogeneity on its demand side. Consequently, the nominal investment share increases. All of a sudden, the measured investment price declines, which themselves are almost the same as in the benchmark case, become more important in the measurement of the economic aggregates. This leads to an increase in the growth rate of real GDP and to substantial decreases in the growth rates of aggregate and final goods sector TFP.

In case (IV) a decrease in the returns to scale in the machine producing sector leads to a reduction in the nominal investment ratio. This has the opposite effect on the measured growth rates of economic aggregates as case (III).

In case (V) we double the growth rate of embodied technological change. The most remarkable result for this case is that this increase in embodied technological change is actually mostly captured through a doubling of the growth rates of TFP in both the aggregate economy as well as the final goods sector. That is, even though the observed acceleration in output growth in this case would be completely embodied in machines, existing growth accounting techniques would attribute the majority of it to an acceleration in factor neutral technological progress.

The cases that we considered here are definitely not exhaustive. What they do show, however, is how deviations from the assumptions underlying the measurements of economic activity and productivity growth, can result in these measures being distorted by factors that should, in principle, not affect them.

6 Implications

So far, we have presented a theoretical example to illustrate how the construction of an aggregate investment price index for a non-existent capital aggregate can lead to misleading inference about the trend properties of the macroeconomy. That is, we revisited the Cambridge Capital Controversy and showed how it might apply to our measures of economic growth and productivity.

In this section we do two things. First of all, we discuss a set of facts that are inconsistent with the assumption of the existence of a proper capital aggregate. Secondly, we discuss a set of recent empirical results for which the assumption of the existence of such aggregates is very relevant, because they are especially driven by the capital price indices that are applied.

6.1 Does an aggregate capital stock exist?

We have shown, in our analysis here, that the application of a capital price index in the absence of a capital aggregate can potentially lead to misleading inference about growth in economic aggregates.

Contrary to the spurious regressions in Granger and Newbold (1974), for which their identifying assumption can be tested, there is no statistical method that allows us to test for the existence of an aggregate capital stock. However, from Fisher (1969) we know that the only case in which an aggregate capital stock representation exists is when all vintage production functions are Cobb-Douglas. In that case, the aggregate production function representation is the Cobb-Douglas representation used in, for example, Greenwood, Hercowitz, and Krusell (1997). This specification implies several testable empirical implications that can potentially be falsified.

On the aggregate level, the transitional dynamics in the model in Greenwood, Hercowitz, and Krusell (1997) are basically the same as those in the Solow growth model with a depreciation rate that is adjusted for the relative price declines of capital goods. Gilchrist and Williams (2000,2001) argue, however, that the actual transitional dynamics of the U.S. economy, and that of Germany and Japan, are probably better described by a putty-clay vintage capital model in which an aggregate capital stock does not exist than by the conventional Solow model.

At the disaggregate level, there is actually some relevant information in the cross sectional behavior of prices across models sold. The Cobb-Douglas model has very stark predictions about the prices of different capital vintages. As we show in Appendix A, it implies that relative prices reflect relative quality differences across machines, no matter what the human capital level of the worker is that they are matched with.

Formally, it implies that for all vintages τ for which there is non-zero investment

$$\widehat{P}_{t,\tau} = \frac{P_{t,\tau}}{A_{t-\tau}} = \widetilde{\widehat{P}}_t \text{ and does not depend on } \tau \tag{59}$$

Hence, the price per efficiency unit is the same for all vintages for which there is strictly positive demand.

In such a model, the introduction of a new machine does not necessarily imply that the prices of the other models decline. Furthermore, assuming that the quality of each particular vintage is constant over time, such a model implies that the prices of all machines decline at the same rate to maintain their relative quality ratios.

In our model, the relative prices of machines depend both on their relative quality levels as well as their production costs and the workers that they are matched up with. Consequently, our model implies that the prices of older models decline when a new and better model is introduced. This is because the assortative matching between machines and workers implies that the older models are now sold to workers with lower human capital levels that are less productive using them.

This implication of our model is consistent with the behavior of prices of high-tech goods, like computers, printers, and microprocessors. For example, in May 2004 Intel introduced three faster Pentium M chips and reduced the price of older Pentium M chips by as much as 30 percent[10]. Similarly, in April 2003 Hewlett-Packard introduced new models of its LaserJet printers and reduced the price of older models by as much as 20 percent[11].

The price contours of semiconductors plotted in Aizcorbe and Kortum (2004) also suggest that not all models exhibit the same percentage price decline at each point in time. This is not consistent with the Cobb-Douglas assumption needed for the existence of an aggregate capital stock and suggests that relative prices reflect more than only relative quality differences.

[10] Source: "Daily Briefing." *The Atlanta Journal Constitution.* May 11, 2004. Business Section. Page 2D.

[11] Source: HP News Release. "HP Announces Innovative New Products and Services for Small- and Medium-sized Business." April 2, 2003.

Hence, both at the aggregate and the disaggregate level, there is evidence that the assumption of the existence of an aggregate capital stock might not be valid and that, thus, the application of a capital price index might be misleading.

6.2 Investment price index driven results

In recent years, there have been many studies that have heavily relied on investment price indices. The reason for this is the increased importance of information technology equipment and the inherent problem of accounting for the quality improvements embodied in it. Two strands of the literature stand out in particular in this respect.

The first is that on investment specific technological change, i.e. higher productivity growth in the capital goods producing sector than in the consumption goods producing sector, as initiated by Greenwood, Hercowitz, and Krusell (1997).

Greenwood, Hercowitz, and Krusell (1997) were the first to use the changes in a quality adjusted capital price index, in particular one based on Gordon (1990), relative to the changes in the consumption price index as a measure of investment specific technological change in a general equilibrium framework. They use the capital price index to decompose productivity growth into disembodied TFP growth and growth induced by the decline of the quality adjusted relative price of capital goods, known as investment specific technological change.

Their analysis yields the result that, since the middle of the 1970's, the quality adjusted relative price declines of investment goods have accelerated, increasing the contribution of investment specific technological change to U.S. output growth. This result is consistent with the observation that quality improvements in computers and other IT capital goods have accelerated since the middle of the 1970's.

Another aspect of the Greenwood, Hercowitz, and Krusell (1997) results is more difficult to interpret. Their study finds that the rate of investment specific technological change measured using a quality adjusted investment price index implies that TFP growth in the U.S. has been persistently negative between 1973 and 1990. The average annual decline in TFP for the period between 1973 and 1990 reported in their analysis is 0.9%. We would argue that this technological regress might be an artifact of the capital good price index overstating the contribution of embodied technological change to economic growth.

Subsequently, many other empirical studies of technological change have used changes in the relative price of investment goods with respect to consumption goods as a measure of the degree of investment specific or embodied technological change. These studies include, among others, Greenwood, Hercowitz, and Krusell (1997,2000), Violante, Ohanian, Ríos-Rull, and Krusell (2000), Cummins and Violante (2002), Fisher (2002), Altig, Christiano, Eichenbaum, and Linde (2005), and Ireland and Shuh (2006).

The second strand of the literature is made up of growth accounting studies that emphasize the importance of IT capital deepening for U.S. output growth. These studies also have the potential to suffer from the same measurement problem introduced in this paper. That is, just like Greenwood, Hercowitz, and Krusell (1997), these growth accounting studies, like Jorgenson and Stiroh (2000) and Oliner and Sichel (2000), also assume the existence of an aggregate IT capital stock. Therefore, the same issue raised by us in this paper might lead them to overestimate the contribution of IT goods to U.S. output growth.

IT capital seems to be especially subject to the issue raised in this paper, because, as documented by Aizcorbe and Kortum (2004), the price contours that we observe for these capital goods are much more similar to those implied by our model than those implied by the Cobb-Douglas case in which an IT capital aggregate would be well-defined.

7 Conclusion

This paper is a note of caution on the use of capital price indices. We use a vintage capital model in which workers are matched with machines of increasing quality to illustrate a potential problem with the application of such price indices. The most important feature of our model economy, for the results in this paper, is that there does not exist an aggregate production function representation in terms of a single capital aggregate. Hence, a capital price index in this economy is a spurious measure of something that is not defined.

However, the nonexistence of a capital stock does not mean that one cannot apply a capital price index to obtain measures of the trend properties of aggregate economic activity.

We show that when one does so in our model economy, these measures yield misleading results on productivity and economic growth. Besides technological progress, measured growth rates in our model economy also depend on the competitive structure of the capital goods producing sector, which is irrelevant for the economy's long-term growth rate.

We use a numerical example to show that a shift in this competitive structure might lead to spurious increases in the measured rate of decline in the relative price of investment, and the perceived growth rates of real investment, real GDP, and investment specific technological change.

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A Proofs

Proof of equations (12) and (13):

To see why (12) is true, consider $h' > h$ and $\tau' > \tau$, then $h \in D(\tau, \mathbf{P}_t, \mathbf{A}_t)$ implies that

$$\forall\ s \in \mathbf{T}_t :\ A_{t-\tau}h - P_{t,\tau} \geq A_{t-s}h - P_{t,s} \tag{60}$$

or, equivalently, in terms of marginal benefits and costs

$$\forall\ s \in \mathbf{T}_t :\ \left(A_{t-\tau} - A_{t-s}\right) h \geq P_{t,\tau} - P_{t,s} \tag{61}$$

Consequently, because for all $\tau' > \tau$ strictly positive technological progress implies $A_{t-\tau'} > A_{t-\tau}$, the marginal benefits from updating for the worker of type h' exceed those of the worker of type h. That is,

$$\forall \tau' > \tau :\ \left(A_{t-\tau} - A_{t-\tau'}\right) h' > \left(A_{t-\tau} - A_{t-\tau'}\right) h \geq P_{t,\tau} - P_{t,\tau'} \tag{62}$$

This implies that it must thus be true that $h' \notin D_t(\tau', \mathbf{P}_t, \mathbf{A}_t)$ for all $\tau' > \tau$.

The result of equation (12) implies that the demand sets are connected for the following reason. Suppose there would be a demand set that was not connected, then there exist $h'' > h' > h$ such that $h'' \in D_t(\tau, \mathbf{P}_t, \mathbf{A}_t)$, $h' \in D_t(\tau', \mathbf{P}_t, \mathbf{A}_t)$, and $h \in D_t(\tau, \mathbf{P}_t, \mathbf{A}_t)$ where $\tau \neq \tau'$. However, if $\tau > \tau'$, then the choices of h'' and h' do not satisfy assortative matching. On the other hand, if $\tau' > \tau$, then the choices of h' and h do not satisfy assortative matching. Hence, the demand sets need to be connected.

If the demand sets are connected and subsets of the interval $(\underline{h}, \overline{h}]$, then they have to be of the form given in equation (13).

The proof that the set of all workers that is indifferent between two machines is negligible is a bit more involved. Let $\mathcal{H}_t$ denote the set of all human capital levels for which the workers are indifferent between two vintages of machines at time t. Since the human capital levels are uniformly distributed, it suffices to prove that $\mathcal{H}_t$ contains a finite number of elements. Since we have already derived that workers will only use technologies $\{0, \ldots, M\}$ there are only a finite number of combinations between which workers can be indifferent.

We will show that, if a worker of type h is indifferent between two intermediate goods, then no other worker will be. That is, define the set

$$\mathcal{H}_t^*(\tau, \tau') = \left\{h \in (\underline{h}, \overline{h} \mid h \in D_t(\tau) \wedge h \in D_t(\tau')\right\} \tag{63}$$

such that

$$\mathcal{H}_t = \bigcup_{\tau=0}^{M-1} \bigcup_{\tau'=\tau+1}^{M} \mathcal{H}_t^*(\tau, \tau') \tag{64}$$

and, denoting the Lebesque measure as $\mu(.)$, we obtain

$$\mu(\mathcal{H}_t) \leq \frac{1}{\overline{h}} \sum_{\tau=0}^{M-1} \sum_{\tau'=\tau+1}^{M} \mu(\mathcal{H}_t^*(\tau,\tau')) \tag{65}$$

We will simply show that $\forall \tau' > \tau : \mu(\mathcal{H}_t^*(\tau,\tau')) = 0$. Let $h \in (\underline{h}, \overline{h}]$ be such that $h \in D_t(\tau)$ as well as $h \in D_t(\tau')$ for $\tau' > \tau$. In that case

$$A_{t-\tau}h - P_{t,\tau} = A_{t-\tau'}h - P_{t,\tau'} \tag{66}$$

or equivalently

$$(A_{t-\tau} - A_{t-\tau'})\,h = P_{t,\tau} - P_{t,\tau'} \tag{67}$$

This, however implies that for all $h' > h > h''$

$$(A_{t-\tau} - A_{t-\tau'})\,h' > P_{t,\tau} - P_{t,\tau'} > (A_{t-\tau} - A_{t-\tau'})\,h'' \tag{68}$$

such that the workers of type $h' > h$ will prefer τ over τ', while workers of type $h'' < h$ will do the opposite. Hence, $\mathcal{H}_t^*(\tau,\tau') = \{h\}$ and is of measure zero.

Proof of equations (19) through (22):

We will prove these equations in three steps. In the first step, we prove equation (19) and show that, irrespective of $\mathbf{A}_t$, M, and c, the suppliers will set their prices such that there is demand for each of the vintages. In the second step, we derive the first order conditions that, given that it is interior, determine the optimal price schedule and show that the suppliers make strictly positive profits of the supply of each of the patented vintages. That is, we prove equation (20). In the final step, we prove the properties of the price schedule per efficiency unit that are formalized in equations (21) and (22).

Strictly positive demand for all M newest vintages: In order to prove equation (19), it turns out to be useful to introduce the function that relates a vintage back to its supplier. We denote this function by $\iota(\tau)$. It is equal to the index number of the supplier that supplies machines of vintage τ.

Furthermore, to keep track of which vintages are supplied by the same supplier and which are not, we define the indicator function

$$I[a=b] = \left\{ \begin{array}{l} 1 \text{ if } a=b \\ 0 \text{ if } a \neq b \end{array} \right\} \tag{69}$$

so that $I[\iota(\tau) = \iota(\tau')]$ is equal to one if vintages τ and τ' are supplied by the same supplier and zero otherwise.

For this proof we will consider the supplier of vintage τ and consider the effect of its price setting on the profits made from the supply of vintage τ, as well as that of vintage ages $\tau-1$ and $\tau+1$. Here we assume, without loss of generality that these adjacent vintages have prices set such that $K_{t,\tau-1}, K_{t,\tau+1} > 0$ in case vintage τ would not be supplied. We will distinguish the cases $\tau = 0$, for which $K_{t,\tau-1}$ is irrelevant, and $\tau = M - 1$, for which we know that there are no profits made of vintage $\tau + 1$.

For this vintage τ, we will show that there exists a price $P_{t,\tau} > 0$ such that the supplier makes strictly positive profits of the supply of vintage τ as well as that this price increases the sum of the profits over all three vintages $(\tau - 1, \tau, \tau + 1)$, or any two of these vintages that include τ. That is, independent of the prices of the adjacent vintages for which there is demand, the supplier of vintage τ can increase its profits, no matter whether it only owns the patent for vintage τ or any of the patents for the adjacent vintages. The assortative matching result implies that looking at three adjacent vintages is enough for this argument, because the price set for vintage τ at the margin only affects the demand for the adjacent vintages.

Let us first determine the reservation price level, above which vintage τ will not be demanded at all. This price level is determined by the type of worker, that, without the availability of vintage τ, is indifferent between vintage $\tau - 1$ and $\tau + 1$. We denote the human capital level of this worker by $\widetilde{h}$. It has to satisfy

$$A_{t-\tau+1}\widetilde{h} - P_{t,\tau-1} = A_{t-\tau-1}\widetilde{h} - P_{t,\tau+1} \tag{70}$$

such that

$$\widetilde{h} = \begin{cases} \overline{h} & \text{for } \tau = 0 \\ \frac{P_{t,\tau-1} - P_{t,\tau+1}}{A_{t-\tau+1} - A_{t-\tau-1}} & \text{for } \tau > 0 \end{cases} \tag{71}$$

Hence, demand for vintage τ implies that its price level much be such that

$$A_{t-\tau}\widetilde{h} - P_{t,\tau} \geq A_{t-\tau+1}\widetilde{h} - P_{t,\tau-1} = A_{t-\tau-1}\widetilde{h} - P_{t,\tau+1} \tag{72}$$

In terms of the price per efficiency unit, this implies that

$$\hat{P}_{t,\tau} \leq \begin{cases} \frac{A_t - A_{t-1}}{A_t}\overline{h} + \frac{A_{t-1}}{A_t}\widehat{P}_{t,1} & \text{for } \tau = 0 \\ \left[\begin{array}{c} \frac{A_{t-\tau+1}}{A_{t-\tau}} \left(\frac{A_{t-\tau} - A_{t-\tau-1}}{A_{t-\tau+1} - A_{t-\tau-1}} \right) \widehat{P}_{t,\tau-1} \\ + \frac{A_{t-\tau-1}}{A_{t-\tau}} \left(\frac{A_{t-\tau+1} - A_{t-\tau}}{A_{t-\tau+1} - A_{t-\tau-1}} \right) \widehat{P}_{t,\tau+1} \end{array} \right] & \text{for } \tau > 0 \end{cases} \tag{73}$$

$$\equiv \widetilde{P}_{t,\tau} \tag{74}$$

Hence, $\widetilde{P}_{t,\tau}$ is the maximum price per efficieny unit at which the supplier of vintage τ faces positive demand.

The supplier of vintage τ has three options. First of all, it can choose to make vintage τ available at minimum cost, in which case $P_{t,\tau} = \underline{h}A_{t-\tau}$ and the firm would make non-positive profits. Secondly, it could choose $P_{t,\tau} \geq A_{t-\tau}\widetilde{P}_{t,\tau}$ at which it faces no demand and profits are zero. Finally, it can choose a price $P_{t,\tau} \geq A_{t-\tau}\left(\widetilde{P}_{t,\tau} - \varepsilon\right)$ where $0 < \varepsilon < \widetilde{P}_{t,\tau}$.

The firm will choose the third option, whenever that option increases the profits its makes over all the vintages its supplies. In the following we will show that, independent of the prices $P_{t,\tau-1}$ and $P_{t,\tau+1}$, there exists an $\varepsilon > 0$ for which this is the case.

We will consider the profits that the supplier of vintage τ makes when it chooses a price equal to

$$\stackrel{\wedge}{P}_{t,\tau} = \widetilde{P}_{t,\tau} - \varepsilon \text{ for } \varepsilon > 0 \tag{75}$$

For a small enough $\varepsilon > 0$ when $K_{t,\tau-1}, K_{t,\tau+1} > 0$ the choice of $\stackrel{\wedge}{P}_{t,\tau}$ will not affect the demand of vintages other than those of vintage ages $\tau - 1$, τ and $\tau + 1$. Hence, for small $\varepsilon > 0$, which turns out to be the relevant case in this proof, what matters for the supplier of vintage τ and what determines the price it chooses is whether it also supplies vintage $\tau - 1$, and/or $\tau + 1$, or neither of them.

At the price $\stackrel{\wedge}{P}_{t,\tau} = \widetilde{P}_{t,\tau} - \varepsilon$ the demand for vintage τ can be shown to equal

$$K_{t,\tau} = \left(I\left[\tau \neq 0\right] \frac{A_{t-\tau}}{A_{t-\tau+1} - A_{t-\tau}} + \frac{A_{t-\tau}}{A_{t-\tau} - A_{t-\tau-1}} \right) \frac{\varepsilon}{\overline{h} - \underline{h}} \tag{76}$$

The new profits over the three adjacent vintages for the supplier of vintage τ are given by

$$\begin{aligned} & I\left[\iota\left(\tau-1\right) = \iota\left(\tau\right)\right] A_{t-\tau+1} \left(\widehat{P}_{t,\tau-1} - \underline{h} - \frac{c}{2} \left(K_{t,\tau-1} - \frac{A_{t-\tau}}{A_{t-\tau+1} - A_{t-\tau}} \frac{\varepsilon}{\overline{h} - \underline{h}} \right) \right) \times \\ & \left(K_{t,\tau-1} - \frac{A_{t-\tau}}{A_{t-\tau+1} - A_{t-\tau}} \frac{\varepsilon}{\overline{h} - \underline{h}} \right) + \\ & A_{t-\tau} \left(\widetilde{P}_{t,\tau} - \underline{h} - \left[1 + \frac{c}{2} \left(I\left[\tau \neq 0\right] \frac{A_{t-\tau}}{A_{t-\tau+1} - A_{t-\tau}} + \frac{A_{t-\tau}}{A_{t-\tau} - A_{t-\tau-1}} \right) \right] \frac{\varepsilon}{\overline{h} - \underline{h}} \right) \times \\ & \left(I\left[\tau \neq 0\right] \frac{A_{t-\tau}}{A_{t-\tau+1} - A_{t-\tau}} + \frac{A_{t-\tau}}{A_{t-\tau} - A_{t-\tau-1}} \right) \frac{\varepsilon}{\overline{h} - \underline{h}} + \\ & I\left[\iota\left(\tau+1\right) = \iota\left(\tau\right)\right] A_{t-\tau-1} \left(\widehat{P}_{t,\tau+1} - \underline{h} - \frac{c}{2} \left(K_{t,\tau+1} - \frac{A_{t-\tau}}{A_{t-\tau} - A_{t-\tau-1}} \frac{\varepsilon}{\overline{h} - \underline{h}} \right) \right) \times \\ & \left(K_{t,\tau+1} - \frac{A_{t-\tau}}{A_{t-\tau} - A_{t-\tau-1}} \frac{\varepsilon}{\overline{h} - \underline{h}} \right) \end{aligned} \tag{77}$$

Which simplifies to

$$\begin{aligned} & I\left[\iota\left(\tau-1\right) = \iota\left(\tau\right)\right] A_{t-\tau+1} \left(\widehat{P}_{t,\tau-1} - \underline{h} - \frac{c}{2} K_{t,\tau-1} \right) K_{t,\tau-1} + \\ & I\left[\iota\left(\tau+1\right) = \iota\left(\tau\right)\right] A_{t-\tau-1} \left(\widehat{P}_{t,\tau+1} - \underline{h} - \frac{c}{2} K_{t,\tau+1} \right) K_{t,\tau+1} + \\ & a \left(\frac{\varepsilon}{\overline{h} - \underline{h}} \right) - b \left(\frac{\varepsilon}{\overline{h} - \underline{h}} \right)^2 \end{aligned} \tag{78}$$

where $a > 0$ and $b > 0$. In particular, they equal

$$a = I\left[\tau = 0\right] A_t \left(\overline{h} - \underline{h} \right) \tag{79}$$

$$+ I\left[\tau \neq 0\right] \left(1 - I\left[\iota\left(\tau\right) = \iota\left(\tau-1\right)\right]\right) \frac{A_{t-\tau+1} A_{t-\tau}}{A_{t-\tau+1} - A_{t-\tau}} \left(\widehat{P}_{t,\tau-1} - \underline{h} \right) \tag{80}$$

$$+ \left(1 - I\left[\iota\left(\tau+1\right) = \iota\left(\tau\right)\right]\right) \frac{A_{t-\tau} A_{t-\tau-1}}{A_{t-\tau} - A_{t-\tau-1}} \left(\widehat{P}_{t,\tau+1} - \underline{h} \right) \tag{81}$$

$$+ I\left[\iota\left(\tau-1\right) = \iota\left(\tau\right)\right] \frac{A_{t-\tau+1} A_{t-\tau}}{A_{t-\tau+1} - A_{t-\tau}} c K_{t,\tau-1}$$

$$+ I\left[\iota\left(\tau+1\right) = \iota\left(\tau\right)\right] \frac{A_{t-\tau-1} A_{t-\tau}}{A_{t-\tau} - A_{t-\tau-1}} c K_{t,\tau+1} \tag{82}$$

and

$$b = \frac{c}{2} I\left[\iota(\tau-1)=\iota(\tau)\right] A_{t-\tau+1}\left(\frac{A_{t-\tau}}{A_{t-\tau+1}-A_{t-\tau}}\right)^2 \tag{83}$$

$$+\left[1+\frac{c}{2}\left(I\left[\tau\neq 0\right]\frac{A_{t-\tau}}{A_{t-\tau+1}-A_{t-\tau}}+\frac{A_{t-\tau}}{A_{t-\tau}-A_{t-\tau-1}}\right)\right]\times$$
$$\left(I\left[\tau\neq 0\right]\frac{A_{t-\tau}}{A_{t-\tau+1}-A_{t-\tau}}+\frac{A_{t-\tau}}{A_{t-\tau}-A_{t-\tau-1}}\right)+ \tag{84}$$
$$+I\left[\iota(\tau+1)=\iota(\tau)\right] A_{t-\tau-1}\frac{c}{2}\left(\frac{A_{t-\tau}}{A_{t-\tau}-A_{t-\tau-1}}\right)^2$$

Note that the first two terms of equation (78) equal the profits that the supplier of vintage τ would have made of the two adjacent vintages, if it would have owned any of them. The term $a\varepsilon - b\varepsilon^2$ represents the additional profits earned due to the supply of vintage τ at price $\widetilde{P}_{t,\tau}-\varepsilon$. Hence, the supplier of vintage τ would always set a price that generates strictly positive demand for that vintage if there exists an $\varepsilon > 0$ for which this additional profit is strictly positive. Since there always is an $\varepsilon > 0$ for which $a\varepsilon - b\varepsilon^2 > 0$, it always the case that the supplier of vintage τ will supply that vintage at a price that generates strictly positive demand.

Strictly positive profits: This follows as a corollary from the proof above. The supplier of the vintage τ can always choose its price to strictly increase its profits relative to zero.

$\widehat{P}_{t,\tau}$ **is strictly decreasing in** τ**:** This follows from an induction argument. We have proven above that in the equilibrium there must be strictly positive demand for each of the vintages of age $\tau = 0,\ldots,M-1$, i.e. $K_{t,\tau} > 0$ in equilibrium. In terms of the prices per efficiency unit, the demand sets are

$$K_{t,\tau} = \begin{cases} \frac{1}{\overline{h}-\underline{h}}\left[\overline{h}-\frac{A_t}{A_t-A_{t-1}}\widehat{P}_{t,0}+\frac{A_{t-1}}{A_t-A_{t-1}}\widehat{P}_{t,1}\right] & \text{for } \tau=0 \\ \frac{1}{\overline{h}-\underline{h}}\left[\frac{A_{t-\tau+1}}{A_{t-\tau+1}-A_{t-\tau}}\left(\widehat{P}_{t,\tau-1}-\widehat{P}_{t,\tau}\right)\right. & \\ \left. -\frac{A_{t-\tau-1}}{A_{t-\tau}-A_{t-\tau-1}}\left(\widehat{P}_{t,\tau}-\widehat{P}_{t,\tau+1}\right)\right] & \text{for } \tau=1,\ldots,M-1 \end{cases} \tag{85}$$

This implies that for $\tau = 1,\ldots,M-1$

$$\frac{A_{t-\tau+1}}{A_{t-\tau+1}-A_{t-\tau}}\left(\widehat{P}_{t,\tau-1}-\widehat{P}_{t,\tau}\right) > \frac{A_{t-\tau-1}}{A_{t-\tau}-A_{t-\tau-1}}\left(\widehat{P}_{t,\tau}-\widehat{P}_{t,\tau+1}\right) \tag{86}$$

Hence, if the price per efficiency unit for vintage age τ is larger than that for $\tau+1$, then it must be the case that the price per efficiency unit for vintage age $\tau-1$ is higher than that of vintage τ. The only thing we need to proof our claim is a initial result and then we can apply an induction argument.

We do know that in equilibrium the supplier of vintage age $M-1$ will choose a price that yields strictly positive profit, which implies $\widehat{P}_{t,M-1} > \underline{h}$. Furthermore, we know that perfect competition in the supply of vintage M will drive its price to minimum cost, such that $\widehat{P}_{t,M} = \underline{h}$. Hence, we know that $\left(\widehat{P}_{t,M-1}-\widehat{P}_{t,M}\right) > 0$. Applying our induction argument thus yields that this implies that $\left(\widehat{P}_{t,\tau}-\widehat{P}_{t,\tau+1}\right) > 0$ for $\tau = 0,\ldots,M-1$. Hence $\widehat{P}_{t,\tau}$ is strictly decreasing in τ.

$\widehat{\mathbf{P}}_t^* = \widehat{\mathbf{P}}(\mathbf{A}_t, c)$ **where** $\widehat{\mathbf{P}}(.)$ **is homogenous of degree zero in** A_t**:** Supplier i sets the prices of the vintages of machines its supplies to maximize the profits

$$\pi_{t,i} = \sum_{\tau=0}^{M-1} I[\iota(\tau) = i] A_{t-\tau}\left(\widehat{P}_{t,\tau} - \underline{h} - \frac{c_\tau}{2}K_{t,\tau}\right)K_{t,\tau} \tag{87}$$

The necessary and sufficient conditions for profit maximization in equilibrium imply that this supplier will set the price of each vintage τ which it supplies, i.e. $\iota(\tau) = i$, to satisfy the condition

$$0 = K_{t-\tau} + \sum_{s=0}^{M-1} I[\iota(s) = i] \frac{A_{t-s}}{A_{t-\tau}}\left(\widehat{P}_{t,s} - \underline{h} - c_s K_{t,s}\right)\frac{\partial K_{t,s}}{\partial \widehat{P}_{t,\tau}} \tag{88}$$

However, note that these optimality conditions are homogenous of degree zero in $\mathbf{A}_t = \{A_t, \ldots, A_{t-M}\}$. This is because the demand functions that determine $K_{t,\tau}$ are homogenous of degree zero in $\mathbf{A}_t = \{A_t, \ldots, A_{t-M}\}$ and so are $\partial K_{t,\tau}/\partial \widehat{P}_{t,\tau}$. Furthermore, besides the productivity levels in $\mathbf{A}_t$ the only other parameters that show up in these equilibrium conditions are the cost parameter, c, and the bounds of the skill distribution, i.e. $\overline{h}$ and $\underline{h}$. Thus the equilibrium price per efficiency unit profile is only a function of the productivity levels, the cost parameters, and skill distribution and it is homogenous of degree zero in the productivity levels.

Furthermore, the system of equilibrium conditions, implied by the optimality conditions above, is linear in the prices per efficiency unit and it turns out to be straightforward to show that it has one unique solution. That is, the PSN equilibrium exists and it is unique.

Proof of equation (45) :

The following is the proof of equation (45). The matched model Laspeyres index yields a capital price inflation estimate of

$$\pi_t^{(M)} = \frac{\sum_{\tau=1}^{M} P_{t,\tau}K_{t-1,\tau-1}}{\sum_{\tau=0}^{M-1} P_{t-1,\tau}K_{t-1,\tau}} - 1 = \frac{\sum_{\tau=0}^{M-1} P_{t,\tau+1}K_{t-1,\tau}}{\sum_{\tau=0}^{M-1} P_{t-1,\tau}K_{t-1,\tau}} - 1 \tag{89}$$

$$= \sum_{\tau=0}^{M-1} s_{t-1,\tau}^* \widehat{\pi}_{t,\tau} \tag{90}$$

where the shares $s_{t,\tau}^*$ are given by

$$s_{t-1,\tau}^* = \frac{P_{t-1,\tau}K_{t-1,\tau}}{\sum_{s=0}^{M-1} P_{t-1,s}K_{t-1,s}} = \frac{A_{t-1-\tau}\widehat{P}_{t-1,\tau}K_{t-1,\tau}}{\sum_{s=0}^{M-1} A_{t-1-s}\widehat{P}_{t-1,s}K_{t-1,s}} \tag{91}$$

and represent the expenditure share in period $t-1$ of vintage age τ in the expenditures on machines that are also available at time t. The inflation rates $\widehat{\pi}_{t,\tau}$ equal

$$\widehat{\pi}_{t,\tau} = \left(\widehat{P}_{t,\tau+1} - \widehat{P}_{t-1,\tau}\right)\Big/ \widehat{P}_{t-1,\tau} \tag{92}$$

On the balanced growth path both $s_{t-1,\tau}^*$ and $\widehat{\pi}_{t,\tau}$ will be constant over time. Furthermore, because the price per efficiency unit is declining in the vintage age, $\widehat{\pi}_{t,\tau} < 0$ for all τ. And thus $\pi_t^{(M)}$ is constant over time and negative.

Proof of equation (59) :

Instead of the Leontief technology that we consider in our model consider a firm that has hired a worker of skill-level h which it matches with $K_{t,\tau}$ units of the capital good of vintage age τ. Here $K_{t,\tau}$ is not fixed at one but the firm can choose it. The production technology in this case is Cobb-Douglas in the sense that output produced equals

$$Y_t = h\left(A_{t-\tau}K_{t,\tau}\right)^{\alpha} \text{ where } 0 < \alpha < 1 \tag{93}$$

The firm thus demands the amount of capital inputs that maximizes

$$Y_t - P_{t,\tau}K_{t,\tau} \tag{94}$$

The optimal capital input choice for the firm is

$$K_{t,\tau} = \left(\frac{P_{t,\tau}}{\alpha h A_{t-\tau}^{\alpha}}\right) \tag{95}$$

and the resulting level of profits equals

$$Y_t - P_{t,\tau}K_{t,\tau} = \left[\alpha^{\frac{\alpha}{1-\alpha}} - \alpha^{\frac{1}{1-\alpha}}\right] h^{\frac{1}{1-\alpha}} \left(\frac{A_{t-\tau}}{P_{t,\tau}}\right) \tag{96}$$

The firm will choose the technology vintage τ that maximizes these profits. Independently of the skill level, h, of the worker, this turns out to be the technology with the lowest price per efficiency unit $P_{t,\tau}/A_{t-\tau}$. Thus, for this case, i.e. the case in which a proper capital aggregate exists, all technology vintages in which there is positive investment must have the same price per efficiency unit.

Table 1: Numerical results

Parameters	benchmark	I	II	III	IV	V
Growth rate of embodied technological change (g)	3.7%	3.7%	3.7%	3.7%	3.7%	7.3%
Number of models sold (M)	20	20	10	20	20	20
Market structure*	M	MC	M	M	M	M
Cost level (c)	0.5	0.5	0.5	0.5	5	0.5
Upperbound skill distribution ($\overline{h}$)	0.325	0.325	0.325	0.650	0.325	0.325
Lowerbound skill distribution ($\underline{h}$)	0.008	0.008	0.008	0.008	0.008	0.008
Measured variables						
Aggregate labor share	83.6%	89.3%	91.5%	69.2%	89.1%	74.3%
Final goods sector labor share (s_L^f)	89.9%	92.2%	94.4%	80.4%	92.5%	83.8%
Nominal investment ratio (I/Y)	9.4%	7.5%	5.5%	16.8%	7.3%	14.3%
Investment price inflation (Matched model)	-6.3%	-10.7%	-12.5%	-6.3%	-6.2%	-5.7%
Investment price inflation (Hedonic)	-5.8%	-10.5%	-10.3%	-6.1%	-5.0%	-5.4%
Growth rate of real investment	10.6%	16.1%	18.4%	10.7%	10.5%	13.8%
Growth rate of real GDP	4.1%	4.1%	4.1%	4.6%	3.9%	8.1%
Aggregate TFP growth (GHK-methodology)	2.0%	2.1%	2.2%	0.5%	2.6%	3.9%
Final goods sector TFP growth	2.6%	2.5%	2.7%	1.7%	2.9%	5.2%

Note: *M means 'monopolist' case, MC means 'monopolistic competition'.

Cases: (I) Monopolistic competition, (II) Shorter product life cycle, (III) Skill accumulation, (IV) Decreasing returns to scale, (V) Increase in growth rate of embodied technological change.

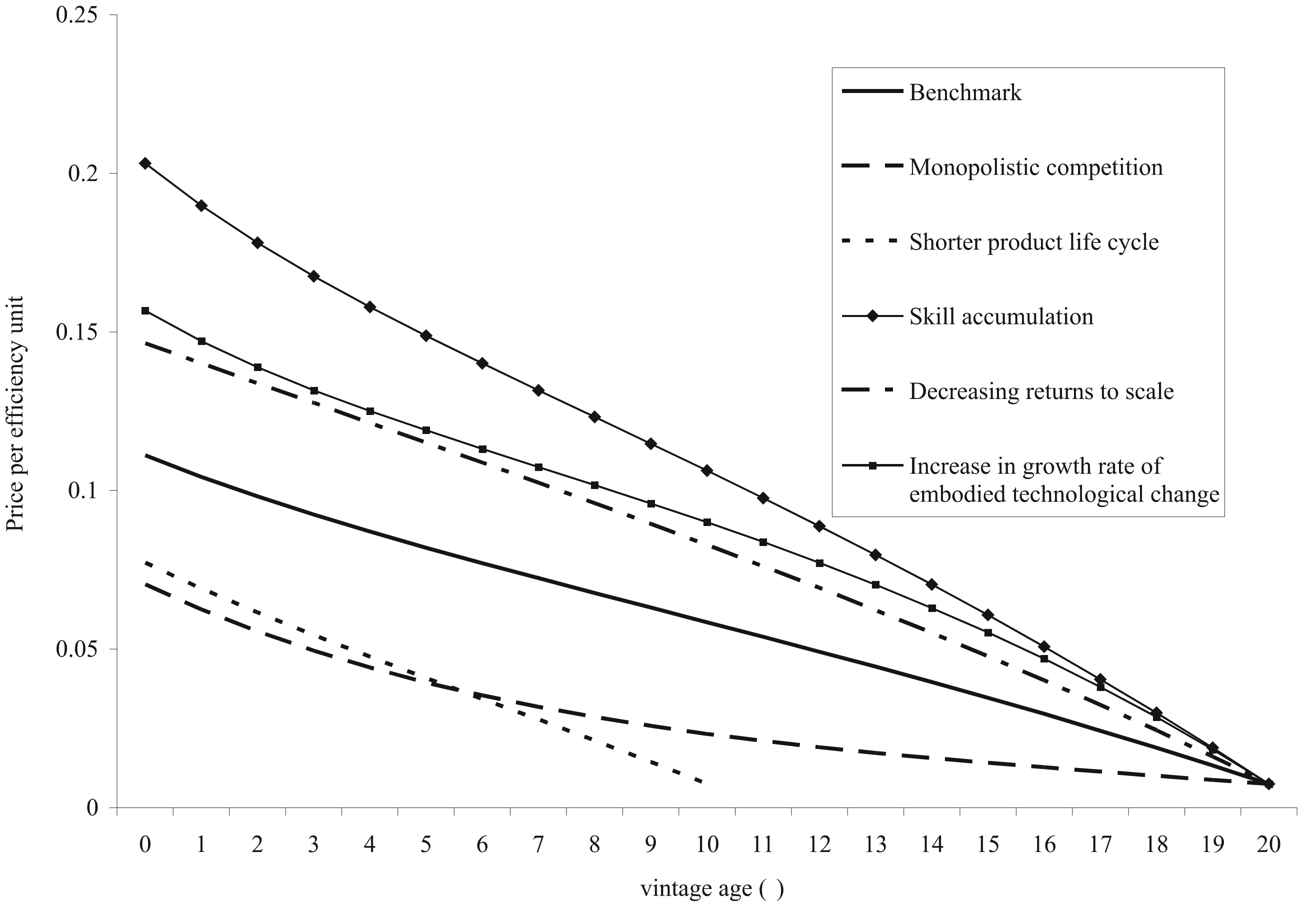

Figure 1: Equilibrium price contours

ON BOTH SIDES OF THE QUALITY BIAS IN PRICE INDEXES

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Abstract

It is often argued that price indexes do not fully capture the quality improvements of new goods in the market. Because of this shortcoming, price indexes are perceived to overestimate the actual price increases that occur. In this paper, I argue that the quality bias in price indexes is just as likely to be upward as it is to be downward. I show how both the sign and the magnitude of the quality bias in the most commonly applied price index methods is determined by the cross-sectional variation of prices per quality unit across the product models sold in the market.

I do so by introducing a model of a market that includes monopolistically competing suppliers of the various product models and a representative consumer with CES (Constant Elasticity of Substitution) preferences. I simulate the model using actual observed CPU prices and find that a large part of the price declines measured for CPUs turns out to be due to a downward quality bias rather than to actual price declines.

Keywords: Price index theory, hedonic price methods, quality bias, monopolistic competition.
JEL-codes: C43, D11, D43, L13.

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1. Introduction

There is a widespread consensus among economists that price index methods tend to overestimate actual inflation in markets where there is a rapid turnover of goods due to technological progress. The Boskin (1996) commission made this point with respect to the U.S. Consumer Price Index, while Gordon (1990) used hedonic price indexes to correct for this bias in equipment price indexes.

There is, however, also a small number of studies that challenge this conventional wisdom. Studies by by Triplett (1972,2002), Feenstra (1995), as well as Hobijn (2001) have each made the point that quality adjustments in price index methods might actually lead to an understatement of inflation.

This paper follows up on the above papers by introducing a parsimonious theoretical model that can generate both a positive as well as a negative quality bias in the most commonly applied price indexes. The value added of this approach is that it allows for the study of the factors that determine the sign and magnitude of the quality bias in a stylized framework. This contrasts strongly with the methodology that is traditionally applied in the price index literature.

A large part of the literature on price indexes compares various price indexes calculated for the same dataset. This is for example the approach of Aizcorbe and Jackman (1993), Manser and McDonald (1988), and Braithwait (1980) when assessing the magnitude of substitution bias as well as of Aizcorbe, Corrado, and Doms (2000) and Silver and Heravi (2002) in the comparison of hedonic and matched model price indexes.

Such an approach allows us to consider the sensitivity of price indexes to the choice of method applied. It does not, however, enable us to make any normative statements about which index method is 'better' than another. Such normative statements on price indexes are all based on the extensive theoretical price index literature, which focuses on properties like idealness, exactness, and superlativity of price index formula.

It turns out that the theoretical results derived in this paper contradict some of the properties of price indexes that are presumed in this applied strand of the literature. Three results stand out in particular.

The most important is that the theoretical model in this paper confirms the claims by Triplett (1972,2002), Feenstra (1995), and Hobijn (2001) that the quality bias in price indexes is not by definition upward. Moreover, the sign and magnitude of the bias turn out *not to depend* on the overall level of inflation. Instead they depend on the cross-sectional behavior of prices per quality unit across models sold in the market during the same period.

Secondly, the existence and sign of this bias does not depend on the specific price index formula applied. I show how the application of the most popular price index formulas, like Laspeyres, Paasche, Geometric Mean, Fisher Ideal, and Tornqvist, all lead to a bias in the same direction.

Finally, hedonic price indexes suffer from the same quality bias as matched model indexes. Hence, the theoretical results here seem to disagree with the presumption that hedonic price indexes do a better job at correcting prices for quality improvements, as made in, among many, Pakes (2002) and Hulten (2002).

The particular theoretical model that I use for my analysis in this paper is that of a market with a representative consumer with CES preferences over a set of models sold. This setup is very similar to Dixit and Stiglitz (1977) and Hornstein (1993). The main difference is that the market that I consider has a countably finite number of models and suppliers. The advantage of this choice of model is that price index theory for CES preferences is extremely well developed. Sato (1976) derived the ideal exact price index for CES preferences when the same models are sold in both the base- and measurement periods. Feenstra (1994) extended Sato's index to an exact matched model index that can be used when the universes of models sold in both periods do not coincide.

The methodology used in this paper is closely related to the Monte Carlo methodology in econometrics. In this sense, I follow Lloyd (1975) who also used simulation methods to quantify certain properties of price indexes. In Lloyd's (1975) study the focus was on the substitution bias in price indexes. Here the focus is on their quality bias. I take Lloyd's (1975) approach one step further by not generating my own data but instead basing my simulation on observed prices of CPU's for PCs. Hence, my simulation results do not only illustrate the existence of the bias but also its empirical relevance.

The structure of the paper is as follows. In the next section I introduce the form of the CES preferences that I consider in the rest of the paper and derive the theoretical price level that price indexes are meant to measure. In Section 3 I illustrate graphically how conventional price index methods might yield a downward quality bias for these preferences. This graphical description is essentially an informal version of the results that are derived in the context of the theoretical model. I introduce this theoretical model in Section 4. I consider its demand and supply side and show how its Pure Strategy Nash equilibrium exists and is unique. In Section 5 I then proceed by deriving some general results for the sign of the quality bias in matched model and hedonic price indexes calculated for a specific parameterization of the model. In Section 6 I present the results of a simulation of the model that is based on actual data on prices and benchmark ratings of CPUs. I illustrate how this simulation confirms the results shown in Sections 3 through 5 and show why these results are empirically relevant. Section 7 concludes.

2. CES-preferences and the theoretical price level

The aim of this paper is to be able to make normative statements about price index methods and to say which ones perform better, in certain situations, than others. In order to make these normative statements we need to define what it is we would like our price index methods to measure. Since Konüs (1939) the main focus of price index theory has been on constructing a cost-of-living index (COLI). The aim of a COLI is to track the (percentage) changes in the minimum expenditures required to reach a certain base-level of utility over time[1].

The minimum amount of expenditures that is necessary to reach a certain utility level crucially depends on the underlying preferences of the consumer. Hence, the theoretical price level that price index methods are after depends on the preferences of the consumer. In reality, a market consists of a spectrum of consumers with different preferences. It turns out that it is not always possible, in such cases, to specify the theoretical price level because aggregate demand does not always behave as if it is generated by a well-behaved aggregate utility representation.

[1] I will focus on consumer price indexes throughout this paper. The theory presented in this paper is also applicable to producer price indexes, which are aimed at tracking the minimum cost required to obtain a base quantity of output over time.

The focus of this paper is not on the conditions for the existence of an aggregate utility representation for aggregate demand. What I will do is simply use one of the best developed aggregate utility representations for which it has been proven that it can be interpreted as the aggregate utility function of a market with a continuum of heterogeneous agents. This aggregate utility representation is Constant Elasticity of Substitution (CES) preferences. Anderson, de Palma, and Thisse (1993) introduced the microfoundations of CES preferences and showed how they can be interpreted as the aggregate utility representation of a market consisting of a continuum of heterogeneous agents.

Let $X_{i,t}$ be the quantity consumed of good i at time t, where I will assume that good $i=0$ is the numeraire good. C_t is the universe of goods sold at time t. I will assume that aggregate demand in the market in the theoretical model behaves as if resulting from the utility maximizing decision of a representative consumer with the utility function

$$U_t = \left(\sum_{i \in C_t \backslash \{0\}} \left(a_i X_{i,t} \right)^{\frac{1}{1+\lambda}} \right)^{1+\lambda} X_{0,t}^{\alpha} \text{ where } \lambda>0 \text{ and } 0<\alpha<1 \tag{1}$$

This is a relatively standard CES utility function, where $\sigma \equiv (1+\lambda)/\lambda$ is the constant elasticity of substitution. The only non-standard features of (1) are that the quantities for goods $i \in C_t \backslash \{0\}$ are multiplied by a quality parameter a_i and that the numeraire good, $i=0$, is included.

Let $p_{i,t}$ be the price of a unit of good i. Since $i=0$ is the numeraire good, I will assume that $p_{0,t}=1$ for all t. In the rest of this paper, I will focus on the construction of a price index for the set of goods, which I will call models in the future, that are contained in the CES part of (1). That is, my focus is on the measurement of the price level of the set of models $i \in C^*_t$ where $C^*_t = C_t \backslash \{0\}$.

We are thus confronted with two sets of goods, i.e. the numeraire good and the models for which we would like to measure an aggregate price level. Diewert (2001) shows that, because the preferences in (1) are separable between $X_{0,t}$ and the other goods, the aggregate price level for the models $i \in C^*_t$ is well defined. In particular, aggregate demand for the models $i \in C^*_t$ will be as if it was generated by the representative agent maximizing the amount of utility obtained from these models for the expenditures solely on these models. This implies that the theoretical price aggregate for the set of models $i \in C^*_t$ is the CES price aggregate as applied in, among many, Dixit and Stiglitz (1977), Hornstein (1993), and Feenstra (1994). This aggregate, the value at time t of which I will denote by P_t^T, reads

$$P_t^T = \left[\sum_{i \in C_t^*} \left(p_{it} / a_i \right)^{-1/\lambda} \right]^{-\lambda} \tag{2}$$

It is a CES aggregate of the prices per quality unit for all models that are traded in the market. This price aggregate represents the money cost of a unit of utility obtained from the consumption of the competing varieties in the set of models C_t^*. This money cost does not depend on the base-level of utility because the preferences are homothetic.

The aim of price index methods is to construct an index that approximates, up to a constant, the path of P_t^T. In particular, the index methods are meant to estimate the period by period percentage change in P_t^T.

Throughout this paper, I will focus on the percentage change in P_t^T between periods $t=0$ and $t=1$. I will refer to the percentage change in P_t^T between those two periods as the theoretical inflation rate and will denote it as

$$\pi^T = \frac{P_1^T - P_0^T}{P_0^T} \tag{3}$$

It represents the percentage change of the money cost of a unit of utility between periods 0 and 1.

If one would know all the preference parameters in (1) then it would not be difficult to calculate the theoretical inflation rate in (3). In practice, however, the preference parameters are not observed. That is, we do not exactly know the elasticity of substition, i.e. $\sigma\equiv(1+\lambda)/\lambda$. Neither do we know the quality embodied in each unit sold for each model, i.e. a_i. In fact, when we apply price index methods we do not even know by what preference representation aggregate demand is generated. There are basically two lines of thought here, which I will both pursue in this paper.

The first line assumes that aggregate preferences belong to a certain class and then uses this restriction to obtain an estimate of (3). For the CES preferences the index that exactly measures the theoretical inflation rate is the one derived by Sato (1976). The details of this index are described in Table 1. Sato's index is valid under the assumption that the universes of models sold in both periods are the same, such that $C_0^* = C_1^*$. It is a proper price index in the sense that it only depends on observables, namely expenditure shares and prices.

The requirement of coinciding sets of models being sold in both periods renders the Sato (1976) index inapplicable at many lower levels of aggregation. Many markets have a high rate of product turnover, as illustrated in Aizcorbe, Corrado, and Doms (2000) for the market for Intel CPU units and in Silver and Heravi (2002) for the market for laundry machines. Hence, it is thus essential to develop price index methods that allow for dynamic universes of models that change over time, i.e. $C_0^* \neq C_1^*$. Feenstra (1994) extends Sato's result to a quasi-index that is exact for CES preferences with non-overlapping universes of models. Feenstra's is a quasi-index because it depends on the unobserved elasticity of substitution, which has to be estimated to implement the index. It is described in Table 1 and I will discuss its intuition in more detail later on.

Price index theory is thus very well developed for CES preferences. We know the form of the exact indexes both when the universe of models is static as well as when it is dynamic. The problem is that in many practical cases it is a big leap to assume that demand is generated by aggregate CES preferences. This brings us to the second line of thought. This line is to construct price indexes that do not exactly measure (3) but instead reasonably approximate it for a very broad class of preferences.

This is the approach most commonly chosen for the calculation of aggregate statistics. Classical price index theory, among others Konüs (1939), Frisch (1936) and Fisher (1922), yielded many important results for the case in which the universe of models is static. Konüs (1939) introduced the concepts of a cost-of-living index and substitution bias in price indexes that price a fixed basket of goods. Frisch (1936) showed how Konüs's substitution bias result implied that for homothetic preferences the change in the true cost of living is bounded from above by the Laspeyres index and from below by the Paasche index. Fisher (1922) showed how the geometric mean of the Laspeyres and Paasche indexes constitutes an ideal index in the sense that both the price and quantity indexes have the same functional form.

A large part of the literature has focused on the question which price index formula approximates (3) in the 'most reasonable' way. Examples of studies along this line are Fisher (1922), Diewert (1976), as well as Lloyd (1975), Braithwait (1980), Manser and McDonald (1988), and Aizcorbe and Jackman (1993).

A much smaller part of the literature has focused on the construction of 'reasonable' approximations to (3) in case of dynamic universes of models. The problem when the universes of models are dynamic is that the prices of new goods are not observed in the first period, while the prices of obsolete goods are not observed in the second period. It is thus not possible to measure the percentage change in the prices between both periods for new and obsolete goods.

Two approaches are generally considered when dealing with this problem. The first, known as matched model indexes, makes specific assumptions about the relative price per quality unit of the new models versus the old models. These assumptions are such that they imply that the change in the overall price level can be estimated solely as a function of the price changes of the models that are sold in both periods, i.e. that are matched. Triplett (2002) contains an overview of the different matched model methods and the possible biases that they induce.

The second, known as hedonic price indexes, uses a regression model that relates the price of a model in a certain period to its characteristics to impute the unobserved prices for the new and obsolete models. This imputation completes the set of prices needed to apply conventional price index methods developed for overlapping universes of models. After the price imputation of the missing price observations, indexes are then constructed using conventional price index methods.

3. A graphical illustration of the main argument

The conventional wisdom is that the introduction and obsolescence of goods in a market would cause standard price index methods to overstate the actual inflation rate. The Boskin (1996) commission report as well as its recent reassessment by Lebow and Rudd (2001) both contain extensive descriptions of this conventional wisdom. There are three main reasons why this is argued to be the case. The first reason, designated *quality bias* by the Boskin (1996) commission, is that current price indexes do not properly capture the quality improvements embodied in new (or improved) models. By underestimating these quality improvements, price indexes will attribute too much of changes in expenditures to changes in prices rather than to changes in quantities. The second reason, designated *product bias* by the Boskin (1996) commission, argues that prices of new goods tend to drop faster than those of established models. Because new goods and models are only included in the sample of goods used to calculate the price index with a certain delay, the initial price drops early in the product cycle are not captured by current price indexes. Finally, there is the *substitution bias*. This bias is due to some price indexes, including the CPI and most price indexes calculated in Europe, being fixed weighted price indexes which do not capture the increases in welfare from consumers being able to substitute new goods for goods that they were previously consuming.

In the rest of this paper I will mainly focus on the *quality bias* and ignore issues related to the latter two sources of bias. In general it is hard to argue against statistical agencies including new models and goods more timely in their samples and reducing the potential sources of product bias. Furthermore, the issue of substitution bias is currently being addressed, at least for the U.S. CPI, by the joint publication of a fixed weighted as well as a chain weighted price index. The latter is meant to account for the substitution bias. See Bureau of Labor Statistics (2002) for a detailed description.

The main point of this paper is that the *quality bias* in price indexes is not solely a source of upward bias. Instead, the quality bias induced by most commonly applied price index methods can be both upward as well as downward. Before I illustrate this in a formal mathematical economic framework, I first describe the main intuition of the argument graphically in this section. The graphical description in this section is based on Figure 1 through Figure 3.

The top panel of Figure 1 depicts two hypothetical price schedules, for $t=0$ and $t=1$, of a set of models that differ according to their quality levels, a_i. I will assume that a_i is not directly observed. Therefore, the researcher observes the price of each model, i.e. p_{it}, but does not know its relative position on the x-axis. As explained in the previous section, what is important for the price level associated with the CES preferences that I consider is not the actual price levels, p_{it}, but the price per unit of quality, p_{it}/a_i, for each model. Panel (b) of Figure 1 depicts the associated schedule of prices per quality unit. Panel (c) contains the same price per quality unit schedule and adds some of the notation that I will use in the rest of this paper.

Just like in the previous section C^*_t denotes the set of models sold in period t, while P_t^T denotes the theoretical price level at time t. Note that I have chosen to draw the example such that $P^T_0<P^T_1$. That is, in the graphical example the actual price level increases between periods $t=0$ and $t=1$, such that there is positive inflation. In each period the set of models sold, i.e. C^*_t, consists of a group of models that are not sold in the other period, i.e. the set A_t-B_t, as well as a group of models that are 'matched' in the sense that they are sold in both periods, i.e. the set B_t-D_t.

What I will now illustrate is that, even though the theoretical price inflation is positive for these hypothetical price schedules, most commonly applied price index methods will tend to measure negative inflation instead. That is, in this graphical example standard price index methods will tend to *underestimate* actual inflation rather than overestimate it, as the consensus view suggests. I will illustrate this for both matched model as well as hedonic price indexes.

There are several ways in which matched model indexes are calculated. They each make different identifying assumptions about the relative price per quality unit of the obsolete and new models in the market.

The first method, often referred to as 'direct comparison', assumes that the obsolete and new models can be directly compared in the sense that they embody the same levels of quality. Because this method assumes that there are no quality improvements between the old and new vintages of models, this method is never applied in markets with rapid product turnover due to technological progress, like those for computers and other electronic products for example. Because I will focus on markets with quality improvements in the products sold, I will disregard this method in the rest of this paper.

The second method, known as 'link-to-show-no-price-change', assumes that the price per quality unit is the same for the obsolete and new models. In this case, the relative price of the obsolete and new models is assumed to be fully attributable to quality improvements. Aizcorbe (2001) uses this assumption for example to identify the parts in semiconductor price changes attributable to quality changes and price changes respectively. Note that, as Triplett (2002) describes in more detail, this method *overestimates* inflation only when the price per quality unit of the new models is lower than that of the obsolete models. In that case the method overestimates the price per quality unit for the new models and thus will overestimate inflation. The reverse is true in our graphical example here. In the example the price per quality unit is higher for the new models than for the old models. Consequently, the method will overestimate quality changes and *underestimate* the actual level of inflation.

The final matched model method that is frequently applied is the "Implicit Price – Implicit Quantity"[2] method (IP-IQ). This method is based on the identifying assumption that the overall price change equals the price change in the set of matched models. When one makes this assumption, the price levels of the unmatched models are not needed to measure inflation. Hence, in this case the unmatched models are ignored, i.e. "deleted", and standard price index methods are applied to the set of models that is sold in both periods, i.e. the matched models.

Figure 2 illustrates the application of the IP-IQ method in our graphical example. The set of matched models in the example is the intersection of C^*_0 and C^*_1. Consequently, the IP-IQ method will compare the B_0-D_0 part of the period *0* price schedule with the B_1-D_1 part of that of period *1*. For all models in this range the prices are falling. The IP-IQ method will thus, incorrectly, find a drop in the overall price level. The simplest way to see why the IP-IQ method underestimates inflation in this case is to compare the relative prices of the deleted sections A_0-B_0 and A_1-B_1 with the matched parts of the price schedules.

For the deleted part A_0-B_0 in period *0* we obtain that the prices per quality unit are lower than the prices per quality unit on the matched part of the schedule, B_0-D_0. Consequently, the deletion of the below average prices on the A_0-B_0 part of the price schedule will lead to an inferred price level in period *0* that is higher than the actual level. Similarly, when the above average prices of part A_1-B_1 are deleted in period *1*, the prices of the matched models, i.e. B_1-D_1, reflect a price level that is lower than the actual price level. That is, because the price per quality unit is increasing in quality and the worst models become obsolete while the new models are of the highest quality, the price level in period *0* is overestimated while the price level in period *1* is underestimated. The combination of these two measurement errors leads to an unambiguous downward bias in the measured inflation rate, independent of which price index formula is applied.

One final thing is worth noting about this argument. That is that the bias incurred due to the application of the IP-IQ method does not depend on the overall inflation rate. Instead, it completely depends on the cross-sectional behavior of the prices per quality unit as a function of the quality units embodied in the models sold in the market. I will prove this in a more formal example later on. This result contrasts sharply with the argument in Triplett (2002) who argues that "The errors produced by the IP-IQ method are symmetric, in the sense that when prices are falling the IP-IQ method tends also to miss price declines. … Prices have generally been falling for electronic products, including IT products. When the IP-IQ method is used to construct price indexes for electronic products, the price indexes are biased upward because they do not adequately measure price declines that accompany new introductions". The example here suggests that

[2] This is also often referred to as the "deletion" method.

what matters for the IP-IQ bias in IT product inflation is not whether prices are *declining over time* but rather whether prices per quality unit are *declining in the amount of quality embodied* in the models.

Hedonic price indexes are, in some sense, the opposite of IP-IQ matched model indexes. That is, where the IP-IQ method 'deletes' the observed prices of the unmatched models, hedonic methods 'insert' the unobserved prices of the unmatched models. This insertion, or more correctly 'imputation', is done by estimating a hedonic price equation that relates the price of a model in a particular period to a set of its quality characteristics and then using this equation to predict what the unobserved prices of the unmatched models would have been.

Over the past five years, hedonic price indexes have been implemented for an increasing number of goods for U.S. aggregate statistics. See Landefeld and Grimm (2000) as well as Moulton (2001), for example, for a discussion of the application of hedonic price indexes in the U.S. national accounts. The main reason why hedonic price indexes are adopted for an increasing number of goods is the practical problem that the IP-IQ method ends up not using a large part of the available price quotes in markets where new and obsolete models make up the bulk of models traded. This is particularly a problem for computers and related equipment.

The believe is that by taking the price data for the obsolete and new models into account and relating them directly to quality characteristics, hedonic price indexes more properly adjust for quality and are less subject to quality bias. This seems to be confirmed by the fact that hedonic price indexes tend to find less inflation for most of the goods to which they are applied[3] than standard matched model indexes, which are said to overestimate inflation.

Is it true that hedonic price indexes have a smaller quality bias than matched model indexes? Not necessarily. In order to see why not, consider Figure 3. Which prices are imputed in a hedonic price index depends on the price index formula applied. The two panels of Figure 3 depict the two most common cases.

The top panel considers a hedonic Laspeyres index, which intends to measure the percentage change in the cost of the models sold in period *0*. The Laspeyres index requires the use of the prices of the models that became obsolete in period *1*. Therefore, a hedonic regression model is used to impute these prices and the price schedule in period *1* is extended by the imputed part D_1-E_1. The Laspeyres index then basically approximates the change in the overall price levels implied by the curves A_0-D_0 and B_1-E_1. The overall price level implied by A_0-D_0 coincides with the actual price level in period *0*, i.e. P_0^T. The price level implied by B_1-D_1, denoted by P_1^{HL} in the figure, is lower than the actual price level in period *1*. The reason is that for the calculation of the Laspeyres index the above average prices per quality unit in the part A_1-B_1 are ignored. Moreover, the imputation adds below average prices per quality unit in the section D_1-E_1. Hence, the inferred price level in period *1* is below the actual price level and inflation is underestimated. In fact, because the A_0-D_0 schedule is above the B_1-E_1 schedule everywhere, in this example the hedonic price index would find spurious price deflation.

The bottom panel depicts the calculation of a hedonic Paasche index. It is meant to approximate the change in the cost of the models sold in period *1*. Therefore it requires the imputation of the D_0-E_0 part of the

[3] See for example Gordon's (1990) hedonic price indexes.

price schedule in period *0* and ignores the part A_0-B_0 in its calculation. The imputed part D_0-E_0 consists of above average prices per quality unit and the ignored part A_0-B_0 of below average prices per quality unit. This leads to the hedonic method overestimating the price level in period *0*. This estimate is denoted by $P^{HP}{}_0$. Again, the hedonic method will find spurious deflation. This time because it overestimates the price level in period *0*, rather than underestimates the price level in period *1*.

Thus, my tentative graphical example illustrates why matched model and hedonic methods might actually result in estimates of inflation that are *too low* rather than *too high*. However, this simple graphical example can only be used for illustration purposes, it does not prove that such biases might occur in the data. In order to show that these biases are likely to occur, I introduce a fairly standard theoretical model in the next section and show how the equilibrium outcome of the model gives rise to biases of the same kind as discussed here.

4. Theoretical model

The aim of this section is to introduce a simple theoretical framework that generates the kind of bias that I discussed in the section above. The theoretical framework introduced here is based on the CES model considered by Anderson, de Palma, and Thisse (1992). Feenstra (1995) applied this model to hedonic price indexes. I will introduce the theoretical model in three subsections. The first explains the demand side of the market, while the second focuses on the supply side of the market. In the final subsection, I will prove existence and uniqueness of the Pure Strategy Nash equilibrium that determines prices and quantities in the market and will derive some of the relevant comparative statics for this equilibrium.

Demand side of the market

Aggregate demand in this market can be represented as generated by a representative agent choosing the demand $\{X_{i,t}\}_{i\in C_t}$ to maximize the aggregate utility function in equation (1). This utility function is maximized subject to the budget constraint

$$Y_t = X_{0,t} + \sum_{i\in C_t^*} p_{i,t} X_{i,t} \tag{4}$$

where Y_t denotes real income in terms of the numeraire commodity and $p_{i,t}$ is the price of commodity i in terms of the numeraire good $X_{0,t}$.

The maximization of this utility function yields the demand functions

$$X_{i,t} = \left(\frac{\widetilde{Y}_t}{p_{i,t}}\right)\left(\frac{\left(p_{i,t}/a_i\right)^{-1/\lambda}}{\sum_{j\in C_t^*}\left(p_{j,t}/a_j\right)^{-1/\lambda}}\right) = \left(\frac{\widetilde{Y}_t}{p_{i,t}}\right)\left(\frac{\left(p_{i,t}/a_i\right)^{-1/\lambda}}{\left(P_t^T\right)^{-1/\lambda}}\right) \tag{5}$$

for the non-numeraire commodity, i.e. $i\in C^*_t$. The variable $\widetilde{Y}_t = Y_t/(1+\alpha)$ is the level of total expenditures on these models. These demand functions are very similar to the ones implied by standard CES preferences where the level of quality for all goods is the same, i.e. $a_i=1$ for all $i\in C^*_t$. The main difference is that the relevant relative price of each good that determines its market share is its price per unit of quality, that is p_{it}/a_i, rather than its unit price, p_{it}.

Supply side of the market

The next concern is the supply side of the market for $i \in C^*_t$. I will assume that the producer of model i at each point in time, t, faces a constant unit production cost c_{it}. I will consider Pure Strategy Nash equilibria in prices for a market with a fixed set of models, $\mathbf{a}_t = \{a_i\}_{i \in C^*_t}$. Such Nash equilibria imply that the supplier of model i takes as given the prices p_{jt} for $j \in C^*_t \backslash \{i\}$ and chooses its price p_{it} to maximize its profits

$$(p_{it} - c_{it})X_{it} \tag{6}$$

subject to the demand function (5). The profit maximizing choice of price p_{it} in this case satisfies the following first order condition.

$$\frac{c_{it}}{p_{it}} = 1 + \left[\frac{\partial X_{it} / \partial p_{it}}{X_{it} / p_{it}}\right]^{-1} \tag{7}$$

This condition implies that the supplier of each model chooses its price such that its cost-price ratio equals one plus the inverse of the own price elasticity of demand for good i.

Since the own price elasticity of demand for good i is negative, this implies that $c_{it}/p_{it}<1$. That is, price exceeds marginal and average cost and the firm charges a markup. For the price elasticity of demand, we obtain that

$$\frac{\partial X_{it} / \partial p_{it}}{X_{it} / p_{it}} = -\left(1 + \frac{1}{\lambda}\left(1 - \frac{(p_{it}/a_{it})^{-1/\lambda}}{\sum_{j \in C^*_t}(p_{jt}/a_{jt})^{-1/\lambda}}\right)\right) = -\theta_i(\mathbf{p}_t, \mathbf{a}_t) \tag{8}$$

where $\theta_i(\mathbf{p}_t, \mathbf{a}_t)>0$ is the negative of the price elasticity of demand for good i and $\mathbf{p}_t = \{p_{it}\}_{i \in C^*_t}$ is the sequence of prices charged in the market. Essential for the results that are to follow is that this elasticity is specific to good i. This is contrary to the setup of monopolistic competition that is often used to model imperfect competition in models with price rigidities, like in Hornstein (1993). These models generally consider a symmetric equilibrium in which each monopolistic competitor is too small to affect the aggregate price level and its own price elasticity of demand.

Using the notation above, the supplier of good i will set its price such that

$$\frac{p_{it}}{c_{it}} = \frac{1}{1 - \theta_i(\mathbf{p}_t, \mathbf{a}_t)^{-1}} = \frac{\theta_i(\mathbf{p}_t, \mathbf{a}_t)}{\theta_i(\mathbf{p}_t, \mathbf{a}_t) - 1} = \mu_i(\mathbf{p}_t, \mathbf{a}_t) \tag{9}$$

where $\mu_i(\mathbf{p}_t, \mathbf{a}_t) > 1$ is the markup charged by the firm. Solving for this markup yields that

$$\mu_i(\mathbf{p}_t,\mathbf{a}_t) = (1+\lambda) + \lambda \frac{(p_{it}/a_i)^{-1/\lambda}}{\sum_{j\in C_t^*\setminus\{i\}} (p_{jt}/a_j)^{-1/\lambda}} \tag{10}$$

This implies that the pure strategy Nash equilibrium in this market satisfies the following system of equations

$$p_{it} = \left[(1+\lambda) + \lambda \frac{(p_{it}/a_i)^{-1/\lambda}}{\sum_{j\in C_t^*\setminus\{i\}} (p_{jt}/a_j)^{-1/\lambda}} \right] c_{it} \text{ for all } i\in C^*_t \tag{11}$$

This system of equations will be the center of attention in what is to follow.

Equilibrium

Now that I have derived the conditions for a Pure Strategy Nash equilibrium in equation (11), the question that remains is whether there exists a set of prices the satisfies this equation. In this section I will not only show that this is the case, but also prove the uniqueness of this equilibrium price schedule. I will then proceed by deriving some of its comparative statics that are relevant for the price index measurement results that I will prove later on.

First and foremost though, it is important to realize that the Pure Strategy Nash equilibrium in prices that I consider actually exists and is unique. This is what I prove in the following proposition.

Proposition 1: ***Existence and uniqueness of equilibrium***

For any $\lambda>0$ and sequences $\mathbf{a}_t = \{a_i\}_{i\in C_t^*}$ and $\mathbf{c}_t = \{c_{it}\}_{i\in C_t^*}$ where $a_i, c_{it}>0$ for all $i\in C^*_t$ there exists a unique Pure Strategy Nash equilibrium in prices.

The benchmark case, and as it turns out the only one in which standard price index methods do not generate a bias, is the case in which each supplier faces the same unit production cost per quality unit. As I show in the proposition below, the price per quality unit is the same for all models in the market in that case.

Proposition 2: ***Symmetric equilibrium***

The market has a symmetric equilibrium in which the price per quality unit is constant across models, i.e. $p_{it}=p^*_t a_i$ for all $i\in C^*_t$, if and only if the producer of each model faces the same marginal unit production cost per quality unit, i.e. $c_{it}=c^*_t a_i$.

In the previous section I argued that the bias that I illustrated graphically was the result of the price per quality unit not being constant across models sold in the market. In fact, in the example, the price per quality unit was higher for better models. In the symmetric equilibrium derived above the price per quality unit is constant and it is thus unlikely that this equilibrium will yield a bias of the sort described before. However, if the marginal production cost per quality unit is not the same across models sold in the market, then neither is the price charged per quality unit. In that case the market equilibrium will be asymmetric in the sense the models will have different market shares. As I show in the following proposition, the suppliers that produce the models with the higher marginal production cost per quality unit will charge a higher price per quality unit and will have a lower market share.

Proposition 3: ***Properties of asymmetric equilibrium***

In the asymmetric equilibrium, producers with higher marginal production costs per efficiency units, i.e. c_{it}/a_i*, (i) charge a higher price per efficiency unit,* p_{it}/a_i*, and (ii) a lower markup,* p_{it}/c_{it}*.*

The above result is important because it suggests that any asymmetric equilibrium exhibits prices per quality unit that are unequal across the models sold in the market and thus has the potential of generating the bias described in the previous section.

5. Price index bias in the theoretical model

Now that I have developed the theoretical model of this market, it is time to consider what conventional price index methods would measure in this market. In order to illustrate the quality bias it is essential to consider dynamic universes of goods such that

$$C_0^* \neq C_1^* \text{ and } C_0^* \cap C_1^* \neq \varnothing \tag{12}$$

In principle, there are many ways in which the set of models traded in the market can change and each of these changes might have a different effect in the theoretical example considered here. Because it is simply impossible to consider all of these different cases, I will limit myself to one specific example. In the first subsection, I will describe the parameterization of this example in detail. Then, in the second subsection, I will consider what happens when standard price index methods are applied in this example.

Parameterization of example

The example that I will consider is one where the model at 'the bottom of the line' in period $t=0$ becomes obsolete in period $t=1$ and in which in period $t=1$ a new 'top of the line' model is introduced. The 'bottom of the line' model at $t=0$ is the lowest-quality model, i.e. the one with the smallest a_i among all $i \in C^*_0$. The 'top of the line' model introduced in period $t=1$ is such that its quality exceeds that of all models traded in period $t=0$.

Consequently, in both periods the same number of models is sold[4]. I will denote this number by $N=N_0=N_1$. I will index the models as $i=1,\ldots,N+1$, where model 1 is the 'bottom of the line' model that becomes obsolete at time $t=1$ and model $N+1$ is the new 'top of the line' model introduced at $t=1$. This indexation implies that $C^*_0=\{1,\ldots,N\}$ and $C^*_1=\{2,\ldots,N+1\}$.

[4] Price indexes also have a problem measuring the increased utility from the availability of more models. This is known as variety bias. I will abstract from variety bias throughout this paper.

Two things are still to be defined. The first is the parameterization of the quality levels $\{a_i\}_{i=1}^{N+1}$. I will assume that quality is increasing in i such that

$$a_i = (1+g)^{i-1} \tag{13}$$

where $g>0$ represents the quality growth rate across models.

The second is the parameterization of the unit production costs $\{c_{it}\}_{i=1}^{N+1}$. The parameterization that I will choose for these unit production costs is

$$c_{it} = c_t^* a_i^{1+\gamma} \tag{14}$$

This parameterization is such that if $\gamma=0$ then the production costs per quality unit are identical across models and the equilibrium is symmetric. If $\gamma<0$ then the production costs per quality unit are lower for better models and their suppliers will charge a lower price per quality unit and a higher markup in equilibrium, as shown in proposition 3. Similarly, if $\gamma>0$ then production costs per quality unit are higher for better models and, as in the graphical example, the price per quality unit is higher for better models. Hence, γ represents the steepness of the cross-model production costs per quality unit schedule. Because of proposition 3, this implies that γ also represents the steepness of the cross-sectional price per quality unit schedule.

I will parameterize the change of c^*_t over time as follows. Let $\bar{a}_t = \prod_{i \in C_t^*} a_t^{1/N}$ then I will assume that

$$c_t^* = \frac{\tilde{c}_t}{\bar{a}_t^{\gamma}} \text{ where } \tilde{c}_1 = (1+\pi)\tilde{c}_0 \tag{15}$$

The reason that I parameterize c^*_t like this is because, in equilibrium, the structural parameter π has a specific interpretation. This is proven in the following proposition.

Proposition 4: ***Interpretation of structural parameter*** $\boldsymbol{\pi}$

In equilibrium, the structural parameter π equals the theoretical inflation rate, i.e. $\pi=\pi^T$.

Given this parameterization, the question is how estimated inflation on the basis of the various price index methods depends on the underlying structural parameters, N, g, π, γ and λ and how it compares to the actual level of inflation, π^T. This question is addressed in the next subsection.

Quality bias

Just like in the graphical example of section 3, I will first address the bias induced by matched model indexes and then consider hedonic price indexes in this theoretical model.

For the matched model price indexes I will solely consider the, most frequently used, IP-IQ method. The following proposition states the properties of the IP-IQ linked matched model indexes in this example.

Proposition 5: ***Matched model index properties***

An IP-IQ linked matched model index yields an estimate of inflation, π^M, that has the following properties:

(i) $\pi^M = \pi^T$ *if* $\gamma=0$.

(ii) $\pi^M > \pi^T$ *if* $\gamma<0$.

(iii) $\pi^M < \pi^T$ *if* $\gamma>0$.

This result does not depend on which of the price index formulas (except the Feenstra (1994) index which is exact) is applied.

This proposition is the formal mathematical proof of the informal argument that I stated with respect to matched model price indexes for the graphical example in section 3. That is, the sign and magnitude of the quality bias in matched model price indexes *does not depend on the sign and magnitude of the overall inflation rate*. Instead, *it depends on the cross-sectional behavior of prices per quality unit for the models sold* in the market.

That the bias does not depend on the sign and magnitude of the overall inflation rate follows directly from the fact that the result in proposition 5 does not depend on the structural parameter π. The dependence of the bias on the steepness of the cross-sectional schedule of prices per quality unit across models is implied by the bias in the matched model indexes only depending on the parameter γ.

That is, if $\gamma>0$ then, according to proposition 3, the price per quality unit is increasing in the level of quality embodied in the model. This is the case depicted in the graphical example of section 3 and is the case that yields a *downward* bias in the measured inflation rate. If $\gamma<0$ then the reverse is true.

So, how do hedonic price methods behave in the theoretical model here? This question can only be answered conditional on the behavior of the imputed price levels. I do so in the next proposition.

Proposition 6: ***Hedonic price index properties***

A hedonic price index yields an estimate of inflation, π^H, that has the following properties:

(i) $\pi^H = \pi^T$ *if* $\gamma=0$.

(ii) $\pi^H > \pi^T$ *if* $\gamma<0$, *if the imputed prices satisfy the property of the equilibrium price schedule that prices per quality unit are decreasing in the quality embodied in the model.*

(iii) $\pi^H < \pi^T$ *if* $\gamma>0$, *if the imputed prices satisfy the property of the equilibrium price schedule that prices per quality unit are increasing in the quality embodied in the model.*

Just like in proposition 5, this result does not depend on which of the price index formulas (except the Feenstra (1994) index which is exact) is applied.

The proof of the proposition above gives some interesting insights. First of all, the hedonic price indexes only yield an unbiased estimate of inflation whenever the equilibrium is such that the price per quality unit is constant across the models traded in the market. However, if the price per quality unit is constant across models, then matched model indexes will do just fine. In fact, if the price per quality unit is constant across the models sold in the market, then one can simply measure overall inflation by considering the percentage price change of a single model. That is, when the price per quality unit is constant across the models sold in the market quality bias is not an issue. This itself is an important observation.

Bils and Klenow (2001) for example use microdata from the Consumer Expenditure Survey to estimate the quality bias in the CPI for several durable consumption goods. They do so by estimating a structural model of durable goods consumption. In order to quantify quality growth in this model, however, they assume that independent of each household's expenditures on a particular durable consumption good, the price paid per quality unit is constant for all households. Hence, no matter what model the households are buying, they are assumed to pay the same price per quality unit. This means that Bils and Klenow (2001) implicitly assume that the price per quality unit is constant across models. However, if this identifying assumption would be true in the data then the BLS would have had no problem quantifying quality growth in the first place.

If the price per quality unit is constant, then relative prices represent relative quality differences. In that case the coefficients in the hedonic regression model will represent the marginal quality coefficients of the quality indicators. Feenstra (1995) shows that when these coefficients represent these marginal values, hedonic price indexes will work properly. In fact, for certain classes of preferences Feenstra (1995) derives exact hedonic indexes. However, when he considers the existence of markups he also observes that when this is not the case then the estimated hedonic regression coefficients might over- or underestimate the quality difference between the models.

This is the case when $\gamma>0$ and $\gamma<0$. In those cases hedonic regression coefficients do not only reflect the marginal quality differences between the models but also reflect the slope of the price per quality unit schedule.

Log-linearized approximation of the bias

In order to consider how the size of the bias depends on the parameters, it turns out to be illustrative to consider the log-linear approximation of the bias around the symmetric equilibrium derived in Proposition 2.

Proposition 7: ***Log-linearization of the bias***

The log-linear approximation of the equilibrium around the symmetric equilibrium of Proposition 2 yields that both matched model as well as hedonic price indexes are subject to a bias equal to -θg, where

$$\theta = \gamma\left[1 - \frac{N}{1+(1+\lambda)N(N-1)}\right] \tag{16}$$

and does not depend on the actual inflation rate π.

What constitutes this bias? The reason for this bias is that the price index methods can not distinguish between a movement in the price per quality unit schedule over time due to an actual change in the overall price level and a move in the schedule because the introduction of a new model shifts the relative competitive advantages (in production and the market) and thus prices of the models sold in the market.

In order to see this, consider Figure 4. It disentangles the various effects on the schedule of the logarithm of the price per quality as a function of the logarithm of the quality. That is, it graphically represents the first difference of (59) over time and the various things that influence it.

Consider model i at time $t=0$. It has price p_{i0}, such that the logarithm of its price per efficiency unit is $ln(p_{i0}/a_i)$. At time $t=0$ it is at point A on the log price per quality unit schedule. For expositional purposes, I have drawn this graph for $\pi<0$ and $\gamma>0$. The drop in the overall price level $\pi<0$ shifts the log of the price per quality unit of model i down from point A to point B. However, something else happens at the same time as well. That is the introduction of the new model $N+1$ and the exit of model 1.

Because of the introduction of the new, superior, model and the fact that production costs are increasing in the quality embodied in the model, the production costs of model i relative to those of its competitors will drop. This allows the supplier of model i to charge a lower price per quality unit than in period $t=0$. In fact, because of the setup of the model, model $i+1$ takes over model i's position in the relative quality ladder in period $t=1$. Therefore, in period $t=1$ model $i+1$ will be sold at the same relative price per quality unit that model i was sold at in period $t=0$. This is depicted in Figure 4 by the horizontal shift from B to C. The slope of the log price per quality unit schedule, i.e. θ, and the length of the horizontal shift, i.e. g, then jointly determine how far below $ln(p_{i0}/a_i)+\pi$ the logarithm of model i's price per quality unit in period $t=1$, i.e. $ln(p_{i1}/a_i)$, ends up.

Hence, in terms of this Figure 4, the problem of the price index methods is that they do not distinguish between the actual change in the overall price level, depicted by the shift from A-B, and the effect of the shift in the relative qualities of the models due to the introduction of a new model, depicted by the movement from B-D.

6. Data based simulation: CPU prices

So far, the point that I made is purely theoretical and I haven't addressed its empirical relevance. This section is intended to do so. In this section I illustrate how the effects described above would lead to, mostly downward, bias in measured inflation for CPU prices.

Experiment setup

The approach that I will take is as follows. I will use weekly price data for CPU prices to obtain an empirical set of prices, p_{it}. Furthermore, I use data on benchmark test performance of these CPUs as a measure of their quality, a_i. Using these two datasources[5], for each week I have a set of data on price per quality unit, p_{it}/a_i. The data do not contain any information on market shares of the different processors. I will simulate these data myself under the assumption that demand in the market is generated by the same CES preferences used in the theoretical model, i.e. equation (1). This requires the choice of the elasticity of substitution $\sigma=(1+\lambda)/\lambda>1$.

[5] The data are described in detail in Appendix B.

The result is that for various elasticities of substitution, I simulate a sequence of weekly cross-sectional samples of CPU prices and CPU market shares that are consistent with the preferences in (1). The nice thing of this simulation is that I know exactly the implied path of the theoretical price level. This then allows me to compare the price path estimated using conventional price index method with the path of the theoretical price level and to assess the sign and magnitude of the bias induced by these index methods.

Before considering the details of the results of this simulation, it turns out to be illuminating to first consider the behavior of prices per quality unit, i.e. p_{it}/a_i, for the CPUs in the sample. The sample covers about one and a half years, starting in 10/28/01 and ending 03/17/03. Figure 5 depicts the empirical equivalent of panel (b) of Figure 1. It plots the price per efficiency unit, i.e. the price per benchmark unit, as a function of the benchmark ratings. As can be seen from Figure 5, the price per efficiency unit schedule is increasing in the quality of the processor, both at the beginning and the end of the sample. These periods are no exception. In fact, the price per benchmark unit is increasing in the benchmark rating for all weeks in the sample.

The general cross-sectional pattern of CPU price declines is as follows. The prices of CPUs with the highest benchmark ratings tend to decline the fastest, while those on the bottom end of the range of CPUs sold barely change. The small price changes of CPUs at the bottom end of the spectrum might be partly induced by the way the data are collected. Probably, prices of cheap CPUs are still quoted for a while after they are sold. Anyway, because of this pattern of price declines we know beforehand that methods that weigh the price changes of the bottom end models more will find less price declines than methods that put a higher weight on the upper-end models.

The two observations above suggest the following. First, the fact that the price per benchmark unit is increasing in the benchmark rating creates the potential for price index methods to underestimate the price changes over the sample period. That is, quality bias for the CPU price index calculated based on these data is most likely *negative* rather than *positive*. Secondly, the different rates of price declines of low- and high-end CPUs suggest that price indexes that use different goods baskets might actually diverge over the sample period.

In order to consider how these effects manifest themselves in the data, I calculate the chained weekly CPU price indexes using the market shares implied by different values of the elasticity of substitution. The value for which the experiment is performed is 1.5, 2, 4, and 8. I have chosen to present two sets of results.

The first set calculates the price indexes for a varying number of models sold in the market. Because the CES preference of equation (1) satisfy Inada conditions, the availability of an additional CPU will always lower the price level. The effect of the number of goods on the price level is often referred to as the variety bias. The number of models in the weekly samples fluctuates between 37 and 50. The number of models at the beginning of the sample period was 46, which is 6 higher than at the end. Because of this, the number of models traded induces an upward trend in the price level that is not captured by the price indexes.

The second set of results calculates the price index for a constant number of models sold in the market. For these results I selected the 37 highest priced models in the market in each period and assumed that the market shares of the other models were zero. Because of this way of constructing the sample, this case does not suffer from the variety bias described above.

Results

Table 2 presents the results obtained for both cases. The first line of the table lists the simulated change in the actual price level over the sample period for the four different elasticities of substitution. As can be seen from this line, the decrease in the number of models reduces the magnitude of the implied price declines. That is, price declines with variety bias are smaller than those without variety bias.

The subsequent twelve lines list the estimated price changes over the sample period for both IP-IQ matched model indexes as well as hedonic price indexes using six commonly used price index formulas. These formulas are explained in detail in Table 1. I will first focus on the results for the matched model indexes and then discuss the results for the hedonic price indexes. Because the topic of this paper is the quality bias rather than variety bias in price indexes, I will mainly rely on the results without variety bias in the following.

When one considers the estimated price changes using the hedonic price indexes for the case without variety bias, one thing is immediately obvious. That is that for most elasticities of substitution all the price index formulas estimate a price decline that is larger than actually observed for the theoretical price level. That is, contrary to the consensus view, all the price indexes tend to underestimate CPU price deflation.

The downward bias is actually quite large. For the six price index formulas applied, the downward bias in the index varied between 18% and 24% in case of an elasticity of substitution of 1.5, between 12% and 21% for an elasticity of substitution of 2, between 1% and 20% for the elasticity of substitution of 4 and between –12% and 24% for the elasticity of substitution of 8. The downward bias in the price indexes is the sum of two different biases, namely the substitution bias and the quality bias.

The substitution bias has been extensively studied in the literature. Ever since Konüs (1939) and Frisch (1936) it is well known that price indexes based on the goods basket at the beginning of the period, like the Laspeyres and Geometric (G0) indexes, tend to overestimate inflation because they do not take into account the possibility of substituting away from the initial goods basket in response to relative price changes. On the other hand, price indexes that are based on the basket of goods at the end of the period, like the Paasche and Geometric (G1) indexes, tend to underestimate the actual inflation rate because they do not take into account the possibility of substituting away from the final goods basket in the initial period if relative prices made this desirable.

In order to minimize the potential substitution bias, superlative price indexes, like the Fisher and Tornqvist indexes, are often used. These indexes are exact indexes for a second order approximation to any arbitrary continuously differentiable utility function.

For the results in Table 2 it is important to realize that the substitution bias does not always work in the same direction. It pushes up the CPU price inflation estimated by the Laspeyres and Geometric (G0) indexes, because of which these find smaller price declines than the other index formulas. I pushes down the CPU price inflation estimate by the Paasche and Geometric (G1) indexes, because of which these tend to find larger CPU price declines than the other index formulas. For the superlative indexes, i.e. the Fisher and Tornqvist indexes, the substitution bias should be relatively small. Therefore, downward bias in the inflation rate estimated by these indexes can reasonably be solely attributed to the quality bias.

Taking the biases in the matched model Fisher and Tornqvist indexes as an estimate of the quality bias suggests that the quality bias in the matched model CPU price index is negative and large. It varies between approximately -20% for an elasticity of substitution of 1.5 to -9% for an elasticity of substitution of 8. This implies that a big chunk of the measured price declines in matched model CPU price indexes is spurious and due to quality bias.

It turns out that hedonic price indexes do not fare any better than the matched model indexes.

The hedonic price index results presented in Table 2 are calculated using imputed prices based on weekly log-log regressions in which the log of the CPU price is regressed on an intercept, an Athlon dummy, the logarithm of the clock speed (if applicable), the logarithm of the Athlon speed variable (if applicable), the logarithm of the bus speed, and the logarithm of the manufacturing process wiring width. The definition of these variables is explained in more detail in Appendix B. The estimated parameters for these hedonic regressions turn out to be fairly constant over the sample period and the R^2 varies between 0.69 and 0.83. The coefficients on the bus speed and the manufacturing process variables are insignificant for almost all of the sample period. Thus, the regression results seem to suggest that it is clock speed that is the driving force behind CPU prices.

This specification of the hedonic regression results in the imputation of prices that imply relatively high price declines for models that become obsolete in a period compared to other models on the lower end. Because of this, the hedonic Laspeyres and Geometric (G0) indexes, which weigh such obsolete models in their index calculation, find higher price declines than their matched model counterparts. The opposite is true for the imputed prices for new models at the high-end of the range of models sold. Consequently, the hedonic Paasche and Geometric (G1) indexes find less price declines than their matched model counterparts. On balance, though, the hedonic measures find slightly larger price declines than the matched model indexes. This can be seen from the two superlative indexes, i.e. the Fisher and Tornqvist.

Thus, just like in many other applications, for the example of CPU prices here the hedonic price indexes tend to measure less inflation than the matched model indexes. Although, the difference is not very large. However, because they measure less inflation, this does not imply that they measure inflation better. In fact, they are subject to a bigger quality bias than matched model indexes.

This is an important result because a large part of the literature on investment specific technological change[6] uses Gordon's (1990) hedonic equipment price index as a measure of the path of the 'true' quality adjusted equipment price. This is based on the premise that standard price indexes tend to overestimate inflation and that hedonic price indexes do a better job adjusting for quality. Consequently, the smaller equipment price inflation implied by Gordon's index relative to more standard indexes like those published by the Bureau of Economic Analysis is interpreted as quality improvements that go unmeasured in the standard indexes. The results in this paper suggest that the premise on which these studies are based might not be correct and that hedonic price indexes might actually exhibit a larger negative quality bias than matched model indexes.

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[6] See, for example, Greenwood, Hercowitz, and Krusell (1997), Cummins and Violante (2001), and Violante, Ohanian, Ríos-Rull, and Krusell (2000).

7. Conclusion

In this paper I argued that the quality bias in price indexes does not necessarily always bias them upwards. I illustrated how the sign and the magnitude of this bias depend on the cross-sectional behavior of prices per quality unit across the models sold in the market. I did so by introducing a theoretical model that generated a quality bias in inflation as measured using the most common price index methods. The three main points that can be taken away from the analysis here are.

First and foremost, the quality bias can be both positive and negative. The sign of the bias does *not* depend on the actual underlying overall inflation rate. Instead, it solely depends on the cross-sectional behavior of prices per quality unit.

Secondly, the bias does not depend on which of the many proposed price index formulas are used to calculate the index. Laspeyres, Paasche, Geometric mean, Fisher Ideal, Tornqvist, and Sato indexes all performed is a similar manner in the theoretical model in this paper.

Finally, hedonic price indexes do not necessarily reduce the quality bias. In the examples in this paper, hedonic methods did just as poorly as matched model indexes. However, other examples, like the one given in Hobijn (2001), suggest that they might actually do worse in some cases.

This result is important because the application of hedonic price indexes seems to gain momentum both with statistical agencies, see Moulton (2001) for example, as well as with researchers. In fact, an extensive recent research agenda, including Greenwood, Hercowitz, and Krusell (1997), Violante, Ohanian, Rios-Rull, and Krusell (2000), and Cummins and Violante (2002), has been using Gordon's (1990) hedonic equipment price index as a measure of the 'true' quality adjusted price change for equipment in the U.S.. However, the results here suggest that one has to be careful in using this hedonic price index as such a benchmark. Simply because it measures less equipment price inflation than price indexes published by the Bureau of Economic Analysis and Bureau of Labor Statistics, does not necessarily mean it adjusts better for quality.

The results in this paper provide additional insights in which type of competitive circumstances are suspect to generating a bias, up or down, in the price indexes we calculate. Future research could focus on empirical tests of these conditions and on identifying in which markets what bias is the most likely to occur. At least it seems that the conventional wisdom that the quality bias biases measured inflation upward deserves a more thorough empirical verification.

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A. Proofs of propositions

Proof of proposition 1: (*existence*) Existence of the pure strategy Nash equilibrium follows from the application of Brouwer's fixed point theorem. In order to see how Brouwer's fixed point theorem applies here, it is most convenient to define $c^*_{it}=c_{it}/a_i$. Rewrite the system of equations that defines the equilibrium, i.e. (11), in the form

$$\left(\frac{p_{it}}{c^*_{it}a_i}\right)=\left[1+\lambda\frac{\sum_{j\in C^*_t}\left(c^*_{jt}\right)^{-1/\lambda}\left(p_{jt}/c^*_{jt}a_j\right)^{-1/\lambda}}{\sum_{j\in C^*_t}\left(c^*_{jt}\right)^{-1/\lambda}\left(p_{jt}/c^*_{jt}a_j\right)^{-1/\lambda}-\left(c^*_{it}\right)^{-1/\lambda}\left(p_{it}/c^*_{it}a_i\right)^{-1/\lambda}}\right]\quad \text{for all } i\in C^*_t \tag{17}$$

which implies that

$$\left(\frac{p_{it}}{c_{it}}\right)^{-1/\lambda}=\left[1+\lambda\frac{\sum_{j\in C^*_t}\left(c^*_{jt}\right)^{-1/\lambda}\left(p_{jt}/c_{jt}\right)^{-1/\lambda}}{\sum_{j\in C^*_t}\left(c^*_{jt}\right)^{-1/\lambda}\left(p_{jt}/c_{jt}\right)^{-1/\lambda}-\left(c^*_{it}\right)^{-1/\lambda}\left(p_{it}/c_{it}\right)^{-1/\lambda}}\right]^{-1/\lambda}\quad \text{for all } i\in C^*_t \tag{18}$$

Let N_t be the number of elements of C^*_t, i.e. the number of competing models in the market at time t. Define

$$v_{it}=\left(p_{it}/c_{it}\right)^{-1/\lambda} \text{ and the space } V_t=\left\{\{v_{it}\}_{i\in C^*_t}\in \mathbf{R}^{N_t}\middle| 0\le v_{it}\le 1\right\} \tag{19}$$

then (18) defines a continuous mapping from V_t to V_t and thus, according to Brouwer's fixed point theorem must have a fixed point. Hence, there must exist an equilibrium.
(uniqueness) Define

$$z_{it}=\left(c^*_{it}\right)^{-1/\lambda}\ ,\ w_{it}=z_{it}\left(\frac{p_{it}}{c_{it}}\right)^{-1/\lambda}\text{, and } W_t=\sum_{i\in C^*_t}w_{it} \tag{20}$$

then (18) can we rewritten as

$$w_{it}=z_{it}\left[1+\lambda\frac{W_t}{W_t-w_{it}}\right]^{-1/\lambda}\quad \text{for all } i\in C^*_t \tag{21}$$

Given W_t, for all $i\in C^*_t$, there is one unique $w_{it}\in \mathbf{R}_+$ that solves (21). This follows from a straightforward application of the intermediate value theorem to (21). Define the function $f\colon [0,W_t]\rightarrow \mathbf{R}_+$ as

$$f\left(w_{it}\right)=w_{it}-z_{it}\left[1+\lambda\frac{W_t}{W_t-w_{it}}\right]^{-1/\lambda} \tag{22}$$

then $f(w_{it})$ is continuous and strictly increasing. Furthermore, $f(0)=-z_{it}[1+\lambda]^{-1/\lambda}$ and $f(W_t)=W_t$. Hence, the intermediate value theorem implies that there must be a unique $w_{it,}\in [0,W_t]$ for which $f(w_{it})=0$.

Suppose the equilibrium is not unique, then there exist W_t and W'_t such that $W_t > W'_t = (1+\delta)W_t$, where $\delta > 0$, such that

$$W_t = \sum_{i \in C_t^*} w_{it} \text{ and } W'_t = \sum_{i \in C_t^*} w'_{it} \tag{23}$$

and W_t and w_{it} for all $i \in C^*_t$ satisfy (21), which is also true for W'_t and w'_{it} for all $i \in C^*_t$.

Note that the reason that W_t and W'_t can not be the same is because I have shown above that the same W_t will lead to the same best response by the suppliers of all models and thus to the same equilibrium.

What I will show in the following is that if (21) holds for W_t and w_{it} for all $i \in C^*_t$, then for all $i \in C^*_t$ it must be the case that the w'_{it} that satisfies (21) given W'_t has to satisfy $w'_{it} < (1+\delta)w_{it}$. However, this would imply that $W'_t < (1+\delta)W_t = W'_t$ which is a contradiction.

In order to see this, suppose that $w'_{it} \geq (1+\delta)w_{it}$. In that case, equation (21) implies that

$$\begin{aligned} w'_{it} &= z_{it}\left[1+\lambda\frac{W'_t}{W'_t - w'_{it}}\right]^{-1/\lambda} = z_{it}\left[1+\lambda\frac{(1+\delta)W_t}{(1+\delta)W_t - w'_{it}}\right]^{-1/\lambda} \\ &\leq z_{it}\left[1+\lambda\frac{(1+\delta)W_t}{(1+\delta)W_t - (1+\delta)w_{it}}\right]^{-1/\lambda} = z_{it}\left[1+\lambda\frac{W_t}{W_t - w_{it}}\right]^{-1/\lambda} = w_{it} \\ &< (1+\delta)w_{it} \leq w'_{it} \end{aligned} \tag{24}$$

which is a contradiction. Hence, there can only be one equilibrium. ■

Proof of proposition 2: (⇒) If $c_{it}/a_i = c^*_t$ and does not depend on i, then (11) reduces to

$$\left(\frac{p_{it}}{a_i}\right) = \left[1+\lambda+\lambda\frac{(p_{it}/a_i)^{-1/\lambda}}{\sum_{j \in C_t^* \setminus \{i\}} (p_{jt}/a_j)^{-1/\lambda}}\right] c_t^* \text{ for all } i \in C^*_t \tag{25}$$

When we choose $p_{it}/a_t = p^*_t$ for all $i \in C^*_t$ and substitute it in the system of equations (25) we obtain that for all $i \in C^*_t$

$$\begin{aligned} p_t^* &= \left[1+\lambda+\lambda\frac{p_t^{*-1/\lambda}}{\sum_{j \in C_t^* \setminus \{i\}} p_t^{*-1/\lambda}}\right] c_t^* \\ &= \left[1+\lambda\frac{N_t}{N_t - 1}\right] c_t^* \end{aligned} \tag{26}$$

which does not depend on i. Hence, if $c_{it}/a_i = c^*_t$, then $p^*_t = (1+\lambda N_t/(N_t-1))c^*_t$ is the symmetric pure strategy Nash equilibrium in which all suppliers charge the same price per quality unit and all have an equal market share.

(⇐) If there is a symmetric equilibrium, then for all $i \in C^*_t$

$$\left(\frac{p_{it}}{a_i}\right) = p_t^* = \left[1 + \lambda \frac{N_t}{N_t - 1}\right]\left(\frac{c_{it}}{a_i}\right) \tag{27}$$

which implies that

$$\left(\frac{c_{it}}{a_i}\right) = p_t^* \Big/ \left[1 + \lambda \frac{N_t}{N_t - 1}\right] = c_t^* \tag{28}$$

and does not depend on $i \in C^*_t$. ■

Proof of proposition 3: (*i*) Equation (25) implies that, when we define

$$W_t = \sum_{j \in C_t^*} \left(p_{it} / a_i\right)^{-1/\lambda}, \tag{29}$$

then for all $i \in C^*_t$ it must be in equilibrium that

$$\left(\frac{p_{it}}{a_i}\right) = \left[1 + \lambda \frac{W_t}{W_t - \left(p_{it} / a_i\right)^{-1/\lambda}}\right]\left(\frac{c_{it}}{a_i}\right) \tag{30}$$

Applying the implicit function theorem to the above equation yields

$$\frac{\partial\left(p_{it} / a_i\right)}{\partial\left(c_{it} / a_i\right)} > 0 \tag{31}$$

Furthermore, because (p_{it}/a_i) is higher, equation (5) implies that the market share of the model must be lower. (*ii*) In order to prove this part, it is easiest to reconsider (18), which reads

$$\left(\frac{p_{it}}{c_{it}}\right)^{-1/\lambda} = \left[1 + \lambda \frac{\sum_{j \in C_t^*} \left(c_{jt}^*\right)^{-1/\lambda} \left(p_{jt} / c_{jt}\right)^{-1/\lambda}}{\sum_{j \in C_t^*} \left(c_{jt}^*\right)^{-1/\lambda} \left(p_{jt} / c_{jt}\right)^{-1/\lambda} - \left(c_{it}^*\right)^{-1/\lambda} \left(p_{it} / c_{it}\right)^{-1/\lambda}}\right]^{-1/\lambda} \tag{32}$$

Again, redefining

$$v_{it} = \left(p_{it} / c_{it}\right)^{-1/\lambda} \text{ and defining } \widetilde{V}_t = \sum_{i \in C_t^*} \left(c_{it}^*\right)^{-1/\lambda} v_{it} \tag{33}$$

equation (32) boils down to

$$v_{it} = \left[1 + \lambda \frac{\widetilde{V}_t}{\widetilde{V}_t - \left(c_{it}^*\right)^{-1/\lambda} v_{it}}\right]^{-1/\lambda} \quad \text{for all } i \in C^*_t \tag{34}$$

It is straightforward to show that the v_{it} that solves this equation is increasing in c^*_{it}. Since the markup, p_{it}/c_{it}, is decreasing in v_{it}, this implies that the equilibrium markup is decreasing in c^*_{it}, which is what is claimed. ■

Proof of proposition 4: The basis of this proof is the equilibrium equation

$$p_{it} = \left[(1+\lambda) + \lambda \frac{(p_{it}/a_i)^{-1/\lambda}}{\sum_{j \in C_t^* \setminus \{i\}} (p_{jt}/a_j)^{-1/\lambda}} \right] c_{it} \text{ for all } i \in C^*_t \tag{35}$$

when we define

$$\widetilde{p}_{it} = \frac{p_{it}}{a_i \widetilde{c}_t} \tag{36}$$

then (35) can be rewritten as

$$\widetilde{p}_{it} = \left[(1+\lambda) + \lambda \frac{(\widetilde{p}_{it})^{-1/\lambda}}{\sum_{j \in C_t^* \setminus \{i\}} (\widetilde{p}_{jt})^{-1/\lambda}} \right] \left(\frac{a_i}{\overline{a}_t} \right)^{\gamma} \text{ for all } i \in C^*_t \tag{37}$$

what I will show is that

$$a_i / \overline{a}_t = a_{i+1} / \overline{a}_{t+1} \tag{38}$$

If this is the case, then it must be that $\widetilde{p}_{it} = \widetilde{p}_{(i+1)(t+1)}$. We know that

$$C_0^* = \{1, \ldots, N\} \text{ and } C_1^* = \{2, \ldots, N+1\} \tag{39}$$

Given these sets, (13) implies that

$$\overline{a}_0 = (1+g)^{\frac{1}{N}\sum_{i=1}^{N}(i-1)}, \text{ while } \overline{a}_1 = (1+g)^{\frac{1}{N}\sum_{i=2}^{N+1}(i-1)} = (1+g)\left[(1+g)^{\frac{1}{N}\sum_{i=1}^{N}(i-1)} \right] = (1+g)\overline{a}_0 \tag{40}$$

Since $a_{i+1}=(1+g)a_i$, this implies that

$$a_{i+1} / \overline{a}_{t+1} = (1+g)a_i / (1+g)\overline{a}_t = a_i / \overline{a}_t \tag{41}$$

Therefore, $\widetilde{p}_{it} = \widetilde{p}_{(i+1)(t+1)}$. This implies that

$$P_0^T = \left[\sum_{i=1}^{N} (p_{i0}/a_i)^{-1/\lambda} \right]^{-\lambda} = \widetilde{c}_0 \left[\sum_{i=1}^{N} \widetilde{p}_{i0}^{-1/\lambda} \right]^{-\lambda} \tag{42}$$

and

$$P_1^T = \left[\sum_{i=2}^{N+1} (p_{i1}/a_i)^{-1/\lambda} \right]^{-\lambda} = \widetilde{c}_1 \left[\sum_{i=2}^{N+1} \widetilde{p}_{i1}^{-1/\lambda} \right]^{-\lambda} = \widetilde{c}_1 \left[\sum_{i=1}^{N} \widetilde{p}_{(i+1)1}^{-1/\lambda} \right]^{-\lambda} = \frac{\widetilde{c}_1}{\widetilde{c}_0} P_0^T \tag{43}$$

Hence,

$$\pi^T = \frac{P_1^T - P_0^T}{P_0^T} = \frac{\tilde{c}_1 - \tilde{c}_0}{\tilde{c}_0} = \pi \tag{44}$$

Thus, π represents the theoretical inflation rate that is supposed to be approximated by the empirical price index methods. ■

Proof of proposition 5: I will prove part *(iii)* of the proposition in detail. The other two parts follow directly from the proof below. Note that the inflation rate of good *i* between *t=0* and *t=1* is given by

$$\begin{aligned}\pi_i &= \frac{p_{i1} - p_{i0}}{p_{i0}} = \frac{\tilde{c}_1 \tilde{p}_{i1} - \tilde{c}_0 \tilde{p}_{i0}}{\tilde{c}_0 \tilde{p}_{i0}} \\ &= \frac{\tilde{c}_1 - \tilde{c}_0}{\tilde{c}_0} + \frac{\tilde{c}_1}{\tilde{c}_0}\left[\tilde{p}_{i1} - \tilde{p}_{i0}\right] \\ &= \pi^T + \left(1 + \pi^T\right)\left(\tilde{p}_{i1} - \tilde{p}_{i0}\right)\end{aligned} \tag{45}$$

Hence, what will be essential in the rest of this proof is the property of $\tilde{p}_{i1} - \tilde{p}_{i0}$. It turns out that $\tilde{p}_{it}$ is increasing in $\left(a_i / \overline{a}_t\right)^{\gamma}$. In order to see why, it is useful to rewrite (37) as

$$\tilde{p}_{it} = \left[1 + \lambda \frac{\tilde{P}_t}{\tilde{P}_t - \left(\tilde{p}_{it}\right)^{-1/\lambda}}\right]\left(\frac{a_i}{\overline{a}_t}\right)^{\gamma} \quad \text{for all } i \in C^*_t \tag{46}$$

where

$$\tilde{P}_t = \sum_{i \in C_t^*} \tilde{p}_{it}^{-1/\lambda} \tag{47}$$

Applying the implicit function theorem to the above two equations yields in a straightforward manner that

$$\frac{\partial \tilde{p}_{it}}{\partial \left(a_i / \overline{a}_t\right)^{\gamma}} > 0 \tag{48}$$

Therefore $\tilde{p}_{it}$ is strictly increasing in $\left(a_i / \overline{a}_t\right)^{\gamma}$. Furthermore, note that if $\gamma=0$, then the equilibrium is symmetric and $\tilde{p}_{it}$ is equal for all $i \in C^*_t$.

Consequently, if $\gamma>0$ then $\left(a_i / \overline{a}_t\right)^{\gamma}$ is increasing in *i* and models of higher quality have a higher $\tilde{p}_{it}$. This implies that if $\gamma>0$ then

$$\tilde{p}_{(i+1)0} > \tilde{p}_{i0} = \tilde{p}_{(i+1)1} \tag{49}$$

where the second equality follows from the proof of proposition 4. Hence, if $\gamma>0$ then for all $i \in C^*_t$,

$$\pi_i = \pi^T + \left(1 + \pi^T\right)\left(\tilde{p}_{i1} - \tilde{p}_{i0}\right) < \pi^T \tag{50}$$

Because this inequality holds for all models sold in the market in periods *t=0* and *t=1*, matched model price indexes calculated using the Laspeyres, Paasche, Geometric mean, Fischer, Tornqvist, and Sato formula will all underestimate inflation. The reason is that all these price index formula have the property that measured inflation is in the range of inflation rates of the individual models. Since the actual inflation rate is above the maximum inflation rate measured for the models it must be that the actual inflation rate is understated by the matched model indexes.

A reverse but similar argument yields that the matched model indexes overstate inflation whenever $\gamma<0$.

■

Proof of proposition 6: *(i):* If $\gamma=0$ then the equilibrium price schedule satisfies

$$\frac{p_{it}}{a_i} = p_t^* \text{ for all } i\in C^*_t \text{ and for } t=0,1 \tag{51}$$

If the imputed prices, $\hat{p}_{N+1,0}$ and $\hat{p}_{1,1}$,from the hedonic regression model also satisfy this property such that

$$\frac{\hat{p}_{N+1,0}}{a_{N+1}} = p_0^* \text{ as well as } \frac{\hat{p}_{1,1}}{a_1} = p_1^* \tag{52}$$

then we find that the observed and imputed inflation rates satisfy

$$\pi_i = \begin{cases} \dfrac{p_{i,1} - p_{i,0}}{p_{i,0}} = \dfrac{p_1^* - p_0^*}{p_0^*} = \pi^T & \text{for } i = 2,\dots,N \\ \dfrac{p_{N+1,1} - \hat{p}_{N+1,0}}{\hat{p}_{N+1,0}} = \dfrac{p_1^* - p_0^*}{p_0^*} = \pi^T & \text{for } i = N+1 \\ \dfrac{\hat{p}_{1,1} - p_{1,0}}{p_{1,0}} = \dfrac{p_1^* - p_0^*}{p_0^*} = \pi^T & \text{for } i = 1 \end{cases} \tag{53}$$

Consequently, no matter what type of weighted average one takes of the observed and imputed inflation rates across models to calculate π^H, this average will always equal π^T.

(ii): If $\gamma<0$ then the equilibrium price schedule satisfies

$$\frac{p_{it}}{a_i} < \frac{p_{i-1t}}{a_i} \text{ for all } i,i\text{-}1\in C^*_t \text{ and for } t=0,1 \tag{54}$$

If the imputed prices in the hedonic regression model also satisfy this property, such that

$$\frac{\hat{p}_{N+1,0}}{a_{N+1}} < \frac{p_{N,0}}{a_N} \text{ and } \frac{\hat{p}_{1,1}}{a_1} > \frac{p_{2,1}}{a_1} \tag{55}$$

then, in terms of the notation of proposition 5, the observed and imputed prices obey

$$\tilde{p}_{i,0} < \tilde{p}_{i-1,0} = \tilde{p}_{i,1} \text{ for } i=2,\dots,N, \text{ as well as } \hat{\tilde{p}}_{N+1,0} < \tilde{p}_{N,0} = \tilde{p}_{N+1,1} \text{ and } \hat{\tilde{p}}_{1,1} > \tilde{p}_{2,1} = \tilde{p}_{1,0} \tag{56}$$

This means that the observed and imputed inflation rates satisfy

$$\pi_i = \begin{cases} \pi^T + (1+\pi^T)(\tilde{p}_{i,1} - \tilde{p}_{i,0}) > \pi^T & \text{for } i = 2,\ldots,N \\ \pi^T + (1+\pi^T)(\tilde{p}_{N+1,1} - \hat{\tilde{p}}_{N+1,0}) > \pi^T & \text{for } i = N+1 \\ \pi^T + (1+\pi^T)(\hat{\tilde{p}}_{1,1} - \tilde{p}_{1,0}) > \pi^T & \text{for } i = 1 \end{cases} \tag{57}$$

Hence, no matter what weighted average one takes of these inflation rates across models to calculate the hedonic inflation rate π^H, it will always yield $\pi^H > \pi^T$.

(iii): This follows in the same way as the proof of part *(ii)*. The only thing that is different is that in this case the equilibrium price schedule is such that the prices per quality unit are increasing in the quality levels of the models, which yields a reversal of the inequality signs. ■

Proof of proposition 7: Log-linearization of equation (30) around the symmetric equilibrium derived in Proposition 2 yields

$$\begin{aligned} \ln p_{it} &\approx \ln \tilde{c}_t + \ln\left(1+\lambda\frac{N}{N-1}\right) + \ln a_i + \gamma\left[1 - \frac{N}{1+(1+\lambda)N(N-1)}\right](\ln a_i - \ln \bar{a}_t) \\ &= \left[\ln \tilde{c}_t + \ln\left(1+\lambda\frac{N}{N-1}\right) - \gamma\left[1-\frac{N}{1+(1+\lambda)N(N-1)}\right]\ln \bar{a}_t\right] + \left[1+\gamma-\gamma\frac{N}{1+(1+\lambda)N(N-1)}\right]\ln a_i \\ &= \beta_{0t} + \beta_{1t}\ln a_i \end{aligned} \tag{58}$$

This implies that the logarithm of the price per quality unit satisfies

$$\ln(p_{it}/a_i) \approx \ln \mu_{it} + \ln \tilde{c}_t + \gamma(\ln a_i - \ln \bar{a}_t) \tag{59}$$

where $ln\mu_{it}$ is the logarithm of the markup charged on model i in period t which equals approximately

$$\ln \mu_{it} \approx \ln\left(1+\lambda\frac{N}{N-1}\right) - \gamma\left[\frac{N}{1+(1+\lambda)N(N-1)}\right](\ln a_i - \ln \bar{a}_t) \tag{60}$$

When we take the first difference, over time, of (59), then we obtain that the percentage price change in the price of model i between $t=0$ and $t=1$ can be approximated by

$$\begin{aligned} \pi_i &\approx \Delta \ln p_{i1} \approx \Delta \ln \tilde{c}_1 + \Delta \ln \mu_{i1} - \gamma\Delta \ln \bar{a}_1 \approx \pi^T + \Delta \ln \mu_{i1} - \gamma \Delta \ln \bar{a}_1 \\ &\approx \pi^T - \gamma\left[1 - \frac{N}{1+(1+\lambda)N(N-1)}\right]g = \pi^T - \theta g \end{aligned} \tag{61}$$

Thus, we obtain that the inflation rates of each of the matched models deviate from the actual inflation rate by approximately the same amount, namely $-\theta g$. Because the hedonic regression extrapolates this approximately linear relationship for the imputation of the unobserved prices, it also finds that the imputed inflation levels $\pi_{N+1}=\pi_1=\pi^T-\theta g$. Therefore, the results for the hedonic price indexes do not differ much from the matched model indexes and both are biased by approximately $-\theta g$. ■

B. Data on CPU prices and benchmark tests

The data are constructed from two main sources. The first contains the data on the CPU prices. The second contains the data on the benchmark test results and technical specifications of the various chips.

CPU price data: Weekly data on CPU prices for the period of October 28 2001 through March 17 2003 are taken from www.sharkyextreme.com. These prices are the lowest online list prices for CPU units of different types[7]. For comparability purposes, I have limited the sample to Original Equipment Manufacturer (OEM) processors, sometimes referred to as tray processors. These are processors that are sold to an OEM manufacturer or distributor intended for installation. These processors generally do not include a heat sink or fan and have a shorter warranty than their retail versions. The OEM versions tend to sell for about 15% less than the retail versions of the same processors.

These price data contain prices for 77 different processors. However, not all of these processors are sold in each week. The number of processors for which there are price quotes in a week varies between 37 and 50.

Benchmark test data: Quality of the CPU's is proxied for by benchmark ratings. The benchmark ratings that I use are taken from www.tomshardware.com[8]. They are taken from a comparative benchmark test of 65 processors varying in clock speed from 100MHz through 3066 MHz. I used data on 58 processors for which I was able to find detailed data on their technical specifications and which were classified as OEM versions. The processor in this sample with the lowest benchmark rating is the Intel Celeron 400MHz with a rating of 39. The highest rated processor, with a rating of 206, is the Pentium 4 3.06 MHz.

Technical specifications: Data on the technical specifications of the chips in my sample are taken from Tom's Hardware for the chips reported on in the benchmark test and from the Intel and AMD web sites for the other chips.

The technical specifications variables that I used in the benchmark imputation as well as the hedonic price regressions can be described as follows. *Athlon dummy*: Equals one if the processor is an AMD Athlon processor for which clock speed is not reported in MHz but instead in the measure AMD uses for speed, zero otherwise. *Speed:* Clock speed in MHz. *Athlon speed:* Clock speed measure published by AMD for most of its Athlon processors. *Bus speed:* Speed in MHz with which the processor communicates with the systems RAM memory. *Process:* Wiring width measured in nanometers used for the manufacturing process. The lower this width the higher the density of transistors on the chip can be.

Merger of price and benchmark data: The price data cover a set of processors that are not included in the benchmark test (and the benchmark test contains some processors that are not in the price data). I imputed a

[7] A detailed description of how SharkExtreme collects these data on CPU prices is available at http://www.sharkyextreme.com/guides/WCPG/article.php/10705_2224371__6.

[8] In particular, the benchmark tests that use are reported at http://www6.tomshardware.com/cpu/20030217/index.html.

benchmark rating for these untested processes using the following regression that aims to explain the benchmark rating as a function of processor specific technical characteristics.

$$\begin{aligned}\ln(\text{benchmark}) = \underset{(0.58)}{2.15} + \underset{(0.65)}{0.75}(\text{Athlon dummy}) \\ + \underset{(0.04)}{0.53}\ln(\text{Speed}) + \underset{(0.08)}{0.43}\ln(\text{Athlon speed}) \\ + \underset{(0.03)}{0.12}\ln(\text{Bus speed}) - \underset{(0.08)}{0.38}\ln(\text{Process})\end{aligned} \tag{62}$$

The sample of this regression consists of the 58 processors from the benchmark data and the R^2 equals 0.98. All technical specification variables come in significant and with the expected sign.

Combining the observed and imputed benchmarks with the price data yields a weekly data set with prices for 77 different processors for the approximately one and a half year between October 28 2001 and March 17 2003. The data set does not only contain data on prices but also on benchmark ratings and technical specifications of the various processors.

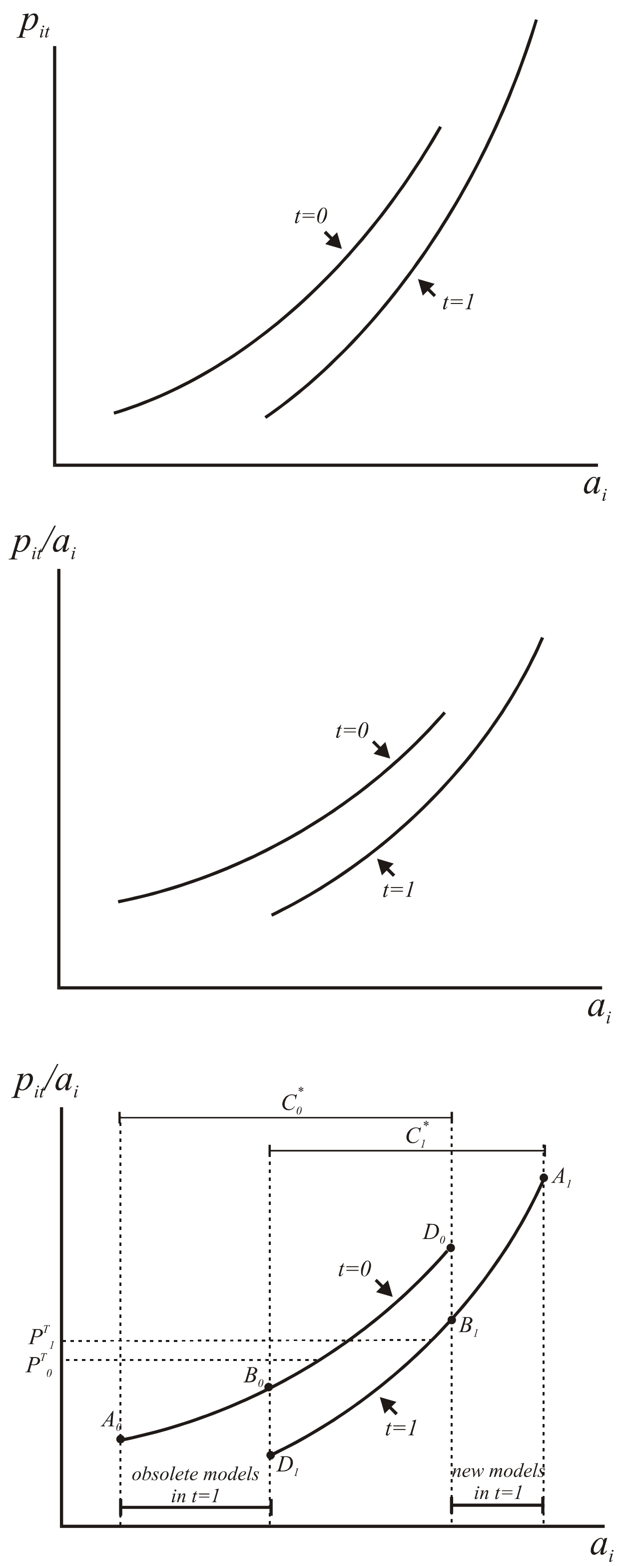
Table 1. Price index formulas applied in this paper

Index	Formula	Applied
Laspeyres	$\pi^L = \sum_{i \in C^*} s_{i0}\pi_i$	• Applied by Bureau of Labor Statistics for the calculation of the Consumer Price Index.
Paasche	$\pi^P = \left[\sum_{i \in C^*} s_{i1}(1+\pi_i)^{-1}\right]^{-1} - 1$	
Geometric mean (G0)	$\pi^{G_0} = \left(\prod_{i \in C^*} (1+\pi_i)^{s_{io}}\right) - 1$	
Geometric mean (G1)	$\pi^{G_1} = \left(\prod_{i \in C^*} (1+\pi_i)^{s_{i1}}\right) - 1$	
Fisher Ideal	$\pi^{FI} = \sqrt{(1+\pi^L)(1+\pi^P)} - 1$	• Applied by Bureau of Economic Analysis for the calculation of chained price indexes in the National Income and Product Accounts.
Tornqvist	$\pi^{TQ} = \sqrt{(1+\pi^{G_0})(1+\pi^{G_1})} - 1$	• Applied by Bureau of Labor Statistics (2002) in chained Consumer Price Index.
Sato	$\pi^S = \left(\prod_{i \in C^*} (1+\pi_i)^{\phi_i}\right) - 1$ where $\phi_i = \theta_i / \sum_{j \in C^*} \theta_j$ $\theta_i = \begin{cases} \dfrac{s_{i1} - s_{i0}}{\ln s_{i1} - \ln s_{i0}} & \text{if} \quad s_{i0} \neq s_{i1} \\ s_{i1} & \text{if} \quad s_{i0} = s_{i1} \end{cases}$	• Exact price index for CES preferences when same goods are sold in both periods.
Feenstra	$\pi^F = \left(\prod_{i \in (C_0^* \cap C_1^*)} (1+\pi_i)^{\phi_i}\right)\left(\dfrac{\varpi_1}{\varpi_0}\right)^{1/(1-\sigma)} - 1$ where ϕ_i is as in the Sato index $\varpi_t = \sum_{i \in (C_0^* \cap C_1^*)} s_{it}$ for $t=0,1$ σ elasticity of substitution ($\sigma=1+\lambda/\lambda$)	• Exact price index for CES with matched model correction in case not all goods are sold in both periods.

Note: All indexes are meant to measure percentage price change between t=0 and t=1. π_i denotes the percentage price change of item i between t=0 and t=1. s_{it} is the expenditure share of good i in period t. C^*_t is the set of items sold in period t. Whenever it is denoted without time index it is assumed that $C^*_0 = C^*_1 = C^*$. $\lambda>0$ is the parameter used in the CES specification of the theoretical model of section 3 and beyond.

Table 2. Actual and estimated levels of CPU price inflation for 10/28/01-03/17/03 for various elasticities of substitution

	With variety bias				Without variety bias			
Elasticity of substitution	1.5	2	4	8	1.5	2	4	8
Theoretical price change	-1%	-16%	-26%	-26%	-31%	-34%	-38%	-40%
			Matched model indexes					
Laspeyres	-42%	-37%	-20%	19%	-49%	-46%	-39%	-28%
Paasche	-46%	-46%	-50%	-59%	-52%	-52%	-56%	-62%
Geometric (G0)	-46%	-42%	-26%	11%	-52%	-49%	-42%	-32%
Geometric (G1)	-50%	-50%	-53%	-62%	-55%	-55%	-58%	-64%
Fisher	-44%	-42%	-36%	-30%	-50%	-49%	-48%	-48%
Tornqvist	-48%	-46%	-41%	-35%	-53%	-52%	-51%	-50%
			Hedonic price indexes					
Laspeyres	-52%	-46%	-23%	30%	-61%	-59%	-52%	-45%
Paasche	-39%	-44%	-51%	-59%	-41%	-46%	-54%	-59%
Geometric (G0)	-58%	-53%	-31%	17%	-66%	-64%	-56%	-49%
Geometric (G1)	-47%	-50%	-55%	-62%	-49%	-52%	-57%	-61%
Fisher	-46%	-45%	-38%	-27%	-52%	-53%	-53%	-52%
Tornqvist	-53%	-51%	-44%	-33%	-59%	-58%	-57%	-55%



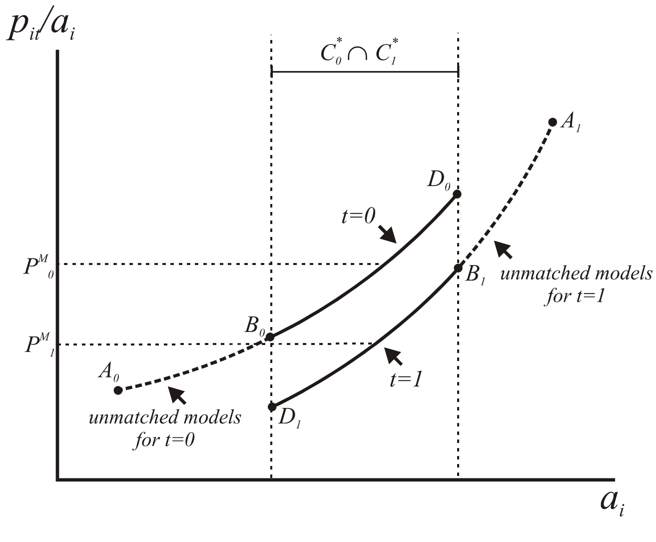
panel (a):
Even though the researcher does not observe the actual quality levels, a_i, the researcher does observe the actual price levels, p_{it}. This panel plots a hypothetical price schedule in two period *t=0,1*.

panel (b):
What is relevant for the price aggregate, the percentage change of which is to be measured, is actually the schedule of price per quality unit across models in the market.
This panel plots this schedule. It plots the implied relationship between p_{it}/a_i and a_i.

panel (c):
This panel is identical to panel *(b)*. What is added are:

C^*_t	set of models sold in market at time t
P^T_t	theoretical price level at time t
B_t-D_t	part of price schedule at time t for models that are sold in both periods
A_t-B_t	part of price schedule at time t for models that are only sold at time t
Note:	$P^T_1 > P^T_0$ such that the theoretical inflation level is positive.

Figure 1. Price per quality unit schedule and related notation.

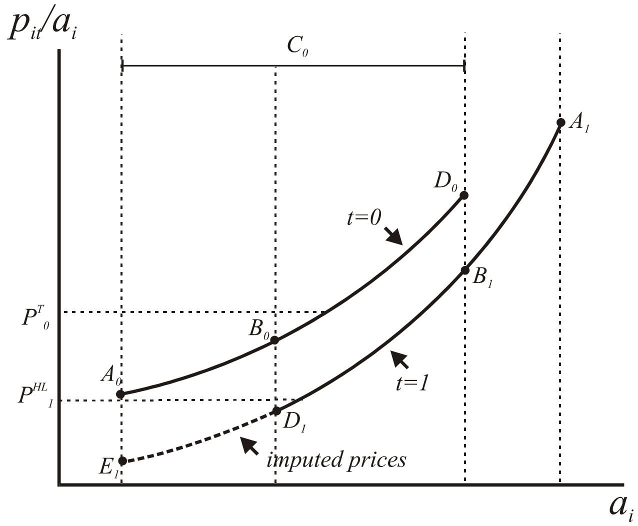


Matched model index: *IP-IQ linking method*
An IP-IQ linked matched model index only compares the prices of models that were sold in both periods and calculates a change in the price index based on these models:

- The set of models sold in both periods is $C^*_1 \cap C^*_2$.
- Calculates price increase between schedules B_0-D_0 and B_1-D_1.
- P^M_t is price level implied by schedule B_t-D_t.
- Measured inflation is change from P^M_0 to P^M_1.

Note: $P^M_1 < P^M_0$ such that the measured inflation level is negative, while the actual level of inflation is positive.

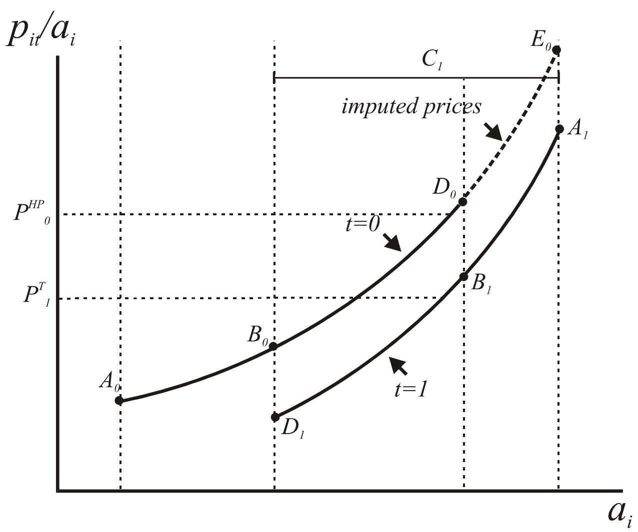
Figure 2. Downward bias in inflation measured using matched model index.



Hedonic index: *Laspeyres index*
A hedonic regression model is used to impute the prices of the unmatched models in period $t=0$ for $t=1$ in order to calculate a Laspeyres index with $t=0$ as baseperiod.

- A Laspeyres index cannot be calculated because price schedule D_1-E_1 is not observed.
- Unobserved part, D_1-E_1, of period $t=1$ price schedule is imputed using hedonic model.
- P^{HL}_1 is price level imputed for schedule B_1-E_1.
- Measured inflation is change from P^T_0 to P^{HL}_1.

Note: $P^{HL}_1 < P^T_0$ such that the measured inflation level is negative, while the actual level of inflation is positive.



Hedonic index: *Paasche index*
A hedonic regression model is used to impute the prices of the unmatched models in period $t=1$ for $t=0$ in order to calculate a Paasche index with $t=1$ as measurement period.

- A Paasche index cannot be calculated because price schedule D_0-E_0 is not observed.
- Unobserved part, D_0-E_0, of period $t=0$ price schedule is imputed using hedonic model.
- P^{HP}_0 is price level imputed for schedule B_0-E_0.
- Measured inflation is change from P^{HP}_0 to P^T_1.

Note: $P^T_1 < P^{HP}_0$ such that the measured inflation level is negative, while the actual level of inflation is positive.

Figure 3. Downward bias in inflation measured using hedonic price index methods.

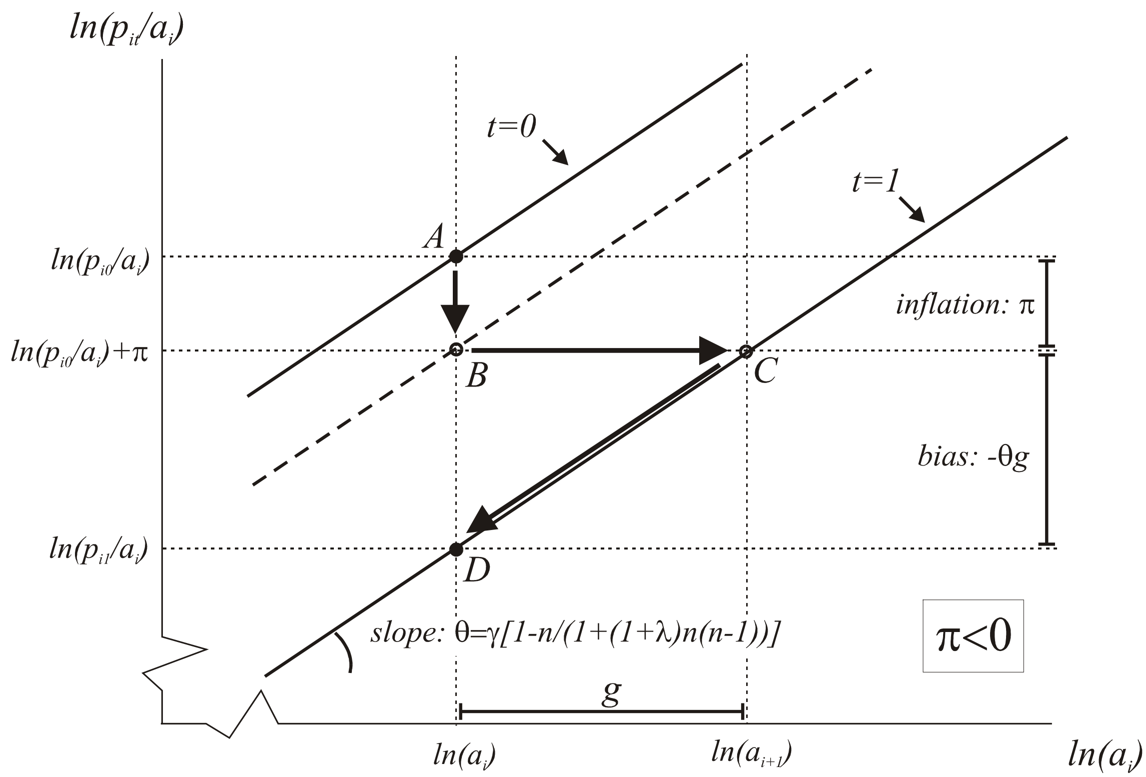


Figure 4. Graphical representation of log-linearization of bias in inflation measures

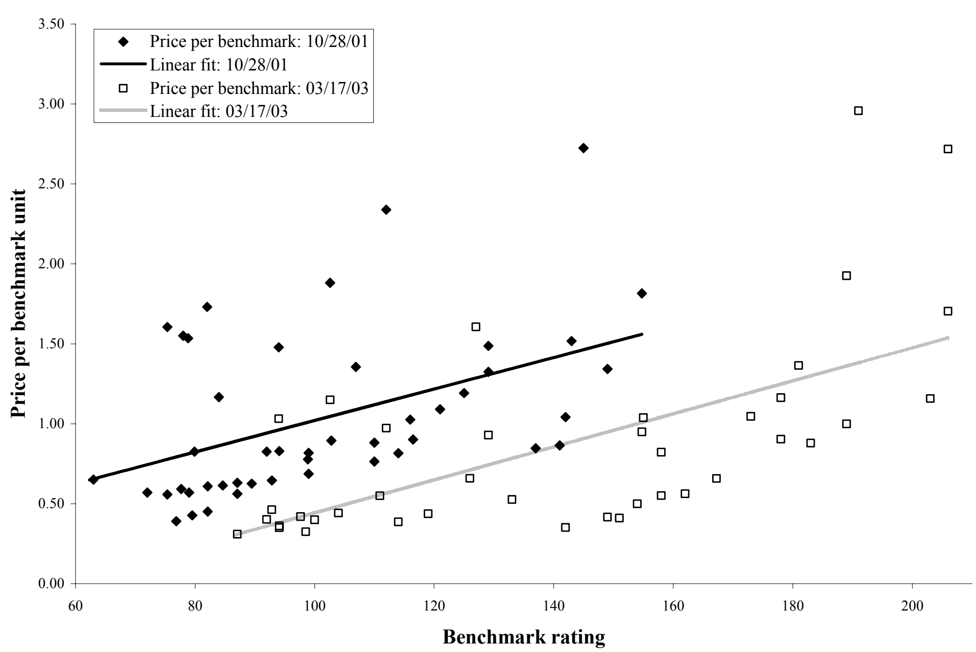


Figure 5. Cross sectional pattern of prices per benchmark unit at beginning and end of sample

Is Equipment Price Deflation a Statistical Artifact?

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Abstract

I argue that equipment price deflation might be overstated because the methods used to measure it rely on the erroneous assumption of perfectly competitive markets. The main intuition behind this argument is that what these price indices might actually capture is not a price decrease but the erosion of the market power of existing vintages of machines. To illustrate my argument, I introduce an endogenous growth model in which heterogeneous final goods producers can choose the technology they will use. The various technologies are supplied by monopolistically competing machine suppliers. This market structure implies that the best machines are marketed to the best workers and are sold at the highest markup. In my model economy, the endogenously determined markups are such that standard methods will tend to find equipment price deflation, even though the model does not exhibit any equipment price deflation.

keywords: imperfect competition, price indices, vintage capital.

JEL-code: O310, O470, C190.

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1 Introduction and motivation

Many recent studies argue that one of the main driving forces behind the economic expansion of the 1990's is the productivity growth in the information technology producing sectors. Virtually all of these studies, including Oliner and Sichel (2000), Gordon (2000), Jorgenson and Stiroh (2000), and Violante, Ohanian, Ríos-Rull, and Krusell (2000) argue that the productivity growth in the IT producing sectors is reflected in the steady decline of their output prices relative to GDP or consumption goods. Their argument is similar to that of Greenwood, Hercowitz and Krusell (1997), who like to refer to it as investment specific technological change. Namely, if the productivity growth rate of the investment goods producing sector is consistently higher than that of the final goods producing sector, then this will lead to a steady decline in the relative price of investment goods.

If all markets would be perfectly competitive, as is assumed by all growth accounting studies referred to above, then this suggests that productivity growth in the IT producing sectors outpaces that of the consumption goods sector. This is a big if, however. Because, if these markets would be perfectly competitive then none of the IT producers would be able to make the profits necessary to recoup the expenses they made on research and development of their products[1].

What I will show in this paper is that the measurement techniques used by the Bureau of Labor Statistics (BLS) and the Bureau of Economic Analysis (BEA) used to construct these investment price indices are likely to overestimate investment price deflation. The reason for this is that these methods, i.e. hedonic price methods and matched-model price indices, implicitely also assume perfectly competitive markets. The conventional assumption is that relative price declines in existing vintages when a new vintages enters the market are due to obsolescence, in the sense that the existing vintages become less productive relative to the frontier. In that case, relative price declines can be used as a measure of quality improvements and technological progress. In this paper I argue that in a market in which there is imperfect competition between the suppliers of the various vintages these relative price declines reflect two things. The first is the conventional obsolescence effect. The second is an erosion of market power of the older vintages. That is, the

[1] That these expenses are often substantial can be seen from Intel's and Hewlett-Packard's \$2 billion in expenses for the development of the Itanium chip, to come on the market in 2001. (Source: BusinessWeek, October 15, 2001, page 84)

existence of better vintages erodes the market power of the supplier of an older vintage. This reduces his ability to charge markups on his product, leading this supplier to reduce its price in reaction to the introduction of better vintages by its competitors.

My argument basically consists of two steps. In the first step I argue that imperfect competition causes markups in a market to be positively correlated with the quality that the various vintages embody. The second step consists of the argument that this positive correlation will cause the BLS and BEA to overestimate equipment price deflation. I have chosen to divide this paper into two parts, each addressing one of these steps.

In the first part, consisting of section 2, I address the second step. That is, I show how a positive correlation between markups and the quality embodied in the different vintages will bias the price indices constructed by the BLS and BEA to find too much price deflation. I illustrate this point using an empirical example for PC microprocessor chips. This part is basically self-contained and can be read without reference to the theoretical model introduced in the second part.

In the second part, consisting of sections 3 and 4, I address the first step and introduce an endogenous growth model in which heterogeneous consumption goods producers can choose what technology to use. The various technologies are supplied by monopolistically competing machine suppliers. This market structure, combined with capital skill complementarities, implies that the best machines are marketed to the best workers and are sold at the highest markup. The endogenously determined markups are positively correlated with the level of productivity embodied in a machine. I use this model to show how in a world where there is *no* technological progress or price deflation in the machine producing sector[2], the methods applied by the Bureau of Labor Statistics as well as the Bureau of Economic Analysis for the construction of the Producer Price Indices and Investment Price Indices will tend to find a steady decline in the relative price of investment goods similar to that observed in the data. In my theoretical model this measured productivity growth is completely spurious, however, and is induced by the structure of the varying markups across different vintages of machines.

My analysis in this paper distinguishes itself from the previous literature in three ways. Most importantly, it is the first to show how markups that

[2] In my model the average price paid and average production cost per constant quality unit (efficiency unit) of equipment are both constant over time.

vary over the product cycle can cause a structural bias in measured price deflation. Secondly, the theoretical setup of monopolistic competition between suppliers of different vintages is new and deviates importantly from the conventional monopolistic competition models in the endogenous growth literature, based on Romer (1990) and Grossman and Helpman (1991). Finally, my analysis emphasizes a new dimension of markups. Previously, the empirical importance of markups had been established by Hall (1988) and the importance of their fluctuations over the business cycle had been emphasized by, among others, Woodford and Rotemberg (1999). My analysis emphasizes that their fluctuations are also important if one considers them over the *product cycle*. I focus on equipment prices, but my argument is equally applicable to consumer price indices. My emphasis on equipment prices is simply because these are the goods for which quality adjustments are considered most relevant.

The structure of this paper is as follows. In section 2 I show how the existence of markups affect equipment price deflation, as measured by matched-model and hedonic price indices. In sections 3 and 4 I introduce my endogenous growth model with imperfect competition between the suppliers of different vintages of machines that generates markups of the form that will bias equipment price indices. Section 5 is the theoretical equivalent of section 2 in the sense that I show how the pattern of equilibrium markups in my theoretical model affects matched-model and hedonic price indices. It also contains a numerical example in which I calculate the estimate equipment price deflation rates that would be measured in my theoretical model. Finally, I conclude in section 6.

2 Markups bias equipment price inflation

In this section I briefly review the BEA's and BLS's methods for the construction of equipment price indices and argue why varying markups over the product cycle would bias these indices. I would like to emphasize the brevity of this review and refer to Dulberger (1989) for a more extensive description of these methods. There are basically two main methods used to construct quality adjusted price indices. The first is a matched-model methodology where price indices are constructed by using the price changes for the machines that were in the market in the previous period as well as in the current one. The second are hedonic price methods where regression analysis is used to estimate which part of price variations can be attributed to variations in the quality of the machines in the market.

This section consists of four parts. In the first part I start by introducing the basic notation that I will use in the rest of this paper and introduce the idea of varying markups over the product cycle. In the second part, I discuss the measurement of mathed-model price indices and how they are affected by the markups. In the third part, I focus my attention on hedonic price methods. Finally, I illustrate my argument with an empirical example concerning PC microprocessor chips.

2.1 Notation and markups

Denote the price of a machine of vintage v at time t as $P_{t,v}$. Let a machine of vintage v embody A_v efficiency units of capital, where $A_v > A_{v-1}$ such that there is technological progress over time. Furthermore, let vintage age, denoted by τ, be given by the difference between the time the vintage was introduced, i.e. v, and the current period, such that $\tau = t - v$. Implicit in this notation is that I will assume that in each period (at maximum) one new vintage is introduced. I will denote the number of machines of vintage v sold at time t as $X_{t,v}$. The implicit assumption in the vintage capital literature is that when $\overline{P}_t$ represents the average price level, then

$$P_{t,v} = \overline{P}_t A_v \exp(u_{t,v}) \tag{1}$$

such that, up to the stochastic term $u_{t,v}$, prices are proportional in the quality that is embodied in the machines. Implicit in this assumption is that there are no markups and that prices reflect marginal (as well as average) production costs which are assumed to be constant per efficiency unit. In this case, relative prices reflect relative quality differences between different vintages, in the sense that

$$(\ln P_{t,v} - \ln P_{t,v'}) = (\ln A_v - \ln A_{v'}) - (u_{t,v} - u_{t,v'})$$

In this paper I argue that, instead of (1), a market with monopolistic competition between the suppliers will naturally lead to a price schedule that satisfies

$$P_{t,v} = \mu_{t,v} \overline{P}_t A_v \exp(u_{t,v}) \tag{2}$$

where $\mu_{t,v}$ is a measure of market power, which I will freely interpret as a markup in the following[3]. Suppliers of superior machines have more market power than suppliers of older ones. For example, the supplier of a superior machine could in principle decide to charge the same price as that of an inferior one and wipe out all the demand for its competitor. Consequently, $\mu_{t,v} > \mu_{t,(v-1)}$ and is thus positively correlated with the level of technology embodied in the various vintages of machines. This implies that the price per efficiency unit, i.e. $P_{t,v}/A_v$ is increasing in v. The purpose of an equipment price index in this economy would basically be to measure the path of the average price paid per efficiency unit, i.e. $\overline{P}_t$. The BLS and BEA generally apply two methods for the approximation of the path of $\overline{P}_t$, namely matched-model and hedonic methods. I will discuss the behavior of these methods under (1) and the alternative (2) below and will show that if (2) is the true underlying data-generating process, then these methods might overestimate the decline in $\overline{P}_t$ over time.

2.2 Matched-model indices

Matched model indices measure the equipment price inflation rate as a weighted average of the percentage price increases in the prices of the models of machines that were in the market both in the current measurement period as well as the previous one. Denote P_t^M as the matched-model price index in this economy, then

$$P_t^M = (1 + \pi_t^M) P_{t-1}^M$$

where equipment price inflation, i.e. π_t^M, equals

$$1 + \pi_t^M = \sum_{v \in \mathbf{M}} \omega_{t,v} \left(\frac{P_{t,v}}{P_{t-1,v}} \right) \tag{3}$$

where $\mathbf{M}$ is the set of vintages of machines sold in the market both in the current as well as in the previous measurement period. The weights, $\omega_{t,v}$, depend on the type of price index chosen. My argument is completely independent on the choice of the weights $\omega_{t,v}$.

Suppose that (1) holds and, additionally, that the $u_{t,v}$'s are independent,

[3] This is not a proper markup because it does not depend on production costs. It has, however, a similar interpretation in the sense that it is a measure of market power.

both across vintages as well as over time, and $E\left[e^{u_{t,v}}\right] = E\left[e^{-u_{t,v}}\right] = 1$[4], then the expected value of the measured equipment price inflation equals

$$E\left[\pi_t^M\right] = \sum_{v \in \mathbf{M}} \omega_{t,v} E\left[\frac{P_{t,v}}{P_{t-1,v}} - 1\right] = \frac{\overline{P}_t}{\overline{P}_{t-1}} - 1$$

where I have assumed, for simplicity, that the weights $\omega_{t,v}$ do not depend on the prices. In this case, the matched model index is a useful method to approximate the average price increase per efficiency unit.

If, however, (2) holds instead of (1) then

$$E\left[\pi_t^M\right] = \sum_{v \in \mathbf{M}} \omega_{t,v} E\left[\frac{P_{t,v}}{P_{t-1,v}} - 1\right] = \sum_{v \in \mathbf{M}} \omega_{t,v} \frac{\mu_{t,v}}{\mu_{t-1,v}} \frac{\overline{P}_t}{\overline{P}_{t-1}} - 1 \tag{4}$$

and the estimated equipment price deflation is not an unbiased estimate of the average price increase per efficiency unit. In fact, the estimated investment price deflation is also going to depend on the rate at which the market power of each vintage erodes, i.e. on the average ratio $\mu_{t,v}/\mu_{t-1,v}$. As I will argue in the rest of this paper, in case of technological progress, i.e. an increasing A_v, the market power of a supplier of a particular vintage will erode in the sense that he will be able to charge lower markups over time, such that $\mu_{t,v}/\mu_{t-1,v} < 1$. Consequently, measured equipment price deflation π_t^M overestimates actual equipment price deflation. Moreover, the faster technological progress, the faster the erosion of market power, i.e. the smaller $\mu_{t,v}/\mu_{t-1,v}$. Hence, this bias is likely to be highest for goods for with the highest rate of embodied technological change, like the PC microprocessors that I study in my empirical example later in this section.

2.3 Hedonic price methods

A problem with matched-model indices is that the set $\mathbf{M}$ can be relatively small for quickly evolving product markets. An alternative is to use regression analysis methods to attribute part of price variations to observed variations in quality indicators. This is the basis of hedonic price methods. In the following I will consider log-log hedonic regressions, though my argument also applies to level-regressions.

[4] For example $u_{t,v}$ is independently log normal with mean $-\sigma^2/2$ and standard deviation σ.

Log-log hedonic regressions are based on the assumption that we can observe a set of quality indicators for a machine, say $Q_{1,v}$ through $Q_{k,v}$, such that

$$A_v = A \prod_{j=1}^{k} Q_{j,v}^{\beta_j}$$

and that, at each point in time, the price level of a cross section of different machines is determined by (1). Under this assumption the log-linear regression model

$$\ln P_{t,v} = \left(\ln A + \ln \overline{P}_t\right) + \sum_{j=1}^{k} \beta_j \ln Q_{j,v} + u_{t,v}$$

can be used to estimate the unknown coefficients β_j which measure the elasticities of efficiency units with respect to the various quality indicators. For simplicity, I will focus on the case where $k = 1$, such that there is a unique indicator of quality. In that case the regression model can be written as

$$\ln P_{t,v} = \left(\ln A + \ln \overline{P}_t\right) + \beta \ln Q_v + u_{t,v} \tag{5}$$

Suppose, however, that instead of (1) the actual data generating process is (2). In that case, the true regression equation reads

$$\ln P_{t,v} = \left(\ln A + \ln \overline{P}_t\right) + \beta \ln Q_v + \ln \mu_{t,v} + u_{t,v} \tag{6}$$

where, as I have argued above, the markup variable $\ln \mu_{t,v}$ is likely to be positively correlated with the quality indicator Q_v. The problem is that $\ln \mu_{t,v}$ is unobserved. Now, if (6) is the underlying price setting schedule, but one applies the regression model based on (5), then the estimate of the elasticity, i.e. β, suffers from a standard omitted variable problem where the omitted variable is positively correlated with the explanatory variable. This means that the elasticity β will be overestimated. If β is overestimated, too large a part of price changes is attributed to improvements in quality rather than to changes in the average price level, $\overline{P}_t$, and equipment price inflation is again underestimated.

2.4 An empirical example: PC microprocessor prices

How relevant is the effect of markups when we look at the data? I order to address this question, I will consider a simple empirical example. The example that I consider is the price schedule for PC microprocessor chips, to which matched-model and hedonic methods have been applied by the BLS and by Aizcorbe, Corrado, and Doms (2000). The latter find that matched-model and hedonic methods yield very similar deflation rates. In my example, I will mainly focus on hedonic price methods.

PC microprocessors have two distinguishing features. First of all, their most important characteristic is their clock speed, measured in MHz. Secondly, the other distinguishing feature is the specific chip architecture used for the chip, like Intel's Pentium vs. Celeron chips. I will consider two cross-sections with prices for various PC microprocessors, one for April 11, 1999 and the other for March 4, 2001. The data and their sources are given in Appendix 2.

Before I actually present the data, it is useful to think about a price schedule for microprocessors. A standard hedonic analysis would be based on the assumption that these prices follow

$$P_{t,v} = \overline{P}_t Q_{architecture,v} \left(MHz_v\right)^{\beta} \exp\left(u_{t,v}\right) \tag{7}$$

where $Q_{architecture,v}$ is a constant that is the same for chips with the same architecture, while MHz_v is the clock speed of the chip of vintage v. Here β represents the elasticity of the efficiency units embodied in a chip with respect to its clock speed. Since I will argue that hedonic methods tend to overestimate this elasticity and this elasticity is unobservable, what I will argue is that the estimate of β based on my data will be too high relative to some upperbound value of β. What is this upperbound value? I would argue that it is 1. That is, it is reasonable to assume that a chip that is twice as fast as another one at maximum only embodies twice as many efficiency units. In fact, β is probably smaller than 1 because chips with twice the clock speed do not interact with peripherals and other computer components at twice the speed. In fact, the speed at which the chip interacts with other components in the computer depends more on its architecture than its clockspeed. Hence, my maintained hypothesis throughout this example will be that the true elasticity β is smaller than or equal to one.

If this hypothesis is true, then (7) implies that the price per MHz, i.e. $P_{t,v}/MHz_v$, should be non-increasing in the processor speed. Figure 1 depicts the relationship between price per MHz and processor speed for both of my datasets. As can be seen from the figure, contrary to what my hypothesis suggests, the price paid per MHz is increasing for all the processor architectures for which I have data in both of my cross sections. This suggests that a large part of this price schedule is determined by the markups $\mu_{t,v}$. In fact, let's assume that the true price setting schedule is

$$P_{t,v} = \mu_{t,v} \overline{P}_t Q_{architecture,v} \left(MHz_v\right)^{\beta} \exp\left(u_{t,v}\right) \tag{8}$$

instead of (7), then, apart from the stochastic term $u_{t,v}$, for processors of the same architecture relative price per MHz measures relative markups. That is, in that case

$$\frac{P_{t,v}/MHz_v}{P_{t,v'}/MHz_{v'}} = \frac{\mu_{t,v}}{\mu_{t,v'}} \left(\frac{MHz_{v'}}{MHz_v}\right)^{1-\beta} \frac{\exp\left(u_{t,v}\right)}{\exp\left(u_{t,v'}\right)}$$

This implies that since price per MHz is increasing in the vintage v, if $\beta \leq 1$ then $\mu_{t,v}$ is indeed increasing in v, as I have assumed throughout my argument.

What would happen if we would apply hedonic price regressions to these data? A hedonic regression based on (7) would be a regression of the logarithm of the price on an intercept, the logarithm of the clock speed, and a set of architecture dummies[5]. The regression results are summarized in Table 1. The estimated elasticity is 3.46 for the 1999 data and 2.41 for the 2001 data. Both are significantly bigger than one. Such estimates would lead to a severe overstatement of the contribution of increases in clock speeds of processors to their price leading to an overestimation of their price deflation.

Is this result due to imperfect competition? I would argue that the answer to this question is affirmative. In fact, the IDG News Service article about Intel's price trimming starts off with the observation that

> "Intel Corp. has shaved a few dollars off the prices of its microprocessor chips for desktop computers, part of a broader effort to accelerate the adoption of its recently launched Pentium 4 chip"

which suggests that Intel's pricing policy is partly based on the cross-vintage market power depreciation considerations that are the central point of my argument.

It is fair to mention, however, that an alternative explanation for this observed price difference is that producers charge price equal to average cost,

[5]The regression results that I report are normalized with respect to the frontier architecture for which no dummy is included.

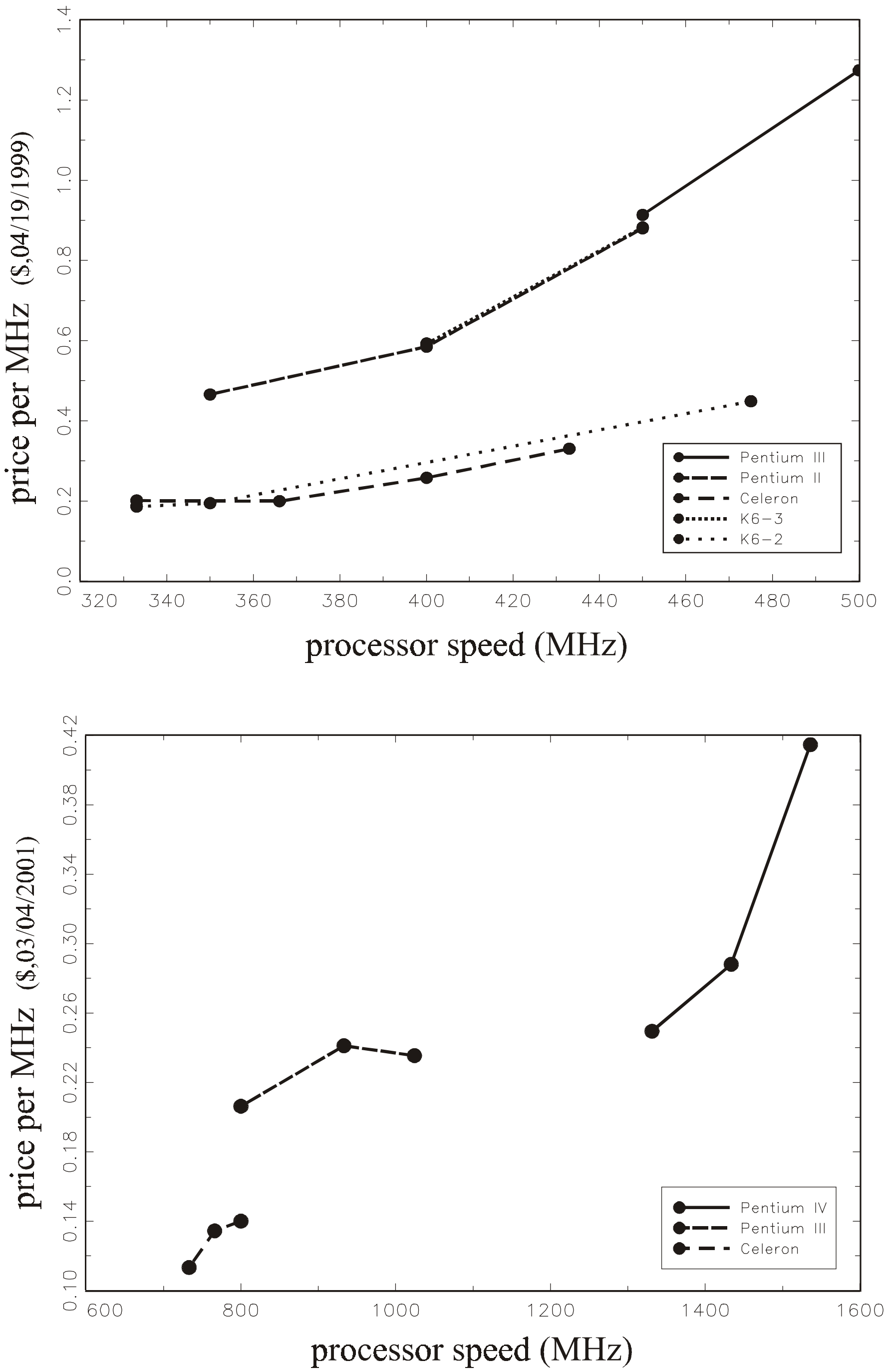

Figure 1: Price per MHz is increasing with speed of processor

Table 1: Hedonic regressions for microprocessors

dependent variable: $\ln(P_{t,v})$				
	04/11/1999		*03/04/2001*	
variable	*coefficient*	*t-stat*	*coefficient*	*t-stat*
$\ln(MHz_v)$	$3.46^{*,\#}$	16.52	$2.41^{*,\#}$	3.87
intercept	-15.10^{*}	-11.68	-11.40	-2.52
	architecture dummies			
P-III	-	-	0.32	1.07
P-II	-0.11	-1.41	-	-
Celeron	-0.96^{*}	-11.23	0.00	0.00
K6-3	-0.13	-1.45	-	-
K6-2	-0.91^{*}	-11.22	-	-
n	14		9	
R^2	0.99		0.97	

*: significantly different from zero at 5% significance level

$^{\#}$: significantly bigger than one at 5% significance level

but that average costs of newer vintages are so much higher because of learning by doing effects. Irwin and Klenow (1994), for example, provide evidence that a doubling of cumulative past output of memory chips leads to a 20 percent drop in production costs. Since cumulative output of previous vintages is much higher than that of the newest vintage, this might explain part of the increase. One has to realize, though, that the increased marginal cost of the newest features is not only observed for capital goods, but is also observed for many consumer goods, like electronics, where it is hard to argue that learning by doing applies to the same extent. An example of such an electronics product for which markups are increasing as a fraction of price in the level of advancement of the model is the Palm Pilot. In the spring of 2001 Palm charged a price of $399 for its high-end model, the Palm V, on which it made a profit of $150, a 38% markup. For its low-end model, the m-100, it charged $149 for a profit of $26, a markup of 17%[6]. There are even more striking examples of how markups affect prices. The most extreme is Cockburn and Anis (1998), who show that generic Arthritis drugs

[6]Source: BusinessWeek, June 4, 2001, "Palm's Market Starts to Melt Down in it's Hands"

are cheaper than patented ones, even though clinical trials suggest that the generic ones are of superior quality.

2.5 Towards a model of market power erosion

Throughout this section I have assumed that prices follow equation (2). Where the essential difference with previous studies is the assumption of the existence of the markup variable $\mu_{t,v}$. I have argued that it is reasonable to assume that the market power of a given vintage erodes over time, such that $\mu_{t,v} > \mu_{t+1,v}$, because better vintages come online. Moreover, I have assumed that better vintages have more market power, such that $\mu_{t,v} > \mu_{t,v-1}$. In order to substantiate these assumptions I will introduce a theoretical vintage capital model, the equilibrium of which exhibits exactly these properties. This is the subject of the next two sections that follow.

3 Market for machines

In this section I introduce the market for machines with imperfect competition that forms the backbone of my theoretical model. This section consists of three parts. In the first part I describe the demand side of the market for machines in which workers of different types decide on their optimal technology choice. In the second part I introduce the supply side of the market in which monopolistic competitors decide on the profit maximizing price of their machines. In the third part I combine the demand and supply sides of the market and define the Pure Strategy Nash equilibrium outcome, prove its existence and uniqueness, and derive its main properties.

3.1 Machine users

I will take a certain degree of heterogeneity on the demand side of the market as given. This heterogeneity takes the form of different productivity types for workers. Each worker's type is denoted by h. I will assume there is a continuum of workers of measure one that is uniformly distributed over the unit-interval, such that $h \backsim unif\,(0,1)$.

Final goods are produced by the combination of one worker, of type h, with one machine, which embodies A_t efficiency units. The output of such a combination is hA_t. In order to avoid having to consider intractable intertemporal optimization problems and having to make assumptions about possible second hand markets, I will assume that machines fully depreciate in one period. This assumption basically implies that the machines considered here are equivalent to intermediate goods in the sense of Aghion and Howitt (1992) and Romer (1990). The workers can not use these machines for nothing. The price of a machine of type $A_{t-\tau}$ at time t is denoted by $P_{t,\tau}$.

I will allow workers to choose the technology that they are using from a menu of available technologies. That is, workers have the choice between all the types of machines that have been introduced so far. Let A_t be associated with the machines introduced at time t, then, at time t, the workers can choose from the 'technology-menu' $\mathbf{A}_t = \{A_t, A_{t-1}, \ldots\}$. The notational convention that I will use in this paper follows Chari and Hopenhayn (1991) in the sense that τ represents 'vintage age'. That is, A_t represents the frontier technology level and $A_{t-\tau}$ is the frontier technology level of τ periods ago. For notational convenience, I will, every once in a while, switch between the notation of technology in its levels, i.e. A_t, and technology growth rates, i.e. $g_t = \frac{A_t - A_{t-1}}{A_{t-1}}$. Throughout, I will assume that there is no technological regress such that $g_t > 0$ for all t.

In order to maximize his income, a worker of type h will choose a technology from the technology choice set, $\Upsilon_t(h)$, which is defined as the set of vintages for which he maximizes his labor income, such that

$$\Upsilon_t(h) = \left\{ \tau \in \mathbb{N} \,\middle|\, \tau \in \underset{s \in \mathbb{N}}{\arg\max} \{hA_{t-s} - P_{t,s}\} \right\}$$

The resulting labor income of a worker of type h at time t equals

$$y_t(h) = hA_{t-\tau} - P_{t,\tau}, \text{ for all } \tau \in \Upsilon_t(h)$$

3.2 Machine producers

Machine designs are assumed to be patented for M periods. During the first M periods of a machine design's life, the particular machine is supplied by a monopolist firm. After the patent expires the machine design is public domain and there is perfect competition in the supply of these machines. I will assume that units of the consumption good are the only input needed in machine production, this to avoid having to deal with the selection of workers across sectors. The production of a continuum of mass X of machines of type

$A_{t-\tau}$ requires the use of $\frac{c_\tau}{2} A_{t-\tau} X^2$ units of the consumption good, where $c_\tau \geq 0$. That is, when $c_\tau > 0$ a machine producer faces decreasing returns to scale. Note that the cost function is scaled by c_τ which depends on the vintage age, in order to allow for learning by doing.

The set of buyers of machines of type $t-\tau$, which I will denote by $D_t(\tau)$, is given by

$$D_t(\tau) = \left\{ h \in [0,1] \left| \tau \in \underset{s \in \{0,1,2,\ldots\}}{\arg\max} \left(hA_{t-s} - P_{t,s}\right) \right. \right\}$$

As I will show in proposition 1 in the next subsection, these sets will be connected intervals of the form

$$D_t(\tau) = \left[\underline{h}_{t,\tau}, \overline{h}_{t,\tau}\right]$$

Total demand for the machine of type $t-\tau$ at time t is then given by the measure of workers demanding the specific vintage, i.e.

$$X_{t,\tau} = \left(\overline{h}_{t,\tau} - \underline{h}_{t,\tau}\right)$$

The question that is left is how these machine producers end up choosing the prices of their machines. Throughout this paper, I will focus on Pure Strategy Nash equilibria. For the particular problem at hand here this implies that, taking the prices of the other machines, i.e.

$$\mathbf{P}'_{t,\tau} = \{P_{t,0}, \ldots, P_{t,\tau-1}, P_{t,\tau+1}, \ldots\},$$

and the levels of the technologies, i.e.

$$\mathbf{A}_t = \{A_t, A_{t-1}, \ldots\},$$

as given, the machine producer to type $A_{t-\tau}$ chooses the price of his machine to maximize profits. This implies that $P_{t,\tau}$ is an element of the best response set

$$BR_t\left(\tau; \mathbf{P}'_{t,\tau}, \mathbf{A}_t, r_t\right) = \left\{ P_{t,\tau} \in \mathbb{R}_+ \left| P_{t,\tau} \in \underset{P \in \mathbb{R}_+}{\arg\max} \left\{ PX_{t,\tau} - \frac{c_\tau}{2} A_{t-\tau} X_{t,\tau}^2 \right\} \right. \right\}$$

Because patents expire after M periods, these best response sets only apply to $\tau = 0, \ldots, M-1$. For machines that were designed M or more periods ago, perfect competition implies that price must equal average cost, and that free entry drives both to zero, such that $P_{t,\tau} = 0$ for $\tau \geq M$. The corresponding profits are

$$\pi_{t,\tau} = P_{t,\tau} X_{t,\tau} - \frac{c_\tau}{2} A_{t-\tau} X_{t,\tau}^2 \text{ for all } P_{t,\tau} \in BR_t\left(\tau; \mathbf{P}'_{t,\tau}, \mathbf{A}_t, r_t\right)$$

for $\tau = 0, \ldots, M-1$.

3.3 Equilibrium and its properties

Now that the demand and supply side of the machine market are well defined, I can focus on the resulting Pure Strategy Nash equilibrium. Such an equilibrium consists of a price schedule, i.e. $\mathbf{P}_t^* = \left\{P_{t,0}^*, \ldots, P_{t,M-1}^*\right\}$ and a collection of corresponding demand sets, i.e. $\{D_t^*(0), D_t^*(1), \ldots\}$. I will derive the equilibrium in two steps. In the first step I show that, independent of the price schedule, the demand sets $D_t(\tau)$ have some important properties. In the second step I use these properties to derive the equilibrium price schedule $\mathbf{P}_t^*$. This equilibrium price schedule is then used to derive equilibrium output, profits, and demand sets.

The main result of the first step is that (i) better workers use better technologies, i.e. there is endogenous assortative matching between workers and machines. Models where this matching also occurred are, for example, Jovanovic (1999) and Sattinger (1975). (ii) Perfect competition implies that machines of a design for which the patent is expired for more than a year are not demanded anymore. They are obsolete. (iii) Demand functions are properly specified in the sense that for almost all workers their technology choice is unique. (iv) If two workers of different types buy the same vintage of machine, then so will the workers of all types in between. These four things are formalized in Proposition 1 below, which is proven in Appendix 1.

Proposition 1 ***Properties of demand sets***
Independent of the technology menu $\mathbf{A}_t$ and the price schedule $\mathbf{P}_t$, the demand sets $D_t(\tau)$ have the following properties:
(i) For $h' > h$, if $h \in D_t(\tau)$ then $h' \notin D_t(\tau')$ for all $\tau' > \tau$.
(ii) $D_t(\tau) = \emptyset$ for all $\tau > M$.
(iii) Define the set of workers for whom the optimal technology choice is not unique as

$$\widetilde{\mathcal{H}}_t = \{h \in [0,1] \,|\, \exists\ \tau \neq \tau' \text{ such that } h \in D_t(\tau) \wedge h \in D_t(\tau')\}$$

then $\widetilde{\mathcal{H}}_t$ is negligible.
(iv) $D_t(\tau)$ is connected for all τ.

The intuition behind this proposition is probably most clear when one considers a graphic example of how these demand sets are determined. Figure 2 depicts the way these demand sets are determined for the case in which $M = 2$. For simplicity, the time subscript, t, is ignored in the figure. The top panel of figure 2 shows the levels of gross output, i.e. hA_τ, that workers of different types get for the three available technologies, i.e. $\tau \in \{0,1,2\}$, the dots mark the points at which the price and gross output levels coincide. The short dashed vertical lines, that extend to the bottom panel, determine the levels of the critical types that get zero income for the various technologies. The bottom panel then depicts the net output levels, i.e. the workers income levels for the three technologies. Workers choose that technology that yields them the highest income level, which implies the demand sets plotted at the bottom.

Now that I have shown that the demand sets have some convenient properties, I can use them to derive the equilibrium price schedule. Before doing so, I first formally define the PSN-equilibrium for the prices in the machine producing sector, which is

Definition 1 ***Equilibrium price schedule***
For a given sequence of technology levels, $\mathbf{A}_t = \{A_t, A_{t-1}, \ldots\}$, *a price schedule* $\mathbf{P}_t^* = \left\{P_{t,0}^*, P_{t,1}^*, \ldots\right\}$ *is a Pure Strategy Nash equilibrium price schedule if*
(i) $P_{t,\tau}^* = 0$ *for all* $\tau \geq M$.
(ii) Define $\mathbf{P}_{t,\tau}^{*\prime} = \left\{P_{t,0}^*, \ldots, P_{t,\tau-1}^*, P_{t,\tau+1}^*, \ldots\right\}$, *then* $P_{t,\tau}^* \in BR\left(\tau; \mathbf{P}_{t,\tau}^{*\prime}, \mathbf{A}_t, r_t\right)$ *for all* $\tau = 0, \ldots, M-1$.

What I will show in this step is that, for all possible technology paths $\mathbf{A}_t = \{A_t, A_{t-1}, \ldots\}$, there exists a unique equilibrium price schedule. This price schedule is such that all technologies of age M or more recent are used. In particular, the core-proposition of this subsection reads

Proposition 2 ***Solution of equilibrium price schedule***
For any sequence of technology levels, $\mathbf{A}_t = \{A_t, A_{t-1}, \ldots\}$, *there exists a unique equilibrium price schedule with the following properties:*
(i) $P_{t,\tau} > \frac{c_\tau}{2} A_{t-\tau} X_{t,\tau}^2 \geq 0$ *for all* $\tau = 0, \ldots, M-1$.
(ii) $D_t(\tau) \neq \emptyset$ *for all* $\tau = 0, \ldots, M$.
(iii) The equilibrium price schedule is unique, and defining the price per efficiency unit as $\widehat{P}_{t,\tau} = P_{t,\tau}/A_{t-\tau}$, *it satisfies*

$$\widehat{P}_{t,\tau} = \begin{cases} \left[\frac{1+c_0\left(1+w_{t,0}^1\right)}{2+c_0\left(1+w_{t,0}^1\right)}\right]\left[\frac{1}{1+w_{t,0}^1} + \frac{w_{t,0}^1}{1+w_{t,0}^1}\widehat{P}_{t,1}\right] & \text{for} \quad \tau = 0 \\ \left[\frac{1+c_\tau\left(w_{t,\tau}^{\tau+1}+w_{t,\tau}^{\tau-1}\right)}{2+c_\tau\left(w_{t,\tau}^{\tau+1}+w_{t,\tau}^{\tau-1}\right)}\right]\left[\frac{w_{t,\tau}^{\tau-1}}{w_{t,\tau}^{\tau+1}+w_{t,\tau}^{\tau-1}}\widehat{P}_{t,\tau-1} + \frac{w_{t,\tau}^{\tau-1}}{w_{t,\tau}^{\tau+1}+w_{t,\tau}^{\tau-1}}\widehat{P}_{t,\tau+1}\right] & \text{for} \quad \tau = 1, \ldots, M-1 \\ 0 & \text{for} \quad \tau = M \end{cases} \tag{9}$$

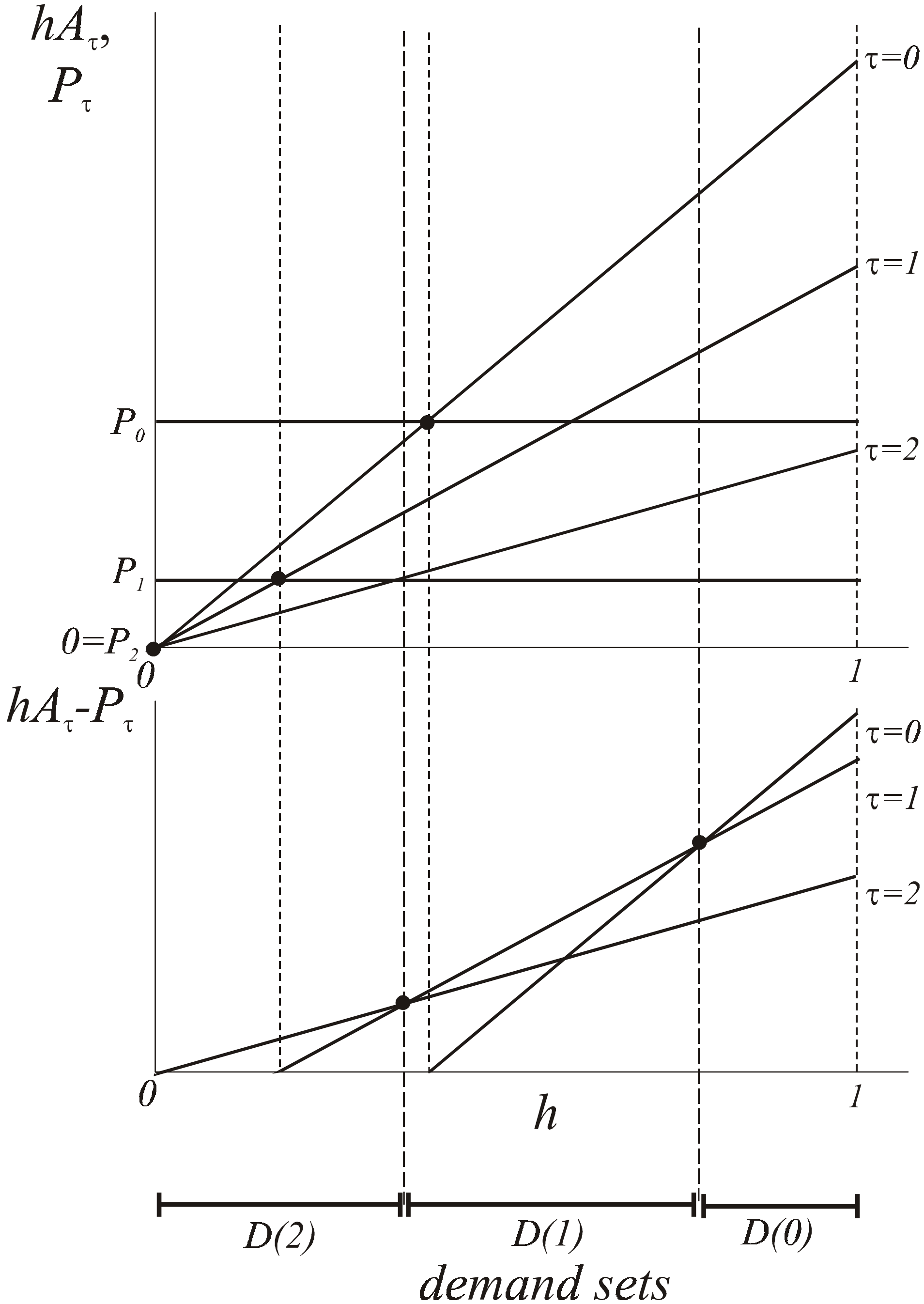

Figure 2: Determination of demand sets

where

$$w_{t,\tau}^{\tau-1} = \frac{A_{t-\tau+1}}{A_{t-\tau+1} - A_{t-\tau}}, \text{ and } w_{t,\tau}^{\tau+1} = \frac{A_{t-\tau-1}}{A_{t-\tau} - A_{t-\tau-1}}$$

(iv) $\widehat{P}_{t,\tau}$ *is strictly decreasing in* τ*.*
(v) The demand sets satisfy

$$X_{t,\tau} = \begin{cases} \left[\frac{1+w_{t,0}^{1}}{1+c_0\left(1+w_{t,0}^{1}\right)}\right]\widehat{P}_{t,0} & \text{for} \quad \tau = 0 \\ \left[\frac{\left(w_{t,\tau}^{\tau+1}+w_{t,\tau}^{\tau-1}\right)}{1+c_\tau\left(w_{t,\tau}^{\tau+1}+w_{t,\tau}^{\tau-1}\right)}\right]\widehat{P}_{t,\tau} & \text{for} \quad \tau = 1,\ldots,M-1 \\ w_{t,M}^{M-1}\widehat{P}_{t,M-1} & \text{for} \quad \tau = M \end{cases}$$

The proof of this proposition is again in Appendix 1. Besides the fact that this proposition proves the existence and uniqueness of the PSN-equilibrium in this market, the most important result of this proposition is *(iv)*. It basically implies that the average cost per efficiency unit is higher for more recent vintages than for older ones. Note that this result is independent of the path of technological progress as well as the cost structure underlying production of the vintages of machines, i.e. $\{c_\tau\}_{\tau=0}^{\infty}$. It is simply due to the imperfect competition between the machine suppliers. This is the result that will underlie the spurious equipment price deflation result that I will present in Section 5. First, however, I will implement the market for machines introduced here in an endogenous growth model.

4 Endogenous Growth Model

The aim of this section is to implement the market introduced above in a general equilibrium framework with endogenous growth. In order to do so, I have to combine the machine buyers and suppliers with a final goods demanding sector, i.e. consumers, as well as with a sector that creates new machine designs and moves the technological frontier outward, i.e. an R&D sector. These two respective additions form the first two subsections of this section. In the third subsection, I combine all sectors of the economy to define a competitive equilibrium and balanced growth path for it.

4.1 Consumers

Consumers and workers in the final goods sector are basically the same. I will assume that workers of all types each maximize the present discounted value of their lifetime utility and have constant relative risk aversion preferences, such that they choose to maximize

$$\frac{1}{1-\sigma}\sum_{s=t}^{\infty}\beta^{s-t}c_s(h)^{1-\sigma}$$

subject to their budget constraint

$$k_t(h) = (1+r_t)\,k_{t-1}(h) + y_t(h) - c_t(h) + \Pi_t$$

where $k_t(h)$ is capital holdings at period t of a worker of type h, which is assumed to be the same for all workers of the same type, r_t is the interest rate, $y_t(h)$ is the labor income of a worker of type h in period t, and $c_t(h)$ is the corresponding consumption level, and $\beta \in (0,1)$ the discount factor. The income Π_t is obtained from innovative activities that each household invests in, which I will explain in the subsection below.

This problem yields the familiar Euler equation

$$\frac{c_{t+1}(h)}{c_t(h)} = [\beta(1+r_t)]^{\frac{1}{\sigma}}$$

This implies that the consumption growth rates of all workers are the same, independent of their type. As described in Caselli and Ventura (2000), this implies that aggregate consumption is consistent with that of a representative consumer that has CRRA preferences himself.

In the following, capital letters, e.g. Y_t, denote aggregates obtained from aggregation over the various types of workers, i.e.

$$Y_t = \int_0^1 y_t(h)\,dh$$

The aggregates K_t, C_t, and Y_t, behave as if they were the solution to a representative consumer solving a utility maximization problem that is identical to that of the workers of each type, but then defined in these aggregates.

4.2 Patent race and innovation

As a simplifying assumption for my general equilibrium framework, I will assume, as do Reinganum (1983), Gilbert and Newbery (1982) and Aghion and Howitt (1992), that the size of the innovation (in each period) is fixed.

In particular, the size of the innovation is $g > 0$, in the sense that $A_{t+1} = (1+g)\,A_t$ for all t, as a result of an innovation[7]. Instead of the size of the innovation, what is determined in equilibrium is the R&D intensity with which the innovation is pursued. This intensity is represented by the amount of output spent on the patent race, which I will denote by $X_{t,R}$. The final good is assumed to be the only input into the R&D process.

If one wins the patent race, then one obtains a patent with a value that is equal to the present discounted value of the monopoly profits made on the particular machine design. I will derive this value in more detail later, but for the moment will simply denote it by V_t. It turns out to be convenient to also consider the value per efficiency unit, V_t^*, which satisfies $V_t^* = V_t/A_t$. The patent race that I consider is one in which the probability of winning per unit of output spent is inversely proportional to the total amount of resources devoted to R&D, i.e. $X_{t,R}$. That is, when $X_{t,R}$ is the total amount of resources spent for R&D purposes, the spender of a unit of output on R&D pays a price equal to one and obtains the expected revenue $V_t/X_{t,R}$. I will assume that there is no advantage for the incumbent, such that incumbents and entrants have an equal chance of winning the patent race. Since there are basically M incumbents and a continuum of researchers, the possibility of an incumbent winning the race is zero. Furthermore, freedom of entry and exit in R&D implies the zero profit equilibrium condition $X_{t,R} = A_t V_t^*$.

So, what is left to derive is the present discounted value of the monopoly profits made off a machine design, i.e. V_t. The assumption that g is constant over time implies some important simplifications for the behavior of prices, output, and the value of an innovation. These implications are derived in the proposition below.

Proposition 3 ***value of innovation, output, etc., at constant*** g
If the technological frontier moves out at the same rate, g, in each period then this implies
(i) prices: *the vintage age specific prices per efficiency unit satisfy*

$$P_{t,\tau}/A_{t-\tau} = \mu_\tau\,(g, \mathbf{c})$$

where $\mu_\tau\,(g) > \mu_{\tau+1}\,(g)$ for all $\tau = 0, \ldots, M-1$, $g > 0$, and $\mathbf{c} = \{c_0, \ldots, c_M\}$ is the sequence of production parameters for machines.

[7]Throughout, I will assume that $g < \left(\frac{1}{\beta}\right)^{\frac{1}{1-\sigma}} - 1$, such that the consumer's objective function will be bounded.

(ii) demand for various vintages: *the demand for different vintages of machines , i.e.* $X_{t,\tau}$*, satisfies*

$$X_{t,\tau} = \widetilde{X}_\tau (g, \mathbf{c})$$

and only depends on vintage age and not on time.
(iii) output: *aggregate output,* Y_t*, can be written as*

$$Y_t^* = Y_t / A_t = \widetilde{Y} (g, \mathbf{c})$$

(iv) profits: *the vintage age specific profits follow*

$$\pi_{t,\tau}^* = \pi_{t,\tau} / A_t = \widetilde{\pi}_\tau (g, \mathbf{c})$$

(v) value of innovation: *for the value of the innovation I will assume that the R&D costs incurred today yield a patent for a machine that comes online only in the next period. Consequently, the value of the innovation at time t can be written as*

$$\begin{aligned} V_t^* &= \sum_{s=1}^{M} \left(\frac{1}{1+g} \right)^{s-1} \left(\prod_{j=0}^{s-1} \left(\frac{1}{1+r_{t+j}} \right) \right) \widetilde{\pi}_{s-1} (g, \mathbf{c}) \\ &= V^* (r_t, \ldots, r_{t+M}; g, \mathbf{c}) \end{aligned}$$

(vi) rents on innovative activities: *For simplicity, I will assume that each household will take an equal share in each research project, such that there is no uncertainty about the return to their expenditures. Consequently, the income earned from innovative activities equals the sum of the flow profits made on the currently patented machine designs minus the current R&D expenditures, such that*

$$\Pi_t = \sum_{\tau=0}^{M} A_{t-\tau} \widetilde{\pi}_\tau (g, \mathbf{c}) - X_{t,R} \tag{10}$$

$$= A_t \sum_{\tau=0}^{M} \left(\frac{1}{1+g} \right)^{\tau} \widetilde{\pi}_\tau (g, \mathbf{c}) - A_t X_{t,R}^* \tag{11}$$

where $X_{t,R}^* = X_{t,R} / A_t$*.*
The functions $\mu_\tau (g, \mathbf{c})$*,* $\widetilde{X}_\tau (g, \mathbf{c})$*,* $\widetilde{Y} (g, \mathbf{c})$ *and* $\widetilde{\pi}_\tau (g, \mathbf{c})$ *are independent of time.*

4.3 Competitive equilibrium

Having derived the solutions to the individual optimization problems of the three sectors in this economy, I am now able to combine these decentralized decisions to define the competitive equilibrium outcome of this economy. Because I have assumed that capital is not used in production, I have abstracted from possible transitional dynamics. Consequently, similar to Romer (1990), the competitive equilibrium defined below constitutes a balanced growth path

Definition 2 ***Competitive equilibrium***
Given $\mathbf{A}_0 = \{A_0, A_{-1}, \ldots\}$*, and* $\{k_0(h)\}_{h=0}^{\overline{h}}$*, a competitive equilibrium in this economy is a path*

$$\left\{\{c_t(h), y_t(h), k_t(h)\}_{h=0}^{1}, \mathbf{P}_t, \{D_t(\tau)\}_{\tau=0}^{\infty}, X_{R,t}, \Pi_t, r_t, A_t\right\}_{t=1}^{\infty}$$

such that
(i) Utility maximization*: Given* $\{y_t(h), r_t, k_0(h)\}_{t=1}^{\infty}$*,* $\{c_t(h)\}_{t=1}^{\infty}$ *solves the utility maximization problem of the workers of all types* $h \in [0,1]$*.*
(ii) Optimal technology choice*: In every period, given* $\mathbf{P}_t$ *and* $\mathbf{A}_t$*, the demand sets* $\{D_t(\tau)\}_{\tau=0}^{\infty}$ *are determined by the workers' optimal technology choice decision introduced in subsection 3.2.*
(iii) Price equilibrium*: In every period, given* $\mathbf{A}_t$*,* $\mathbf{P}_t$ *is the price equilibrium.*
(iv) Patent race equilibrium*: In every period, the research intensity* $X_{R,t}$ *solves the patent race equilibrium.*
(v) Rents on innovative activity*:* Π_t *is determined by (10).*
(vi) Capital market clearing*: In every period, the interest rate* r_t *clears the capital market such that* $K_t = 0$*.*
(vii) Technological progress*: In every period,* $A_{t+1} = (1+g) A_t$*.*

In order to derive the competitive equilibrium of this economy it is easiest to rewrite the competitive equilibrium dynamics in terms of transformations of variables that will be constant on the equilibrium path. These transformations turn out to be output, capital, and consumption per efficiency unit, i.e. $Y_t^* = Y_t/A_t$, $K_t^* = K_t/A_t$, and $C_t^* = C_t/A_t$, prices per efficiency unit, i.e. $\widehat{P}_{t,\tau}$, the interest rate, i.e. r_t, and the research intensity and income per efficiency unit, i.e. $X_{t,R}^* = X_{t,R}/A_t$ and $\Pi_t^* = \Pi_t/A_t$. In terms of these variables, a competitive equilibrium is defined as a combination of variables

$$\left\{Y^*, K^*, C^*, \widehat{P}_\tau, \Pi^*, X_R^*, r\right\}$$

such that

$$
\begin{aligned}
Y^* &= \widetilde{Y}(g,\mathbf{c}) && (12)\\
K^* &= 0 && (13)\\
X_R^* &= V^*(r,\ldots,r;g,\mathbf{c}) && (14)\\
\Pi^* &= \sum_{\tau=0}^{M}\left(\frac{1}{1+g}\right)^{\tau}\widetilde{\pi}_\tau(g,\mathbf{c}) - X_R^* && (15)\\
(1+g)K^* &= (1+r)K^* + Y^* - C^* + \Pi^* && (16)\\
C^*/C^* &= \left(\frac{1}{1+g}\right)[\beta(1+r)]^{\frac{1}{\sigma}} = 1 && (17)
\end{aligned}
$$

and $\widehat{P}_\tau$ is the PSN equilibrium in the machine market. The following proposition establishes its existence and uniqueness.

Proposition 4 ***Existence and uniqueness of competitive equilibrium***
For all $g>0$, $\mathbf{c}\in\mathbb{R}_+^M$, there exists a unique septuple $\left\{Y^*,K^*,C^*,\widehat{P}_\tau,\Pi^*,X_R^*,r\right\}$ that satisfies equations (12) through (17) and where $\widehat{P}_\tau$ is the PSN equilibrium.

In the next section, I will show how equipment price indices, as measured by the BEA and BLS, will behave on the competitive equilibrium path of this economy.

5 Spurious equipment price deflation

So, what would happen if the BEA and BLS would measure equipment price indices in the economy above? Before I analyze this question in detail, I start off by considering what would be a reasonable price index and productivity index in this economy. In order to consider this, it is important to realize that on the balanced growth path

Implication 1 ***Average price paid per efficiency unit is constant over time***
The total number of efficiency units sold in the market equals

$$
\sum_{\tau=0}^{M} A_{t-\tau}X_{t,\tau} = A_t\sum_{\tau=0}^{M}\left(\frac{1}{1+g}\right)^{\tau}\widetilde{X}_\tau(g,\mathbf{c}) = A_t\overline{X}(g,\mathbf{c}) \qquad (18)
$$

while the total revenue in the market equals

$$\sum_{\tau=0}^{M} P_{t,\tau} X_{t,\tau} = A_t \sum_{\tau=0}^{M} \left(\frac{1}{1+g} \right)^{\tau} \mu_{\tau} (g, \mathbf{c}) = A_t \overline{P} (g, \mathbf{c})$$

such that the average price of an efficiency unit equals $\overline{P} (g, \mathbf{c}) / \overline{X} (g, \mathbf{c})$ *and is independent of time.*

Implication 2 ***Average production cost per efficiency unit is constant over time***
The total production costs of all the efficiency units sold in the market equals

$$\sum_{\tau=0}^{M} A_{t-\tau} c_{\tau} X_{t,\tau}^{2} = A_t \sum_{\tau=0}^{M} \left(\frac{1}{1+g} \right)^{\tau} c_{\tau} \widetilde{X}_{\tau}^{2} (g, \mathbf{c}) = A_t \overline{C} (g, \mathbf{c})$$

such that the average production cost per efficiency unit equals $\overline{C} (g, \mathbf{c}) / \overline{X} (g, \mathbf{c})$ *and is again independent of time.*

These two implications are important because they suggest that (i) because the average production cost per efficiency unit is constant there is no productivity growth in the machine producing sector, (ii) because the average price paid per efficiency unit is constant, any reasonable quality adjusted investment price index should be constant over time.

I will now proceed with the following thought experiment in this section. Suppose that the BEA and BLS would observe the quality of machines, i.e. A_t, perfectly and would apply their methods to the construction of an investment price index in this economy, how would the resulting investment price index behave? As it turns out, for all the methods used by the BEA and BLS the resulting price index would not be constant, but would instead be steadily declining.

Similar to Section 2, I will again discuss the implications for both matched-model as well as hedonic price indices.

5.1 Matched-model indices

The application of (3) in my model to construct a matched model price index would yield an estimate of equipment price inflation equal to

$$\pi_t^M = \sum_{\tau=1}^{M} \omega_{t,\tau} \left(\frac{P_{t,\tau}}{P_{t-1,\tau-1}} - 1 \right)$$

$$= \sum_{\tau=1}^{M} \omega_{t,\tau} \left(\frac{\mu_\tau}{\mu_{\tau-1}} - 1 \right) < 0$$

which is the theoretical equivalent of (4). Just like I explained in section 2, equipment price inflation in this model is underestimated due to the continuous erosion of market power of the vintages that are traded, implied by the result that newer vintage have higher markups than older ones such that $\mu_{\tau-1} > \mu_\tau$. In fact, the application of matched-model indices in my model economy would lead to the measurement of spurious equipment price deflation.

5.2 Hedonic price indices

Hedonic price indices are used by the BEA and BLS to quality adjust the price indices for, among others, computer equipment, and software. They apply two types of hedonic price indices. The first type is based on a sequence of separate cross sectional regressions, each for a specific period. This is the methodology that the BLS applies for the construction of some of its Producer Price Indices. Holdway (2001) contains an excellent explanation of the BLS' methodology. The second type consists of hedonic price indices based on pooled cross-sectional regressions. These are applied by the BEA for the construction of its investment price indices used in the National Income and Product Accounts. Wasshausen (2000) contains a detailed description of the evolution of the hedonic regressions used by the BEA over the years.

In order to address the behavior of these two price indices in my theoretical model, I first describe what the model implies for the cross-sectional behavior of prices. Throughout, I will assume that the quality index A_t is observed correctly, which is doubtful in the actual application of hedonic price methods. Prices in my model satisfy

$$\ln P_{t,\tau} = \ln A_{t-\tau} + \ln \mu_\tau \tag{19}$$

Bearing in mind that (19) is the underlying data generating process, I will again consider the application of log-log hedonic regressions.

A cross-sectional hedonic regression, as used by the BLS, in the context of my theoretical model would be for a specific t and of the form

$$\ln P_{t,\tau} = \left(\ln \overline{P}_t \right) + \beta \ln A_{t-\tau}$$

the resulting regression coefficient for quality will then equal

$$\widehat{\beta} = 1 + \frac{\sum_{\tau=0}^{M-1} \left(\ln A_{t-\tau} - \overline{\ln A_{t-\tau}}\right)\left(\ln \mu_\tau - \overline{\ln \mu_\tau}\right)}{\sum_{\tau=0}^{M-1} \left(\ln A_{t-\tau} - \overline{\ln A_{t-\tau}}\right)^2} > 1 \tag{20}$$

where

$$\overline{\ln A_{t-\tau}} = \frac{1}{M-1} \sum_{\tau=0}^{M-1} \ln A_{t-\tau} \text{ and } \overline{\ln \mu_\tau} = \frac{1}{M-1} \sum_{\tau=0}^{M-1} \ln \mu_\tau$$

and the summation runs up till $M-1$, because $P_{t,M} = 0$ and thus can not be taken a logarithm of. The positive bias in the estimate $\widehat{\beta}$ is an example of the omitted variable problem discussed in section 2.

How does this positive bias affect measured investment price inflation? To answer this question, I will compare the implication for the sequence of frontier machines over time. The theoretical model implies that $\ln A_t = \ln(1+g) + \ln A_{t-1}$ and, as can be seen from (19), $\ln P_{t,0} = \ln(1+g) + \ln P_{t-1,0}$. Consider a hedonic price index for the frontier machine, which I will denote by P_t^H. The percentage change in the hedonic price index, i.e. π_t^H, is equal to the percentage change in the prices minus the part that is attributable to the quality change. That is,

$$\pi_t^H \approx \Delta \ln P_t^H = \Delta \ln P_{t,0} - \widehat{\beta} \Delta \ln A_t = \left(1 - \widehat{\beta}\right) \ln(1+g) < 0$$

where Δ is the first difference operator. Hence, the positive bias in the estimate of β leads to spurious investment price deflation. The extent of this deflation is increasing in the correlation between $\ln \mu_\tau$ and $\ln A_{t-\tau}$. Note that if $\beta = 1$ would be estimated correctly, then this method would lead to the proper result that there is no investment price deflation whatsoever. Furthermore, the balanced growth properties of the model imply that $\widehat{\beta}$ and thus π_t^H are independent of time, i.e. $\pi_t^H = \overline{\pi}^H$.

Instead of data on a single cross-section for each year, the BEA pools these cross-sections for the quality adjustment of the price indices used for some types of computer equipment in the NIPA. Wasshausen (2000) contains a detailed description of the hedonic regressions used. In the context of the theoretical model here, the BEA's pooled cross-sectional regressions boil down to the regression of $\ln P_{t,\tau}$ on $\ln A_{t-\tau}$ and time dummies. That is, the regression equation is

$$\ln P_{t,\tau} = \sum_{t=1}^{T} \delta_t D_t + \beta \ln A_{t-\tau}$$

where $D_t = 1$ in period t and 0 otherwise. It is fairly straightforward to show that, because of the balaced growth properties of the model, in this equation the estimated coefficient on quality, i.e. $\widehat{\beta}$, is the same as in (20). Moreover, the estimated time-varying intercepts, i.e. $\hat{\delta}_t$'s, will satisfy

$$\hat{\delta}_t = \hat{\delta}_{t-1} + \left(1 - \widehat{\beta}\right) \ln (1 + g)$$

If quality adjusted inflation is directly measured by the changes in the estimated time-varying intercepts, then the estimated equipment price inflation using this method is

$$\pi_t^H \approx \hat{\delta}_t - \hat{\delta}_{t-1} = \left(1 - \widehat{\beta}\right) \ln (1 + g) < 0$$

which is equal to that measured by the simple cross-sectional method. Again, this method leads to the measurement of spurious equipment price deflation due to imperfect competition.

In the simple theoretical model in this paper, this bias can be eliminated by the inclusion of vintage age dummies. That is, the hedonic regression equation

$$\ln P_{t,\tau} = \sum_{t=1}^{T} \delta_t D_t + \sum_{\tau=0}^{M-1} \theta_\tau D_\tau + \beta_2 \ln A_{t-\tau} \tag{21}$$

where $D_\tau = 1$ if a machine is of vintage age τ and zero otherwise, would lead to the appropriate regression result that $\hat{\delta}_t = \hat{\delta}_{t-1}$ for all $t \geq 1$. Berndt, Griliches and Rapaport (1993) use vintage age dummies in their empirical studies of PC prices and find limited significance. One has to realize, however, that vintage age dummies are a good proxy for the extent of the markup in the theoretical model here, because I have assumed that only one new machine design is invented in each period. Furthermore, the coefficients θ_τ in (21) are constant because I assume that g is constant over time. Though these two assumptions are innocuous for the expositional purpose of the theoretical model introduced in this paper, they would be unrealistic to make for the purpose of an empirical analysis. For example, in the case of PC's, where Berndt, Griliches, and Rapaport (1993) use dummies for vintage age in years, the frequency of introduction of new models is much higher than once a year.

This is not the first paper to point out that markups might affect hedonic price indices. Pakes (2001), for example, contains an illuminating discussion of the same topic. What is different here is that I show, in the specific theoretical context of my model, that these indices have a bias with a known sign and how this bias comes about through imperfect competition.

Table 2: Numerical results

case	g	c_0	γ	$\overline{\pi}^M$	$\overline{\pi}^H$
A	0.587	0	0	-67.2%	-383.8%
B	0.587	30	0.125	-4.8%	-40.3%
C	0.587	30	0.25	-7.0%	-88.2%
D	0.587	60	0.25	-4.5%	-59.2%

5.3 A numerical example

In the two subsections above, I have shown that the methods applied by the BEA and BLS lead to an upward bias in measured investment price deflation in the theoretical model introduced in this paper. I have, however, not addressed the possible magnitude of this bias. In this subsection, I use a simple numerical example to get at this magnitude.

As one can see from the explanation in the previous subsection, the only parameters that are relevant for this bias are the growth rate of the technological frontier, i.e. g, the vintage age dependent cost parameters, collected in the vector $\mathbf{c}$, and the patent length M.

As Jaffe (1999) reports, the standard patent length in the U.S. since 1994 is 20 years, so I will fix $M = 20$. In order to keep my numerical example in line with my empirical example of section 2, I will choose $g = 0.587$. This is the annual growth rate of PC microprocessor speeds implied by Moore's law, i.e. the prediction that the speed of microprocessors will double about once every 18 months.

What is left to choose is the cost structure, $\mathbf{c}$. I will allow for possible learning by doing in the sense that

$$c_\tau = \frac{c_0}{(1+\gamma)^\tau} \text{ where } \gamma \geq 0$$

Here γ reflects the per period percentage gain in efficiency in the production of machines of a particular vintage due to learning by doing. I will illustrate the outcome of the model for different rates of learning by doing and different values of c_0. The values of γ and c_0 that I use are chosen purely for illustrative purposes to show the effect the cost parameters have on the equilibrium outcome and on the deflation bias.

Table 2 lists the parameter combinations and implied measured investment price inflation rates for matched-model and hedonic price indices.

For the calculation of the matched model indices I will choose, as Dulberger (1989) suggests, the weights for the matched model price index as the previous period market share of the vintage relative to the other vintages in the index, such that

$$\omega_{t,\tau} = \frac{\left(\frac{1}{1+g}\right)^{\tau} \mu_{\tau-1}\left(g, \mathbf{c}\right) \widetilde{X}_{\tau-1}\left(g, \mathbf{c}\right)}{\sum_{\tau=1}^{M} \left(\frac{1}{1+g}\right)^{\tau} \mu_{\tau-1}\left(g, \mathbf{c}\right) \widetilde{X}_{\tau-1}\left(g, \mathbf{c}\right)} = \overline{\omega}_{\tau}$$

and only depends on vintage age and not on time. This implies that $\pi_t^M = \overline{\pi}^M$ is also independent of time.

Before I discuss the estimated deflation rates in detail it is useful to first consider the equilibrium results plotted in Figure 3. The figure plots the price as a function of the vintage (upper-left), the price per efficiency units as a function of the number of efficiency units (lower-left), the market share for each vintage (upper-right), and the diffusion curve (lower-right). Diffusion is defined as the percentage of people using vintages that are as good as or better than a particular vintage.

In the case of no production costs, i.e. $c_0 = 0$ (case A), the producer of the frontier vintage is the dominating market force. This producer can basically decide what part of the market he conquers and what part he leaves for his competitors. Consequently, the frontier vintage absorbs more than half of the market. If the supplier of the frontier vintage faces significant production costs then decreasing returns to scale might force him to raise his price and to lower his market share in order to cover costs. This effect can be seen because the supplier of the frontier vintage has a lower market share in the case of D than in C. Learning by doing gives the suppliers of older vintages a relative competitive edge over those of new vintages. Consequently, an increase in the learning by doing rate shifts market share from newer to older vintages. This can be seen when one compares the market share curves for cases B and C, where the increased learning by doing increases the market shares of vintages 6 through 18 at the expense of the other ones. The lower-left hand panel of Figure 3 plots $\mu_{\tau}\left(g, \mathbf{c}\right)$. As you can see, $\mu_{\tau}\left(g, \mathbf{c}\right)$ is relatively flat for the cases B, C, and D and has a lower correlation with $A_{t-\tau}$ plotted on the x-axis of that panel. Consequently, the downward bias in the hedonic price index is much smaller in cases B-D, as is listed in Table 2, than in case A. In fact, in case A the bias in the estimated coefficient in the hedonic regression is so severe that it implies an infeasible equipment deflation rate of -383%.

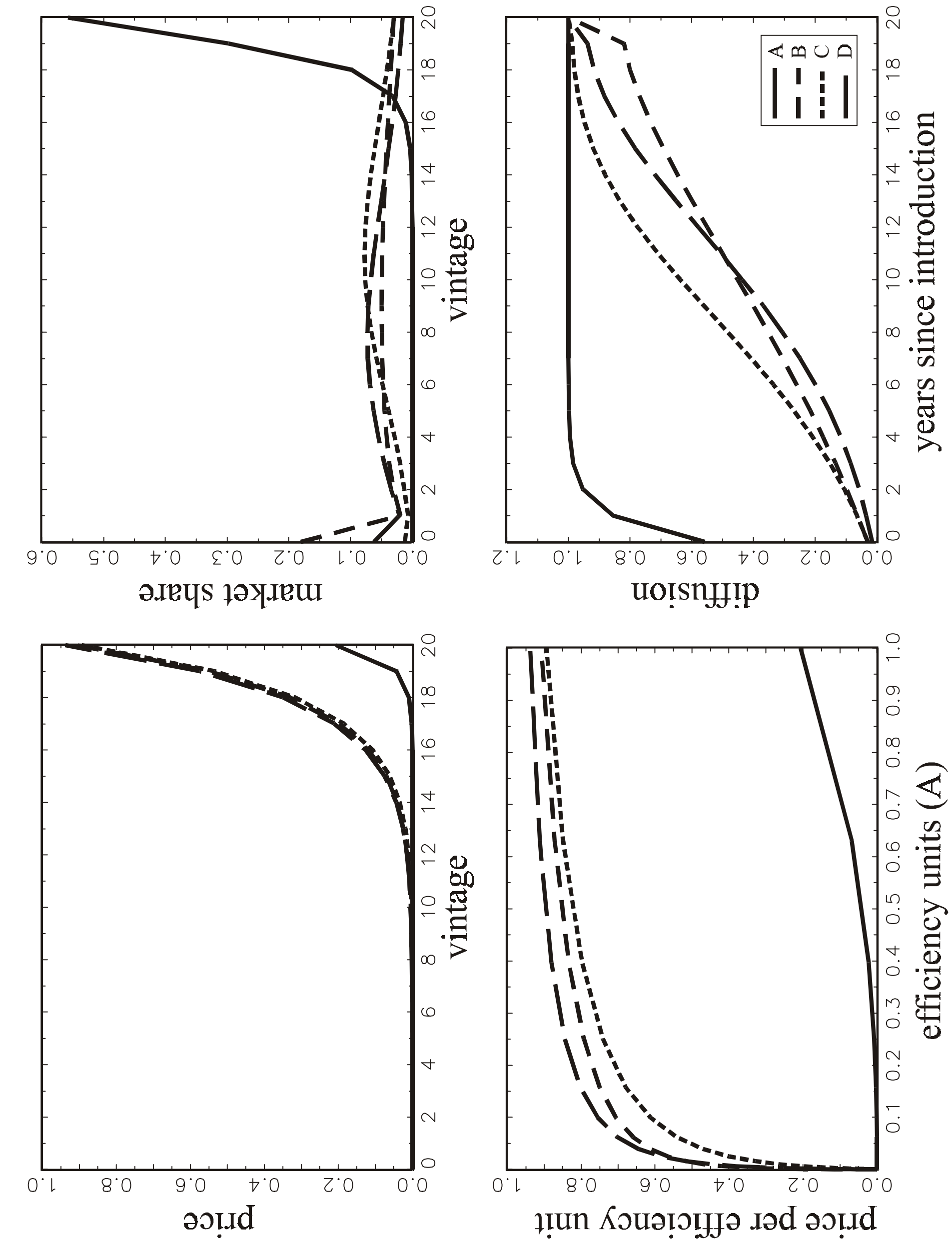

Figure 3: Equilibrium outcome for Moore's law ($g = 0.587$)

The conclusion of this numerical exercise is thus that in my theoretical model the potential biases in measured investment price deflation can be quite large. For hedonic price indices they can even be so large that they lead to infeasible deflation rates. Furthermore, these biases turn out to be very sensitive to the underlying market structure.

6 Conclusion

In this paper I argued that, by not taking into account the fact that equipment markets are not perfectly competitive, the BEA and BLS are likely to overestimate investment price deflation. The main intuition behind this result is that what their price indices might capture is actually not a price decrease but the constant erosion of the market power of existing vintages of machines in the market. To illustrate my argument, I introduced an endogenous growth model in which suppliers of different vintages of machines imperfectly compete for the demand of a heterogeneous set of workers. This market structure results in a price schedule which would lead the BLS and BEA to find investment price deflation, even though the model economy does not exhibit any investment price deflation at all. The measured deflation in the model economy is a complete statistical artifact.

The theoretical example given in this paper is an extreme case. In practice there is good reason to believe that the quality of investment goods has been steadily improving. Quality adjustments of capital goods, however, are currently treated with double standards. On the one hand, there are computer equipment and software to which the BEA and BLS extensively apply the quality adjustment methods discussed in this paper. While on the other hand, there are the other capital goods for which there is no serious effort to quality adjust.

This paper suggests that real investment in computer equipment and software is likely to be overstated because of the bias discussed here. Thus, everything else equal, the results in this paper suggest an overestimation of real output growth and productivity growth in the IT producing sector. However, real output growth and productivity for other capital goods producing sectors is likely to be underestimated because it is virtually not quality adjusted at all.

In this paper, I have focussed my attention on equipment price indices, the bias that I discuss, however, is a potential problem for any product market with rapid technological change to which matched model and hedonic price indices are applied.

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1 Proofs

Proof of proposition 1: Properties of demand sets

(i) Consider $h' > h$ and $\tau' > \tau$, then $h \in D_t(\tau)$ implies that

$$\forall\ s \in \mathbb{N}:\ A_{t-\tau}h - P_{t,\tau} \geq A_{t-s}h - P_{t,s}$$

or, equivalently, in terms of marginal benefits and costs

$$\forall\ s \in \mathbb{N}:\ (A_{t-\tau} - A_{t-s})\, h \geq P_{t,\tau} - P_{t,s}$$

Consequently, because for all $\tau' > \tau$ strictly positive technological progress implies $A_{t-\tau'} > A_{t-\tau}$, the marginal benefits from updating for the worker of type h' exceed those of the worker of type h. That is,

$$\forall \tau' > \tau:\ (A_{t-\tau} - A_{t-\tau'})\, h \geq P_{t,\tau} - P_{t,\tau'}$$

This implies that it must thus be true that $h' \notin D_t(\tau')$ for all $\tau' > \tau$.
(ii) Because patents expire after M periods, all intermediate goods producers of vintages of age M or older face perfect competition. As a result, the price of these vintages is competed down to zero. Consequently all workers will use at least the best technology that is available for free, such that no one will use a technology that is older than M.
(iii) Since $h \backsim unif\,(0,1)$, it suffices to prove that $\widetilde{\mathcal{H}}$ contains a finite number of elements. Since workers will use only technologies $\{0,\ldots,M\}$ there are only a finite number of combinations between which workers can be indifferent. I will show that, if a worker of type h is indifferent between two intermediate goods, then no other worker will be. That is, define the set

$$\widehat{\mathcal{H}}_t(\tau,\tau') = \{h \in [0,1]\,|\, h \in D_t(\tau) \wedge h \in D_t(\tau')\}$$

such that

$$\widetilde{\mathcal{H}}_t = \bigcup_{\tau=0}^{M-1} \bigcup_{\tau'=\tau+1}^{M} \widehat{\mathcal{H}}_t(\tau,\tau')$$

and, denoting the Lebesque measure as $\mu(.)$, we obtain

$$\mu\left(\widetilde{\mathcal{H}}_t\right) \leq \frac{1}{\overline{h}} \sum_{\tau=0}^{M-1} \sum_{\tau'=\tau+1}^{M} \mu\left(\widehat{\mathcal{H}}_t(\tau,\tau')\right)$$

I will simply show that $\forall \tau' > \tau : \mu\left(\widehat{\mathcal{H}}_t(\tau,\tau')\right) = 0$. Let $h \in [0,1]$ be such that $h \in D_t(\tau)$ as well as $h \in D_t(\tau')$ for $\tau' > \tau$. In that case

$$A_{t-\tau}h - P_{t,\tau} = A_{t-\tau'}h - P_{t,\tau'}$$

or equivalently

$$(A_{t-\tau} - A_{t-\tau'})\, h = P_{t,\tau} - P_{t,\tau'}$$

This, however implies that for all $h' > h > h''$

$$\left(A_{t-\tau} - A_{t-\tau'}\right) h' > P_{t,\tau} - P_{t,\tau'} > \left(A_{t-\tau} - A_{t-\tau'}\right) h''$$

such that the workers of type $h' > h$ will prefer τ over τ', while workers of type $h'' < h$ will do the opposite. Hence, $\widehat{\mathcal{H}}_t\left(\tau, \tau'\right) = \{h\}$ and is of measure zero.
(iv) Consider $h'' > h' > h$ such that $h'' \in D_t(\tau)$ as well as $h \in D_t(\tau)$. This implies that

$$\forall\ s \in \mathbb{N}:\ \left(A_{t-\tau} - A_{t-s}\right) h'' > \left(A_{t-\tau} - A_{t-s}\right) h' > \left(A_{t-\tau} - A_{t-s}\right) h \geq P_{t,\tau} - P_{t,s}$$

such that

$$\forall\ s \in \mathbb{N}: A_{t-\tau} h' - P_{t,\tau} > A_{t-s} h' - P_{t,s}$$

and thus $h' \in D_t(\tau)$. Hence, $D_t(\tau)$ is connected. ■

Proof of proposition 2: Solution to equilibrium price schedule

I will prove this proposition in two parts. The first consists of my proof of (i) and (ii). In the second part, I use (i) and (ii), together with results (i) and (iv) of proposition 1 to prove (iii).
Proof of (i) and (ii): I will prove these parts by induction. The proof applies lemmas 5 and 6. Lemma 5 implies that, no matter what the other machine producers do, the frontier machine producer will always set a price $P_{t,0} > \frac{c_0}{2} A_t X_{t,0}$ and make strictly positive profits. This lemma initializes the induction. Lemma 6 then shows that, independently of what the suppliers of older vintages do, if all suppliers of newer vintages charge a markup and make strictly positive profits, then so will the supplier of vintage $\tau \in \{1, \ldots, M-1\}$. Combining these two lemmas implies that, if there is a Pure Strategy Nash equilibrium, then it must be one in which (i) all monopoly suppliers of machines charge a strictly positive markup and make strictly positive profits. That is,

$$(i)\ P_{t,\tau} > \frac{c_\tau}{2} A_{t-\tau} X_{t,\tau} \text{ for all } \tau \in \{0, \ldots, M-1\} \text{ and } (ii)\ \mu\left(D(\tau)\right) > 0 \text{ for all } \tau \in \{0, \ldots, M\}$$

Combining (ii) with parts (i) and (iv) of proposition 1, this implies that if there is a Pure Strategy Nash equilibrium, then there exist $\{h_t(0), \ldots, h_t(M-1)\}$ such that

$$D(\tau) = \begin{cases} \left[h_t(0), \overline{h}\right] & \text{for} \quad \tau = 0 \\ \left[h_t(\tau), h_t(\tau-1)\right] & \text{for} \quad \tau \in \{1, \ldots, M-1\} \\ \left[0, h_t(M-1)\right] & \text{for} \quad \tau = M \end{cases}$$

and

$$h_t(\tau) = \frac{A_{t-\tau}}{A_{t-\tau} - A_{t-\tau-1}} \hat{P}_{t,\tau} - \frac{A_{t-\tau-1}}{A_{t-\tau} - A_{t-\tau-1}} \hat{P}_{t,\tau+1}$$

which is the basis for the derivation of the form of the Pure Strategy Nash equilibrium in part (iii).
Proof of (iii), (iv), and (v): A machine producer of machines of vintage age τ chooses $\hat{P}_{t,\tau}$ to maximize profits

$$\pi_{t,\tau} = A_{t-\tau} \left(\hat{P}_{t,\tau} - \frac{c_\tau}{2} X_{t,\tau}\right) X_{t,\tau}$$

since parts (i) and (ii) have proven that the solution to the PSN-equilibrium is interior, I will simply use standard calculus to find a profit maximizing solution. The associated first order necessary condition for the profit maximization problem is

$$0 = A_{t-\tau}\left[\hat{P}_{t,\tau}\frac{\partial X_{t,\tau}}{\partial \hat{P}_{t,\tau}} + X_{t,\tau} - c_\tau X_{t,\tau}\frac{\partial X_{t,\tau}}{\partial \hat{P}_{t,\tau}}\right]$$

such that if there is an interior solution to this problem, it must satisfy

$$\hat{P}_{t,\tau} = \left[c_\tau - \frac{1}{\partial X_{t,\tau} / \partial \hat{P}_{t,\tau}}\right] X_{t,\tau}$$

For the supplier of a non-frontier vintage, i.e. $\tau \in \{1, \ldots, M-1\}$, the demand set satisfies

$$\begin{aligned} X_{t,\tau} &= \left[\frac{A_{t-\tau+1}}{A_{t-\tau+1} - A_{t-\tau}}\hat{P}_{t,\tau-1} - \frac{A_{t-\tau}}{A_{t-\tau+1} - A_{t-\tau}}\hat{P}_{t,\tau} - \frac{A_{t-\tau}}{A_{t-\tau} - A_{t-\tau-1}}\hat{P}_{t,\tau} + \frac{A_{t-\tau-1}}{A_{t-\tau} - A_{t-\tau-1}}\hat{P}_{t,\tau+1}\right] \\ &= w_{t,\tau}^{\tau-1}\hat{P}_{t,\tau-1} - \left(w_{t,\tau}^{\tau-1} + w_{t,\tau}^{\tau+1}\right)\hat{P}_{t,\tau} + w_{t,\tau}^{\tau+1}\hat{P}_{t,\tau+1} \end{aligned}$$

such that

$$\frac{\partial X_{t,\tau}}{\partial \hat{P}_{t,\tau}} = -\left(w_{t,\tau}^{\tau-1} + w_{t,\tau}^{\tau+1}\right)$$

Using the above two equations to solve the first order necessary condition yields

$$\hat{P}_{t,\tau} = \left[\frac{1 + c_\tau\left(w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}\right)}{2 + c_\tau\left(w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}\right)}\right]\left[\frac{w_{t,\tau}^{\tau-1}}{w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}}\widehat{P}_{t,\tau-1} + \frac{w_{t,\tau}^{\tau-1}}{w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}}\widehat{P}_{t,\tau+1}\right]$$

which is the first part of the second order difference equation in the proposition. Substituting this expression in that for the demand set yields that

$$X_{t,\tau} = \frac{w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}}{1 + c_\tau\left(w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}\right)}\hat{P}_{t,\tau}$$

For the supplier of the frontier vintage, the demand set satisfies

$$X_{t,0} = \left[1 - \left(1 + w_{t,0}^1\right)\hat{P}_{t,0} + w_{t,0}^1\hat{P}_{t,1}\right]$$

such that

$$\frac{\partial X_{t,0}}{\partial \hat{P}_{t,0}} = -\left(1 + w_{t,0}^1\right)$$

Using the above two equations to solve the necessary condition for an interior solution yields

$$\widehat{P}_{t,0} = \left[\frac{1 + c_0\left(1 + w_{t,0}^1\right)}{2 + c_0\left(1 + w_{t,0}^1\right)}\right]\left[\frac{1}{1 + w_{t,0}^1} + \frac{w_{t,0}^1}{1 + w_{t,0}^1}\widehat{P}_{t,1}\right]$$

Substituting this into the expression for the demand set gives

$$X_{t,0} = \frac{1 + w_{t,0}^1}{1 + c_0 \left(1 + w_{t,0}^1\right)} \hat{P}_{t,0}$$

What is left to show is that $\hat{P}_{t,\tau} > \hat{P}_{t,\tau+1}$. This follows from the fact that for $\tau \in \{0, \ldots, M-1\}$, the second order difference equation that has to be satisfied in equilibrium implies

$$\hat{P}_{t,\tau} = \left[1 + c_\tau \left(w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}\right)\right] \left[\frac{w_{t,\tau}^{\tau-1}}{w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}} \left(\widehat{P}_{t,\tau-1} - \widehat{P}_{t,\tau}\right) - \frac{w_{t,\tau}^{\tau-1}}{w_{t,\tau}^{\tau+1} + w_{t,\tau}^{\tau-1}} \left(\widehat{P}_{t,\tau} - \widehat{P}_{t,\tau+1}\right)\right] > 0$$

Since $\left(\widehat{P}_{t,M-1} - \widehat{P}_{t,M}\right) > 0$, a simple induction argument can be used to show that $\widehat{P}_{t,\tau} - \widehat{P}_{t,\tau+1} > 0$ for all $\tau \in \{0, \ldots, M-1\}$.■

Proof of proposition 3: Value of innovation, output, etc., at constant g

(i) Note that, if g is constant, the recursion (9) can be written as a set of linear equations. In matrix form, with the appropriately defined matrices

$$\underset{M+1\times M+1}{\mathbf{F}(g,\mathbf{c})} \; \widehat{\mathbf{P}}_t = \underset{M+1\times 1}{\mathbf{G}(g,\mathbf{c})}$$

such that the equilibrium vector with prices equals

$$\widehat{\mathbf{P}}_t = \left[\left[\mathbf{F}(g,\mathbf{c})\right]^{-1} \mathbf{G}(g,\mathbf{c})\right]$$

which implies that we can write $\widehat{P}_{t,\tau} = \mu_\tau (g,\mathbf{c})$ where $\mu_\tau (g,\mathbf{c}) > \mu_{\tau+1} (g,\mathbf{c})$ simply because $\hat{P}_{t,\tau}$ is decreasing in τ .

(ii) For the demand sets we obtain that we can write

$$X_{t,\tau} = \begin{cases} \frac{1+w_{t,0}^1}{1+c_0\left(1+w_{t,0}^1\right)} \mu_0 (g,\mathbf{c}) & \text{for} \quad \tau = 0 \\ \frac{w_{t,\tau}^{\tau+1}+w_{t,\tau}^{\tau-1}}{1+c_\tau\left(w_{t,\tau}^{\tau+1}+w_{t,\tau}^{\tau-1}\right)} \mu_\tau (g,\mathbf{c}) & \text{for} \quad \tau = 1, \ldots, M-1 \\ 1 - \sum_{\tau=0}^{M-1} X_{t,\tau} & \text{for} \quad \tau = M \end{cases}$$

However, since when g is constant $w_{t,\tau}^{\tau+1}$ and $w_{t,\tau}^{\tau-1}$ only depend on g and not on t for all $\tau = 1, \ldots, M-1$, we can write $X_{t,\tau} = \tilde{X}_\tau(g,\mathbf{c})$ for $\tau = 1, \ldots, M-1$. In that case however, $X_{t,M} = 1 - \sum_{\tau=0}^{M-1} \tilde{X}_\tau(g,\mathbf{c}) = \tilde{X}_M(g,\mathbf{c})$ and is also constant over time.

(iii) Aggregate output in terms of efficiency units can be written as

$$\begin{aligned} Y_t^* &= \frac{Y_t}{A_t} = \frac{1}{A_t} \sum_{\tau=0}^{M} \int_{h\in D_t(\tau)} \left[A_{t-\tau} h - P_{t,\tau}\right] dh \\ &= \frac{1}{A_t} \sum_{\tau=0}^{M} A_{t-\tau} \underbrace{\int_{h\in D_t(\tau)} hdh}_{Z_{t,\tau}} - P_{t,\tau} X_{t,\tau} \end{aligned}$$

Note that

$$\frac{1}{\overline{h}}\int_a^b hdh = \frac{1}{2\overline{h}}\left(a^2-b^2\right) = \frac{1}{2}\left[\frac{1}{\overline{h}}(a-b)\right](a+b)$$

Applying this to the equation for aggregate yields

$$\begin{aligned} Z_{t,\tau} &= \begin{cases} \frac{1}{2}X_{t,\tau}\left[1+h_t(0)\right] & \text{for} \quad \tau=0 \\ \frac{1}{2}X_{t,\tau}\left[h_t(\tau-1)+h_t(\tau)\right] & \text{for} \quad \tau=1,\ldots,M-1 \\ \frac{1}{2}X_{t,\tau}\left[h_t(M-1)\right] & \text{for} \quad \tau=M \end{cases} \\ &= X_{t,\tau}Z^*_{t,\tau} \end{aligned}$$

Using this notation, aggregate output has the representation

$$Y^*_t = \frac{1}{A_t}\sum_{\tau=0}^{M} X_{t,\tau}\left[A_{t-\tau}Z^*_{t,\tau} - P_{t,\tau}\right]$$

where

$$Z^*_{t,\tau} = \begin{cases} \frac{1}{2}\left[1+\left(1+w^1_{t,0}\right)\hat{P}_{t,0} - w^1_{t,0}\hat{P}_{t,1}\right] & \text{for} \quad \tau=0 \\ \frac{1}{2}\left[w^{\tau-1}_{t,\tau}\hat{P}_{t,\tau-1} - \left(2+w^{\tau-1}_{t,\tau}+w^{\tau+1}_{t,\tau}\right)\hat{P}_{t,\tau} - w^{\tau+1}_{t,\tau}\hat{P}_{t,\tau+1}\right] & \text{for} \quad \tau=1,\ldots,M-1 \\ \frac{1}{2}w^{M-1}_{t,M}\hat{P}_{t,\tau-1} & \text{for} \quad \tau=M \end{cases}$$

Since again $w^{\tau+1}_{t,\tau}$ and $w^{\tau-1}_{t,\tau}$ only depend on g and not on t and $\hat{P}_{t,\tau} = \mu_\tau(g,\mathbf{c})$, we can write $Z^*_{t,\tau} = \tilde{z}_\tau(g,\mathbf{c})$. This means that output can be represented as

$$\begin{aligned} Y^*_t &= \frac{1}{A_t}\sum_{\tau=0}^{M}\tilde{X}_\tau(g,\mathbf{c})\left[A_{t-\tau}\tilde{z}_\tau(g,\mathbf{c}) - \mu_\tau(g,\mathbf{c})\right] \\ &= \sum_{\tau=0}^{M}\tilde{X}_\tau(g,\mathbf{c})\left[\left(\frac{1}{1+g}\right)^\tau\tilde{z}_\tau(g,\mathbf{c}) - \mu_\tau(g,\mathbf{c})\right] \\ &= \tilde{Y}(g,\mathbf{c}) \end{aligned}$$

(iv) For the profits we obtain that

$$\begin{aligned} \pi^*_{t,\tau} &= \frac{\pi_{t,\tau}}{A_{t-\tau}} = \left(\hat{P}_{t,\tau} - \frac{c_\tau}{2}X_{t,\tau}\right)X_{t,\tau} \\ &= \left(p_\tau(g,\mathbf{c}) - \frac{c_\tau}{2}\tilde{X}_\tau(g,\mathbf{c})\right)\tilde{X}_\tau(g,\mathbf{c}) \\ &= \tilde{\pi}_\tau(g,\mathbf{c}) \end{aligned}$$

(v) and (vi) Follow directly from the explanation in the main text.■

Proof of proposition 4: Existence and uniqueness of competitive equilibrium

The competitive equilibrium equations (12) through (17) can be solved sequentially. That is, (17) pins down the equilibrium interest rate as

$$r = \frac{1}{\beta}\left(1+g\right)^{\sigma} - 1 > 0$$

At this constant interest rate the value of a new innovation equals

$$V^*\left(r,\ldots,r;g,\mathbf{c}\right) = \left(\frac{1}{1+r}\right)\sum_{s=0}^{M-1}\left(\frac{1}{1+g}\right)^{s}\left(\frac{1}{1+r}\right)^{s}\tilde{\pi}_s\left(g,\mathbf{c}\right) = X_R^*$$

and the profits from the innovative activities equal

$$\Pi^* = \sum_{s=0}^{M-1}\left(\frac{1}{1+g}\right)^{s}\left[1-\left(\frac{1}{1+r}\right)^{s+1}\right]\tilde{\pi}_s\left(g,\mathbf{c}\right) > 0$$

From proposition 3 we know that for any $g>0$, $Y^* = \tilde{Y}\left(g,\mathbf{c}\right) > 0$ is unique, which yields that steady state consumption equals

$$C^* = Y^* + \Pi^* > 0$$

Hence a competitive equilibrium path exists and is unique.■

Lemma 5 *Independent of $\mathbf{P}_{t,0}^{'}$, the supplier of the frontier vintage will choose $P_{t,0} > \frac{c_0}{2}A_t\mu\left(D_t\left(0\right)\right)$.*

Proof: In order to prove this and the following lemma, it is easiest to consider

$$z_\tau\left(h\right) = \max_{s\in\{0,\ldots,M\}\setminus T}\left(A_{t-s}h - P_{t,s}\right)$$

and

$$\overline{z}_\tau\left(h\right) = \max_{s<T}\left(A_{t-s}h - P_{t,s}\right),\ \underline{z}_\tau\left(h\right) = \max_{s>T}\left(A_{t-s}h - P_{t,s}\right),\text{ and } W_\tau\left(h\right) = A_{t-\tau}h - P_{t,\tau}$$

then

$$D_t\left(\tau\right) = \left\{h\in[0,1]\,|W_\tau\left(h\right)\geq z_\tau\left(h\right)\right\}$$

The properties of $\overline{z}_\tau\left(h\right)$ and $\underline{z}_\tau\left(h\right)$, which I will not prove here in detail, are (i) $\overline{z}_\tau\left(h\right)$ and $\underline{z}_\tau\left(h\right)$ are continuous on $[0,1]$, (ii) $\underline{z}_\tau\left(0\right)=0$, (iii) if $P_{t,s}>0$ for all $s>\tau$, then $\overline{z}_\tau\left(0\right)<0$, and (iii) let $h'>h$, then

$$\overline{z}_\tau\left(h'\right) - \overline{z}_\tau\left(h\right) \geq A_{t-(\tau-1)}\left(h'-h\right) \text{ and } \underline{z}_\tau\left(h'\right) - \underline{z}_\tau\left(h\right) \leq A_{t-(\tau+1)}\left(h'-h\right)$$

For the frontier vintage, let the producer choose $h' = 1-\varepsilon$ such that all workers of type h' and higher will choose the frontier vintage. Independent of $\mathbf{P}_{t,0}^{'}$, this can be done by choosing

$$P_{t,0} \geq \left[A_t - A_{t-1}\right]h' = \left[A_t - A_{t-1}\right]\left(1-\varepsilon\right) > 0$$

In that case demand for the frontier vintage equals $\mu(D_t(\tau)) = \varepsilon$, while profits equal

$$\begin{aligned}
\pi_{t,0} &= P_{t,0}\varepsilon - \frac{c_0}{2}A_t\varepsilon^2 \\
&\geq \left[[A_t - A_{t-1}](1-\varepsilon) - \frac{c_0}{2}A_t\varepsilon\right]\varepsilon \\
&= \left[[A_t - A_{t-1}] - \left[\left(1+\frac{c_0}{2}\right)A_t - A_{t-1}\right]\varepsilon\right]\varepsilon
\end{aligned}$$

such that the producer of the frontier vintage makes strictly positive profits, i.e. $\pi_{t,0} > 0$, whenever it chooses

$$0 < \varepsilon < [A_t - A_{t-1}] / \left[\left(1+\frac{c_0}{2}\right)A_t - A_{t-1}\right]$$

which is always feasible.■

Lemma 6 *If $P_{t,s} > 0$ for all $s < \tau$, then, independent of $P_{t,\tau+1}, \ldots, P_{t,M}$, the supplier of the vintage of age τ will choose $P_{t,\tau} > \frac{c_\tau}{2}A_{t-\tau}\mu(D_t(\tau))$.*

Proof: If $P_{t,s} > 0$, then we know that $\overline{z}_\tau(0) < 0$, and we can distinguish two cases:
(i) $\overline{z}_\tau(1) \leq \underline{z}_\tau(1)$: in that case the suppliers of the more recent vintages than that of age τ have chosen their prices so high that they are being competed out of the market by suppliers of vintages older than τ. In this case the vintage of age τ is essentially in the same situation as the supplier of the frontier vintage in Lemma 5 and the proof of can be applied Lemma 5 again.
(ii) $\overline{z}_\tau(1) > \underline{z}_\tau(1)$: Because $\overline{z}_\tau(0) < 0 = \underline{z}_\tau(0)$ and both $\overline{z}_\tau(h)$ and $\underline{z}_\tau(h)$ are continuous, we know that in this case there must exist an $h' \in (0,1)$ such that $\overline{z}_\tau(h') = \underline{z}_\tau(h')$. Hence, by choosing $P_{t,\tau} = A_{t-\tau}h' - \overline{z}_\tau(h')$, the worker would be indifferent between at least three vintages of machine and would obtain an income level of $\overline{z}_\tau(h') = \underline{z}_\tau(h')$. If a worker of type $h < h'$ would use the machine of age τ, then he would obtain

$$A_{t-\tau}h - P_{t,\tau} = \underline{z}_\tau(h') - A_{t-\tau}(h - h') < \underline{z}_\tau(h')$$

and if a worker of type $h > h'$ would the machine of age τ, then he would obtain

$$A_{t-\tau}h - P_{t,\tau} = \overline{z}_\tau(h') + A_{t-\tau}(h - h') < \overline{z}_\tau(h')$$

Hence, the choice of $P_{t,\tau} = A_{t-\tau}h' - \overline{z}_\tau(h')$ is the knife-edge case in which the demand set for vintage τ is a singleton. Now, if the supplier of vintage τ chooses

$$P_{t,\tau} = A_{t-\tau}h' - \overline{z}_\tau(h') - \delta$$

then it can be easily shown that

$$0 < \mu(D_t(\tau)) \leq \delta\left(\frac{1}{A_{t-\tau} - A_{t-\tau-1}} - \frac{1}{A_{t-\tau+1} - A_{t-\tau}}\right)$$

and that the resulting profits satisfy

$$\pi_{t,\tau} = \left[\{A_{t-\tau}h' - \overline{z}_\tau(h')\} - \delta\left[1 + \frac{1}{2}\frac{c_\tau A_{t-\tau}}{A_{t-\tau} - A_{t-\tau-1}} + \frac{1}{2}\frac{c_\tau A_{t-\tau}}{A_{t-\tau+1} - A_{t-\tau}}\right]\right]\mu(D_t(\tau))$$

Hence, $\pi_{t,\tau}$ is strictly positive whenever

$$0 < \delta < \{A_{t-\tau}h' - \overline{z}_\tau(h')\} / \left[1 + \frac{1}{2}\frac{c_\tau A_{t-\tau}}{A_{t-\tau} - A_{t-\tau-1}} + \frac{1}{2}\frac{c_\tau A_{t-\tau}}{A_{t-\tau+1} - A_{t-\tau}}\right]$$

which again is a feasible choice independent of the prices chosen by the suppliers of vintages older than age τ.■

2 Data

This appendix contains the data on microprocessor prices that I used in my empirical examples. They are taken from two sources[8] and cover prices for Intel's Pentium 4, 3, and 2, and Celeron processors, as well as Advance Micro Devices' K6-3 and -2 chips. The data are for two points in time, namely April 11 1999 and March 4 2001, and are listed in Table 3.

Table 3: Processor price data

04/11/1999			03/04/2001		
processor	speed MHz	price	processor	speed MHz/GHz*	price
P-III	500	637	P-IV	1.5*	637
P-III	450	411	P-IV	1.4*	413
P-II	450	396	P-IV	1.3*	332
P-II	400	234	P-III	1.0*	241
P-II	350	163	P-III	933	225
Celeron	433	143	P-III	800	165
Celeron	400	103	Celeron	800	112
Celeron	366	73	Celeron	766	103
Celeron	333	67	Celeron	733	83
K6-3	450	397			
K6-3	400	237			
K6-2	475	213			
K6-2	350	68			
K6-2	333	62			

[8]Electronic Engineering Times, April 19, 1999, "Intel, AMD Slash Processor Prices" and IDG News Service, "Intel Trims Prices on Desktop PC Chips", March 05, 2001.